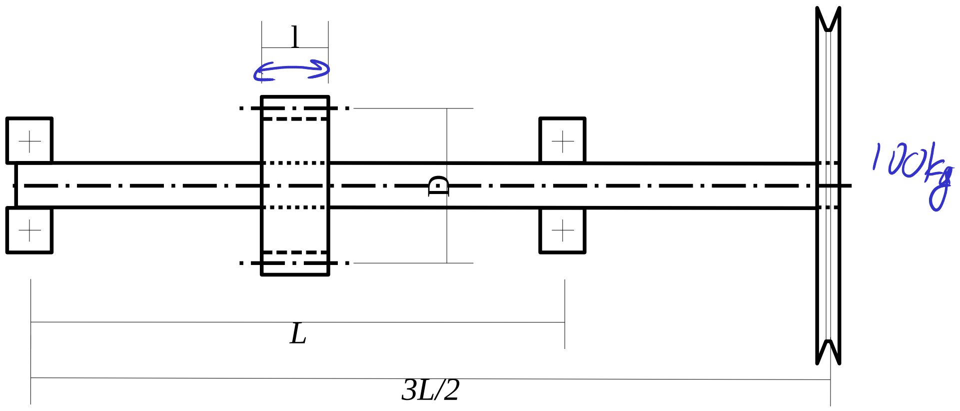
Données

$N = 1000 \quad tr.min^{-1}$
$P = 20000 \quad W$
$M = 100 \quad Kg$
$g = 9.81 \quad m.s^{-2}$
$L = 0.2 \quad m$
$D = 0.1 \quad m$
$\alpha = 20° = 0.3490 \quad rad$
$l = 0.05 \quad m$
$\rho = 7800 \quad kg.m^{-3}$
$E = 210.10^{9} \quad Pa$
$R = 50.10^{6} \quad Pa$

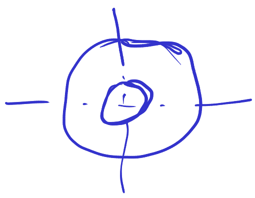
$d = 0,04\ m$



Calcul des vitesses critiques de rotation

Calcul des inerties

$$I_y = \frac{\pi d^4}{64} = 1,257 . 10^{-7}\ m^4 \quad (= 125\,700\ mm^4)$$



Volume et masse de la roue dentée

$$V_R = l . \frac{\pi (D^2 - d^2)}{4} = 3\ 299 . 10^{-4}\ m^3$$

$$m_r = \rho . V_R = 2.573\ kg.$$

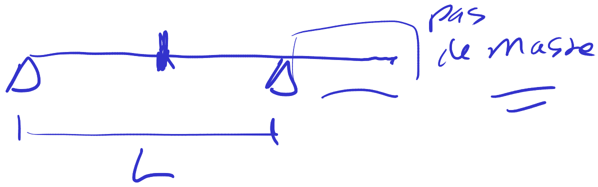
Masse de l'arbre

$$V_a = \frac{3L}{2} . \frac{\pi d^2}{4} = 3,77 . 10^{-4}\ m^3$$

$$m_a = 2.94\ kg.$$

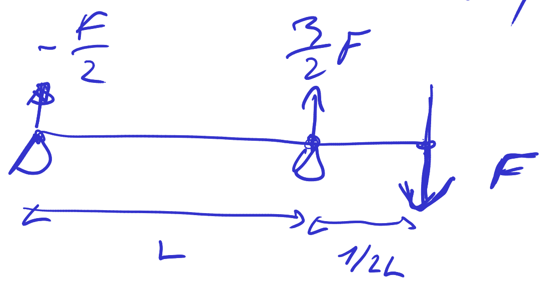
Masse linéique de l'arbre

$$m_{la} = \frac{m_a . 2}{3L} = 9,8\ Kg . m^{-1}$$



Pulsation propre de la roue dentée seule

$$\omega_r = \sqrt{\frac{48EI}{m_r \cdot L^3}} = 7864 \text{ rad/s} \rightarrow N_r = 75\,000 \text{ tr/min}$$



Pulsation propre du poids en bout d’arbre

calcul de la déflexion en bout d'arbre.

$$y\left(\frac{3L}{2}\right) = \frac{\partial E_c}{\partial F} \qquad E_c = \int_0^{3L/2} \frac{Mf^2}{2EI}.$$

$$Mf(x) = \frac{F}{2}x - \frac{3}{2}F(x-L)^+ = \frac{F}{2}\left(x - 3(x-L)^+\right)$$

$$\frac{Mf^2}{2EI} = \frac{F^2}{8EI}\left(x - 3(x-L)^+\right)^2$$

$$\int_0^{3L/2} \frac{Mf^2}{2EI} = \frac{F^2}{8EI}\left[\int_0^L x^2\,dx + \int_L^{3L/2} \underbrace{[x - 3(x-L)]^2}_{(3L-2x)^2}\,dx\right]$$

$$= \frac{F^2}{8EI}\left[\left[\frac{x^3}{3}\right]_0^L + \int_L^{3L/2} 9L^2 - 12Lx + 4x^2\right] = \frac{F^2}{8EI}\left(\frac{L^3}{3} + \left[9L^2x - 6Lx^2 + \frac{4x^3}{3}\right]_L^{3L/2}\right)$$

$$= \frac{F^2}{8EI}\left[\frac{L^3}{3} + \frac{27L^3}{2} - \frac{54L^3}{4} + \frac{108L^3}{24} - 9L^3 + 6L^3 - \frac{4}{3}L^3\right]$$

$$\frac{F^2L^3}{8EI\cdot 24}\left(8 + 12\cdot 27 - 6\cdot 54 + 108 - 9\cdot 24 + 6\cdot 24 - 32\right)$$

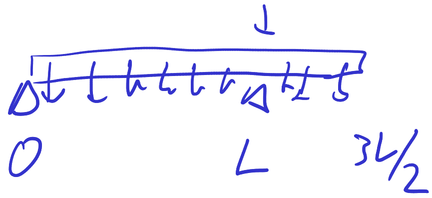
$$\rightarrow \frac{F^2L^3}{8EI}\cdot\frac{12}{24} \qquad \frac{\partial E_c}{\partial F} = \frac{\cancel{2}FL^3}{8EI}\cdot\frac{1}{\cancel{2}} = \frac{FL^3}{8EI}$$

$$\boxed{\omega_p = \sqrt{\frac{8EI}{ML^3}}}$$

$$\omega_p = 513.7 \text{ rad/s}.$$

$$N_p = 4900 \text{ tr/min}$$

Pulsation propre de l'arbre



$y(0) = 0$

$y''(0) = 0$

$y(L) = 0$

$y''\left(\frac{3L}{2}\right) = 0$

$$y(x) = C_1 Y_1(ax) + C_2 Y_2(ax) + C_3 Y_3(ax) + C_4 Y_4(ax)$$

$$K \quad \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ Y_1(aL) & Y_2(aL) & Y_3(aL) & Y_4(aL) \\ Y_2\left(\frac{3aL}{2}\right) & Y_1\left(\frac{3aL}{2}\right) & Y_4\left(\frac{3aL}{2}\right) & Y_3\left(\frac{3aL}{2}\right) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$Y_1 = \sinh + \sin$

$Y_2 = \sinh - \sin$

$Y_3 = \cosh + \cos$

$Y_4 = \cosh - \cos$

$\det(K) = 0 \qquad C_3 = C_4 = 0$

$$\left(\sinh(aL) + \sin(aL)\right)\left(\sinh\left(\tfrac{3aL}{2}\right) + \sin\left(\tfrac{3aL}{2}\right)\right) - \left(\sinh(aL) - \sin(aL)\right)\left(\sinh\left(\tfrac{3aL}{2}\right) - \sin\left(\tfrac{3aL}{2}\right)\right) = 0$$

$$\boxed{\sin aL \,\sinh\frac{3aL}{2} + \sin\frac{3aL}{2}\,\sinh aL = 0} = H(aL)$$

$D(aL)$

1er nul de $\dfrac{M(aL)}{D(aL)} \rightarrow aL = 0{,}9297\,\pi$

$$a^4 = \frac{m_{lu}\,\omega^2}{EI}$$

$$\rightarrow \omega^2 = \frac{EI(aL)^4}{m_{lu}} \rightarrow \omega_a = \sqrt{\frac{EI}{m_{lu}}}\,\frac{(aL)^2}{L^2}$$

$\omega_a = 11067$ rad/s

$N_a = 105\,000$ tr/min

Première pulsation propre de l'ensemble

avec Dunkerley

$$\frac{1}{\Omega^2} = \sum_i \frac{1}{\omega_i} \Rightarrow \Omega = \sqrt{\frac{1}{\sum \frac{1}{\omega_i^2}}}$$

$$\Omega = 512 \text{ rad/s.}$$

$$\boxed{N_c = 4890 \text{ tr/min}}$$