Vehicle Performance

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Lesson 1:
Equation of Longitudinal Motion

Outline

- ASSUMPTIONS
- DESCRIPTION OF VEHICLE MOTION
  - Longitudinal motion

References

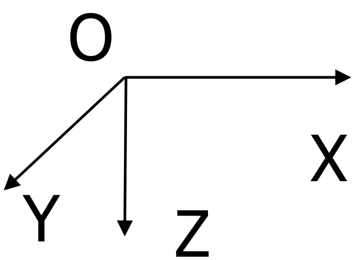
- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- R. Bosch. « Automotive Handbook ». 5th edition. 2002. Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).
- W.H. Hucho. « Aerodynamics of Road Vehicles ». 4th edition. SAE International. 1998.
- M. Eshani, Y. Gao & A. Emadi. Modern Electric, Hybrid Electric and Fuel Cell Vehicles. Fundamentals, Theory and Design. 2nd Edition. CRC Press.

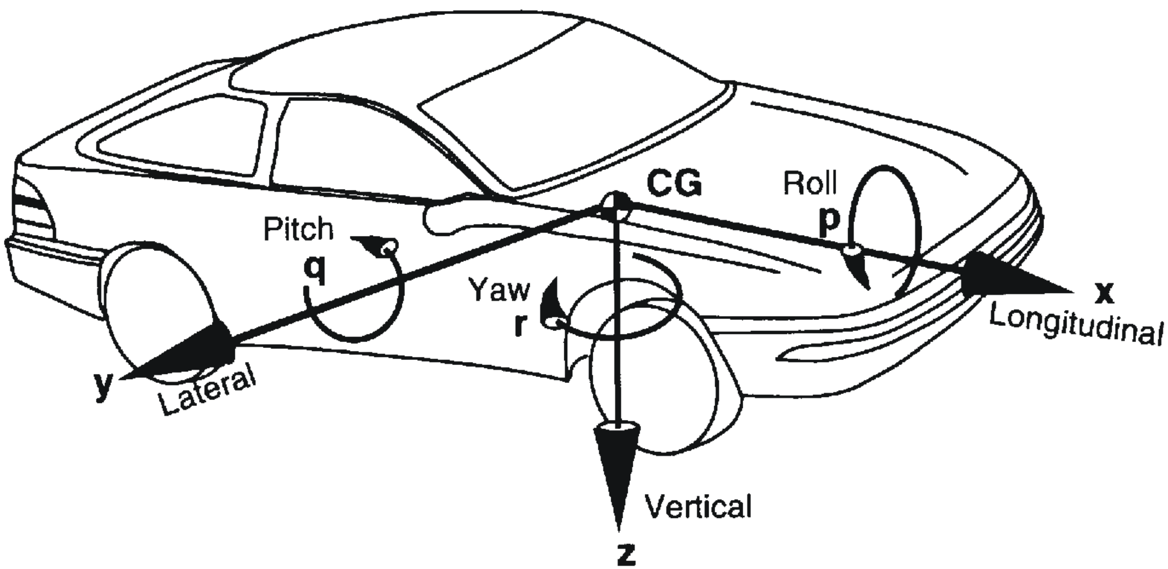
Assumptions and definitions

Assumptions

- The vehicle is made of several components or subsystems
- We consider the motion of the system as a whole
- During acceleration, braking, turn, maneuvers, the vehicle is considered as a rigid body motion and is characterized by its geometry, its mass and inertia properties

Reference frames





Inertial coordinate system OXYZ

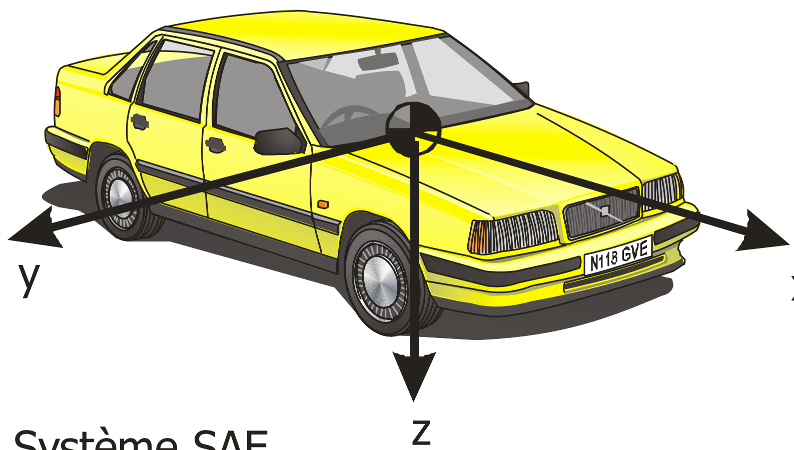
Local reference frame oxyz attached to the vehicle body - SAE (Gillespie, fig. 1.4)

Reference frames

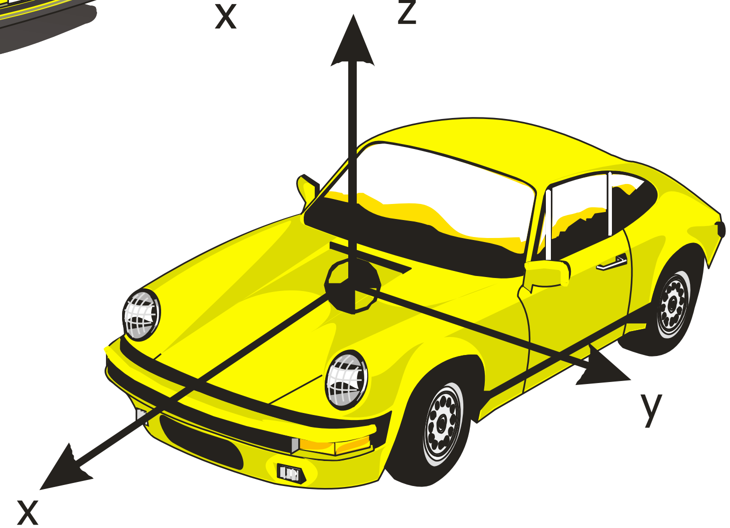
- Inertial reference frame
  - X direction of initial displacement or reference direction
  - Y right side travel
  - Z towards downward vertical direction

- Vehicle reference frame (SAE):
  - x along motion direction and vehicle symmetry plane
  - z along vertical direction pointing to the center of the earth
  - y in the lateral direction on the right-hand side of the driver
  - o, origin at the center of mass

Reference frames



Système SAE

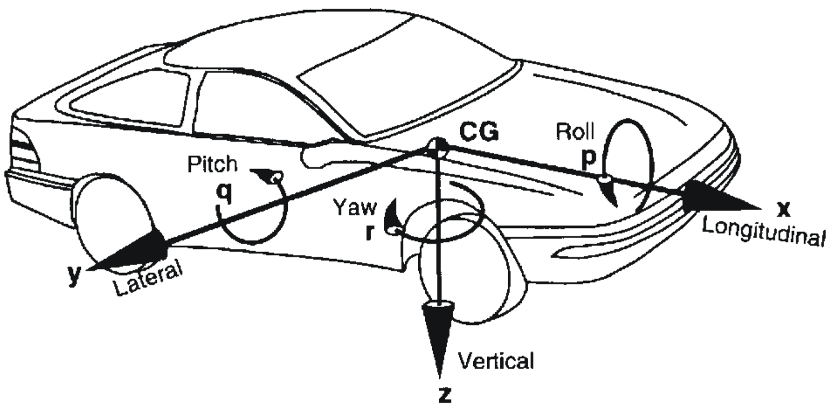


Système ISO

Comparison of conventions of SAE and ISO/DIN reference frames

Local velocity vectors

- Vehicle motion is often studied in car-body local systems
  - u: forward speed (+ if in front)
  - v: side speed (+ to the right)
  - w: vertical speed (+ downward)
  - p ($=\omega_x$): rotation speed about x axis (roll speed)
  - q ($=\omega_y$): rotation speed about y (pitch)
  - r ($=\omega_z$): rotation speed about z (yaw)



Forces

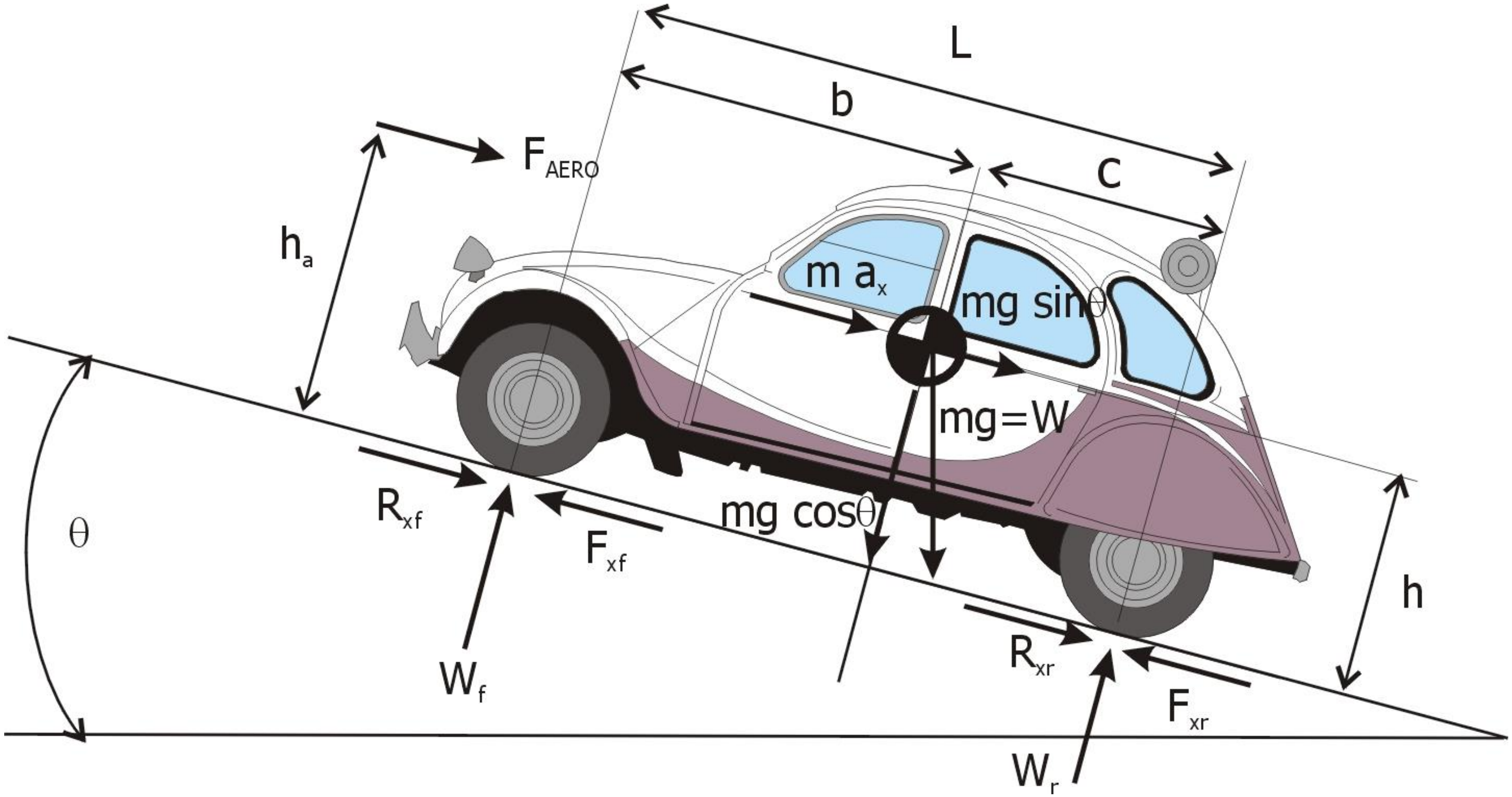
- *Forces and moments are accounted positively when acting onto the vehicle and the positive direction with respect to the considered frame*

- Corollary
  - A positive F_x force is propelling the vehicle forward
  - The reaction force of the ground onto the wheels is accounted negatively.

- Because of the inconveniency of this definition, the SAEJ670e « Vehicle Dynamics Terminology » names as normal force a force acting downward while vertical forces are referring to upward forces

Equilibrium of longitudinal motion

Longitudinal motion



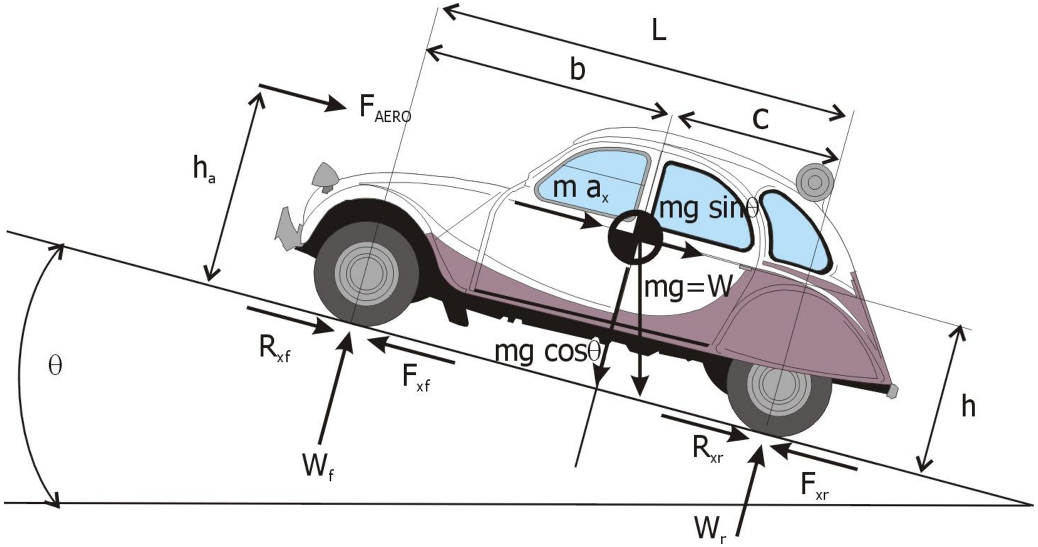
Longitudinal equilibrium

- Newton-Euler equations

$$\sum F_x = m\frac{d}{dt}(u) = ma_x$$

$$\sum F_z = m\frac{d}{dt}(w) = 0$$

$$\sum M_y = \frac{d}{dt}(\mathbf{J}\omega)_y = 0$$



- Equilibrium

$$ma_x = F_{xf} + F_{xr} - R_{xf} - R_{xr} - mg\sin\theta - F_{AERO}$$

$$0 = mg\cos\theta - W_f - W_r$$

$$0 = (F_{xf} + F_{xr})h - (R_{xf} + R_{xr})h + W_f b - W_r c + F_{AERO}(h_A - h)$$

Longitudinal equilibrium

- Equilibrium along forward x direction

$$ma_x = (F_{xf} + F_{xr}) - (R_{xf} + R_{xr}) - mg\sin\theta - F_{AERO}$$

- Equation of vehicle longitudinal motion

$$ma_x = F_x - F_{RR} - mg\sin\theta - F_{AERO}$$

- Or

$$ma_x = F_x - F_{RES}$$

  - Total tractive force $F_x = F_{xf} + F_{xr}$
  - Total road resistance force $F_{RES} = F_{RR} + mg\sin\theta + F_{AERO}$

Longitudinal equilibrium

- In energy form, the equilibrium equation

$$ma_x = F_x - F_{RR} - mg\sin\theta - F_{AERO}$$

- Becomes

$$\boxed{\mathcal{P}_x = m\frac{du}{dt}u + \mathcal{P}_{RES} = \frac{d}{dt}(\frac{1}{2}mu^2) + \mathcal{P}_{RES}}$$

- With
  - The propulsion power $\mathcal{P}_x = F_x.u$
  - The power dissipated by the road resistance forces

$$\mathcal{P}_{RES} = F_{RES}.u$$

Longitudinal equilibrium

- Weight under the wheel sets

$$
\begin{aligned}
0 &= mg\cos\theta - W_f - W_r \\
0 &= (F_{xf} + F_{xr})h - (R_{xf} + R_{xr})h + W_f b - W_r c + F_{AERO}(h_A - h)
\end{aligned}
$$

- Solve for W_f

$$W_f + W_r = mg\cos\theta$$

$$W_r = mg\cos\theta - W_f$$

$$W_f = mg\cos\theta\frac{c}{L} - (F_x - F_{RR})\frac{h}{L} - F_{AERO}\frac{h_A - h}{L}$$

Longitudinal equilibrium

- Solve for W_r:

$$W_f = mg\cos\theta - W_r$$

$$W_r = mg\cos\theta\frac{b}{L} + (F_x - F_{RR})\frac{h}{L} + F_{AERO}\frac{h_A - h}{L}$$

- Generally tractive forces are not known, and the acceleration is preferred (kinematic variables are easier to measure)

$$ma_x = F_x - F_{RR} - mg\sin\theta - F_{AERO}$$

Longitudinal equilibrium

- The final expression of the weight under the wheel sets is:

$$W_f = mg\cos\theta\frac{c}{L} - ma_x\frac{h}{L} - F_{AERO}\frac{h_A}{L} - mg\sin\theta\frac{h}{L}$$
$$W_r = mg\cos\theta\frac{b}{L} + ma_x\frac{h}{L} + F_{AER}\frac{h_A}{L} + mg\sin\theta\frac{h}{L}$$

- The weight under the front (rear) wheels
  - Decreases (increases) with the acceleration, the height of the center of gravity, the slope, the aerodynamics loads
  - Increases (decreases) when breaking or driving down

Longitudinal equilibrium

- Static weight distribution

$$
\begin{aligned}
W_f &= mg\frac{c}{L} \\
W_r &= mg\frac{b}{L}
\end{aligned}
$$

- Low speed weight distribution when accelerating

$$
\begin{aligned}
W_f &= mg\frac{c}{L} - ma_x\frac{h}{L} = mg(\frac{c}{L} - \frac{a_x}{g}\frac{h}{L}) \\
W_r &= mg\frac{b}{L} + ma_x\frac{h}{L} = mg(\frac{b}{L} + \frac{a_x}{g}\frac{h}{L})
\end{aligned}
$$

Longitudinal equilibrium

- Low speed weight distribution when hill climbing

$$W_f = mg\cos\theta\frac{c}{L} - mg\sin\theta\frac{h}{L}$$
$$W_r = mg\cos\theta\frac{b}{L} + mg\sin\theta\frac{h}{L}$$

- If θ is small, $\sin\theta \simeq \tan\theta \simeq \theta \quad \cos\theta \simeq 1$

$$W_f = mg(\frac{c}{L} - \frac{h}{L}\theta)$$
$$W_r = mg(\frac{b}{L} + \frac{h}{L}\theta)$$

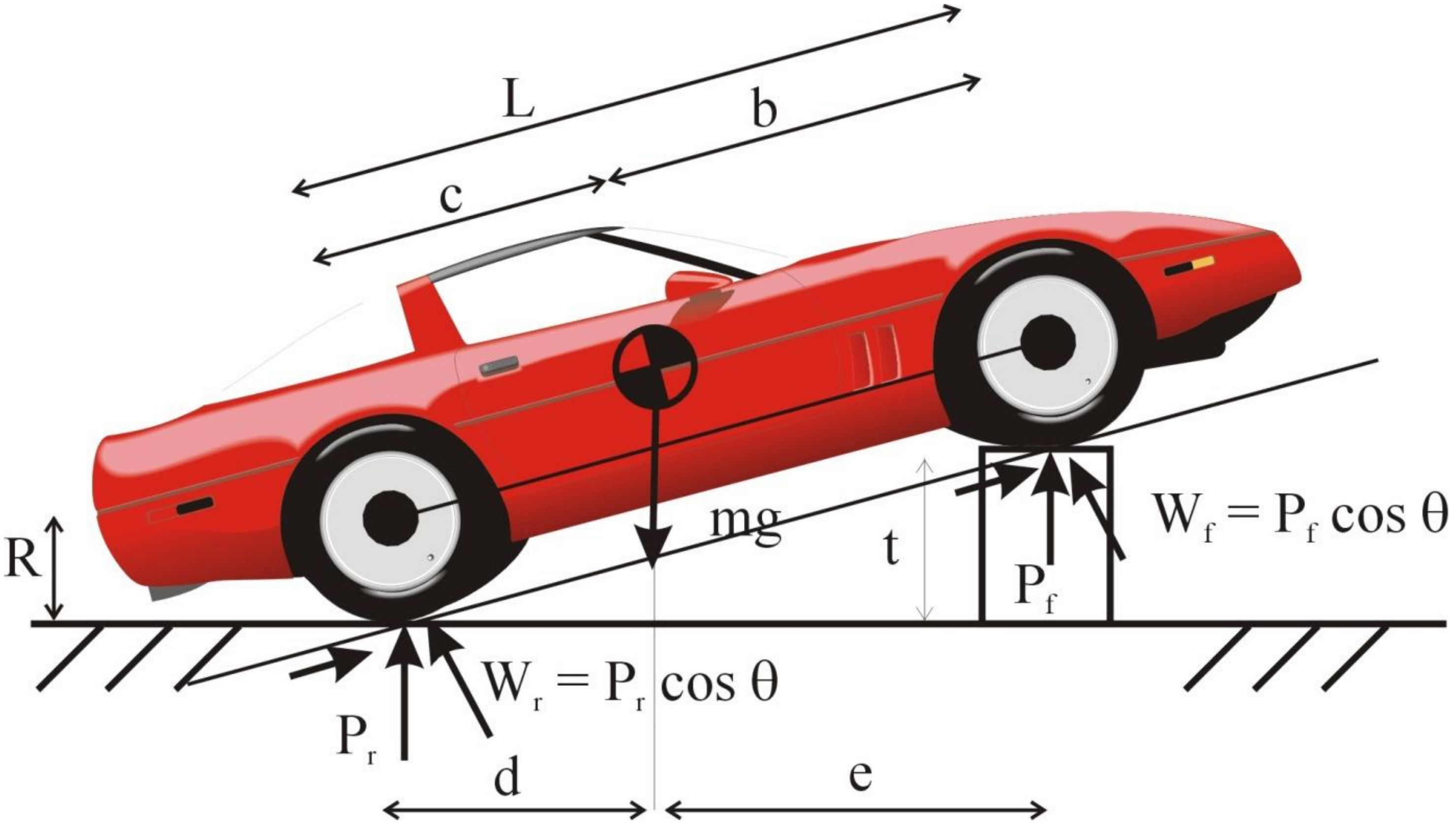
Application : position of CoG

- Horizontal position of CoG: b and c
  - Measure the weight under front and rear axles in horizontal position
  - It comes

$$W_f = mg\frac{c}{L}$$
$$W_r = mg\frac{b}{L}$$

$$\Rightarrow \left\{ \begin{aligned} m\,g &= W_f + W_r \\ \frac{b}{L} &= \frac{W_r}{W_f + W_r} \\ \frac{c}{L} &= \frac{W_f}{W_f + W_r} \end{aligned} \right.$$

Application : position of CoG



Application : position of CoG

- Vertical position h of CoG:
  - Measure the weight under the front and rear axles with predefined inclined position
  - Slope

$$\sin\theta = \frac{t}{L}$$

  - Relation between the weight P_f and P_r measured under the wheel sets and the effective normal reaction forces W_f and W_r :

$$W_f = P_f \, \cos\theta \quad \text{and} \quad W_r = P_r \, \cos\theta$$

Application : position of CoG

- The normal forces, perpendicular to the level plan of the car are given by:

$$\begin{aligned} W_f &= mg\cos\theta\frac{c}{L} - mg\sin\theta\frac{h}{L} \\ W_r &= mg\cos\theta\frac{b}{L} + mg\sin\theta\frac{h}{L} \end{aligned}$$

- It comes

$$W_f + W_r = m\,g\,\cos\theta$$

$$c\,W_r - b\,W_f = m\,g\,\sin\theta\,\frac{h}{L}\,(b+c)$$

Application : position of CoG

- The vertical position of the center of gravity is given by

$$\frac{h}{L} = \frac{c\,W_r - b\,W_f}{m\,g}\,\frac{1}{\sin\theta}$$

- In terms of the measured weight under the axles, it writes:

$$P_f + P_r = m\,g$$

$$\boxed{\frac{h}{L} = \frac{c\,P_r - b\,P_f}{P_f + P_r}\,\cot\theta}$$

Vehicle with trailer

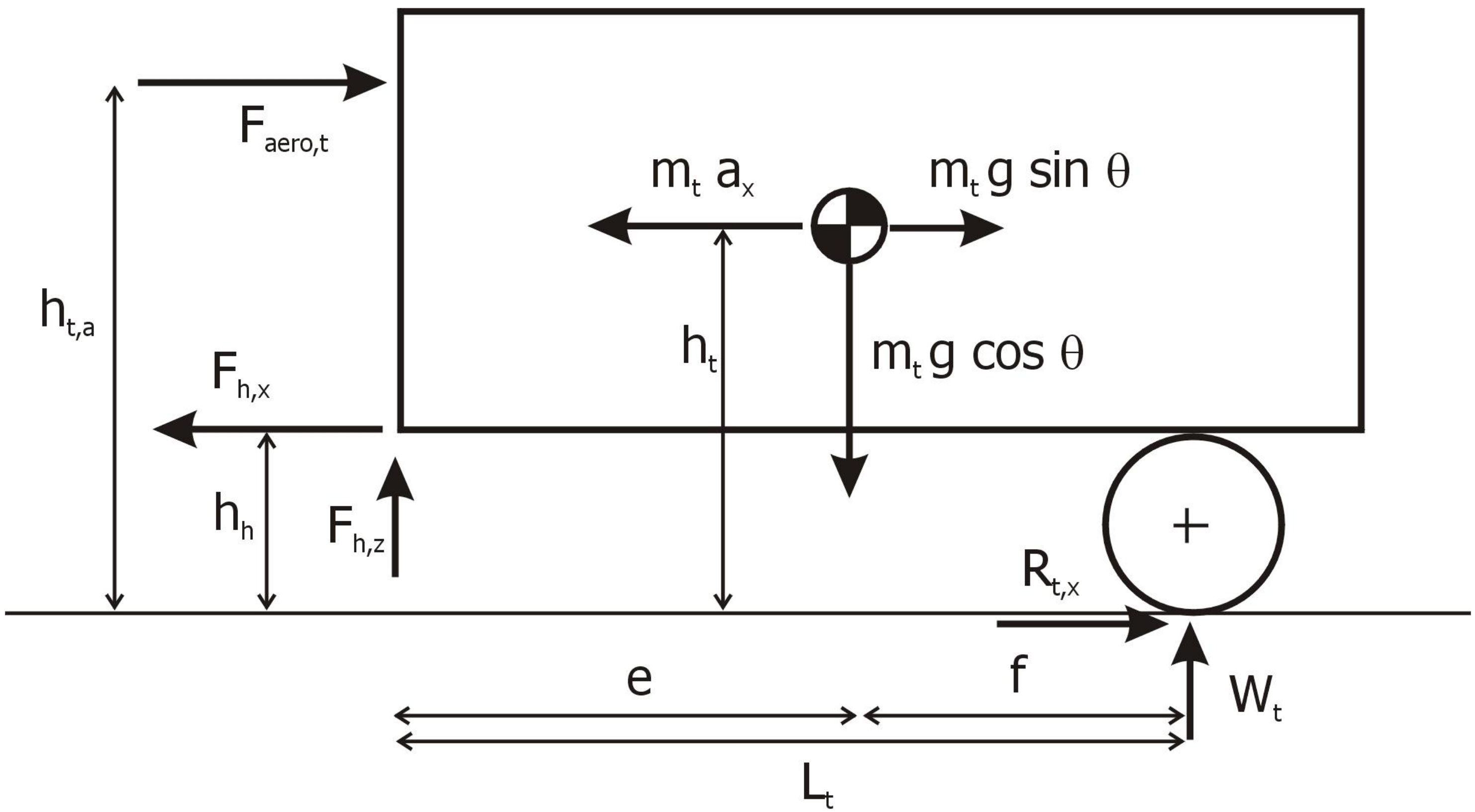
- In the Newton-Euler approach, kinematic joints and connections can be cut and replaced by unknown reaction forces so that the system cab be broken up into isostatic components.

- Then the free body diagrams of each part is investigated separately: the tractor, the trailer, each carriage…

- To study the motion of vehicle and its trailer, one can write down the equations of the trailer equilibrium as well as the equation of the pulling car.

- The car equations being already known, we focus now on the trailer alone.

Vehicle with trailer



Vehicle with trailer

- Equations of equilibrium of the trailer write

$$\begin{aligned} ma_x &= F_{h,x} - F_{AERO,t} - R_{t,x} - m_t g \sin\theta \\ 0 &= m_t g \cos\theta - W_t - F_{h,z} \\ 0 &= F_{AERO,t}\,(h_{t,A} - h_t) - W_t\, f - R_{t,x}\, h_t \\ &\quad + F_{h,x}(h_t - h_h) + F_{h,z}(L_t - f) \end{aligned}$$

- The horizontal motion enables to determine the pulling force at the hook

$$\boxed{F_{h,x} = ma_x + F_{AERO,t} + R_{t,x} + m_t g \sin\theta}$$

Vehicle with trailer

- Vertical and rotation equilibrium
  - To determine the weight under the trailer wheel set and the vertical reaction at the hook, one has to substitute these two unknowns between the two equations

$$\left\{\begin{array}{rcl} F_{h,z} & = & m_t g \cos\theta - W_t \\ W_t\, f & = & F_{AERO,t}\,(h_{t,A} - h_t) - R_{t,x}\, h_t + F_{h,x}(h_t - h_h) + F_{h,z}\,(L_t - f) \end{array}\right.$$

  - It comes

$$\left\{\begin{array}{rcl} W_t & = & m_t\, g\, \cos\theta\, \dfrac{e}{L_t} + F_{AERO,t}\, \dfrac{h_{t,A} - h_t}{L_t} - R_{t,x}\, \dfrac{h_t}{L_t} + F_{h,x} \dfrac{h_t - h_h}{L_t} \\ F_{h,z} & = & m_t\, g\, \cos\theta\, \dfrac{f}{L_t} - F_{AERO,t}\, \dfrac{h_{t,A} - h_t}{L_t} + R_{t,x}\, \dfrac{h_t}{L_t} - F_{h,x} \dfrac{h_t - h_h}{L_t} \end{array}\right.$$

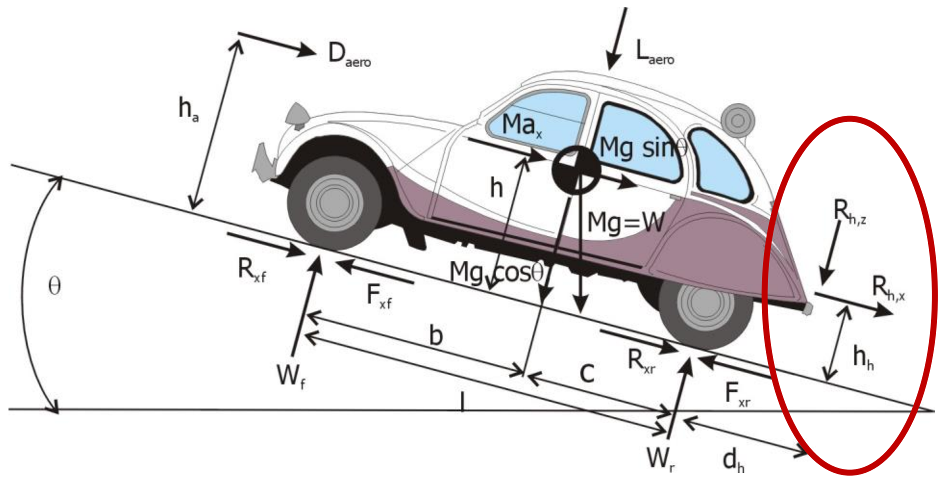
Vehicle with trailer

- It is usual to use the acceleration of the trailer.
- It comes

$$
\begin{aligned}
W_t &= m_t\, g\, \cos\theta\, \frac{e}{L_t} + m_t\, a_x\, \frac{h_t - h_h}{L_t} + m_t\, g\, \sin\theta\, \frac{h_t - h_h}{L_t} \\
&\quad + F_{AERO,t}\, \frac{h_{t,A} - h_h}{L_t} - R_{t,x}\, \frac{h_h}{L_t} \\
F_{h,z} &= m_t\, g\, \cos\theta\, \frac{f}{L_t} - m_t\, a_x\, \frac{h_t - h_h}{L_t} - m_t\, g\, \sin\theta\, \frac{h_t - h_h}{L_t} \\
&\quad - F_{AERO,t}\, \frac{h_{t,A} - h_h}{L_t} + R_{t,x}\, \frac{h_h}{L_t}
\end{aligned}
$$

Vehicle with trailer

- It is also possible to generalize the equations of the motion of the vehicle with hook.



$$
\begin{aligned}
ma_x &= F_{xf} + F_{xr} - R_{xf} - R_{xr} - mg\sin\theta - F_{AERO} - R_{h,x} \\
0 &= mg\cos\theta - W_f - W_r + L_{AERO} + R_{h,z} \\
0 &= (F_{xf} + F_{xr})h - (R_{xf} + R_{xr})h + W_f b - W_r c \\
&\quad + F_{AERO}(h_A - h) + R_{h,x}(h_h - h) + R_{h,z}(d_h + c)
\end{aligned}
$$

Vehicle with trailer

- Replaying the developments, it comes

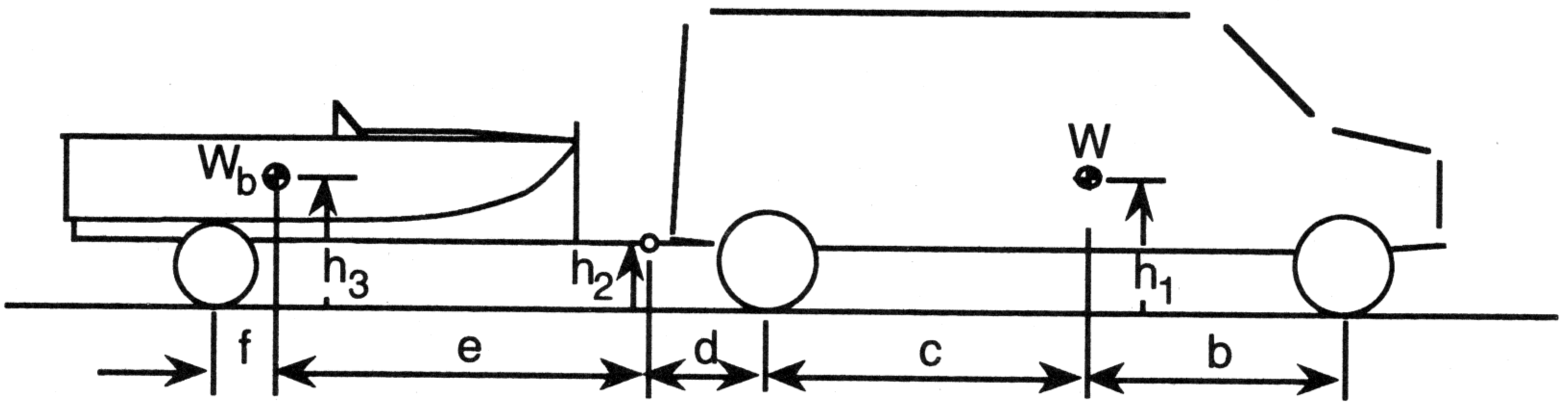
$$ma_x = (F_{xf} + F_{xr}) - (R_{xf} + R_{xr}) - mg\sin\theta - F_{AERO} - R_{h,x}$$

$$W_f = mg\cos\theta\frac{c}{L} - (F_x - R_x)\frac{h}{L} - F_{AERO}\frac{h_A - h}{L} + L_{AERO}\frac{c}{L} - R_{h,x}\frac{h_h - h}{L} - R_{h,z}\frac{d_h}{L}$$

$$W_r = mg\cos\theta\frac{b}{L} + (F_x - R_{RR})\frac{h}{L} + F_{AERO}\frac{h_A - h}{L} L_{AERO}\frac{b}{L} + R_{h,x}\frac{h_h - h}{L} + R_{h,z}\frac{d_h + L}{L}$$

Exercise

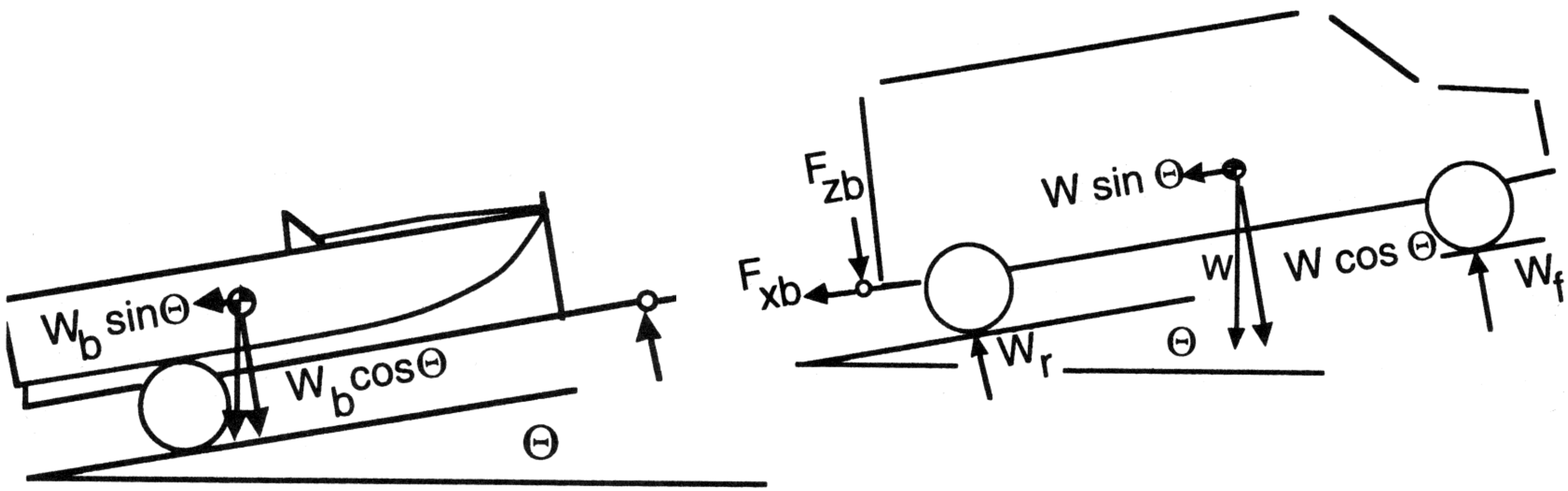
- Compute the maximum slope that the van and its boat trailer can climb with sliding ?



Exercise

- Data of the van:
  - W_f = 760 kg
  - W_r = 573 kg
  - Height of CoG h = 61 cm
  - Height of hook h_h = 35 cm
  - Horizontal position of the hook d_h=57,5 cm
  - Wheelbase L = 300 cm
- Data of trailer and its payload
  - W_t = 600 kg
  - $F_{h,z}$ = 125 kg
  - Height CoG of the boat h_t=27,5cm
  - Wheelbase of the trailer L_t=275 cm

Exercise



Free body diagram of the van + trailer