Vehicle Performance

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Lesson 7:
Braking Performance

Bibliography

- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).

- R. Bosch. « Automotive Handbook ». 5th edition. 2002. Society of Automotive Engineers (SAE)
- R. Bosch. « Automotive Brake Systems ». R. Bosch Publishers. 1995.
- R. Bosch. « Safety, Comfort, and Convenience Systems. Function regulation and components. » Bentley Publishers 2006.

- « An Introduction to Modern Vehicle Design ». J. Happian-Smith ed. Butterworth-Heinemann. 2002
- "Automotive Engineering: Powertain, Chassis System, and Vehicle Body". D. Crolla ed. Butterworth-Heinemann. 2009

Braking system architecture

- Introduction

- Braking performance
  - Weight transfer
  - Optimal braking distribution
  - Non ideal braking
  - Braking distance

- Brake devices
  - Drum brakes
  - Disk brakes

- Braking systems

➔ MECA0063 Vehicle architecture and components

INTRODUCTION

Introduction

- Brakes are primarily used to decelerate a vehicle beyond its road resistance and the braking drag of the engine

- Brakes generally transform the kinetic energy of the vehicle into heat

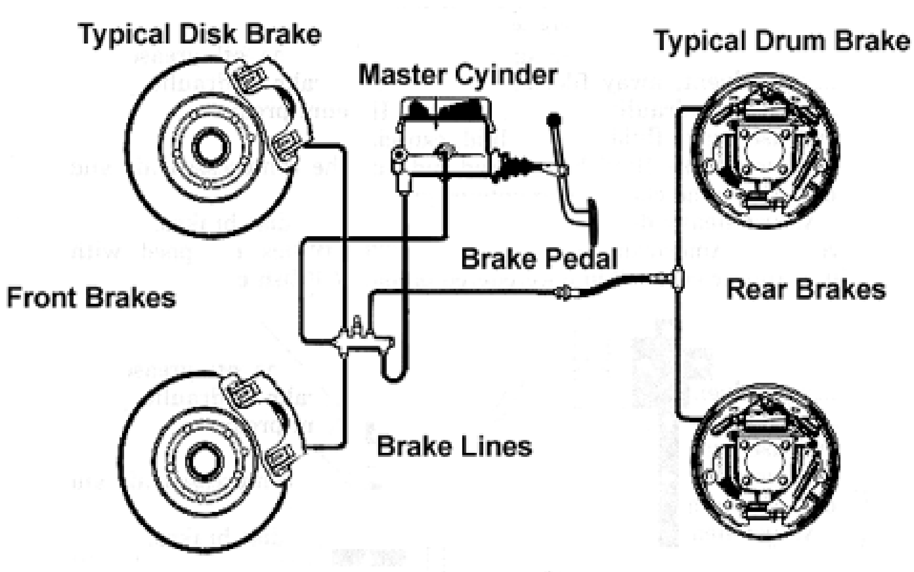
- Brakes can also be used to:
  - Keep a constant speed
  - Keep the vehicle at standstill

Introduction

- One distinguishes the different categories of braking systems
  - Service brake system: generally decreases the speed while driving
  - Emergency brake system: has to take over the function of the service brake system when failing
  - Parking brake system: prevents unwanted motion of the vehicle when parked
  - Continuous service braking systems: for longer uninterrupted braking and frequent stops for instance in urban heavy vehicles

- The service, emergency, and parking brake systems directly work on the wheels
- The brake elements of the continuous service generally act on the driveline

Introduction



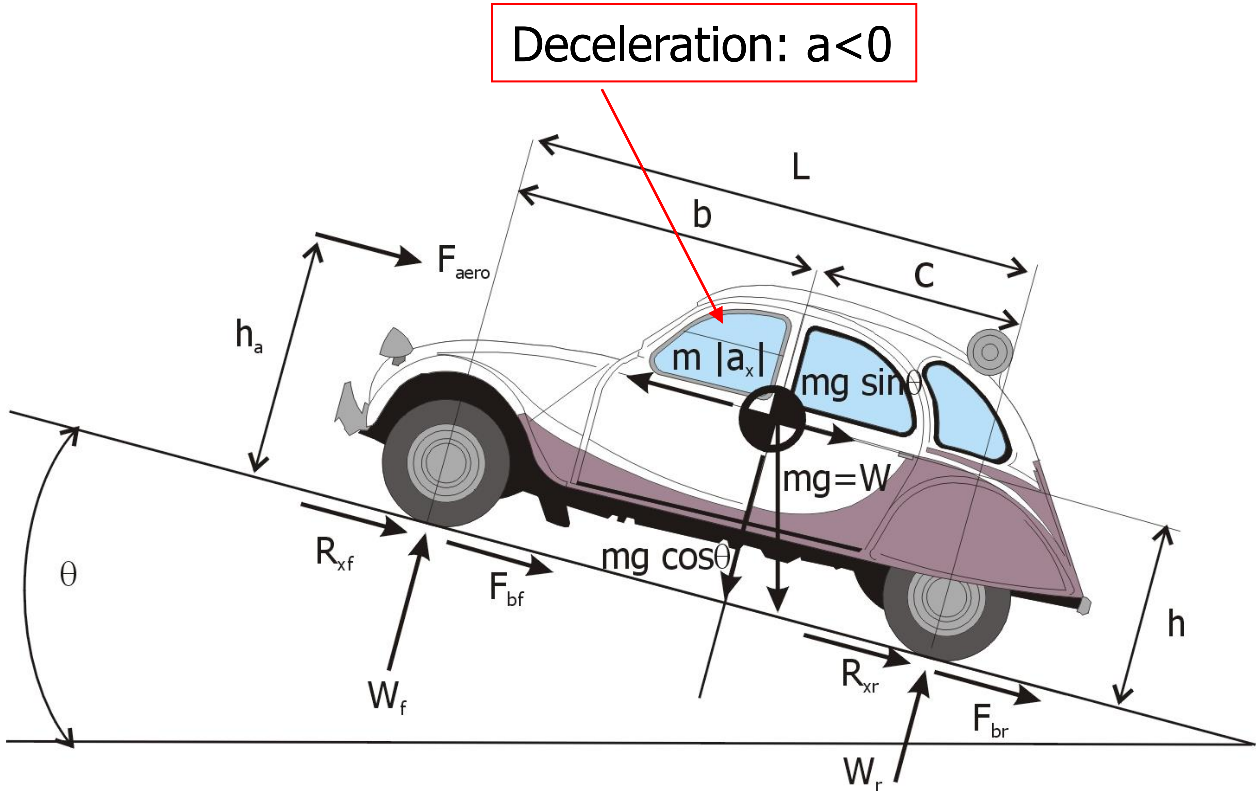
Typical Automotive Braking System

- A common brake system includes
  - Control device: pedals / hand-brake lever
  - An energy source which generates, stores, and releases the energy required by the braking system
  - Transmission device: components between the control device and the brake
  - The wheel brake or foundation brakes generate the forces opposed to the vehicle motion

BRAKING PERFORMANCE

Longitudinal Equation of Motion

EQUILIBRIUM WHILE BRAKING



Equilibrium while braking

EQUILIBRIUM WHILE BRAKING

- Newton's second law

$$\boxed{ma_x = -(F_{bf} + F_{br}) - F_{RR} - F_{aero} \mp W \sin\theta - R_t}$$

Acceleration:
<0 if braking

Front and rear wheel braking

Rolling resistance

$$F_{RR} = f\ m\ g\ \cos\theta$$

$$\frac{F_{RR}}{m} = f\ g \sim 0.01\ g$$

Grading resistance

$$F_g = W \sin\theta \simeq W\ \theta$$

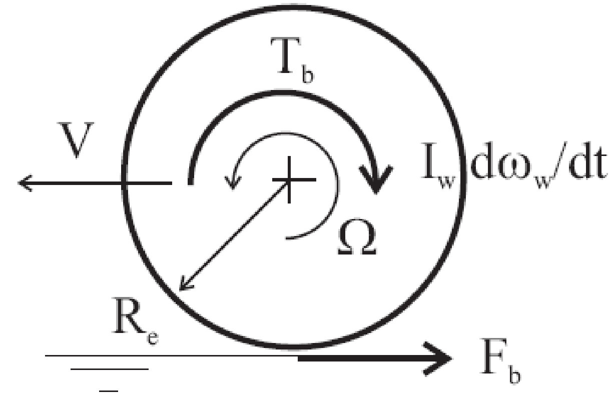
$$\frac{F_g}{m} = g\ \theta \sim 0.03\ g$$

Resistance due to friction in drivetrain

Aerodynamic drag

$$F_{aero} = \frac{1}{2}\rho V^2 S C_x$$

$$\frac{F_{aero}}{m} = \frac{0.5\rho V^2 S C_x}{m} \sim 0.04\ g$$



Braking forces

- Braking forces developed by the braking system

$$\sum I_w\,\dot{\omega}_w = -T_b + F_b\,R_e \quad \Longrightarrow \quad F_b = \frac{T_b - \sum I_w\,|\dot{\omega}_w|}{R_e}$$

  - The brake must also absorb the rotation inertia of the wheels and of the rotating parts (driveline).
  - When there is no slip of the tyres, the inertia of the wheels and the rotating components can be modelled as an additional fictitious translational mass i.e. an effective mass. The correction factors is about 1.03 to 1.05

$$F_b = \frac{T_b}{R_e} \qquad m_{\mathrm{e,b}}\,a_x = -F_b \qquad \gamma_b = \frac{m_{\mathrm{e,b}}}{m}$$

Simplified braking motion

Simplified braking motion

- If the adhesion friction is constant, the braking forces is constant and

$$a_x = -F_x^{tot}/m = \frac{dv}{dt}$$

- Speed and distance as a function of time

$$v(t) = v_0 - a_x\, t \qquad\qquad x(t) = v_0\, t - a_x\, \frac{t^2}{2}$$

- Reduction of the kinetic energy of the vehicle and the work dissipated by the brakes

$$\frac{1}{2} m v_0^2 - \frac{1}{2} m v^2(t) = F_x^{tot}\, x(t)$$

Simplified braking motion

- Motion till rest V_2=0:
  - Time to stop

$$v_0 = a_x \, t_{stop} \quad \Leftrightarrow \quad t_{stop} = v_0 / a_x$$

  - Stopping distance

$$L_{stop} = \frac{v_0^2}{a_x} - \frac{v_0^2}{2\, a_x} = \frac{v_0^2}{2\, a_x} = \frac{v_0 \, t_{stop}}{2}$$

  - Taking care of the reaction time of the driver and the braking system

$$L_{stop} = v_0 \, (t_a + t_d) + \frac{v_0^2}{2\, a_x}$$

t_a + t_d takes into account for the reaction time of the driver (from 0.5 to 2 s) and for the development of the braking forces in the braking system

  - Energy dissipated during braking

$$E_b = \frac{1}{2} \, m v_0^2 = F_x^{tot} \, L_{stop} = F_x^{tot} \, \frac{v_0^2}{2\, a_x}$$

Example

- Passenger car: mass 1400 kg, v_0 = 120 km/h, a_x=6 m/s²

Energy to be absorbed by the braking system

$$E_b = \frac{1}{2} m v_0^2 = \frac{1}{2}\, 1400\, (33,3)^2 = 776,223\, kJ$$

Time and distance to stop

$$t_{stop} = v_0 / a_x = 33,3/6 = 5,5\, s \qquad L_{stop} = v_0^2/(2.a_x) = 90\, m$$

Average power dissipated by the braking

$$P_b = E_b / t_{stop} = 776.223/5,5 = 141,131\, kW \simeq 190\, CV$$

Peak power = 2 * average power = 282 kW

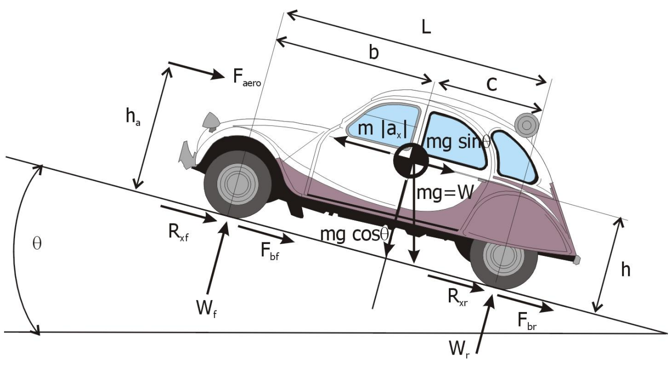
$$P_b^{max} = 2\, E_b / t_{stop} = 282,262\, kW$$

Ideal distribution of braking forces

Distribution of braking forces

- Pitch equilibrium: weight transfer

$$W_f = mg\frac{c}{L}\cos\theta - \frac{h}{L}\left(m\,a_x + F_{aero} \pm W\ \sin\theta\right)$$
$$W_r = mg\frac{b}{L}\cos\theta + \frac{h}{L}\left(m\,a_x + F_{aero} \pm W\ \sin\theta\right)$$



- Longitudinal equilibrium

$$m\,a_x = -F_{bf} - F_{br} - f\,W - F_{aero} \mp W\sin\theta$$

gives

$$F_b + f\,W\ \cos\theta = F_{bf} + F_{br} + f\,W = +m\,|a_x| - F_{aero} \mp W\sin\theta$$

$F_b>0$ if deceleration $a<0$ if deceleration

Distribution of braking forces

- Weight under the axles (one assumes θ=0 for sake of simplicity)

$$\begin{aligned} W_f &= mg\frac{c}{L} + \frac{h}{L}\,(F_b + f\,W) \\ W_r &= mg\frac{b}{L} - \frac{h}{L}\,(F_b + f\,W) \end{aligned}$$

F_b>0 if braking

- Or using weight transfer ΔW

$$\begin{aligned} W_f &= mg\frac{c}{L} + \Delta W \\ W_r &= mg\frac{b}{L} - \Delta W \end{aligned}$$

$$\Delta W = \frac{h}{L}\,(F_b + f\,W)$$

Distribution of braking forces

- The maximum braking forces depends on the weight on the wheels and on the friction coefficient μ:

Assumption!

$$\begin{cases} F_{bf} = \mu\, W_f \\ F_{br} = \mu\, W_r \end{cases}$$

$$F_{bf} + F_{br} = F_b = \mu\, W$$

- Developing the relations

$$\begin{aligned} W_f &= W\frac{c}{L} + \frac{h}{L}\,(F_b + f\,W) = W\frac{c}{L} + \frac{h}{L}\,(\mu W + f\,W) \\ W_r &= W\frac{b}{L} - \frac{h}{L}\,(F_b + f\,W) = W\frac{b}{L} - \frac{h}{L}\,(\mu W + f\,W) \end{aligned}$$

Distribution of braking forces

- The maximum braking forces depends on the weight on the wheels and on the friction coefficient μ:

$$
\begin{aligned}
F_{bf}^{max} &= \mu\, W_f = \frac{\mu\, W\,(c + h(\mu + f))}{L} \\
F_{br}^{max} &= \mu\, W_r = \frac{\mu\, W\,(b - h(\mu + f))}{L}
\end{aligned}
$$

- Ideal braking distribution: both axles reach simultaneously the friction limits, which happens for a unique front / rear braking distribution

$$
\frac{k_{bf}}{k_{br}} = \frac{F_{bf}^{max}}{F_{br}^{max}} = \frac{c + h(\mu + f)}{b - h(\mu + f)}
$$

Distribution of braking forces

- Example

  Light duty vehicle: 68% of the weight on the rear axles

$$b/L = 0.68 \quad c/L = 0.32$$
$$h/L = 0.18$$
$$\mu = 0.85 \qquad f = 0.01$$

The ideal braking distribution is:

$$\frac{k_{bf}}{k_{br}} = \frac{F_{bf}^{max}}{F_{br}^{max}} = \frac{c + h(\mu + f)}{b - h(\mu + f)}$$

$$\frac{k_{bf}}{k_{br}} = \frac{0.32 + 0.18\,(0.85 + 0.01)}{0.68 - 0.18\,(0.85 + 0.01)} = \frac{47}{53}$$

Ideal distribution of braking forces
-
The I curve

The vehicle braking characteristics

- The braking characteristics ('I' curve) is the relation between the maximum braking forces on the front and rear wheels in ideal conditions
- Distribution of braking forces

$$F_{bf}^{max} = \mu W_f = \frac{\mu W (c + h(\mu + f))}{L}$$
$$F_{br}^{max} = \mu W_r = \frac{\mu W (b - h(\mu + f))}{L}$$

- Let's neglect the rolling resistance forces (f=0)

$$F_{bf} = \mu\, mg \left(\frac{c}{L} + \frac{h}{L}\mu \right)$$
$$F_{br} = \mu\, mg \left(\frac{b}{L} - \frac{h}{L}\mu \right)$$

The vehicle braking characteristics

- Let's eliminate the friction coefficient μ:

$$F_{bf} + F_{br} = \mu\, mg \qquad\qquad \mu = \frac{F_{bf} + F_{br}}{mg}$$

- It comes

$$F_{bf} = \left(\frac{F_{bf} + F_{br}}{mg} \right) \, mg \, \frac{c}{L} + \left(\frac{F_{bf} + F_{br}}{mg} \right)^2 \, mg \frac{h}{L}$$

- Reorganizing the terms

$$\frac{h}{L} \left(F_{bf} + F_{br} \right)^2 - mg \left(F_{bf} \frac{b}{L} - F_{br} \frac{c}{L} \right) = 0$$

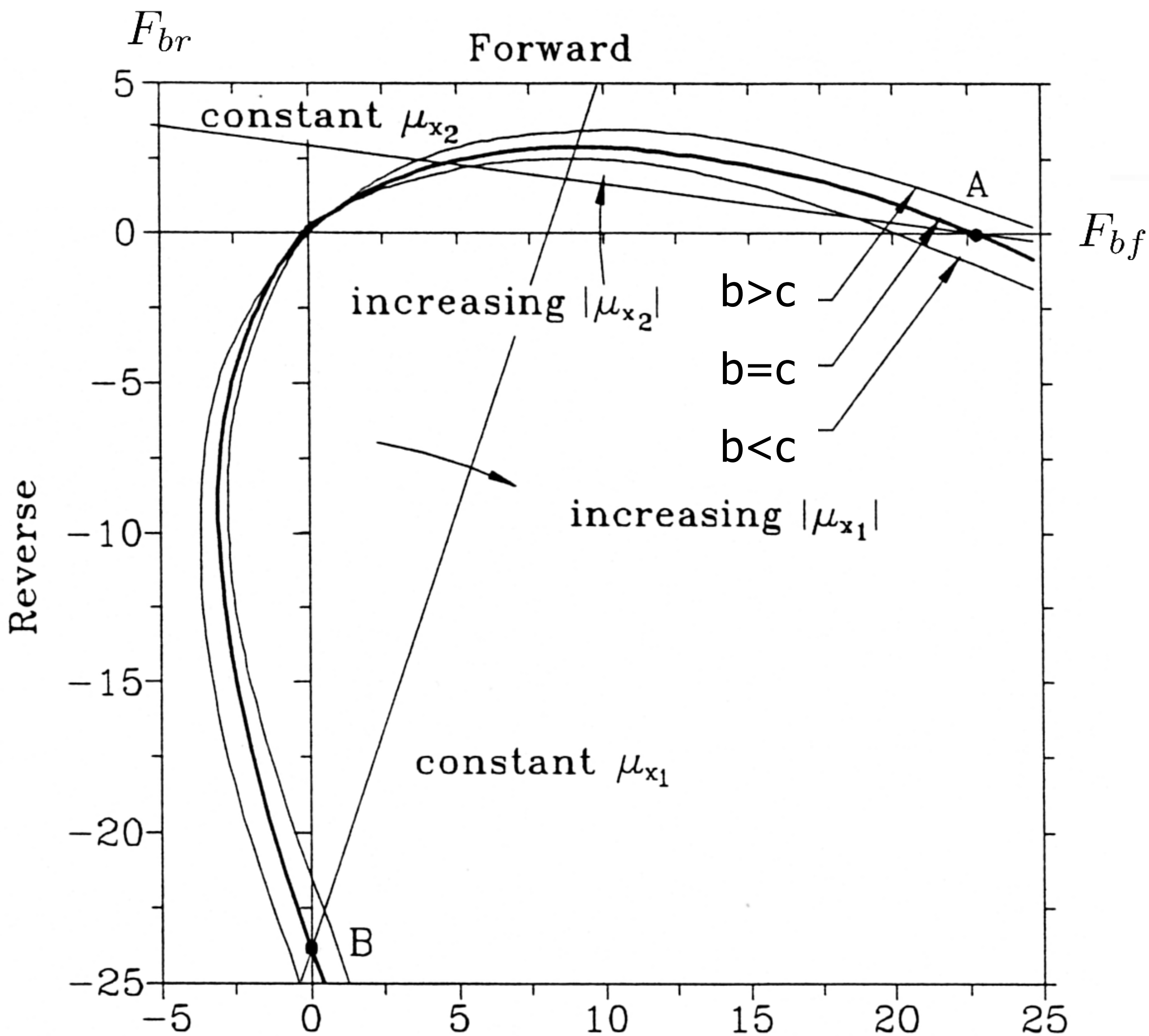
The vehicle braking characteristics

- The equation

$$\frac{h}{L}\left(F_{bf}+F_{br}\right)^2 - mg\left(F_{bf}\frac{b}{L} - F_{br}\frac{c}{L}\right) = 0$$

  is a parabola in the braking forces plane F_{bf} et F_{br} whose major axes are the bisectors
- Intersection with the axes
  - With axis F_{br}=0 $$\frac{h}{L}F_{bf}^2 - mg\ F_{bf}\frac{b}{L} = 0 \qquad \begin{cases} F_{bf} = 0 \\ F_{bf} = mg\ b/h \end{cases}$$
  - With axis F_{bf}=0 $$\frac{h}{L}F_{br}^2 + mg\ F_{br}\frac{c}{L} = 0 \qquad \begin{cases} F_{br} = 0 \\ F_{br} = -mg\ c/h \end{cases}$$

$$\frac{h}{L}\left(F_{bf}+F_{br}\right)^2 - mg\left(F_{bf}\frac{b}{L} - F_{br}\frac{c}{L}\right) = 0$$



Reminder:
$F_b>0$ if braking

Source: Genta Fig. 4-27

The vehicle braking characteristics

- It is interesting to calculate the maximum braking force on the front axles for a given rear braking force $\bar{F}_{br}$

$$W_f = mg\frac{c}{L} + \frac{h}{L}\ (F_{bf} + \bar{F}_{br})$$

- That is

$$F_{bf}^{max} = \mu\ W_f = \mu\left(mg\frac{c}{L} + \frac{h}{L}\ (F_{bf}^{max} + \bar{F}_{br})\right)$$

- And

$$\left(1 - \mu\frac{h}{L}\right) F_{bf}^{max} = \mu\left(mg\frac{c}{L} + \frac{h}{L}\ \bar{F}_{br}\right)$$

$$\boxed{F_{bf}^{max} = \mu\frac{mg\frac{c}{L} + \frac{h}{L}\ \bar{F}_{br}}{1 - \mu\frac{h}{L}}}$$

The vehicle braking characteristics

- Similarly one gets the maximum braking force on the rear axle in terms of a prescribed front wheel braking force $\bar{F}_{bf}$

$$W_r = mg\frac{b}{L} - \frac{h}{L}\left(\bar{F}_{bf} + F_{br}\right)$$

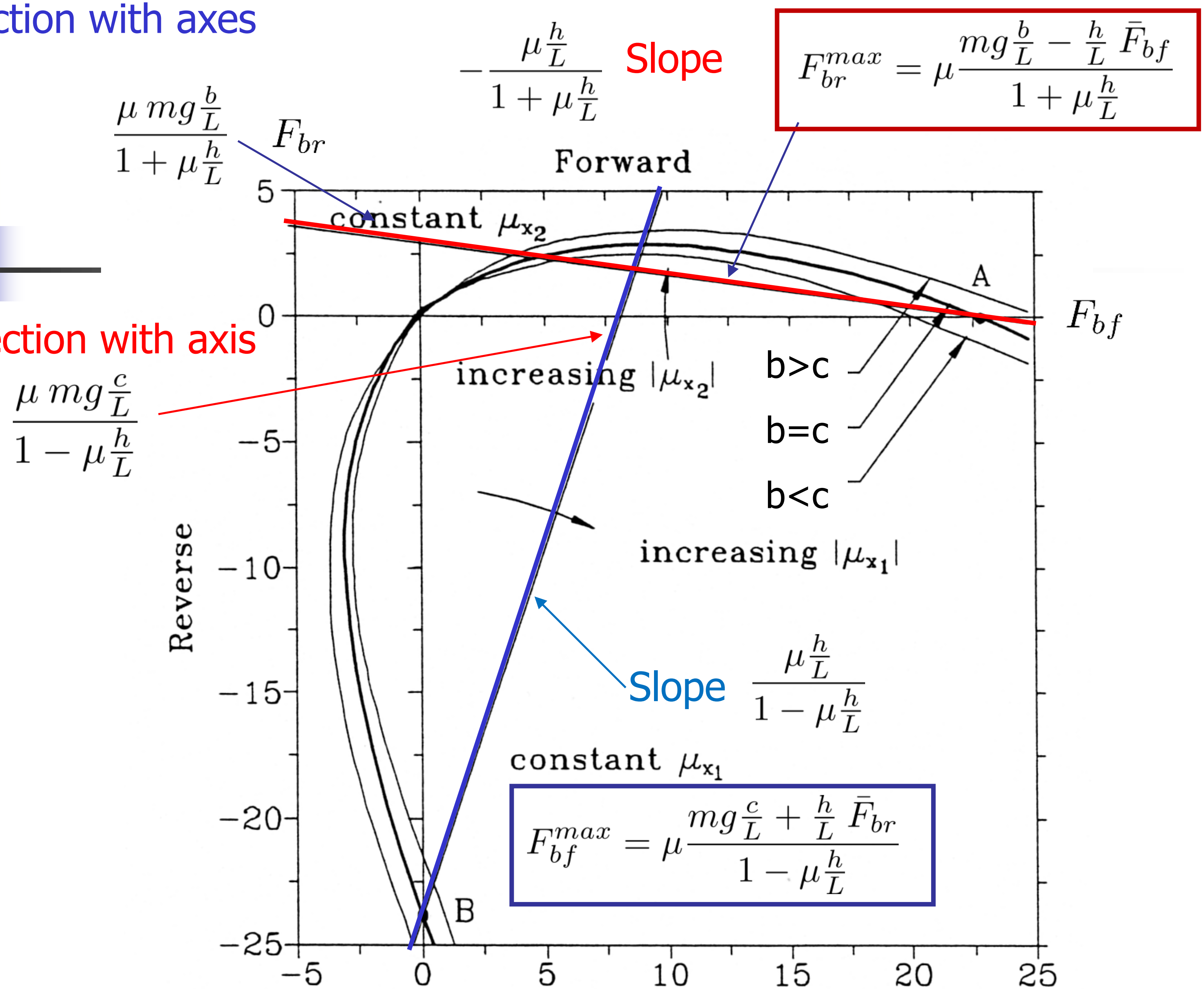
- So

$$F_{br}^{max} = \mu\, W_r = \mu\left(mg\frac{b}{L} - \frac{h}{L}\left(\bar{F}_{bf} + F_{br}^{max}\right)\right)$$

- And

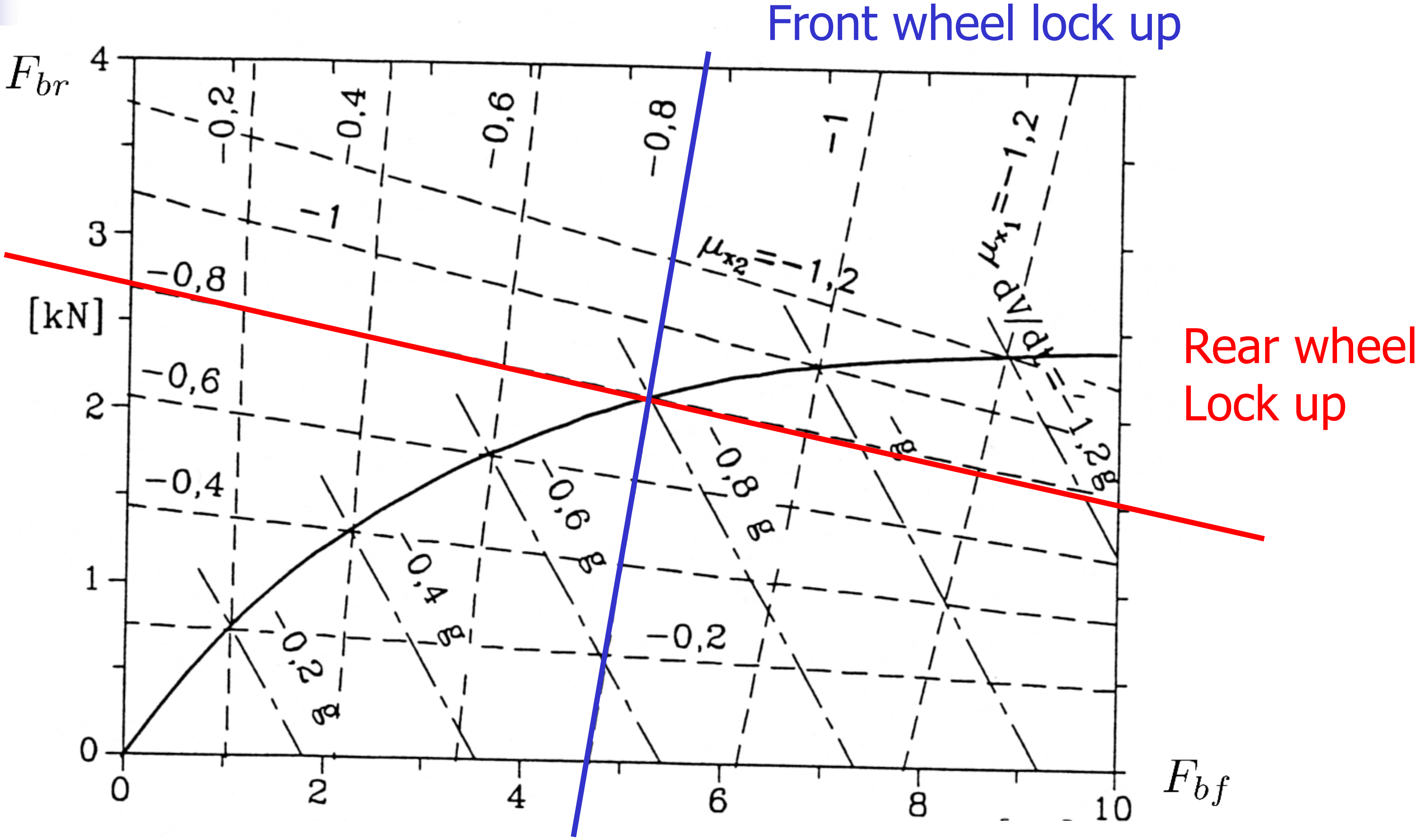
$$\boxed{F_{br}^{max} = \mu\frac{mg\frac{b}{L} - \frac{h}{L}\,\bar{F}_{bf}}{1 + \mu\frac{h}{L}}}$$

Intersection with axes



Source Genta Fig. 4-27

The vehicle braking characteristics



Source Genta Fig. 4-28

The vehicle braking characteristics

- The straight lines of the max braking forces are intersecting on the characteristic parabola (I curve)
- The intersection point is a function of the friction coefficient μ
- The intersection gives the ideal ratio between front and rear wheels.
- Iso-value of the deceleration is related to the friction coefficient

$$m|a_x| = F_{bf} + F_{br} = \mu(W_r + W_f) = \mu\ mg$$

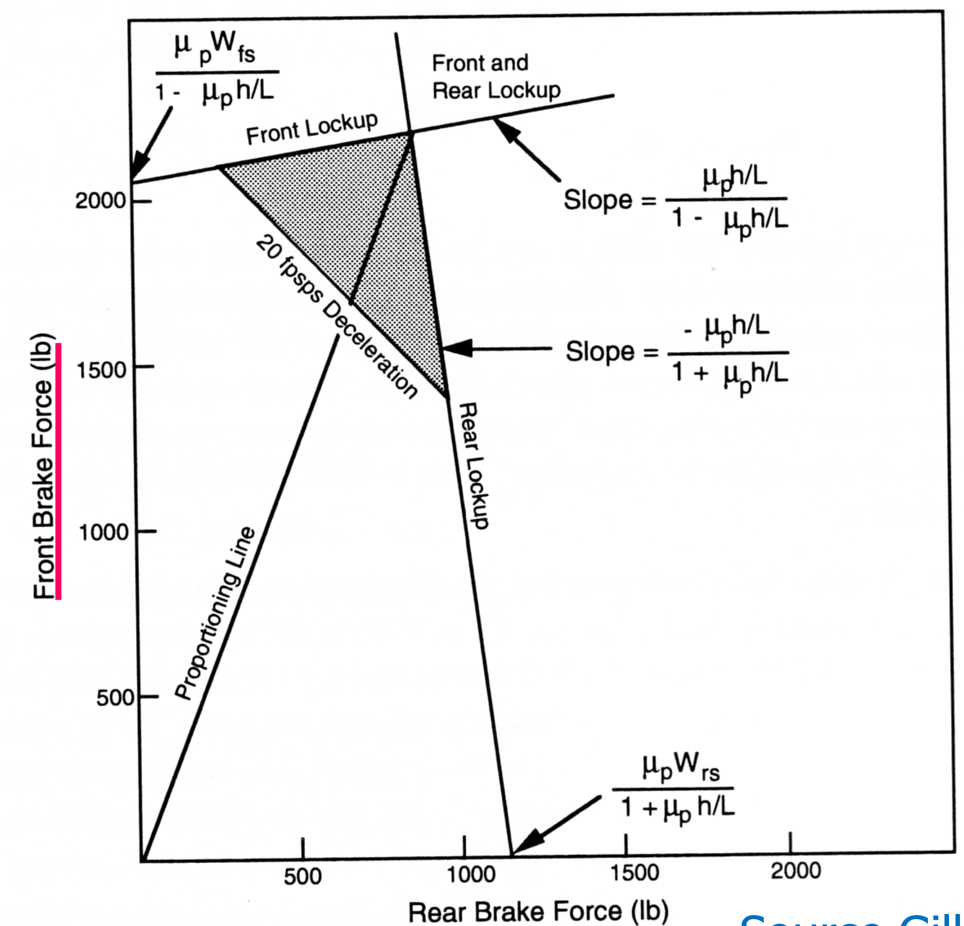
- Constant deceleration along the line

$$F_{bf} + F_{br} = \mu\ mg = m|a_x|$$

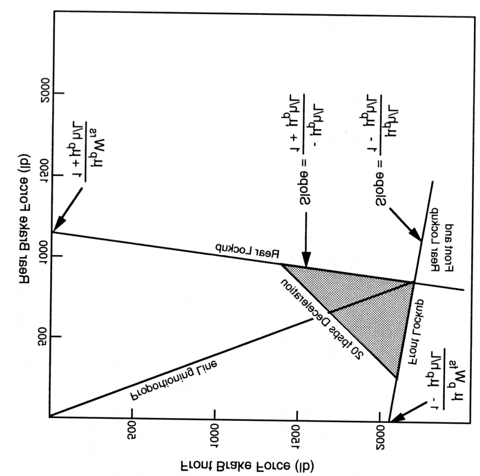
- The deceleration rate is ruled by the friction coefficient μ

$$|a_x|/g = \mu$$

The vehicle braking characteristics



Other presentation of the same results (see Gillespie)

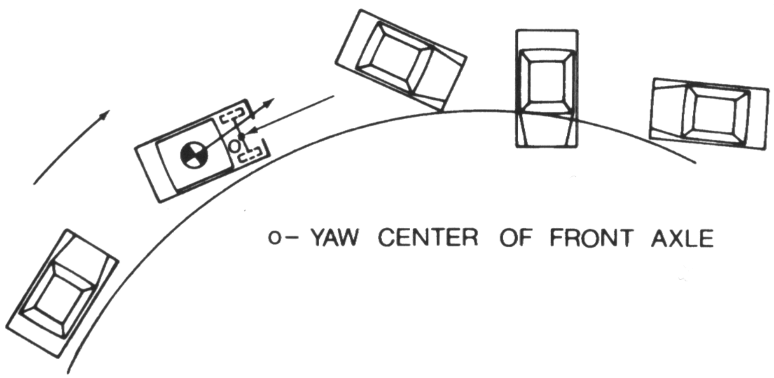


Source Gillespie Fig. 3-8

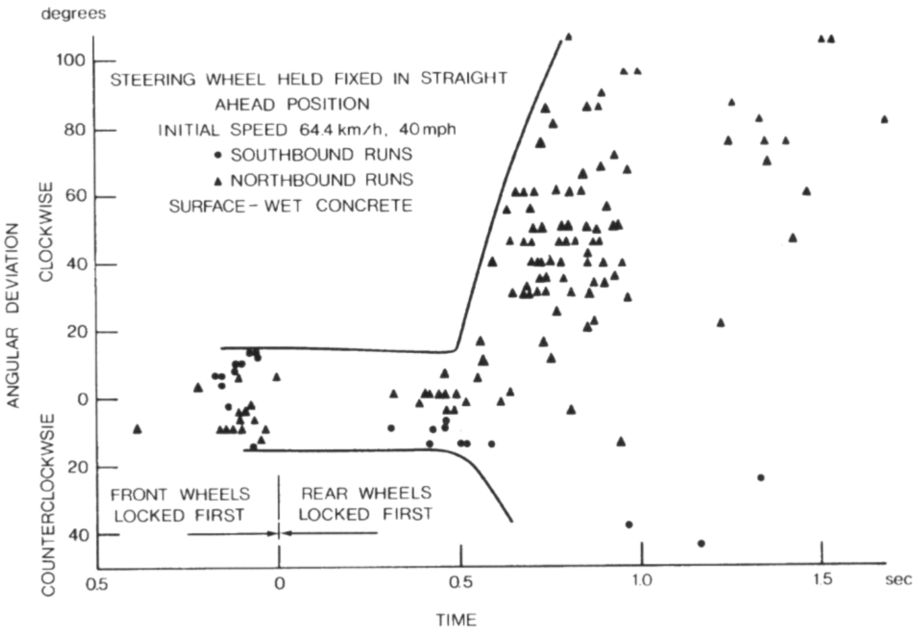
Braking under no ideal conditions

Braking under non ideal conditions

- One generally does not brake under ideal conditions. So what happens?



Wong. Fig 3.48. Loss of control with rear wheels lock-up



Wong. Fig 3.49: Angular yaw deviation for front and rear wheel lock-up

Braking under non ideal conditions

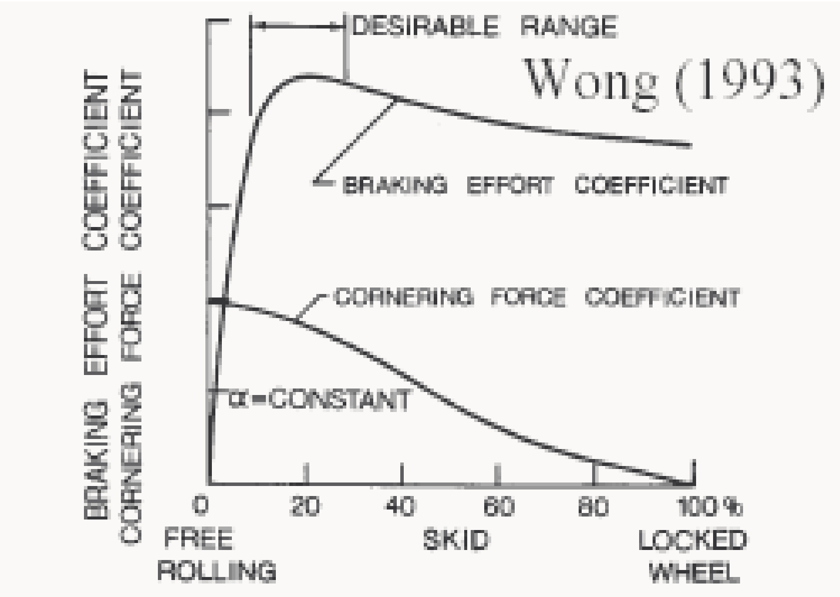
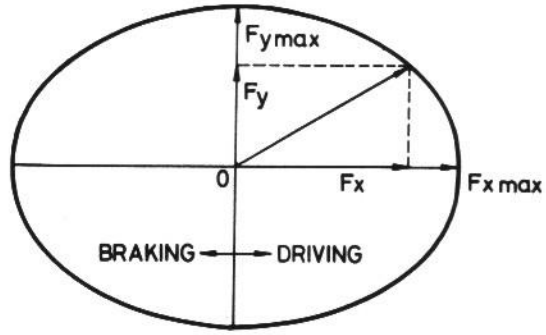
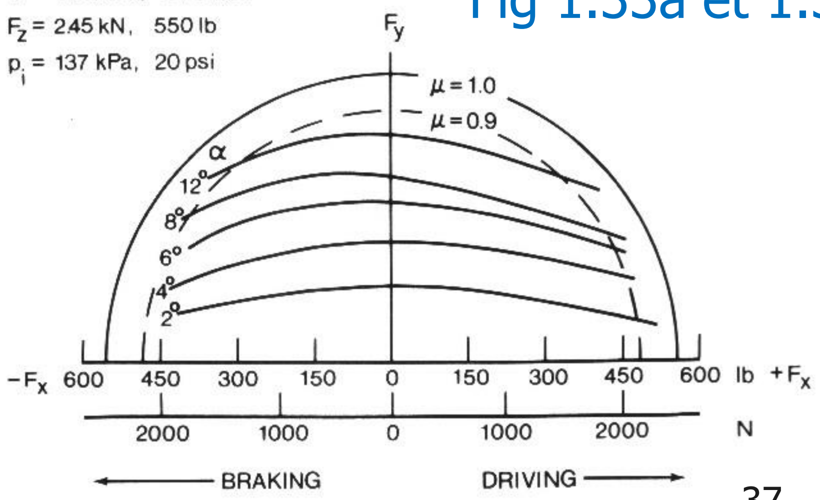


Fig. 3.54 Effect of skid on cornering force coefficient of a tire.



Wong (1993)
Fig 1.33a et 1.34



Braking under non ideal conditions

- If the front wheels lock first, we have a loss of directional control
  - The vehicle slides following a straight line and the centrifugal accelerations are naturally reduced so that the driver can recover the control of its machine ➔ non (lesser) dangerous vehicle

- The rear wheels lock first: Loss of stability
  - The rear of the vehicle loses its ability to develop any lateral forces and the lateral acceleration leads to an uncontrolled increase of the yaw speed ➔ jack knifing
  - This is a dangerous behaviour to avoid…

Braking under non ideal conditions

- Prediction of the wheel locking under non ideal braking conditions and the resulting deceleration

- Neglect the aerodynamic forces and the grading forces
- It comes

$$F_b + f\,W = F_{bf} + F_{br} + f\,W = -m\,a_x = m\,|a_x|$$

$\boxed{a_x<0}$

$$\begin{aligned} W_f &= mg\frac{c}{L} - m\,a_x\,\frac{h}{L} = mg\frac{c}{L} + m\,|a_x|\,\frac{h}{L} \\ W_r &= mg\frac{b}{L} + m\,a_x\,\frac{h}{L} = mg\frac{b}{L} - m\,|a_x|\,\frac{h}{L} \end{aligned}$$

Braking under non ideal conditions

- For a fixed braking distribution between the front and rear wheels, let's calculate which wheels are subject to the locking first

$$F_{bf} = k_{bf}\, F_b \qquad\qquad F_{br} = k_{br}\, F_b = (1 - k_{bf})\, F_b$$

- The braking efforts on the front wheels

$$F_b + f\,W = m\,|a_x| \qquad\qquad F_{bf} = k_{bf}\, F_b = k_{bf}\, W \left(\frac{|a_x|}{g} - f\right)$$

And rear wheels

$$F_{br} = k_{br}\, F_b = (1 - k_{bf})\, F_b = (1 - k_{bf})\, W \left(\frac{|a_x|}{g} - f\right)$$

Braking under non ideal conditions

- Lock-up of the front wheels if

$$F_{bf} = \mu\, W_f$$

- Locking condition of the front wheels

$$k_{bf}\, W\, \left(\frac{|a_x|}{g} - f\right) = \mu\, \left(mg\frac{c}{L} + m\, |a_x|\, \frac{h}{L}\right)$$

- So

$$\boxed{\left(\frac{|a_x|}{g}\right)_f = \frac{\mu\, c/L + k_{bf}\, f}{k_{bf} - \mu\, h/L}}$$

Braking under non ideal conditions

- Similarly, the locking condition of the rear wheels

$$\boxed{\left(\frac{|a_x|}{g}\right)_r = \frac{\mu\, b/L + (1 - k_{bf})\, f}{(1 - k_{bf}) + \mu\, h/L}}$$

- The front wheels are locking before the rear wheels if

$$\left(\frac{|a_x|}{g}\right)_f < \left(\frac{|a_x|}{g}\right)_r$$

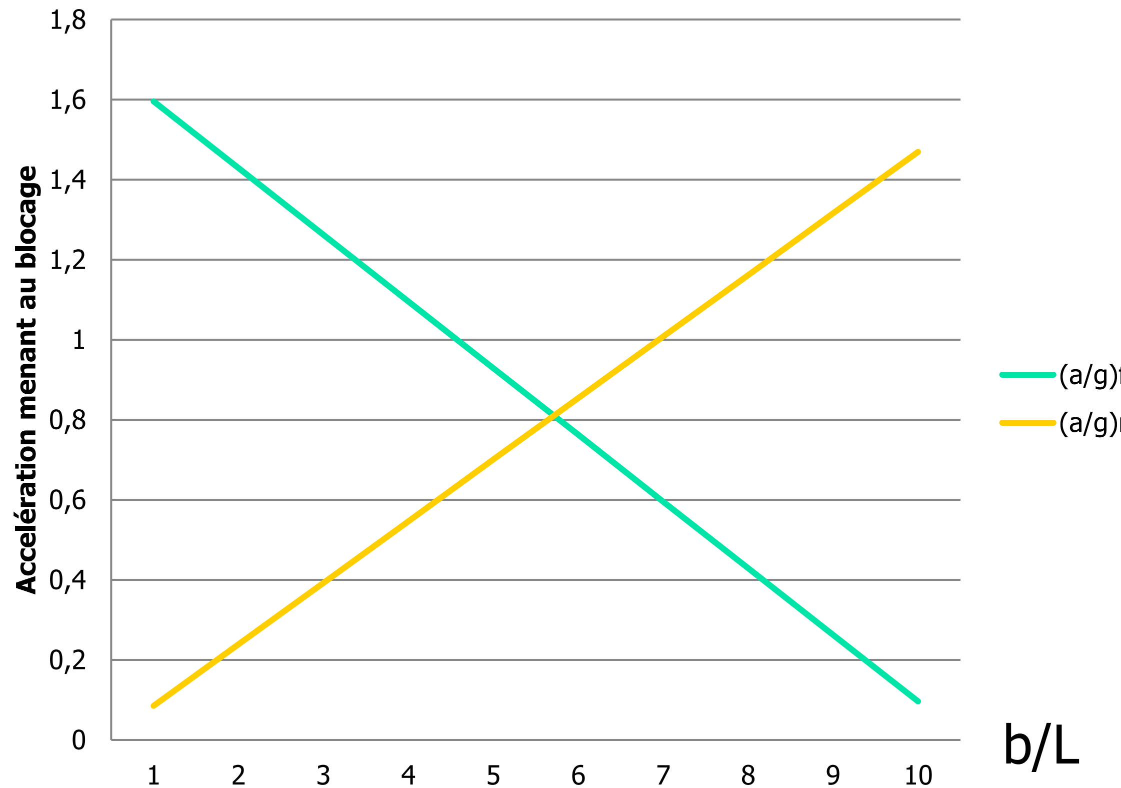
- Or vice-versa

$$\left(\frac{|a_x|}{g}\right)_r < \left(\frac{|a_x|}{g}\right)_f$$

Braking under non ideal conditions

- Example:

$$
\begin{aligned}
\mu &= 0.8 \\
f &= 0.01 \\
h/L &= 0.15 \\
k_{bf} &= 0.6 \\
k_{br} &= 1 - k_{kf} = 0.4 \\
b/L &= x \\
c/L &= 1 - x
\end{aligned}
$$

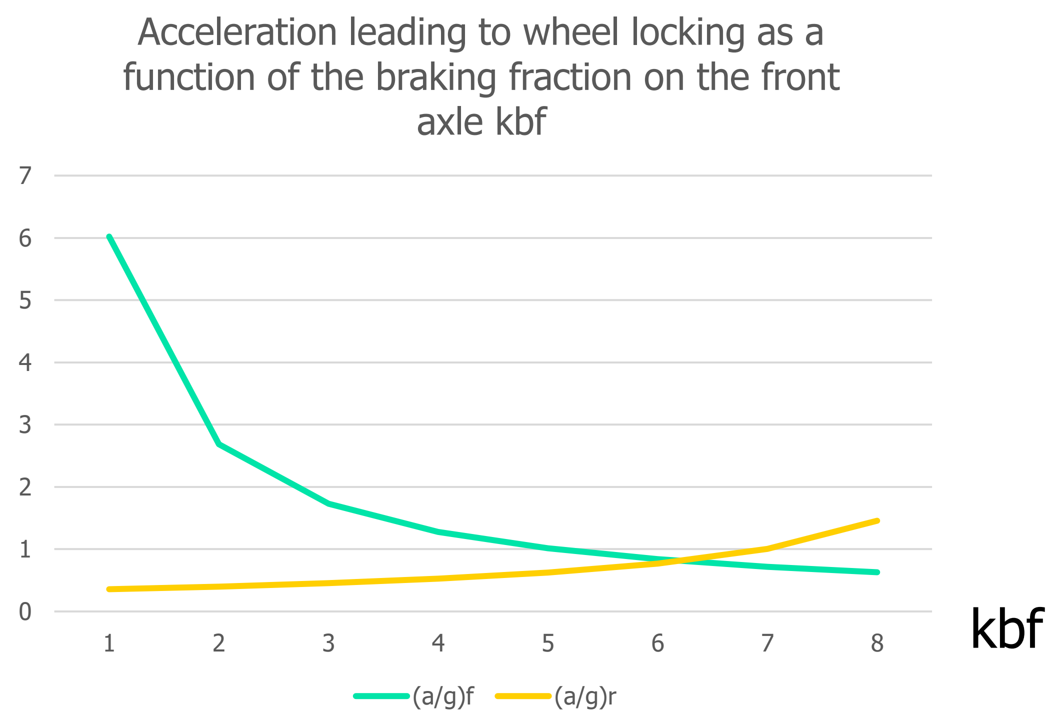


$$
\left(\frac{|a_x|}{g}\right)_f = \frac{\mu\, c/L + k_{bf}\, f}{k_{bf} - \mu\, h/L} \qquad \left(\frac{|a_x|}{g}\right)_r = \frac{\mu\, b/L + (1 - k_{bf})\, f}{(1 - k_{bf}) + \mu\, h/L}
$$

Braking under non ideal conditions

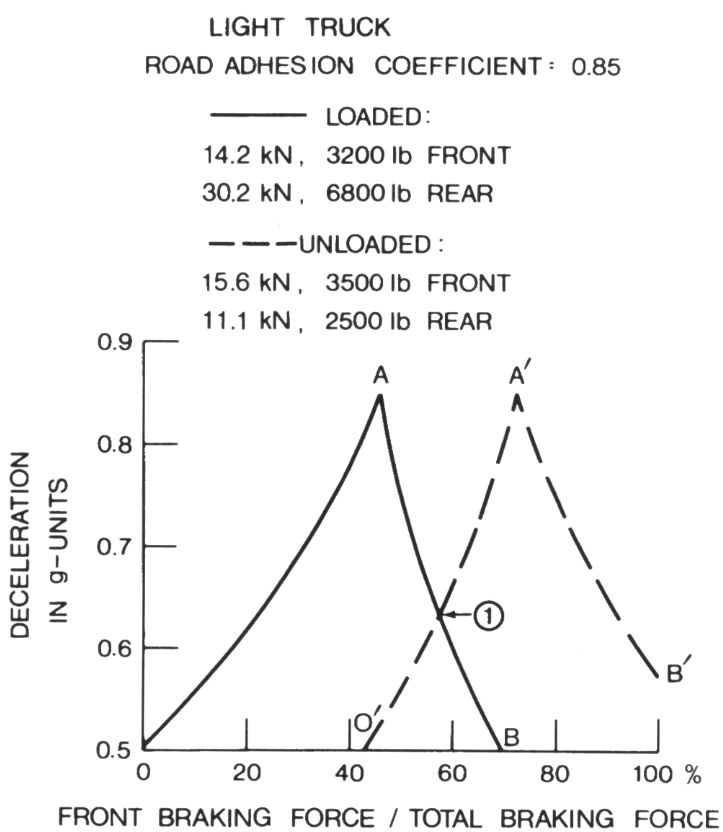
- Example:

$$\mu = 0.8$$
$$f = 0.01$$
$$h/L = 0.15$$
$$k_{bf} = x$$
$$k_{br} = 1 - x$$
$$b/L = 0.4$$
$$c/L = 0.6$$



$$\left(\frac{|a_x|}{g}\right)_f = \frac{\mu\, c/L + k_{bf}\, f}{k_{bf} - \mu\, h/L} \qquad \left(\frac{|a_x|}{g}\right)_r = \frac{\mu\, b/L + (1 - k_{bf})\, f}{(1 - k_{bf}) + \mu\, h/L}$$

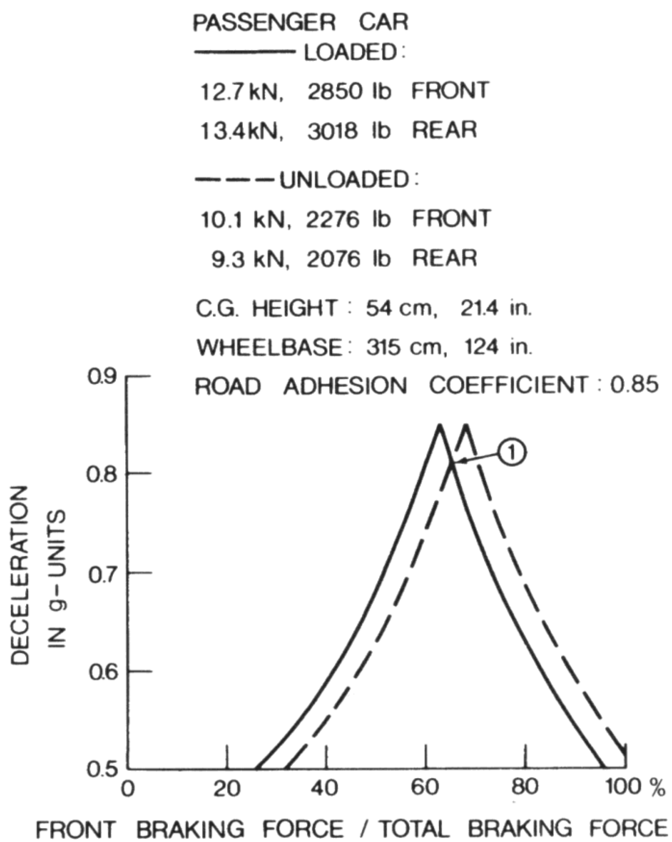
Braking under non ideal conditions



Source Wong: Fig 3.50

- These formulae show the large influence of the weight distribution (and position of CoG) over the optimal braking distribution
- Vehicle with no freight: braking distribution is stronger on the front to have a wheel blocking in the front first
- Design approach: find the right compromise → point 1

Braking under non ideal conditions

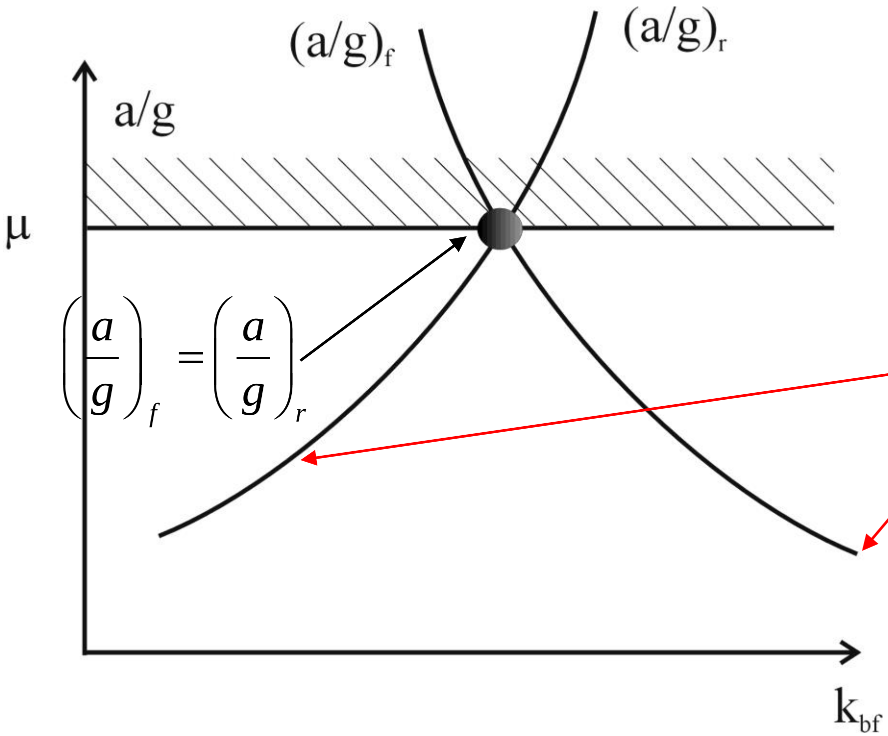


Source Wong: Fig 3.51

- For passenger cars, the influence is lesser than on duty vehicle
- Design approach: find the right compromise → point 1

Braking under non ideal conditions

- Remark: One recovers the ideal braking conditions on the braking ratio by assuming :



Issue: these curves depend strongly on the geometry, the position of the CoG, and the friction coefficient

Estimating the braking distance

Braking efficiency

- Our reference: all wheels are reaching simultaneously the friction limit:

$$ma_x = -(F_{bf} + F_{br}) = -\mu\, W_f - \mu\, W_r = -\mu W$$

- One obtains the maximum deceleration:

$$a_x^{max} = \mu\, g$$

- In these ideal conditions, one compares the actual braking deceleration that is measured to the reference deceleration rate:

$$\boxed{\eta_b = \frac{|a_x|/g}{\mu}}$$

Braking distance

- To calculate the braking distance, we start from Newton equation

$$\gamma_b \, m \, \frac{dv}{dt} = F_b + F_{RES} \qquad\qquad dx = \frac{\gamma_b \, m}{F_b + F_{RES}} \, v \, dv$$

- The effective mass factor is γ_b which is between 1.03 and 1.05 in braking since the clutch is open

$$S_{V_1 \to V_2} = \int_{V_1}^{V_2} \gamma_b \, m \frac{v \, dv}{F_b + F_{RES}}$$

$$S_{V_1 \to V_2} = \gamma_b \, m \int_{V_1}^{V_2} \frac{v \, dv}{F_b + f \, W \; \cos\theta \pm W \; \sin\theta + F_{aero}}$$

Braking distance

- The aerodynamic forces write:

$$F_{aero} = \frac{1}{2}\,\rho\, C_x\, S\, v^2 = C_{aero}\, v^2$$

- It comes

$$S_{V_1 \to V_2} = \frac{\gamma_b\, m}{2\, C_{aero}}\, \ln\left(\frac{F_b + f\, W\ \cos\theta \pm W\ \sin\theta + C_{aero}\, V_1^2}{F_b + f\, W\ \cos\theta \pm W\ \sin\theta + C_{aero}\, V_2^2}\right)$$

- The stopping distance till rest (V_2=0)

$$S_{Stop} = \frac{\gamma_b\, m}{2\, C_{aero}}\, \ln\left(1 + \frac{C_{aero}\, V_1^2}{F_b + f\, W\ \cos\theta \pm W\ \sin\theta}\right)$$

Braking distance

- The best stopping distance: the brakes are producing the forces which are just necessary to reach the friction coefficient (as well as the force to absorb the braking of the driveline)

$$S_{Stop}^{min} = \frac{m}{2\,C_{aero}} \ln\left(1 + \frac{C_{aero}\,V_1^2}{\mu\,W + f\,W\;\cos\theta \pm W\;\sin\theta}\right)$$

- If we have a lower braking efficiency, one can use the coefficient η_b

$$\boxed{S_{Stop} = \frac{m}{2\,C_{aero}} \ln\left(1 + \frac{C_{aero}\,V_1^2}{\eta_b\,\mu\,W + f\,W\;\cos\theta \pm W\;\sin\theta}\right)}$$

Braking distance

- One can further add some time related to :
  - Reaction time necessary to the driver to react t_c: between 0.5 and 2 s generally
  - The lead time of the braking system t_d,
  - The rise time of the braking system to develop full braking forces, generally around t_r = 0.3 s

- During this time, the vehicle is still driving at initial speed so that the stopping distance gets longer:

$$S_a = (t_a + t_d)\, V_1$$