Vehicle Performance

Pierre Duysinx

Research Center in Sustainable Automotive Technologies of University of Liege

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Lesson 5
Performance criteria

Outline

- STEADY STATE PERFORMANCES
  - Maximum speed
  - Gradeability and maximum slope
    - At negligible speed
    - At high speed

- ACCELARATION AND ELASTICITY
  - Effective mass
  - Acceleration time and distance

References

- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- R. Bosch. « Automotive Handbook ». 5th edition. 2002. Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).
- W.H. Hucho. « Aerodynamics of Road Vehicles ». 4th edition. SAE International. 1998.
- M. Eshani, Y. Gao & A. Emadi. Modern Electric, Hybrid Electric and Fuel Cell Vehicles. Fundamentals, Theory and Design. 2nd Edition. CRC Press.

Max speed and gradeability

Vehicle performances

- Vehicle performance are dominated by two major factors:
  - The maximum power available to overcome the power dissipated by the road resistance forces
  - The capability to transmit the tractive force to the ground (limitation of tire-road friction)

- Performance indices are generally sorted into three categories:
  - Steady state criteria: max speed, gradeability
  - Acceleration and braking
  - Fuel consumption and emissions

Study of performances with tractive force diagrams

- The steady state performances can be studied using the tractive forces / road resistance forces diagrams with respect to the vehicle speed
- Newton equation

$$F_T - F_{AERO} - F_{RR} - F_{SLOPE} = m\frac{dv}{dt}$$

- Stationary condition

$$a_x = \frac{dv}{dt} = 0$$

- Then equilibrium writes

$$F_T = F_{RES} = F_{AERO} + F_{RR} + F_{SLOPE}$$

$$\mathcal{P}_T = \mathcal{P}_{RES} = F_{RES}\, v$$

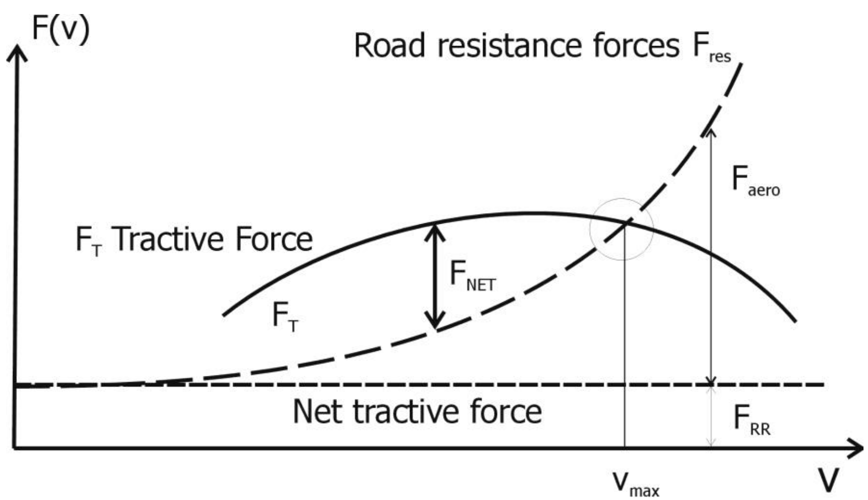
Study of performances with tractive force diagrams

- One generally defines the net tractive force

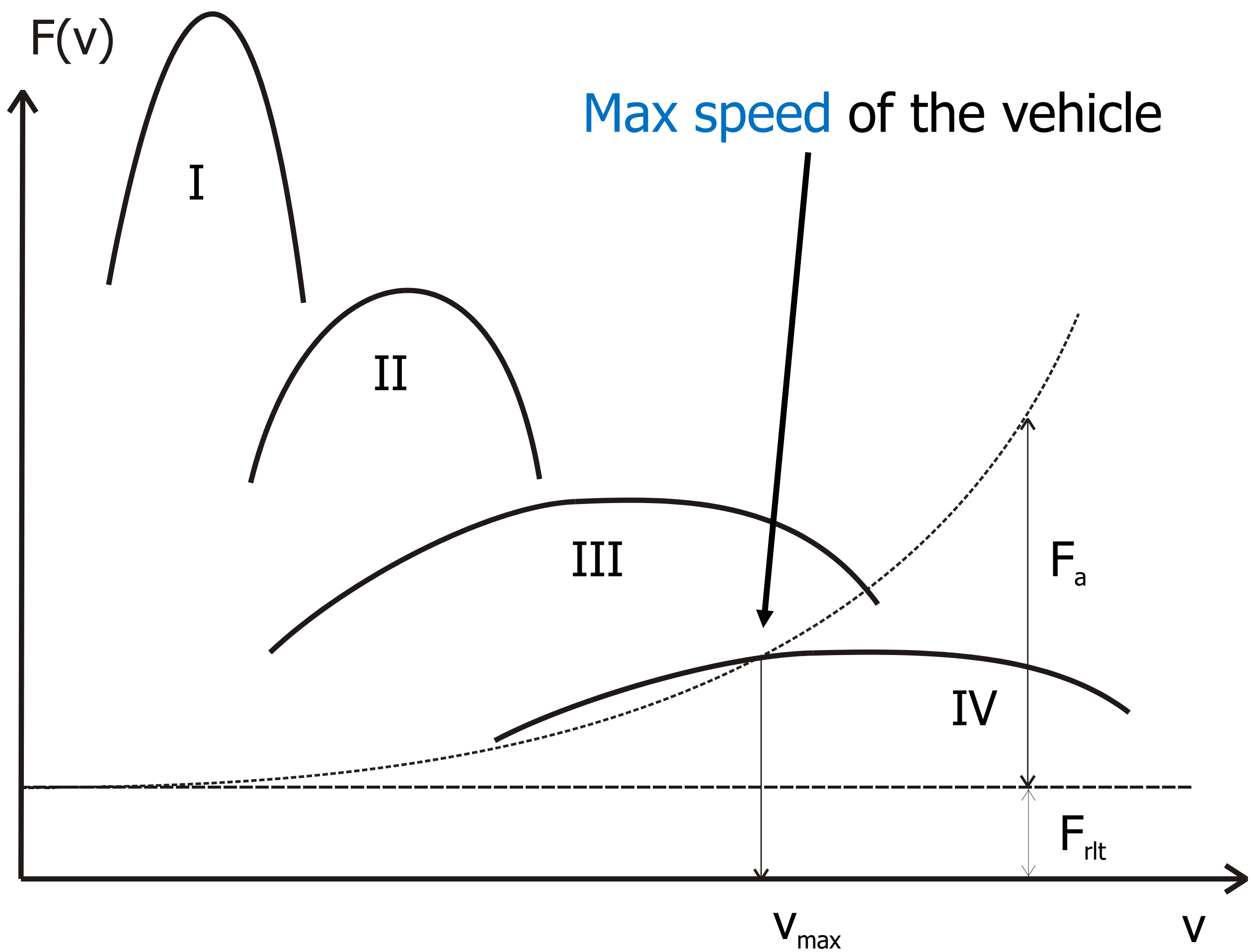
$$F_{NET} = F_T - F_{AERO} - F_{RR} - F_{GRADING}$$

- One also can use the net force diagram to calculate
  - The maximum speed
  - The maximum slope
  - The reserve acceleration available

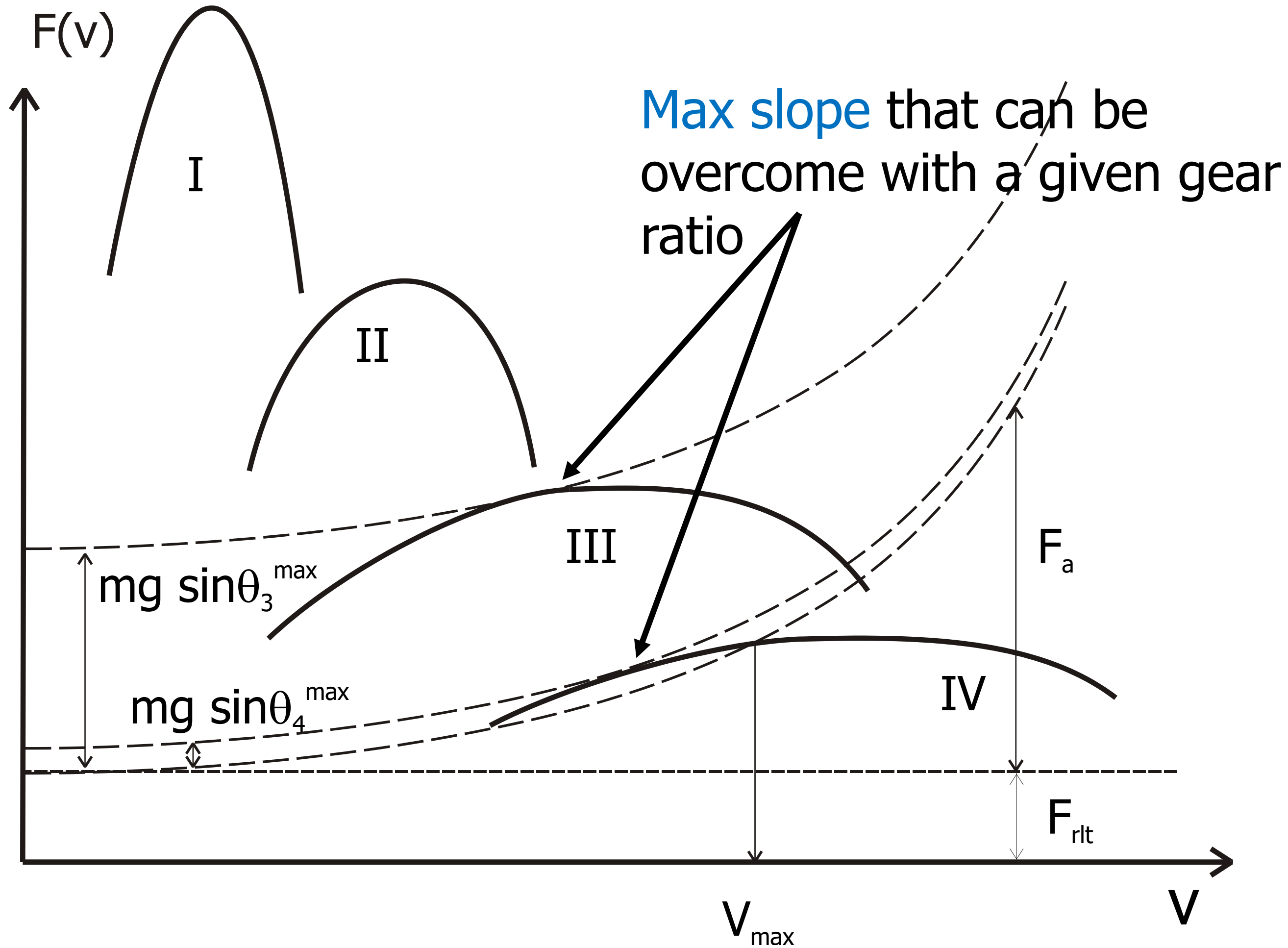
$$a_x = \frac{F_{NET}}{m}$$



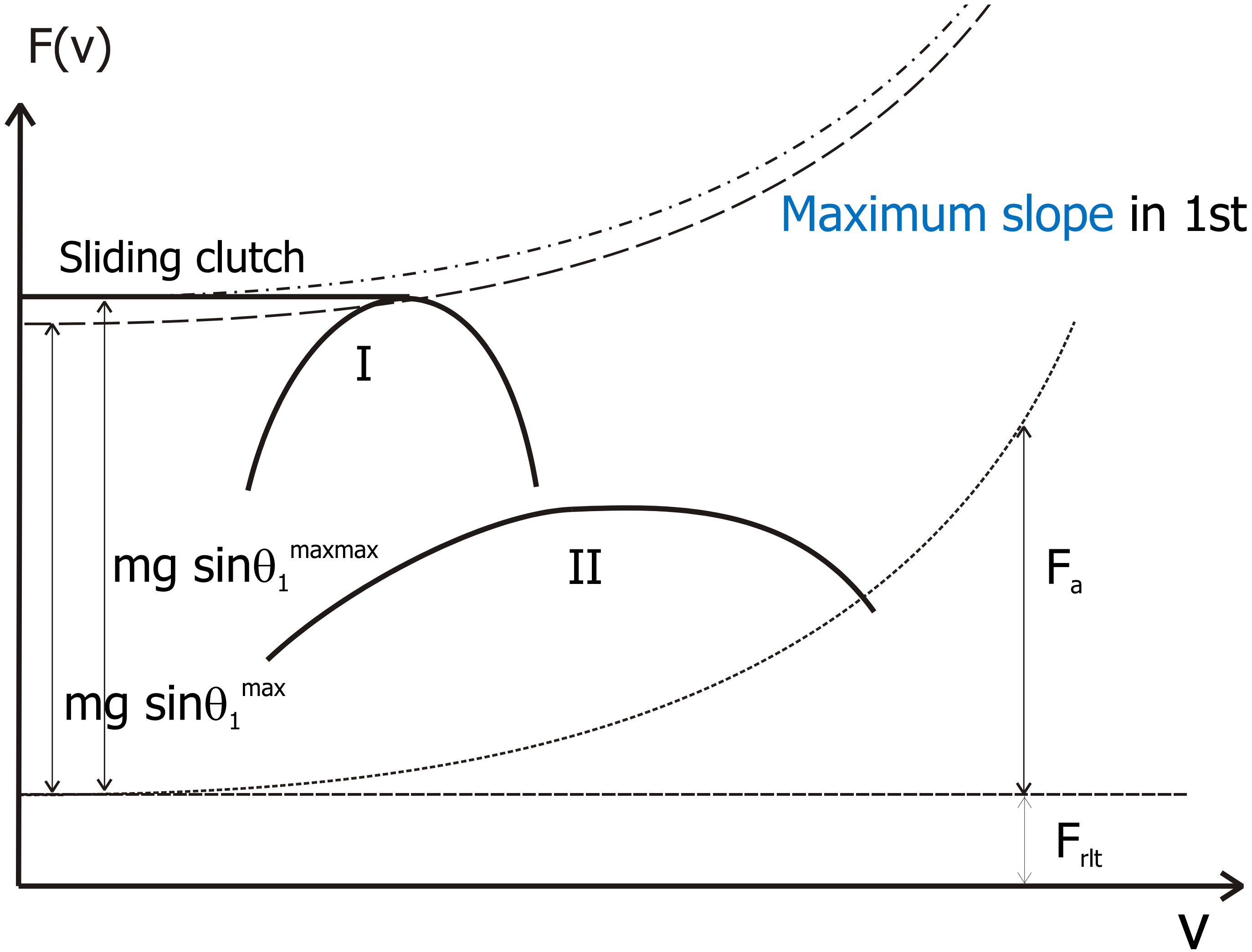
Study of performances with tractive force diagrams



Study of performances with tractive force diagrams



Study of performances with tractive force diagrams



Maximum speed

- For a given vehicle, tires, and engine, calculate the transmission ratio that gives rise to the greatest maximum speed
- Solve equality of tractive power and dissipative power of road resistance

$$\mathcal{P}_w = \mathcal{P}_{RES}$$

  - with

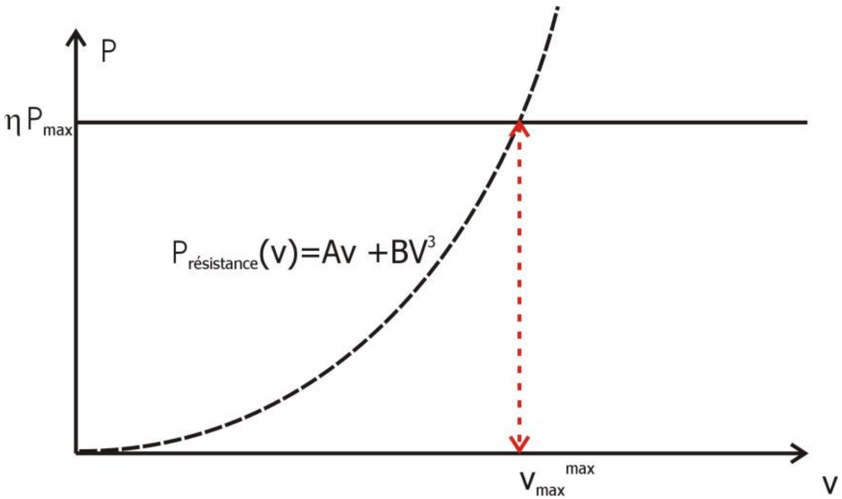
$$\mathcal{P}_{RES} = Av + Bv^3 \quad A,\ B > 0$$

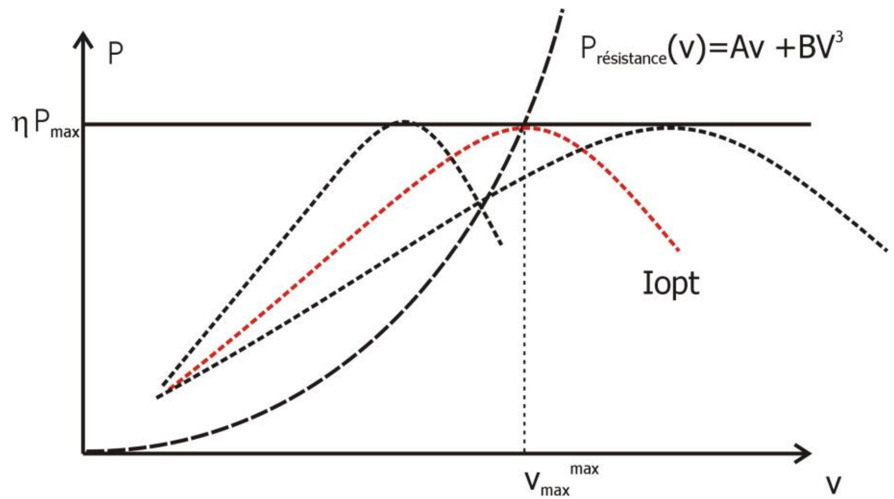
$$\mathcal{P}_w = \eta_t \mathcal{P}_p$$

- As the power of resistance forces is steadily increasing, the maximum speed is obtained when using the maximum power of the power plant

$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

Maximum speed





$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

$$\mathcal{P}_w = \mathcal{P}_{res}$$

$$\left(\frac{R}{i}\right)^* = \frac{v_{max}^{max}}{\omega_{nom}}$$

Maximum speed

- Iterative scheme to solve a nonlinear equation as a fixed-point Picard algorithm

$$F(x) = 0$$

- Reformulate the equation as

$$x = f(x)$$

- Iterative scheme

$$x^{(k+1)} = f(x^{(k)})$$

- Convergence if

$$|f'(x)| < 1$$

Maximum speed

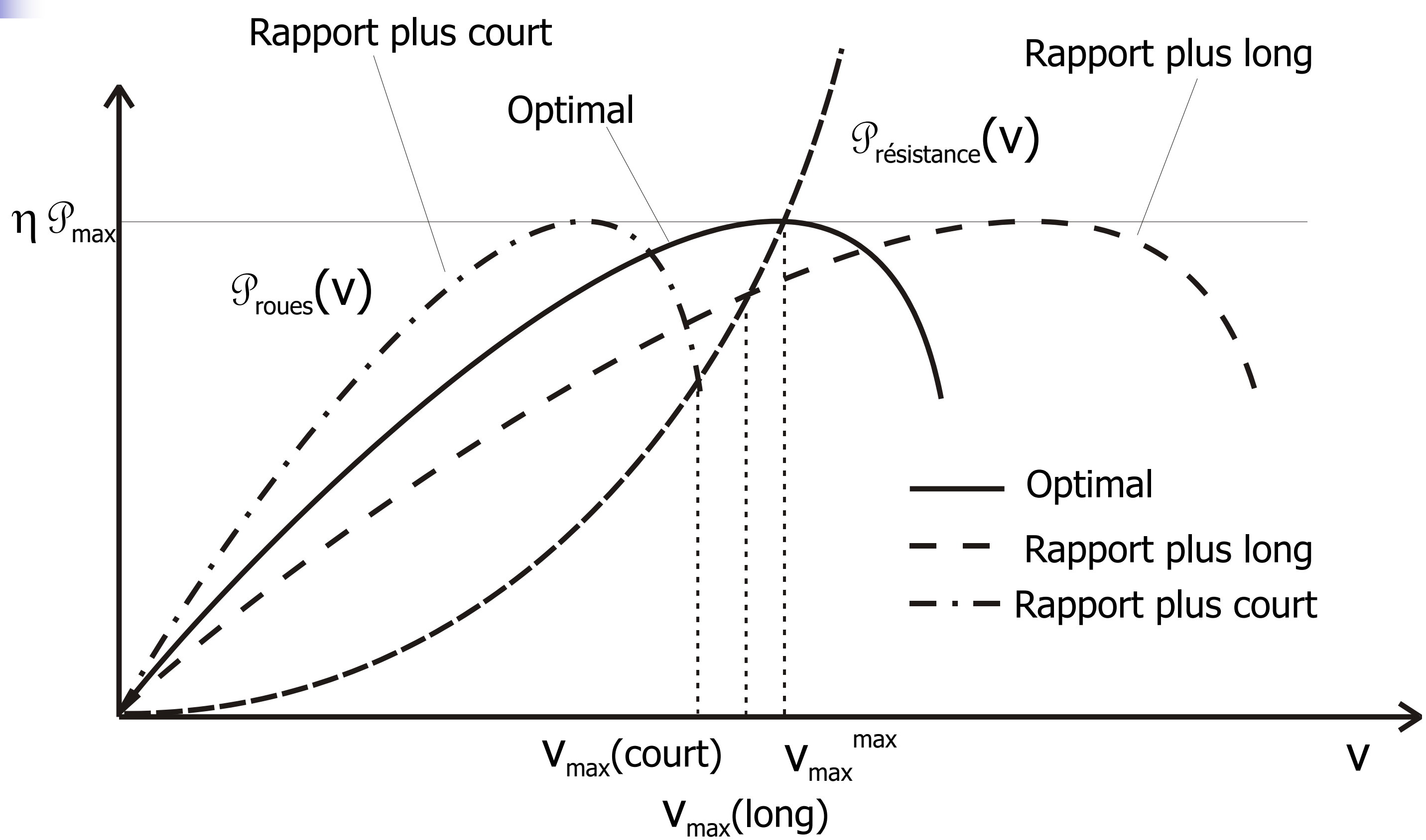
- Iterative scheme to solve the third order equation (fixed point algorithm of Picard)

$$\begin{aligned} v^{(0)} &= 0 \\ v^{(n+1)} &= \left(\frac{\eta \mathcal{P}_{max} - A v^{(n)}}{B} \right)^{1/3} \end{aligned}$$

- Once the maximum speed is determined the optimal transmission ratio can be easily calculated by since it occurs for the nom rotation speed:

$$\left(\frac{R}{i} \right)^* = \frac{v_{max}^{max}}{\omega_{nom}} \qquad i^* = \frac{\omega_{nom} \cdot R_e}{v_{max}^{max}}$$

Max speed for given reduction ratio



Max speed is always reduced compared to v_{max}^{max}

Max speed for given reduction ratio

- If the gear ratio is given, solve equation of equality of tractive and resistance power, but this time, the plant rotation speed is also unknown.

$$\begin{cases} \eta\,\mathcal{P}(\omega) \;=\; \mathcal{P}_{RES} \;=\; Av_{max} + Bv_{max}^3 \\ \omega \;=\; v\,\frac{\bar{i}}{R_e} \end{cases}$$

- Let's eliminate the rotation speed of the engine to find a single nonlinear equation to solve

$$\mathcal{P}_{RES} \;=\; Av_{max} + Bv_{max}^3 \;=\; \eta\,\mathcal{P}(\frac{\bar{i}}{R}v_{max})$$

Max speed for given reduction ratio

- Solve equation of equality of tactive and resistance power, but this time, the plant rotation speed is also varying.

$$\mathcal{P}_{RES} \; = \; Av_{max} \, + \, Bv_{max}^3 \; = \; \eta\,\mathcal{P}(\frac{\bar{i}}{R}v_{max})$$

- Numerical solution using a fixed-point algorithm (Picard iteration scheme)

$$\begin{aligned}
v^{(0)} &= 0 \quad \text{ou} \quad v^{(0)} = v_{max}^{max} \\
\omega^{(k)} &= v^{(k)}\,\frac{\bar{i}}{R_e} \\
\mathcal{P}^{(k)} &= \eta\mathcal{P}(\omega^{(k)}) \\
v^{(k+1)} &= \left(\frac{\mathcal{P}^{(k)} - Av^{(k)}}{B}\right)^{1/3}
\end{aligned}$$

Selection of gear ratio to achieve a given max speed

- We search for the gear ratio i/R to satisfy the power equation:

$$\bar{\mathcal{P}}_p = \frac{\mathcal{P}_{RES}}{\eta} = \frac{1}{\eta}\left(A\bar{v}_{max} + B\bar{v}^3_{max}\right) \qquad \bar{v}_{max} \leq v^{max}_{max}$$

- Let's assume that the curve power of the engine is given by:

$$\frac{\bar{\mathcal{P}}_p}{\mathcal{P}_1} = 1 - \left(1 - \frac{\mathcal{P}_2}{\mathcal{P}_1}\right)\left|\frac{1-\frac{\omega_p}{\omega_1}}{1-\frac{\omega_2}{\omega_1}}\right|^b \qquad \mathcal{P}_1 = \mathcal{P}_{max} \quad \text{and} \quad \omega_1 = \omega_{nom}$$

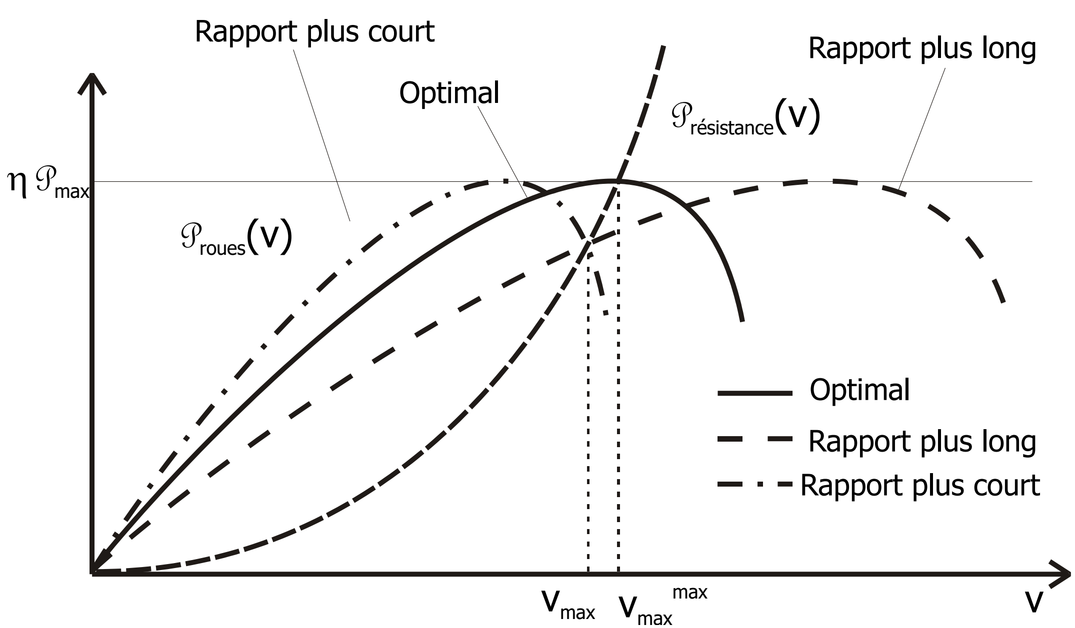
- The solution writes

$$\left|1 - \frac{\omega_p}{\omega_{nom}}\right| = \left(1 - \frac{\omega_2}{\omega_{nom}}\right)\left(\frac{1-\frac{\bar{\mathcal{P}}_p}{\mathcal{P}_{max}}}{1-\frac{\mathcal{P}_2}{\mathcal{P}_{max}}}\right)^{1/b} = \alpha$$

Selection of gear ratio to achieve a given max speed

- We have two solutions (because of the module function) : one is characterized by an engine speed greater than to ω_{nom} and the other one lower

$$\left|1 - \frac{\omega_p}{\omega_{nom}}\right| = \alpha > 0 \quad \Leftrightarrow \quad \omega_p = \omega_{nom}(1 \pm \alpha)$$



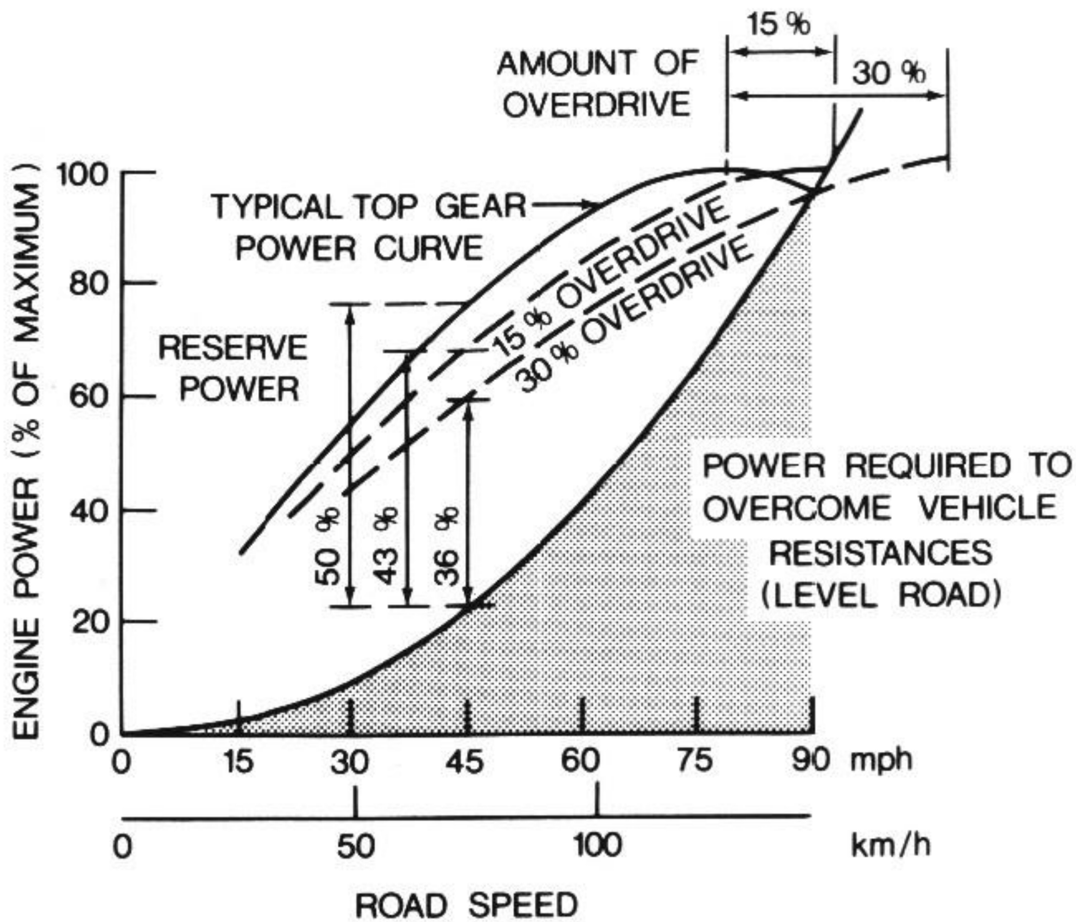
Selection of the top gear ratio

- Design specifications for the top gear ratio in connection with the top speed criteria (from Wong)
  - To be able to reach a given top speed with the given engine
  - To be able to maintain a given constant speed (from 88 to 96 km/h) while overcoming a slope of at least 3% with the selected top gear ratio

- These specifications enable to select a proper top gear ratio
  - The first requirement enables to select a highest gear ratio
  - Then because of the second one, one has to select a gear ratio that gives rise to an engine rotation speed that is just above the nominal rotation speed (and the max power) in order to save a sufficient power reserve to keep a constant speed while climbing a small slope, overcome wind gusts or accounting for loss of engine performance with ageing.

Selection of the top gear ratio

Impact of selected gear ratio on vehicle performance
Wong, Fig. 3.26



Maximum slope

- For the maximum slope the vehicle can climb, two criteria must be checked:

- The maximum tractive force available at wheel to balance the grading force

$$F_w \geq F_{RES} \simeq F_{GRADE} = mg \sin \theta$$

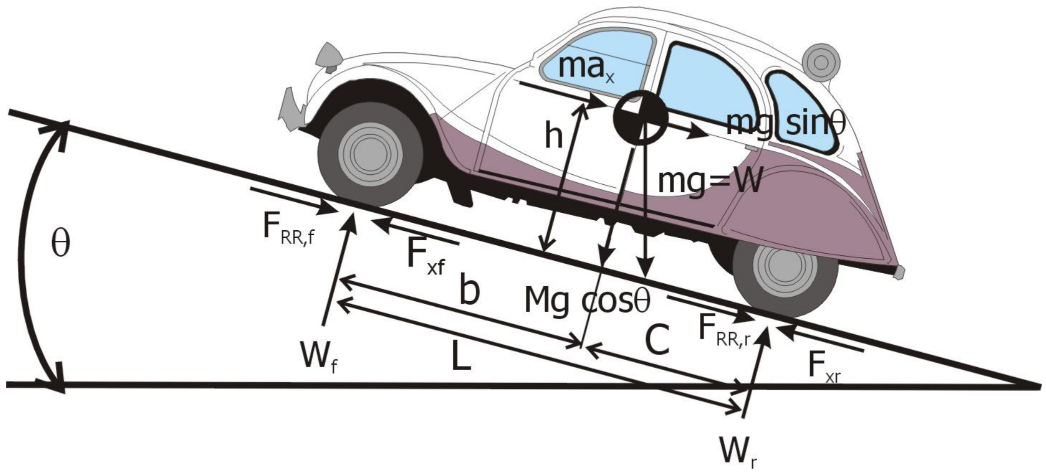
- The maximum force that can be transmitted to the road because of tire friction and weight transfer

$$F_{w,f} \leq \mu W_f \qquad F_{w,r} \leq \mu W_r$$

Maximum slope

Conditions	Coefficient de friction μ
Route sèche	0,8 à 1,2
Route mouillée - 0,2 mm d'eau	0,5 - 0,8
Gravier	0,4
Route mouillée - 2 mm d'eau	0,05 - 0,5
Neige	0,2
Glace	0,1 ou moins

Maximum slope



- Vertical equilibrium

$$m\,g\cos\theta = W_f + W_r$$

- Rotational equilibrium about rear wheels contact point

$$W_f\,L + mg\ \sin\theta\,h + m\,a_x\,h = mg\ \cos\theta\,c$$

- Rotational equilibrium about rear wheels contact point

$$W_r\,L = mg\ \cos\theta\,b + mg\ \sin\theta\,h + m\,a_x\,h$$

Maximum slope

- Limitation due to the friction coefficient

$$F_{w,f} \leq \mu W_f \qquad\qquad F_{w,r} \leq \mu W_r$$

- Normal forces under the front and rear wheel sets

$$W_f = mg\ \cos\theta\ \frac{c}{L} - mg\ \sin\theta\ \frac{h}{L} - m\,a_x\ \frac{h}{L}$$

$$W_r = mg\ \cos\theta\ \frac{b}{L} + mg\ \sin\theta\ \frac{h}{L} + m\,a_x\ \frac{h}{L}$$

- At low speed and constant speed (a_x=0)

$$W_f = mg\ \cos\theta\ \frac{c}{L} - mg\ \sin\theta\ \frac{h}{L}$$

$$W_r = mg\ \cos\theta\ \frac{b}{L} + mg\ \sin\theta\ \frac{h}{L}$$

Maximum slope

FOUR-WHEEL DRIVE with electronic power split

$$F_p = F_{w,f} + F_{w,r} \leq \mu \left(W_f + W_r\right)$$

$$mg\ \sin\theta + mg\ \cos\theta\ f \leq \mu\, mg \cos\theta$$

Maximum slope $\boxed{\tan\theta\ \leq\ \mu - f}$

Maximum slope

FRONT WHEEL DRIVE

$$F_{w,f} \leq \mu \, W_f$$

$$mg \ \sin\theta + mg \ \cos\theta \ f \leq \mu \, mg(\cos\theta \frac{c}{L} - \sin\theta \frac{h}{L})$$

Maximum slope

$$\boxed{\tan\theta \ \leq \ \frac{\mu c/L - f}{1 + \mu h/L}}$$

Maximum slope

REAR WHEEL DRIVE

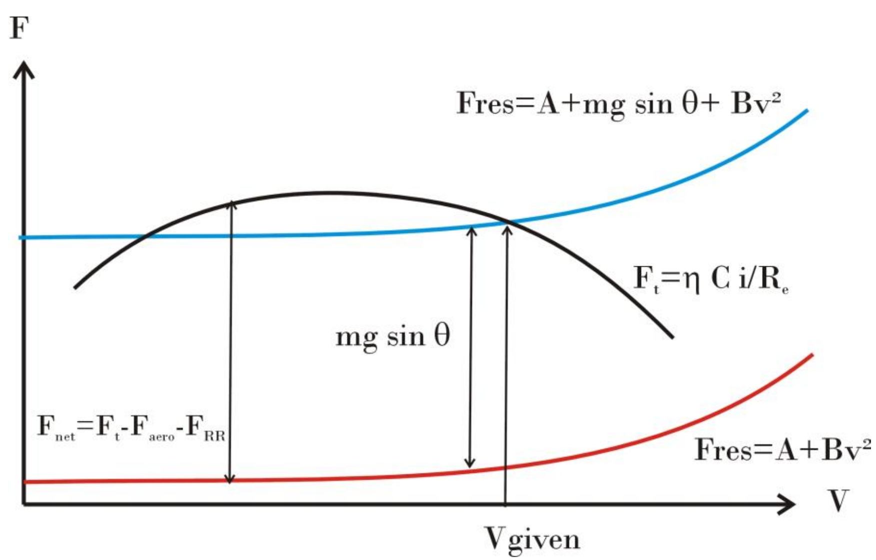
$$F_{w,r} \leq \mu \, W_r$$

$$mg \; \sin\theta + mg \; \cos\theta \; f \leq \mu \, mg(\cos\theta \frac{b}{L} + \sin\theta \frac{h}{L})$$

Maximum slope

$$\boxed{\tan\theta \;\; \leq \;\; \frac{\mu b/L - f}{1 - \mu h/L}}$$

Maximum slope at high speed



- What is the maximum grade that can be overcome at a given speed V is:

$$\sin\theta = \frac{F_t - F_{RR} - F_{aero}}{mg} = \frac{F_t^{net}}{mg}$$

- Gradeability is ruled by the net tractive force available

$$F_t^{net} = F_t - F_{RR} - F_{aero}$$
$$= F_t - mgf\cos\theta - 0,5\,\rho\,SC_x\,V^2$$

- The tractive force is given by the speed

$$F_t(v) = \eta_t C(\omega = \frac{v}{R_e} i)\frac{i}{R_e}$$

- Check also nonslip condition

$$F_t \leq \mu W_f \qquad F_t \leq \mu W_r$$

Maximum slope at high speed

- The maximum slope can be evaluated as follows

$$F_t - F_{aero} - F_{RR} - mg\sin\theta = 0$$
$$\iff F_t - F_{aero} - mg\cos\theta\, f - mg\sin\theta = 0$$

- If we define

$$d = (F_t - F_{aero})/mg$$

- It comes :

$$d - \sin\theta = f\ \cos\theta$$
$$(d - \sin\theta)^2 = f^2\ \cos^2\theta$$
$$\iff d^2 - 2d\sin\theta + \sin^2\theta = f^2(1 - \sin^2\theta)$$
$$\iff (1 + f^2)\ \sin^2\theta - 2d\ \sin\theta + (d^2 - f^2) = 0$$

Maximum slope at high speed

- It a second order equation in sinθ

$$(1+f^2)\ \sin^2\theta - 2d\ \sin\theta + (d^2 - f^2) = 0$$

- Solving for sinθ gives

$$\begin{aligned}\sin\theta &= \frac{2d \pm \sqrt{4d^2 - 4\left(1+f^2\right)\left(d^2-f^2\right)}}{2(1+f^2)} \\ &= \frac{d \pm \sqrt{d^2 - d^2 + f^2 - f^2 d^2 + f^4}}{1+f^2} \\ &= \frac{d \pm \sqrt{f^2 - d^2\ f^2 + f^4}}{1+f^2} = \frac{d \pm f\ \sqrt{1 - d^2 + f^2}}{1+f^2}\end{aligned}$$

- It comes :

$$\sin\theta = \frac{d - f\sqrt{1+f^2-d^2}}{1+f^2} \qquad d = (F_t - F_{aero})/mg$$

Selection of first gear ration

- Maximum slope to be overcome, for instance θ_{max} = 25%

$$F_{\mathrm{RES}} = mg\ \sin\theta_{max} + mg\ f_{RR}\ \cos\theta_{max}$$

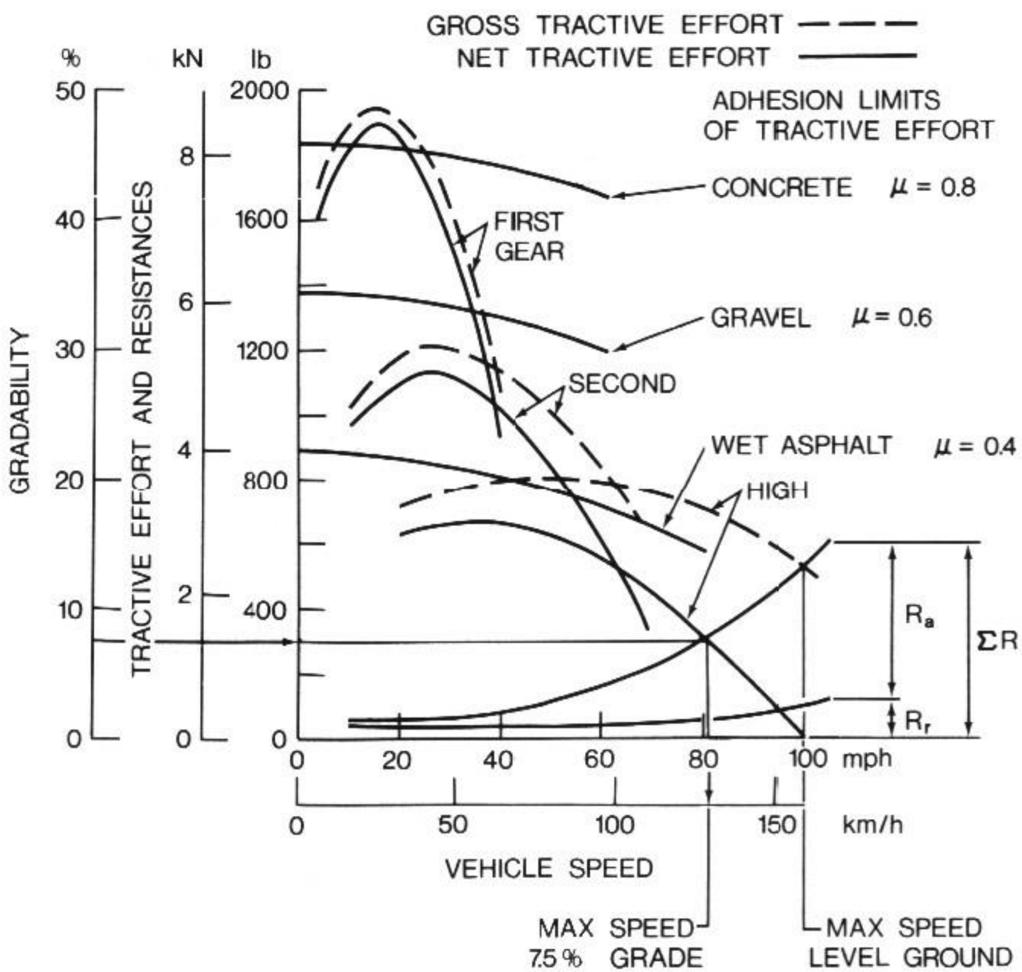
- Tractive force at wheels

$$F_w \;=\; \eta\frac{i}{R_e}C_p$$

- Sizing of first gear ration

$$i_{max} \;=\; \frac{R_e\, F_{res}}{\eta\, C_{max}} \qquad i_{max} \;=\; \frac{R_e\, mg\ \sin\theta_{max}}{\eta\, C_{max}}$$

Gradeability

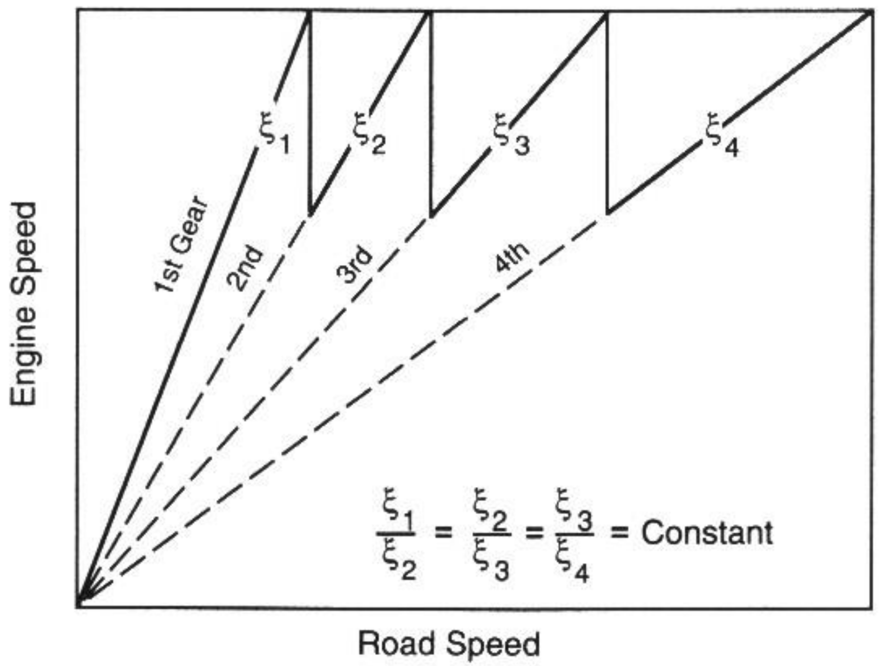


Characteristic curves a 3-gear vehicle
Wong, Fig. 3.29

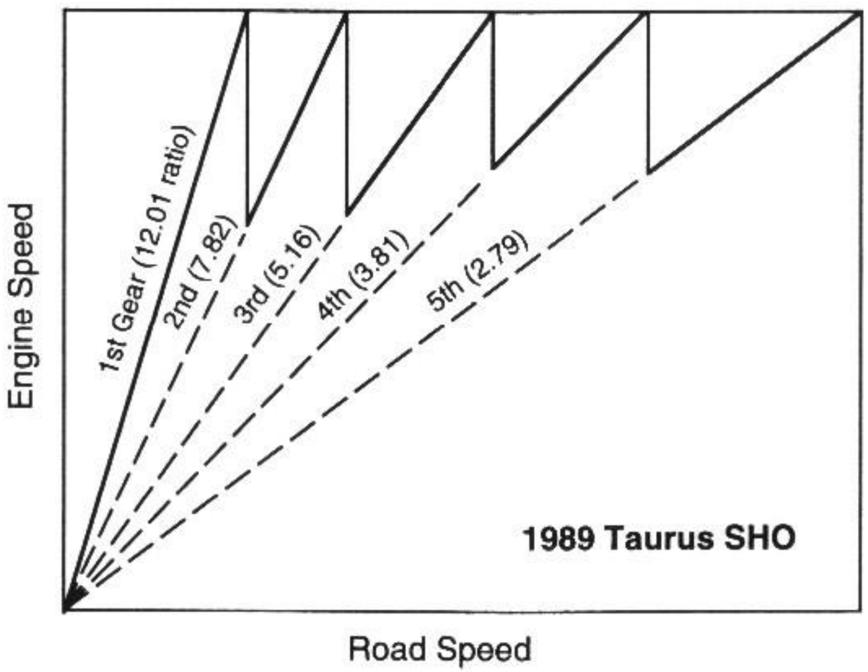
Selection of intermediate gear ratios

- Goal of the selected gear ratio: to adapt the characteristics of engine operation (rotation speed, torque) to the vehicle speed.
- The top and lowest gear ratios are selected
  - To match a given top speed
  - To be able to drive over given grading conditions, that is to develop sufficiently high tractive forces at wheels
- Find the distribution of intermediate gear ratios in between the top and lowest gear ratio
- In principle, the different gear ratios should render as much as possible the maximum power curve
- Practically the selection of intermediate gear ratios is made to span the full range of operating speeds more or less smoothly.

Selection of intermediate gear ratios



Gillespie. Fig. 2.7 Selection of gear ratio following a geometrical rule



Gillespie. Fig. 2.8 Selection of gear ratios for a Ford Taurus

Selection of gear ratios

- Depending on the number of gear ratios in the gear box, one has typically the following gear ratios

Nombre de vitesses	Plage de rapport de réduction
4	3,5 :1 à 3,9 :1
5	4,3 :1 à 5,2 :1
6	6 :1

Selection of intermediate gear ratios

- As a first approximation, one can assume that engine always operates in the same range of rotation speed: between low regime N_L and an upper regime N_H.

- Gear change between ratio 1 and 2 happens at speed :

$$v_{1\to 2} = \omega_H \frac{R_e}{i_1} = \omega_L \frac{R_e}{i_2}$$

So

$$\frac{i_2}{i_1} = \frac{\omega_L}{\omega_H} = K$$

Selection of intermediate gear ratios

- It comes for other ratios

$$\frac{i_3}{i_2} = \frac{\omega_L}{\omega_H} = K$$

$$\frac{i_4}{i_3} = \frac{\omega_L}{\omega_H} = K$$

etc.

So that

$$i_2 = K\, i_1$$

$$i_3 = K\, i_2 = K^2\, i_1$$

$$i_4 = K\, i_3 = K^3\, i_1 \quad \text{etc.}$$

- This shows up that the gear ratios follows a *geometrical progression* of reason $K = N_H/N_L$:

$$i_k = K^{k-1}\, i_1$$

Selection of intermediate gear ratios

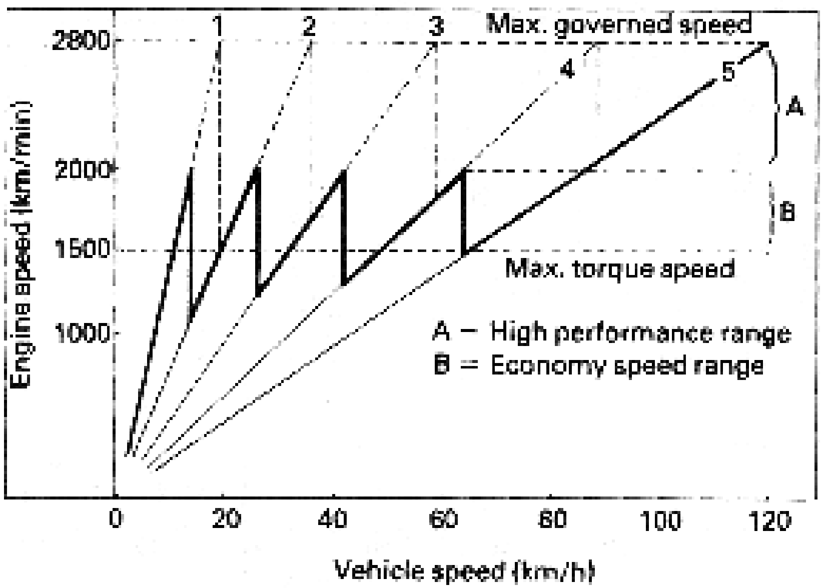
- If we know the lowest and the largest gear ratios, as well as the expected number of gear ratio, one can determine the reason:

$$i_n = K^{n-1}\, i_1 \qquad\qquad \boxed{K = \sqrt[n-1]{\frac{i_n}{i_1}}}$$

- This rule is generally followed in commercial vehicles that are equipped with gear boxes with a high number of gear ratios.

- However, for passenger cars with a moderate number of gear ratios, it is not the case. There are significant discrepancies between the geometrical rules and the actual gear ratio. The reason is that it is preferred in these vehicles to stress the gear ratios to have a more continuous operations at higher speeds.

Selection of intermediate gear ratios



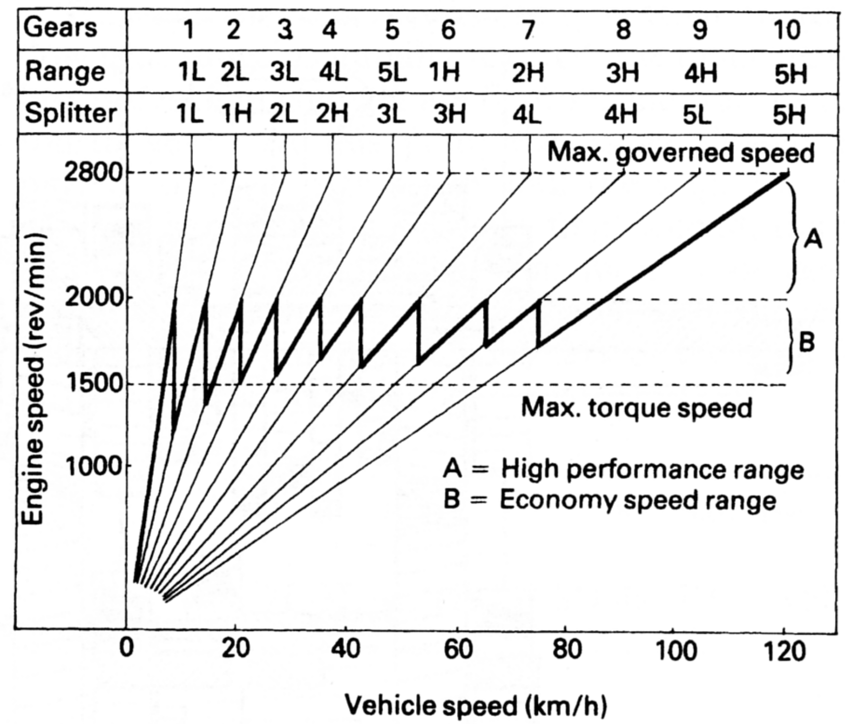


Fig 3.17 & 3.18 Heisler Advanced Vehicle technology:
Gear box ratios of vehicle with 5 to 10 speeds

Selection of intermediate gear ratios

- In addition, nowadays, the emphasis on the fuel consumption is such that the gear box selection is strongly optimized to cope with other constraints like maximum fuel economy or minimization of pollutant emissions.

- This leads to the necessary integrated optimization of the engine and its driveline.

Selection of intermediate gear ratios

TABLE 3.4 Gear Ratios of Transmissions for Heavy Commercial Vehicles

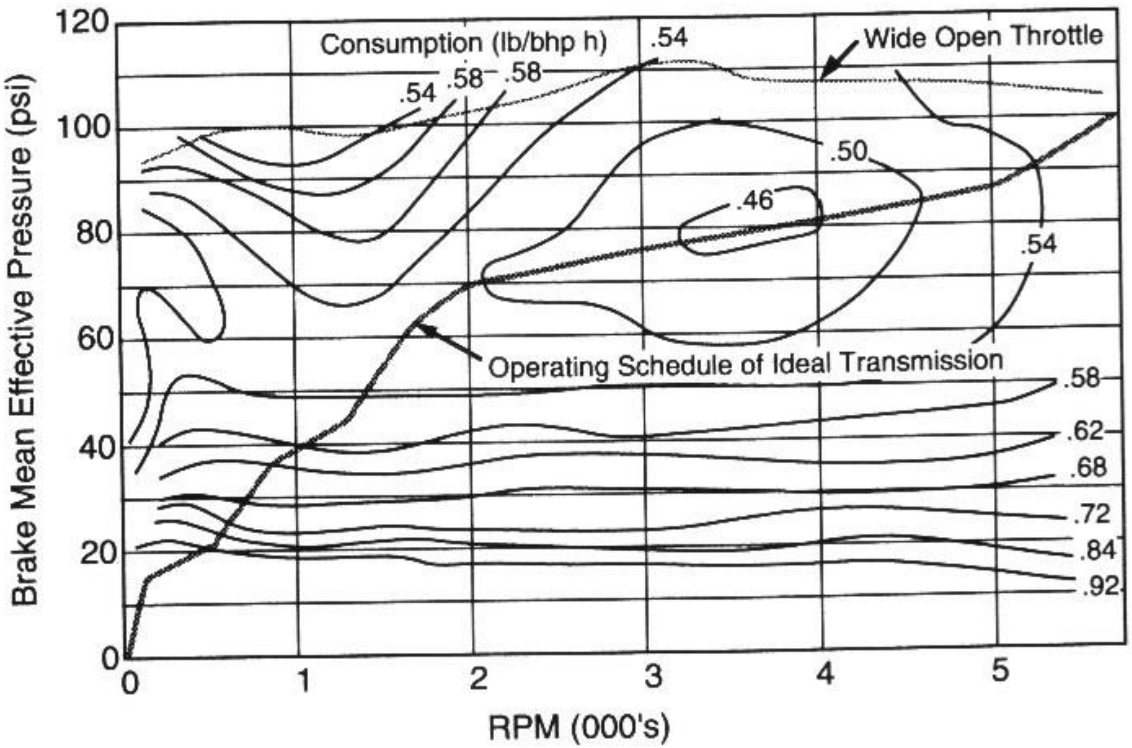
Gear	Allison HT70	Eaton Fuller RT-11608	Eaton Fuller RT/RTO-15615	Eaton Fuller RT-6613	ZF Ecomid 16S 109
1	3.0	10.23	7.83	17.93	11.86
2	2.28	7.23	6.00	14.04	10.07
3	1.73	5.24	4.63	10.96	8.40
4	1.31	3.82	3.57	8.61	7.13
5	1.00	2.67	2.80	6.74	5.71
6	0.76	1.89	2.19	5.26	4.85
7		1.37	1.68	4.11	3.97
8		1.00	1.30	3.29	3.37
9			1.00	2.61	2.99
10			0.78	2.05	2.54
11				1.60	2.12
12				1.25	1.80
13				1.00	1.44
14					1.22
15					1.00
16					0.85
Value of K_g Calculated from Eq. 3.29	0.76	0.717	0.774	0.786	0.839

TABLE 3.5 Gear Ratios of Transmissions for Passenger Cars

Vehicle	Transmission Type	Transmission Ratios 1st	2nd	3rd	4th	5th	Final Drive Ratio
Audi 80 1.8S	Manual	3.545	1.857	1.156	0.838	0.683	4.111
100	Manual	3.545	2.105	1.429	1.029	0.838	4.111
100 Quattro 2.8E	Manual	3.500	1.842	1.300	0.943	0.789	4.111
BMW 325i	Manual	4.202	2.49	1.67	1.24	1.00	3.15
535i	Manual	3.83	2.20	1.40	1.00	0.81	3.64
750i	Automatic	2.48	1.48	1.00	0.73		3.15
Buick Park Avenue	Automatic	2.92	1.57	1.00	0.70		2.84
Cadillac Seville	Automatic	2.92	1.57	1.00	0.70		2.97
Chrysler Voyager SE	Automatic	2.84	1.57	1.00	0.69		3.47
Ford Mustang GT	Manual	3.97	2.34	1.46	1.00	0.79	3.45
Crown Victoria	Automatic	2.40	1.47	1.00	0.67		3.08
Honda Accord GT2.2i	Manual	3.307	1.809	1.230	0.933	0.757	4.266
Mazda 323 1.6i GLX	Manual	3.42	1.84	1.29	0.92	0.73	4.11
929 3.0i GLX	Manual	3.48	2.02	1.39	1.00	0.76	3.73
Mercedes-Benz 230CE	Manual	3.91	2.17	1.37	1.00	0.81	3.46
300E	Automatic	3.87	2.25	1.44	1.00		3.27
600SEL	Automatic	3.87	2.25	1.44	1.00		2.65
Mercury Cougar LS	Manual	2.40	1.47	1.00	0.67		3.27
Nissan Micra LX	Manual	3.41	1.96	1.26	0.92	0.72	3.81
Toyota Camry 2.0G i	Manual	3.285	2.041	1.322	1.028	0.820	3.944
Volkswagen Passat GT	Manual	3.78	2.12	1.43	1.03	0.84	3.68
Volvo 960	Automatic	2.80	1.53	1.00	0.75		3.73

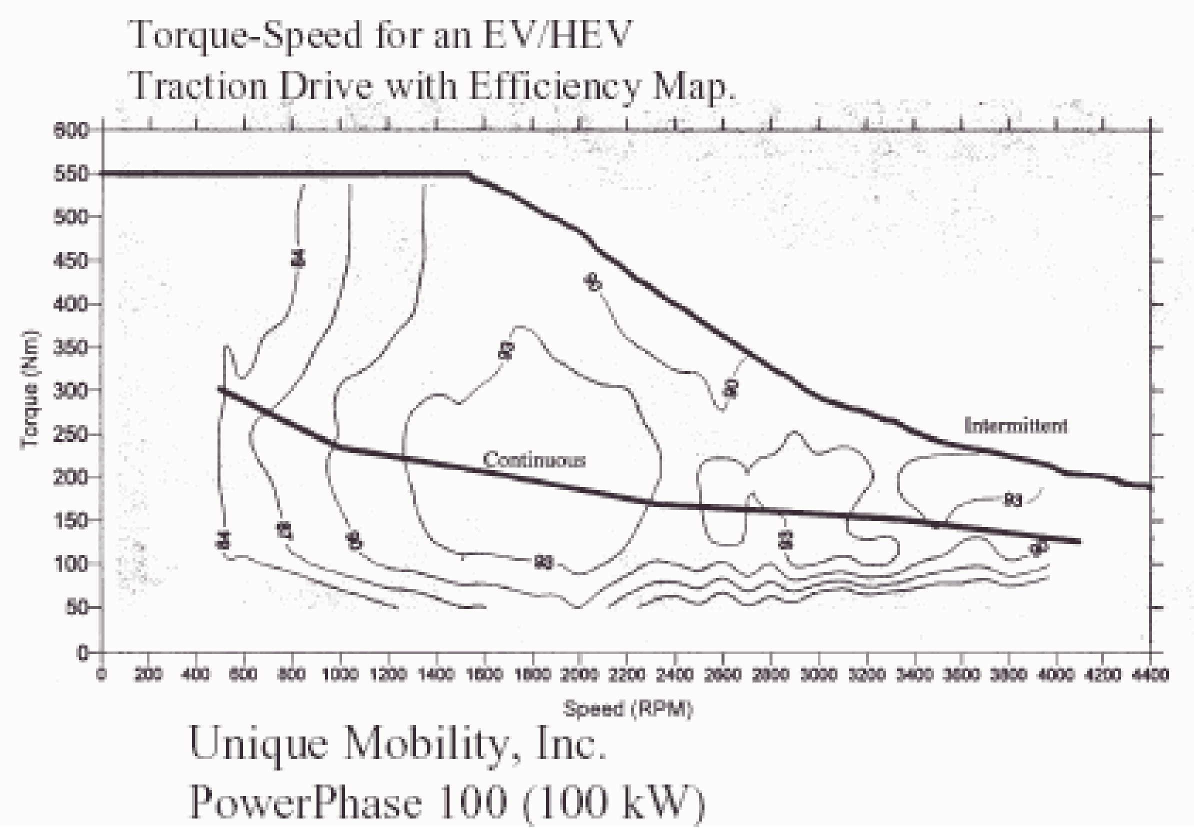
Wong Table 3.4 & 3.5:
typical gear ratios

Selection of intermediate gear ratios



Gillespie. Fig. 2.9 Selection of intermediate gear ratios usually aims at following the curve of minimum fuel consumption while carrying out official driving cycles to minimize emissions and fuel consumption

Selection of other gear ratios



For an electric vehicle

Accelerations and elasticity

Acceleration performance

- Estimation of acceleration and elasticity is based on the second Newton law

$$F_w - \sum F_{res} = F_{net} = m\frac{dV}{dt}$$

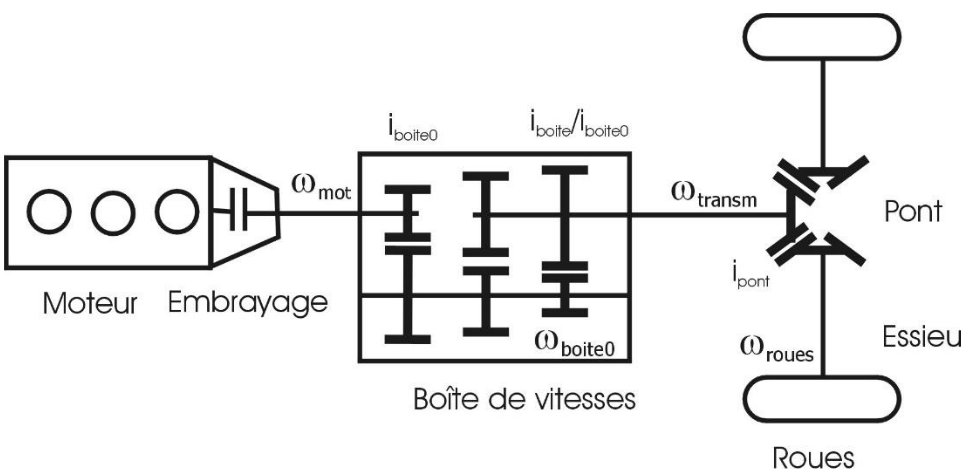
- Warning: when accelerating, the rotation speed of all driveline and transmission components is increasing : wheel sets, transmission shafts, gear boxes and differential, engine…

➔ Effective mass to account for the kinetic energy of all components (translation + rotation)

Effective mass

- Total kinetic energy of the vehicle and its driveline :

$$
\begin{aligned}
T =& 1/2m\, v^2 + 1/2(\sum I_{\mathrm{w}} + I_{\mathrm{axle}})\, \omega_{\mathrm{w}}^2 \\
&+ 1/2(I_{\mathrm{transm}} + I_{\mathrm{box2}})\, \omega_{\mathrm{transm}}^2 \\
&+ 1/2(I_{\mathrm{box0}})\, \omega_{\mathrm{box0}}^2 \\
&+ 1/2(I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, \omega_{\mathrm{p}}^2
\end{aligned}
$$



Effective mass

- The rotation speed of the driveline components is linked to the longitudinal speed of the vehicle

$$\omega_{\mathrm{w}} = v/R_e$$

$$\begin{aligned}\omega_{\mathrm{w}} &= \omega_{\mathrm{transm}}/i_{\mathrm{dif}}\\ \omega_{\mathrm{w}} &= \omega_{\mathrm{box0}}/(i_{\mathrm{dif}} * i_{\mathrm{box}}/i_{\mathrm{box0}})\\ \omega_{\mathrm{w}} &= \omega_{\mathrm{p}}/(i_{\mathrm{dif}} * i_{\mathrm{box}})\end{aligned}$$

- The kinetic energy writes

$$\begin{aligned}T =& 1/2 m v^2 + 1/2(\sum I_{\mathrm{w}} + I_{\mathrm{axle}})\, v^2/R_e^2\\ &+ 1/2(I_{\mathrm{transm}} + I_{\mathrm{box2}})\, v^2 i_{\mathrm{dif}}^2/R_e^2\\ &+ 1/2(I_{\mathrm{box0}})\, v^2 (i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2/i_{\mathrm{box0}}^2)/R_e^2\\ &+ 1/2(I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, v^2 i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2/R_e^2\end{aligned}$$

Effective mass

- One defines an effective mass $T = 1/2\, m_{\mathrm{e}}\, v^2$

$$m_{\mathrm{e}} = m + \frac{\sum I_{\mathrm{w}} + I_{\mathrm{axle}}}{R_e^2} + \frac{(I_{\mathrm{transm}} + I_{\mathrm{box2}})\, i_{\mathrm{dif}}^2}{R_e^2} + \frac{(I_{\mathrm{box0}})\, i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2}{i_{\mathrm{box0}}^2\, R_e^2} + \frac{(I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, i_{\mathrm{dif}}^2\, i_{\mathrm{box}}^2}{R_e^2}$$

- The calculation of the effective mass requires the knowledge of the geometry of all the driveline components

- Empirical formula for preliminary design of cars by Wong

$$m_{\mathrm{e}} = m_0 + m_1\, i^2 \qquad\qquad i = i_{\mathrm{dif}} * i_{\mathrm{box}}$$

Effective mass

- Empirical correction formula to estimate the effective mass of passenger car propelled by piston engines (Wong 2001)

$$\boxed{\gamma_m = \frac{m_{\mathrm{e}}}{m} = 1.04 \, + \, 0.0025 \, i^2}$$

- This estimation formula puts forward the major factors of the corrections :
  - Nearly negligible for low reduction ratios (4th and 5th gear ratios)
  - Rather important for high gear ratios : 1st and 2nd gear ratios
- For railway systems, γ is of an order of magnitude 1,02 to 1,30 for classical train and from 1,30 to 3,50 for rack trains)

Effective mass

- Example: Peugeot 308 1.6 HDi with 5 gear ratios

	i_{boite}	i	γ_m
1	3,95	13,63	1,5043
2	1,87	7,39	1,1764
3	1,16	4,58	1,0925
4	0,82	3,24	1,0662
5	0,66	2,61	1,0570

i_{pont}=3,95

Velocity as a function of time

- We now proceed to time integration of Newton equation.

$$m_e \frac{dv}{dt} = F_w - \sum F_{res} = F_{net}(v)$$

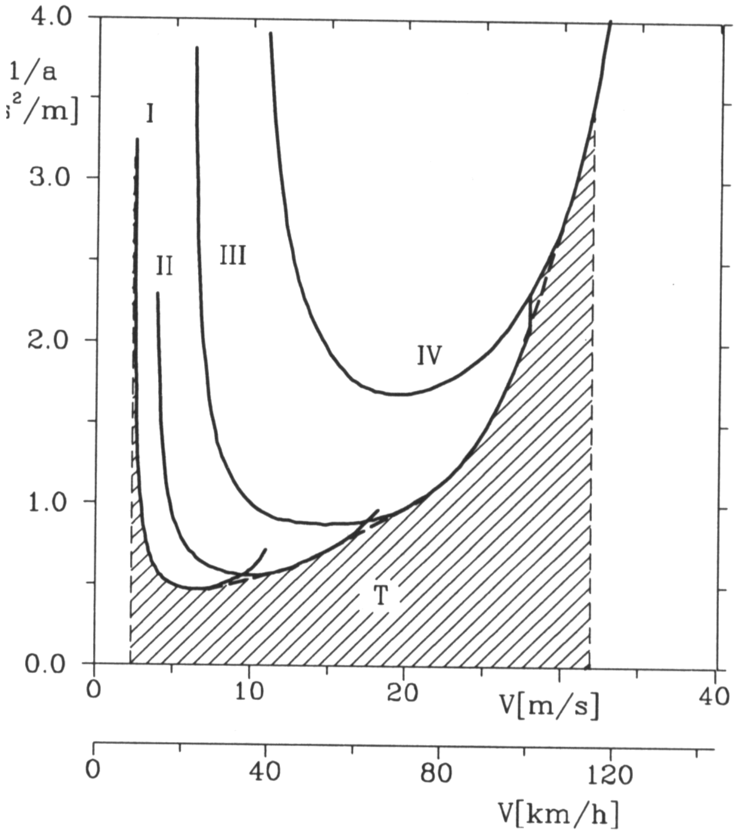
- Time to accelerate form V_1 to V_2.

$$dt = \frac{m_\mathrm{e}\, dv}{F_{net}(v)}$$

$$\Delta t_{V_1 \to V_2} = m_\mathrm{e} \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)}$$

Velocity as a function of time

$$1/a = m_e/F_{net}$$



Genta Fig 4.20 : 1/F as function of time

- Time to accelerate from V_1 to V_2:

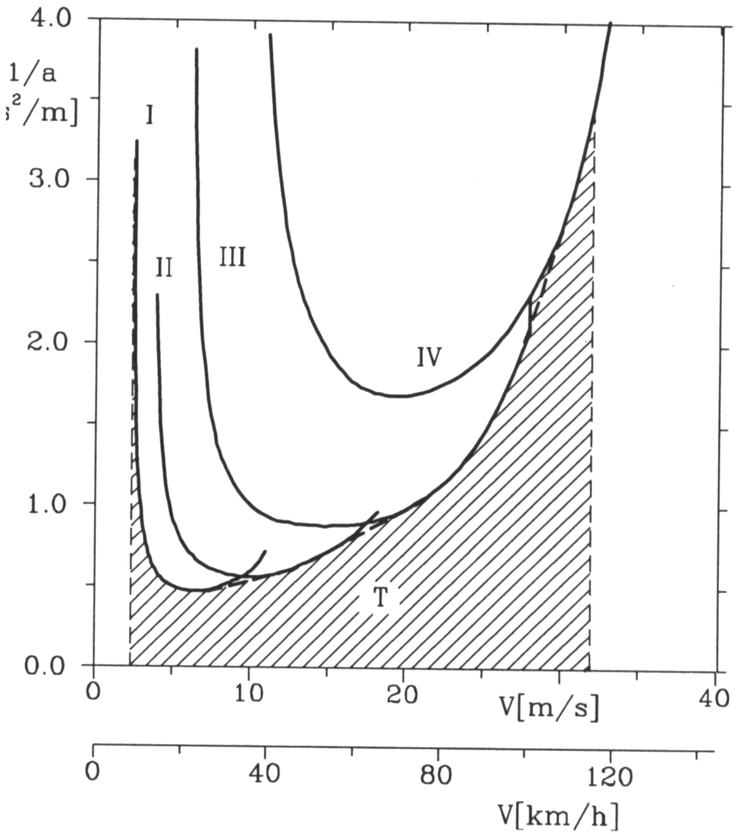
$$\Delta t_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)}$$

- Alternatively

$$F_{net}(v) = \mathcal{P}_{net}(v)/v$$

$$\Delta t_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{v\, dv}{\mathcal{P}_{net}(v)}$$

Velocity as a function of time



Genta Fig 4.20 : 1/F as function of time

- Criteria for gear ratio up shift in order to minimize the acceleration time
- If two curves intersects each other: **change the ratio at curve intersection**
- If there is no intersection, then it is necessary to push the ratio up to maximum rotation speed
- Lower limit is given by an infinite number of gear ratios, that is a Continuous Variables Transmission (CVT)

Velocity as a function of time

- The solution of differential equation yields the time t as a function of the velocity

$$t = f(v)$$

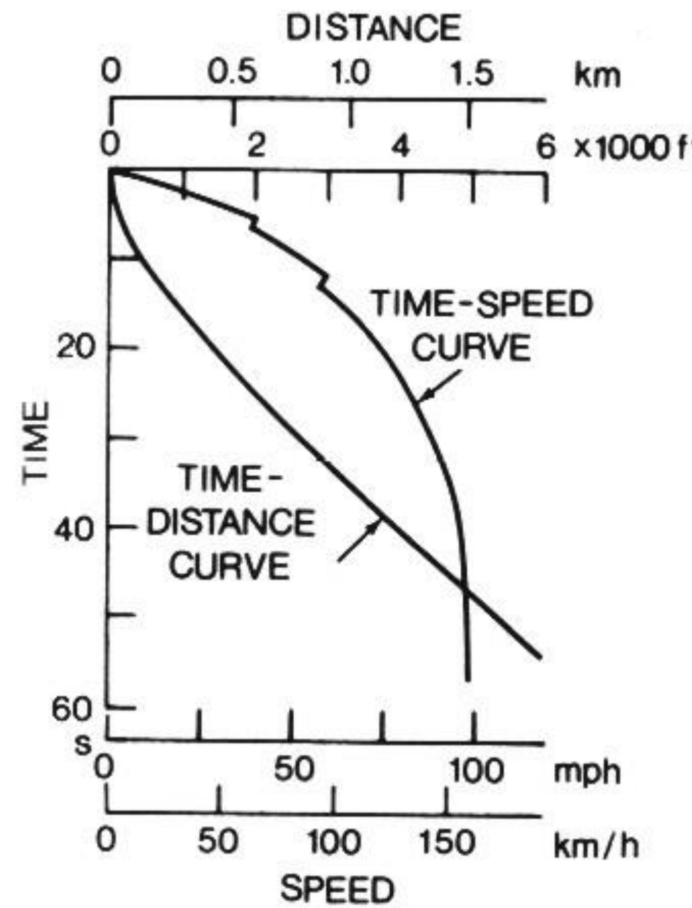
- The reciprocal function

$$v = g(t)$$

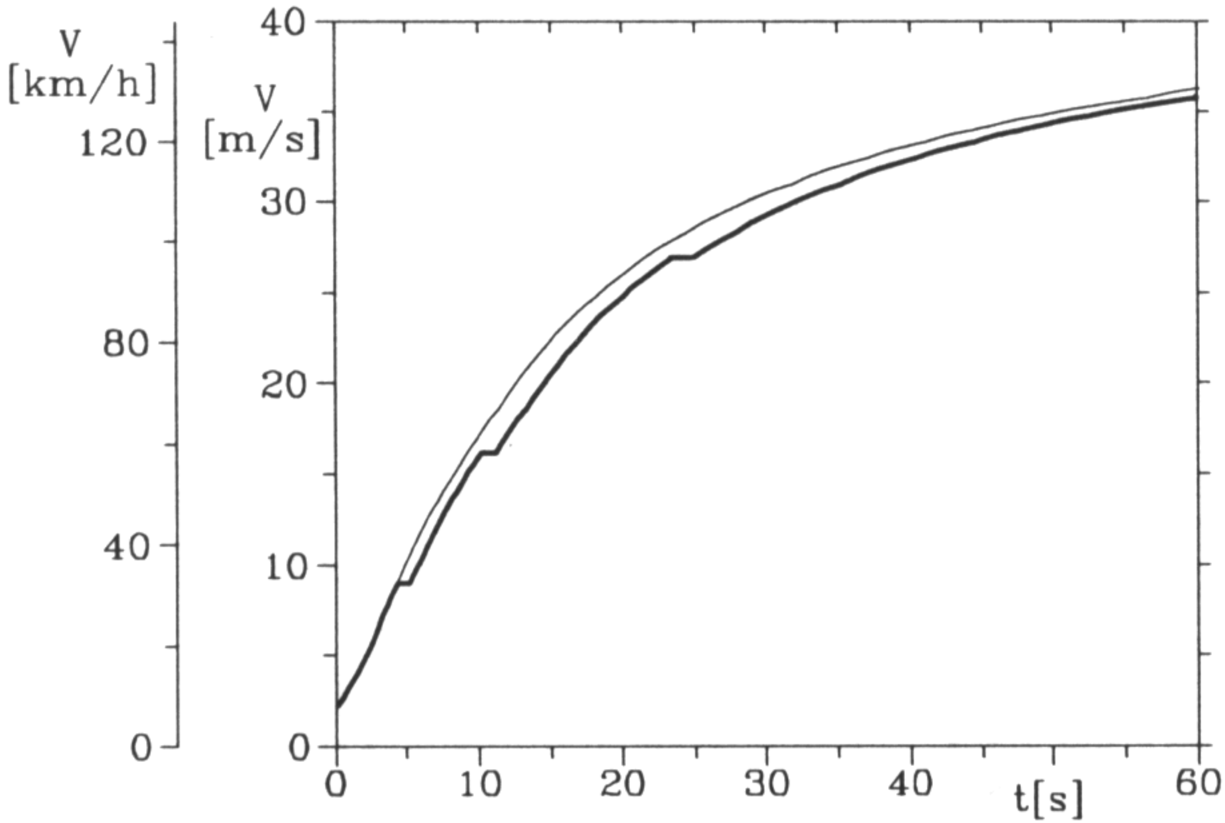
- requires to invert the relation

$$g = f^{-1}$$

- The changes of gear ratio must be taken into account



Velocity as a function of time



G. Genta Fig 4.21: speed vs time during acceleration

Distance as a function of the speed

- The distance from start can be evaluated by a second integration of the Newton equation

- Velocity and distance are linked by the kinematic relation

$$dx = v\, dt$$

- It comes

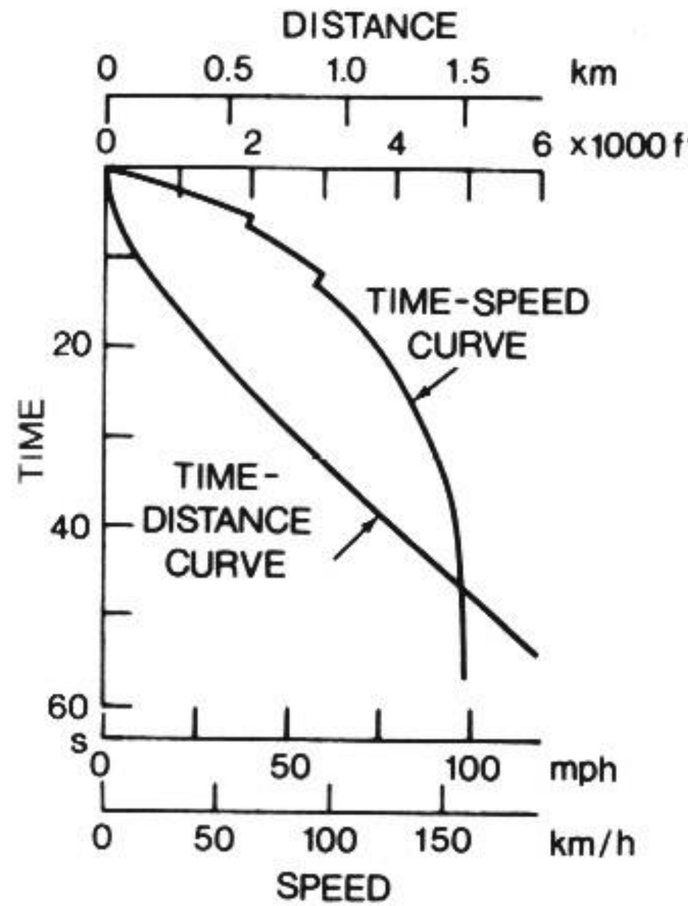
$$\Delta x_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{v\, dv}{F_{net}(v)}$$

Distance as a function of the time

- One can eliminate the velocity V between the two curves Δt=f(V) and Δx=h(V)
- On gets the distance as a function of the time:

$$\Delta t = f(\Delta v) \qquad \Delta x = h(\Delta v)$$

$$\Delta x = h(f^{-1}(\Delta t))$$



Change of gear ratio

- Criteria for changing the gear ratio.
- Gear ratio changing is a delicate operation that needs being studied in details:
  - Changing the gear box ratio takes some time
  - Tractive force is interrupted
  - The vehicle is coasting and slows down
- For an expert driver
  - Small time to change the gear

$$\Delta t \approx 0,8s$$

  - Reduction of the velocity can be estimated by the first order approximation

$$\Delta v \approx -\frac{F_{rés}(v)}{m_{\text{eff}}} \Delta t$$

Change of gear ratio

- When several gear change are necessary, the integration needs to be carried out by parts
- For instance

$$T_{V_1 \to V_2} = \int_{V_1}^{V_{I\to II}} \frac{m_e(i_1)\,dv}{F_{net}(v)} + \Delta t + \int_{V_{II}}^{V_{II\to III}} \frac{m_e(i_2)\,dv}{F_{net}(v)} + \Delta t + \int_{V_{III}}^{V_2} \frac{m_e(i_3)\,dv}{F_{net}(v)}$$

- with

$$V_{II} = V_{I\to II} - \frac{F_{rés}(V_{I\to II})}{m_{\mathrm{e}}}\,\Delta t$$

$$V_{III} = V_{II\to III} - \frac{F_{rés}(V_{II\to III})}{m_{\mathrm{e}}}\,\Delta t$$

Start from standstill

- If we start from rest, one must start with clutch open because of the idle speed of engine
- One has to play with the clutch and let a certain clutch slippage up to the point where the speed is sufficient to close it because the engine rotation speed is equal to the wheel rotation speed.
- Several control strategies are possible with various impact on the start performance:
  - Start with an engine regime that is close to idle speed. Minimization of clutch wear but at the price of a higher acceleration time.
  - Start with a regime close to the maximum torque: this increases the clutch slippage and so the wear, but it leads to minimize the acceleration time.

Start from standstill

Smooth start A->B: start with an engine regime close to idle speed

Fast start A′->B′: start with an engine rotation speed close to the max torque