INTRODUCTION TO TYRE MECHANICS

Part I: Introduction and Longitudinal Force Generation

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Layout

- Introduction
- Tyre construction
- Classification: size, load and velocity indices
- Adhesion mechanisms
- Rolling resistance
- Generation of longitudinal forces
  - Brush model
  - Tractive and braking forces
  - Longitudinal slip ratio
  - Tractive and braking force curves

Layout (2)

- Lateral forces
  - Gough experiment
  - Lateral force as a function of the side slip angle
  - Cornering coefficient
  - Cornering stiffness
- Self aligning torque and pneumatic trail
- Camber thrust
  - Definition and mechanism
  - Camber coefficient
- Combined operations
  - Sakai experiment
  - Friction ellipse
- Modelling: Pacejka magic formula

Introduction

Introduction

- Key role of tyres:
  - Apart from the aerodynamic loads, all control forces act through the tyre contact patches and the tyre ground interface which is not larger than a hand shape
  - Knowing and understanding the physical phenomena at work in the contact patch is a key issue to tackle vehicle dynamics
  - Tyre are essential elements:
    - To insure directional control and stability of the vehicle
    - To guarantee road holding
    - To preserve passenger comfort

Introduction

- Tyre has three main functions:
  - Sustain the vertical load while insuring a first damping against the loading from the road
  - Develop longitudinal forces to enable acceleration and braking
  - Develop lateral forces to act against centrifugal loads in turning and dynamic manoeuvres

- Meanwhile, the tyre is subject to severe operational constraints
  - To be able to operate during a long life time i.e. a high mileage with a high level of reliability
  - To exhibit the smallest rolling resistance and save energy in various travel conditions

Introduction

- Construction complexity of tyres:
  - Geometry : toroidal body
  - Material : rubber is viscoelastic material reinforced with fibres making a complex composite material system
  - Refinement and optimisation of its material and geometrical properties make it a complex and non linear structural system which is difficult to model and to identify its parameters
- The behaviour of the tyre strongly depends on many parameters:
  - Operating conditions : speed, inflating pressure, presence of water, texture and nature of the road, etc.
  - Construction characteristics

Introduction

- Two approaches to investigate tyre properties:
  - Analysing experimental data results
    - Phenological approach
  - Modelling to deduce the properties
    - Require numerical models with a high level of complexity to predict quantitatively the behaviour and the properties
    - Simple analytical models are able to predict only the trends of the properties and the main influence of the parameters and to reveal physical explanation of the observed behaviour, but they are not able to match exact experimental values

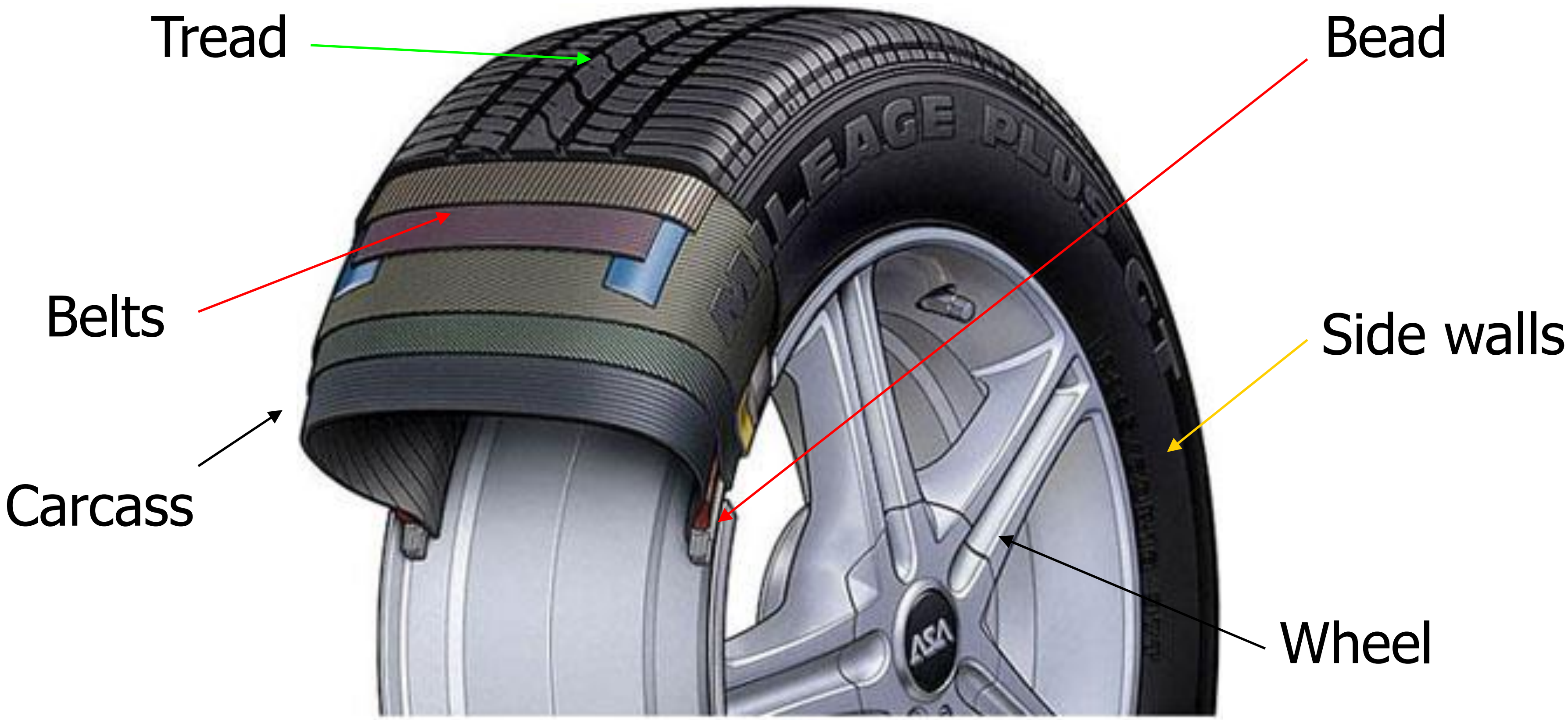
Tyre construction

Tyre construction

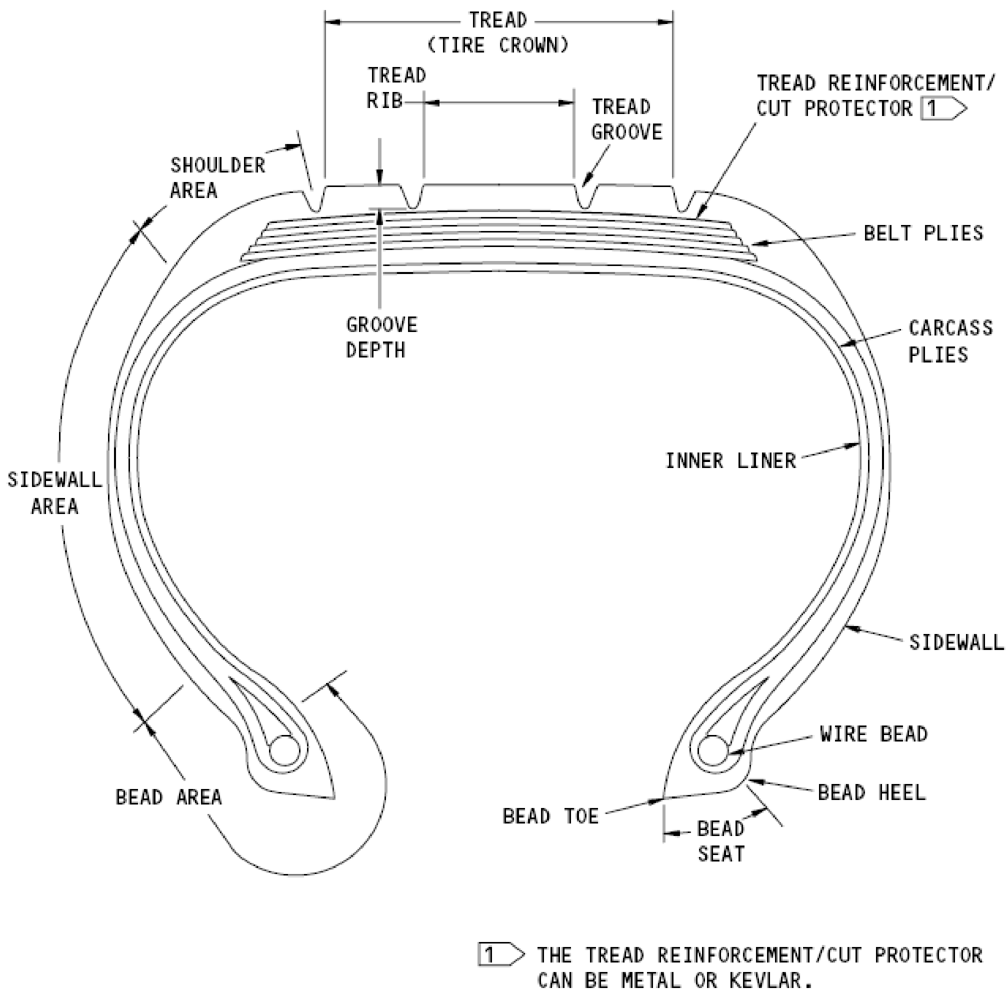
- Mechanical layout:
  - Toroidal hollow shell composed of:
    - The carcass: a flexible shell made of rubber reinforced with several plies of high modulus fibre layers
    - Some stiff beads made of steel cables keeping the contact between the tyre and the wheel rim
    - The tread, which offers high wear resistant layers in contact with the ground, insuring a good life time, water and snow evacuation, thermal cooling between the tyre and the external flow.

- The internal pressure due to inflation
  - Set the structure into a prestressed state so as it maintains the shell in a tension state in any time, whatever should be the deformation state.

Tyre construction



Tyre construction



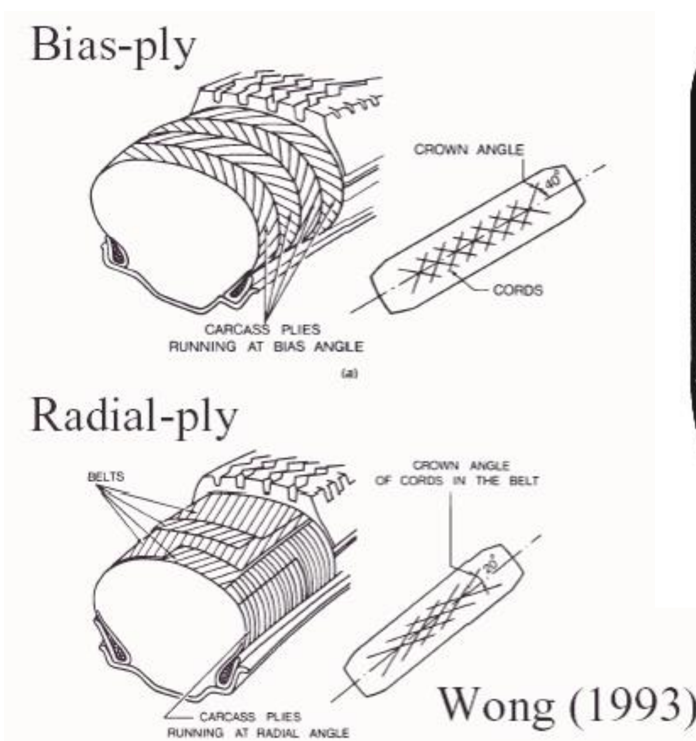
Tyre construction

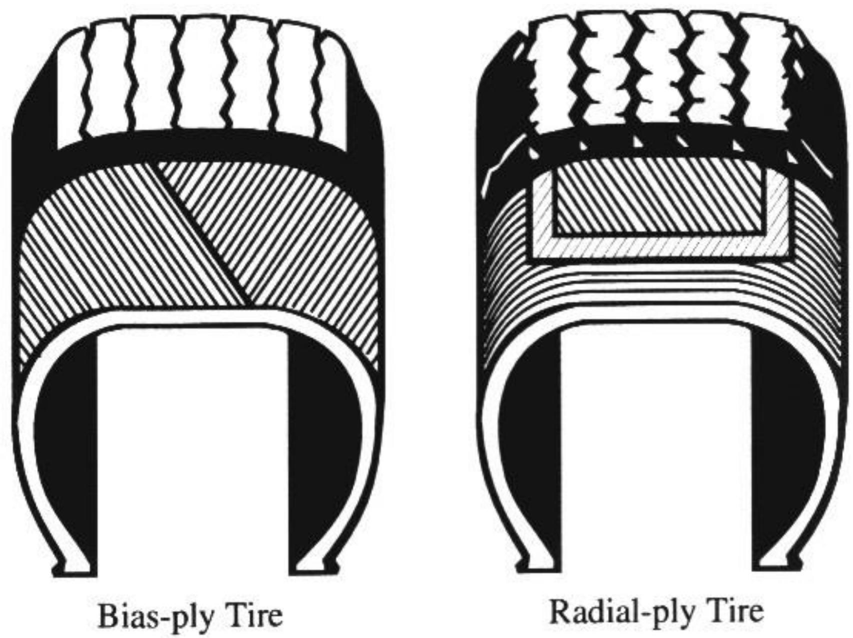
- Tyre is made of rubber mixed with other compounds such as oil and black carbon.
  - A typical truck tyre = 12 kg among which 4 kg of rubber, 2kg of black carbon, 2 kg of oil, 3 kg of steel fibres, and 1 kg of rayon fibres or other fibres

- Rubber density in tyres: 1200 kg/m^3
- Heat capacity: c_p=1200 J/kg.K
- Heat conductivity is too small so that it requires black carbon to increase the conductivity:
  - $k_{20°}$ = 0.23 W/mK
  - $k = k_{20°}$ (293/T) (between 0° and 150°)

Tyre construction

- Two basic construction layouts
  - Radial tyre
  - Bias-ply tyre

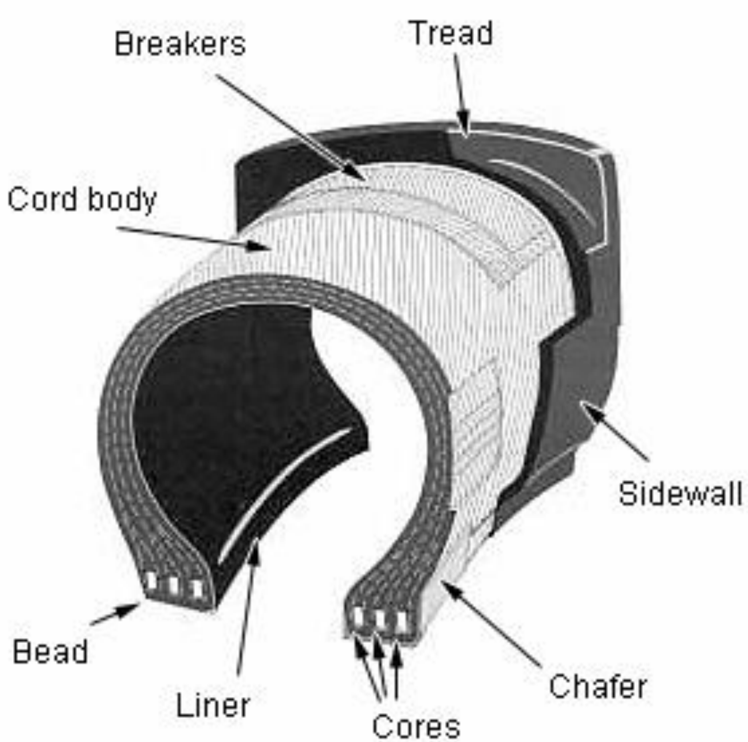




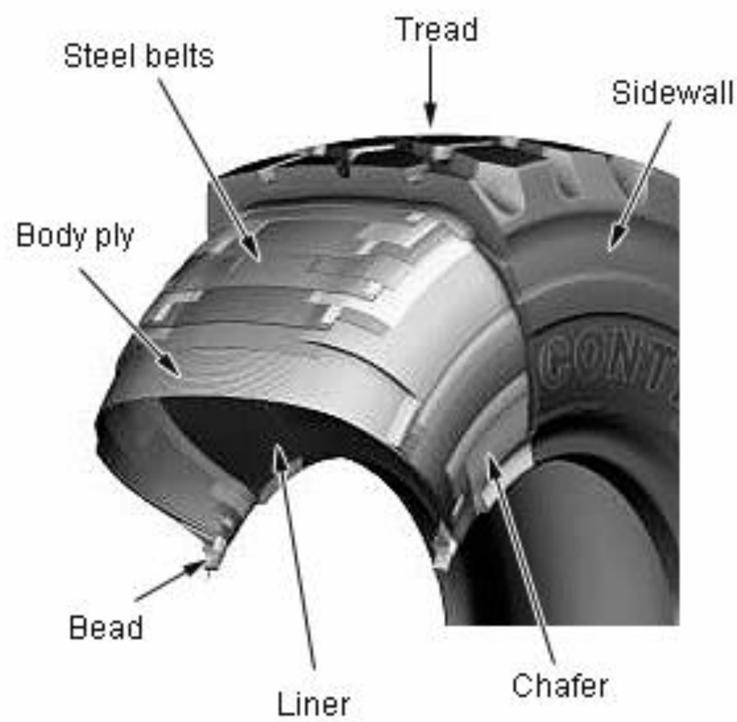
Gillespie Fig. 10.1

Tyre construction

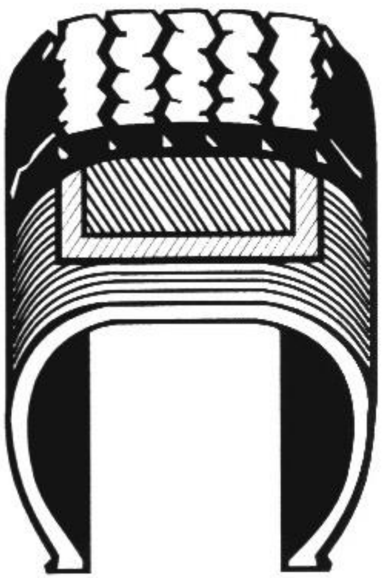
Bias / cross-ply Construction.



Radial Construction.



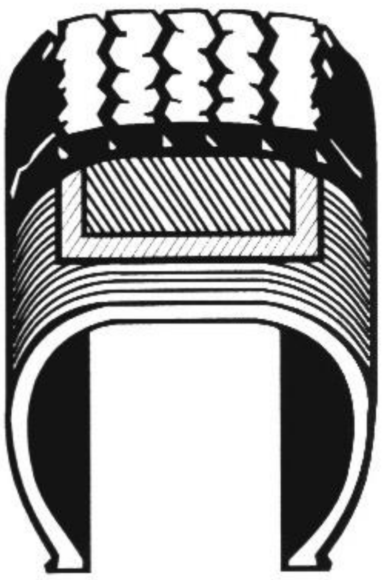
Radial tyre



Radial-ply Tire

- The carcass = layers of parallel fibres laminate of rubber materials reinforced with nylon fibres, rayon, polyester, or glass fibres. Fibres run from one bead to the other with an angle close to 90° with respect to the tyre mid plane and circumferential direction.
- Compliant side wall providing a soft suspension but they are unable to insure the directional control
- Stiff reinforcement belts made of steel or composite plies running around the circumferential direction between the carcass and the tread. The usual angle of the belt plies with the circumferential direction is typically 20° with respect to the tread.

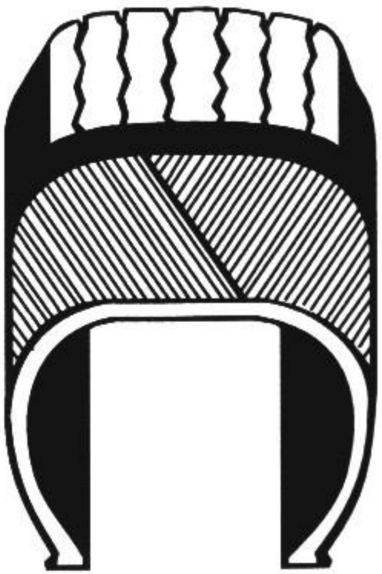
Radial tyre



Radial-ply Tire

- The belts provide the directional control. The belts stabilize the tread and keep it flat with respect to the road surface even in case of lateral deflection.
- Most of tyres for passenger cars have 2 plies on the side walls and 1 or 2 plies of steel fibres or 2 to 6 plies of rayon.

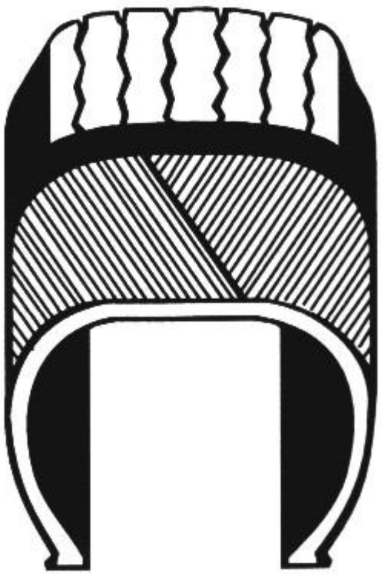
Bias-ply tyre



Bias-ply Tire

- The carcass of bias ply tyres is made of an even number (2 or more) balanced plies, running from one bead to the other but crossing the circumferential direction with an angle between 35° to 40°.
- The ply angle results from a compromise between high angles (90°) which would give a soft behavior and good vertical comfort, and on the other hand values close to 0° with would give the best directional control.

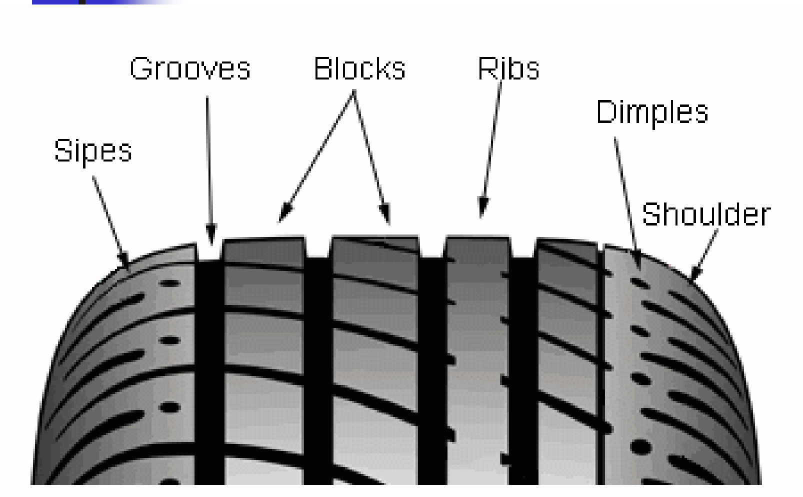
Bias-ply tyre



Bias-ply Tire

- Even if the bias ply carcasses are stiffer that radial carcasses, the tread of bias ply tyre can twist with respect to road during the turn. Because of the twist deformation of the tread, the load is prone to be transferred onto the external shoulder of the contact patch, reducing the uniformity of the contact pressure. This is unfavourable to generate large cornering forces.
- The bias ply construction gives rise to a larger twist of the contact patch and the toroidal shape can be more deformed and elongated.
- They generally lead to a lower performance than radial tyres.

Tread design



Definition of tread parts

Rain tyre

Types of tread geometries

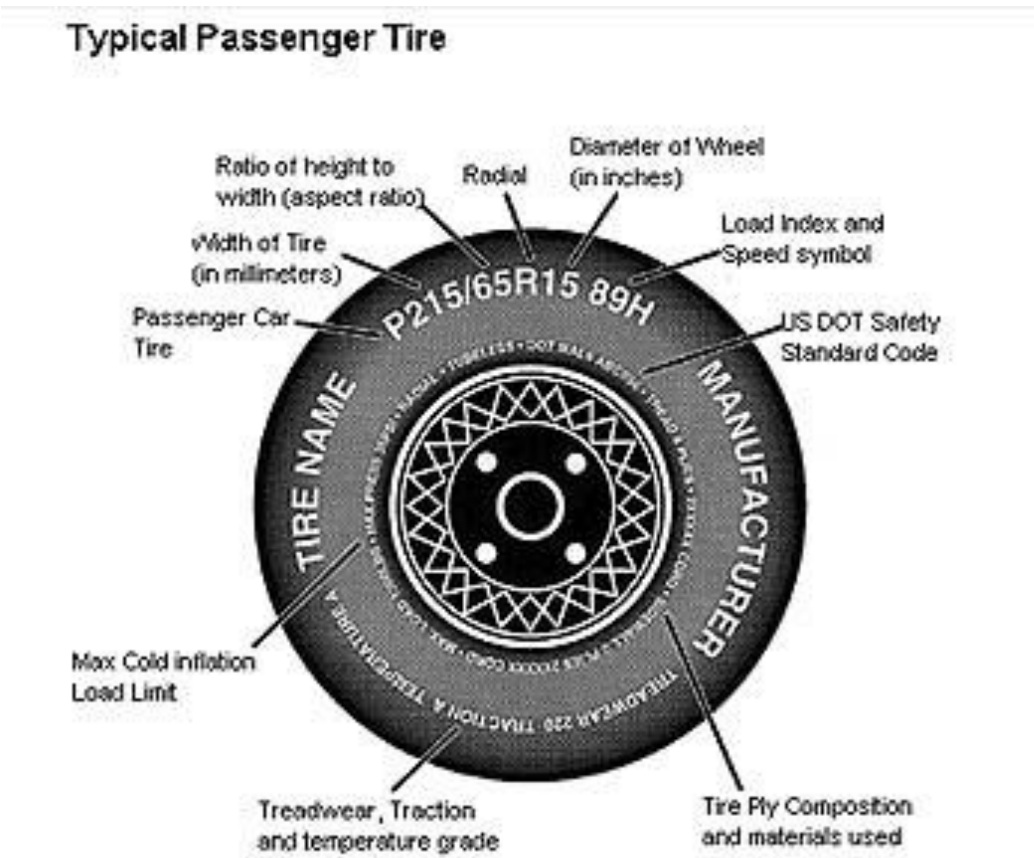
Tread design

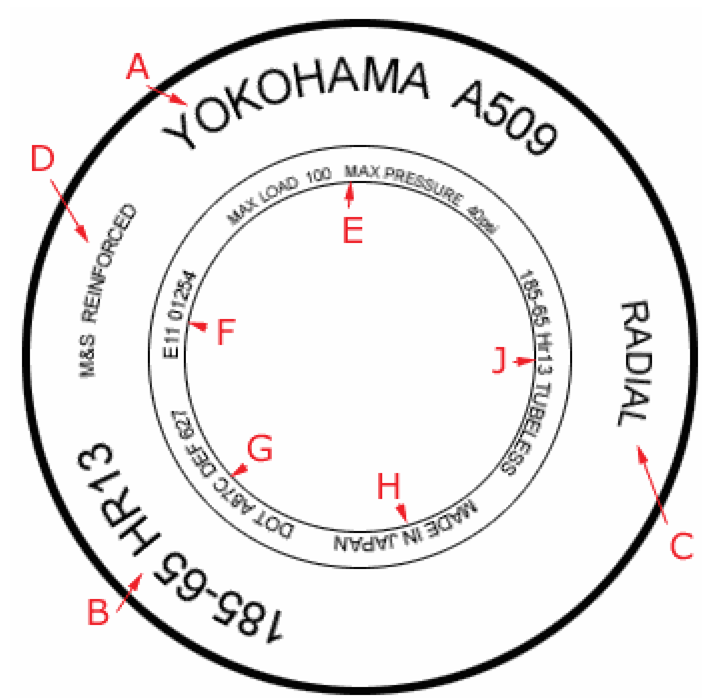
- Tread has three main functions:
  - To insure a high wear resistance
  - Water and snow evacuation…
  - Heat exchange with the air external flow to insure a sufficient cooling of the tyre rubber

- In general, tread has not a significant structural mission

Codes and dimension

Size, load, and velocity indices





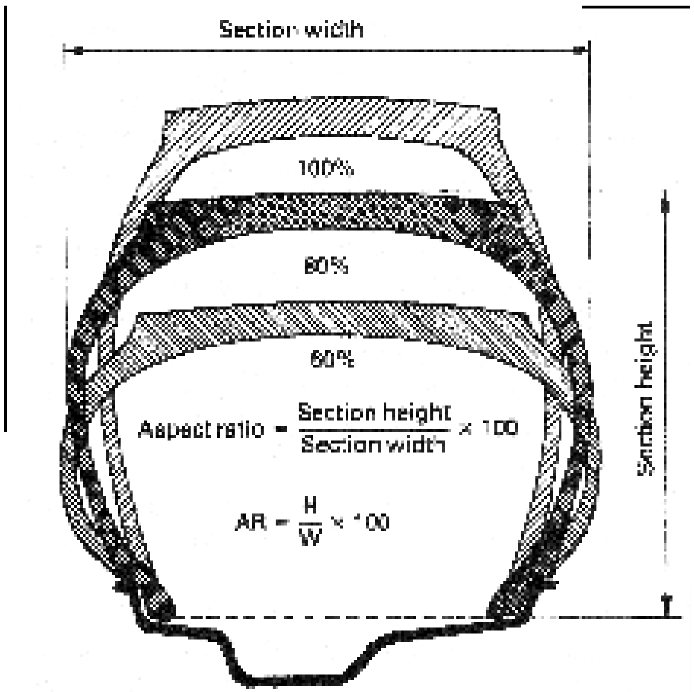
Size, load, and velocity indices

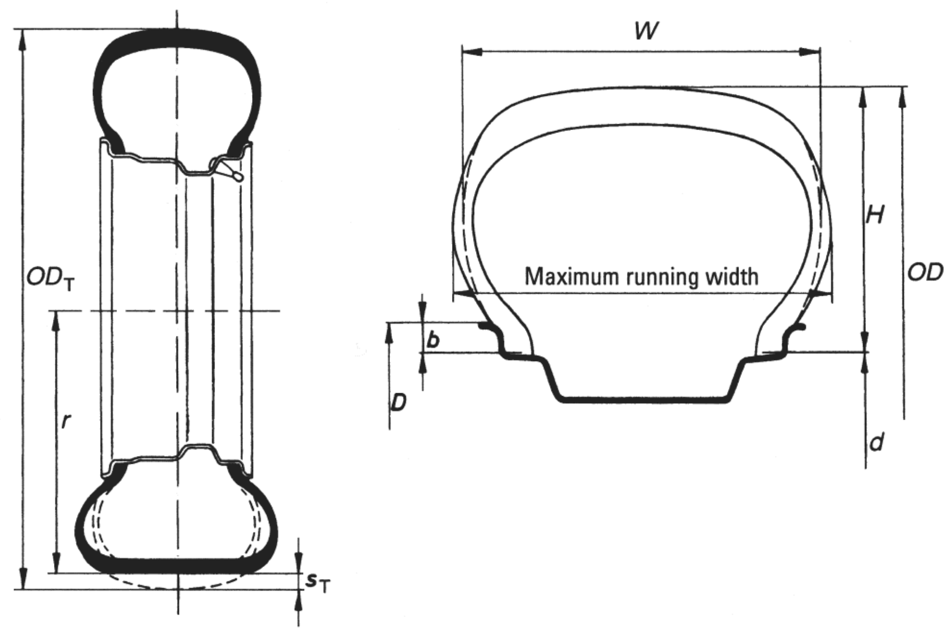
Size description of the tyre

WW / HH R Dd   Li Vm

- WW: Rubber width (generally in mm)
- HH: Height of the rubber tyre: aspect ratio (%) height / width
- ‘R’: radial tyre
- Dd: Diameter at the wheel rim (often expressed in inches)
- Li: The load index tells what is the maximum vertical load that can be sustained by the tyre rolling at high speed.
- Vm: Maximum speed symbol: a letter indicating the maximum speed for the tyre.

Size, load, and velocity indices





Heisler Advanced vehicle Technology Fig 8.28

Reimpel: Fig 2.11

Size, load, and velocity indices

Symbole de vitesse	Vitesse maximale	
	km/h	mph
N	140	87
P	150	93
Q	160	99
R	170	106
S	180	112
T	190	118
H	210	130
V	240	149
W	270	168
Y	300	186
Catégorie de vitesse Code de construction	Vitesse maximale	
	km/h	mph
ZR	240	149
	et plus	

Li	kg	Li	kg	Li	kg	Li	kg
65	290	79	437	93	650	107	975
66	300	80	450	94	670	108	1000
67	307	81	462	95	690	109	1030
68	315	82	475	96	710	110	1060
69	325	83	487	97	730	111	1090
70	335	84	500	98	750	112	1120
71	345	85	515	99	775	113	1150
72	355	86	530	100	800	114	1180
73	365	87	545	101	825	115	1215
74	375	88	560	102	850	116	1250
75	387	89	580	103	875	117	1285
76	400	90	600	104	900	118	1320
77	412	91	615	105	925	119	1360
78	425	92	630	106	950		

Size, load, and velocity indices

- Example:
  - 195/60 R 15 91V

    Width 195 mm,

    aspect ratio 60%, that is side wall height

    195*0,6=117 mm

    R= radial,

    Wheel diameter 15 inch,

    Index of the velocity V=240 km/h, 615 kg @max speed

    Outer diameter: 15*25,4 + 2*117 = 615 mm

    (R=307,5 mm)

Size, load, and velocity indices

The wheel sizes

- Characterized by
  - Diameter of the wheel (in inches)
  - Its width
  - The iron profile (normalized)

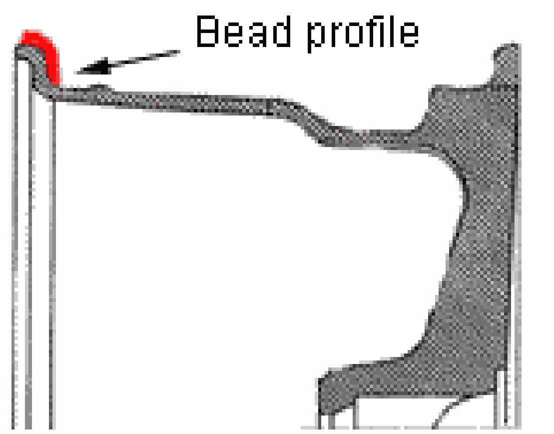
- Example: 61/2 J x 14

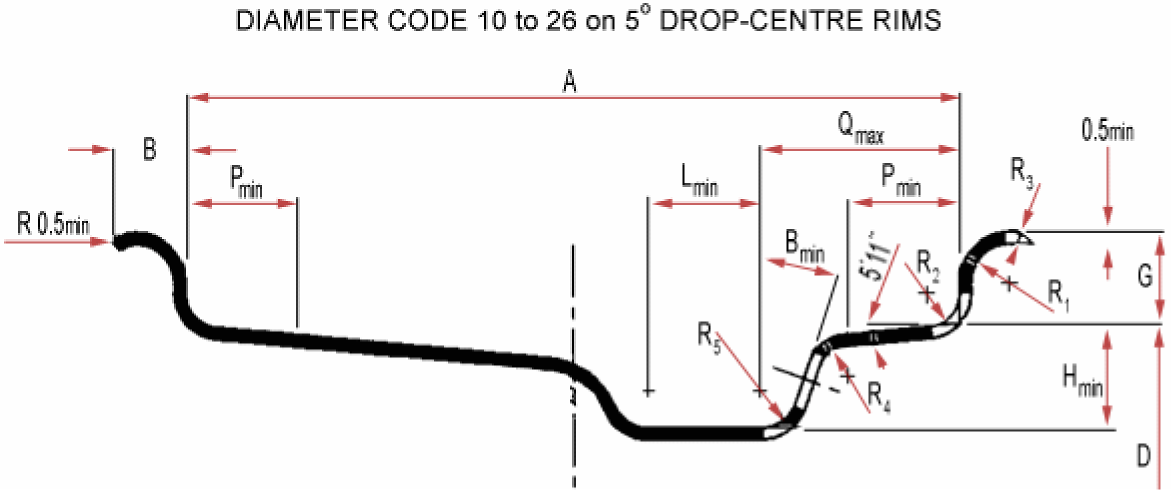
  Width 6,5 inches, J = type, diameter of the wheel = 14 inches,

Size, load, and velocity indices

The wheel sizes

- The details of the rim profile (normalized)

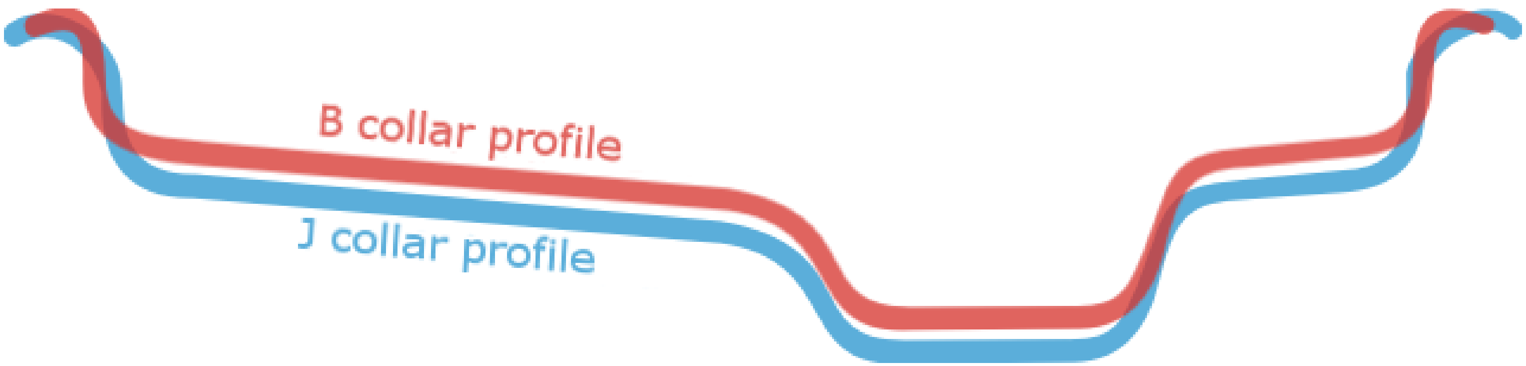




Size, load, and velocity indices

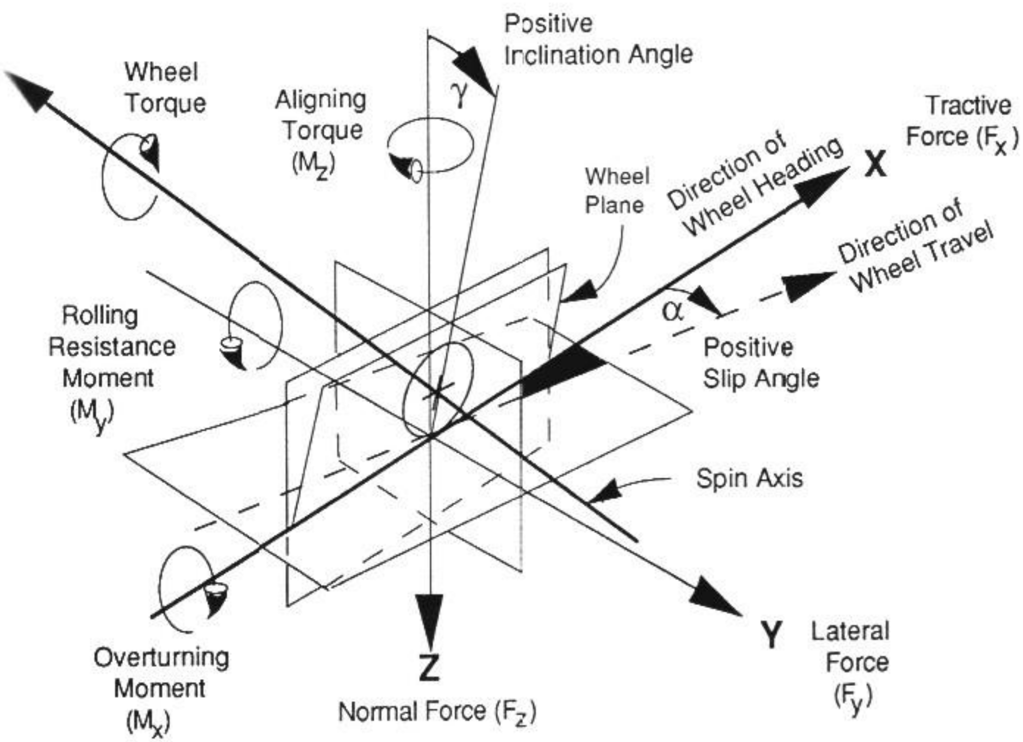
The wheel sizes

- Comparison of B and J profiles for passenger cars



Terminology

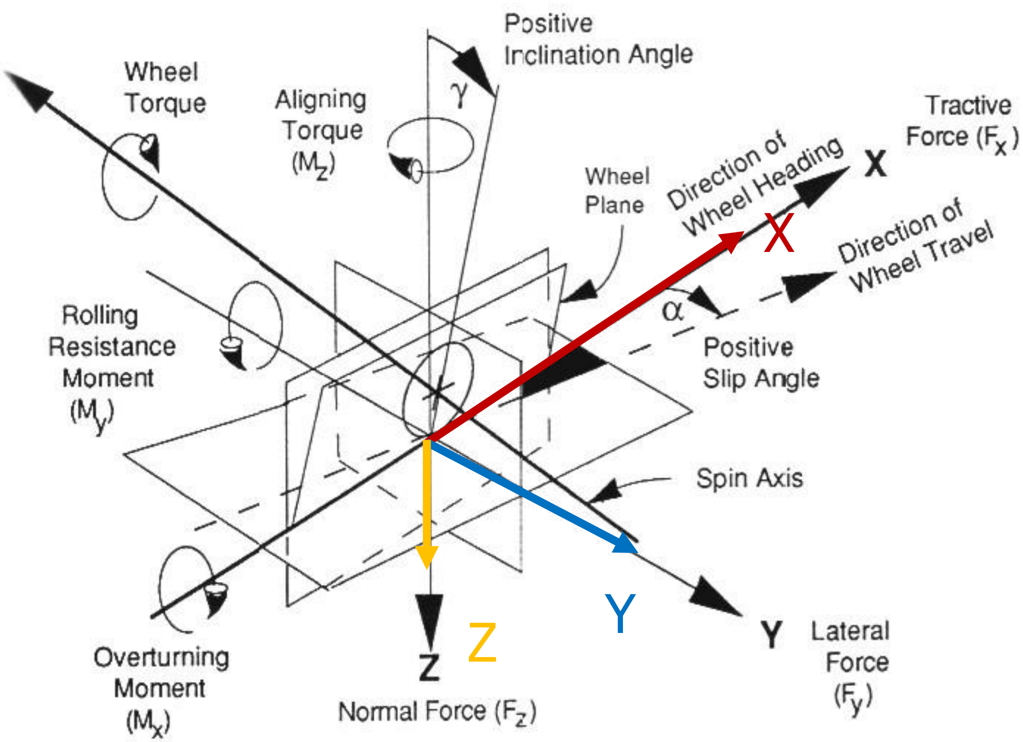
Terminology and axis system (SAE)



Gillespie Fig. 10.3

- <u>Wheel plane:</u> central plane of the tire normal to the axis of rotation
- <u>Wheel centre:</u> intersection of the spin axis with the wheel plane
- <u>Centre of Tire Contact:</u> intersection of the wheel plane and the projection of the spin axis onto the road plane
- <u>Loaded radius:</u> distance from the centre of tire contact to the wheel centre in the wheel plane

Terminology and axis system (SAE)



Gillespie Fig. 10.3

- <u>X-axis :</u> is the intersection of the wheel plane and the road plane, with a positive direction pointing into forward direction.
- <u>Z-axis:</u> is perpendicular to the road plane with a positive direction downward.
- <u>Y-axis:</u> is in the road plane; it is perpendicular to both X-axis and Z-axis and chosen to make an orthogonal right hand reference system. Y-axis points to the right hand side of the driver.
- <u>Origin O:</u> is located in the wheel centre

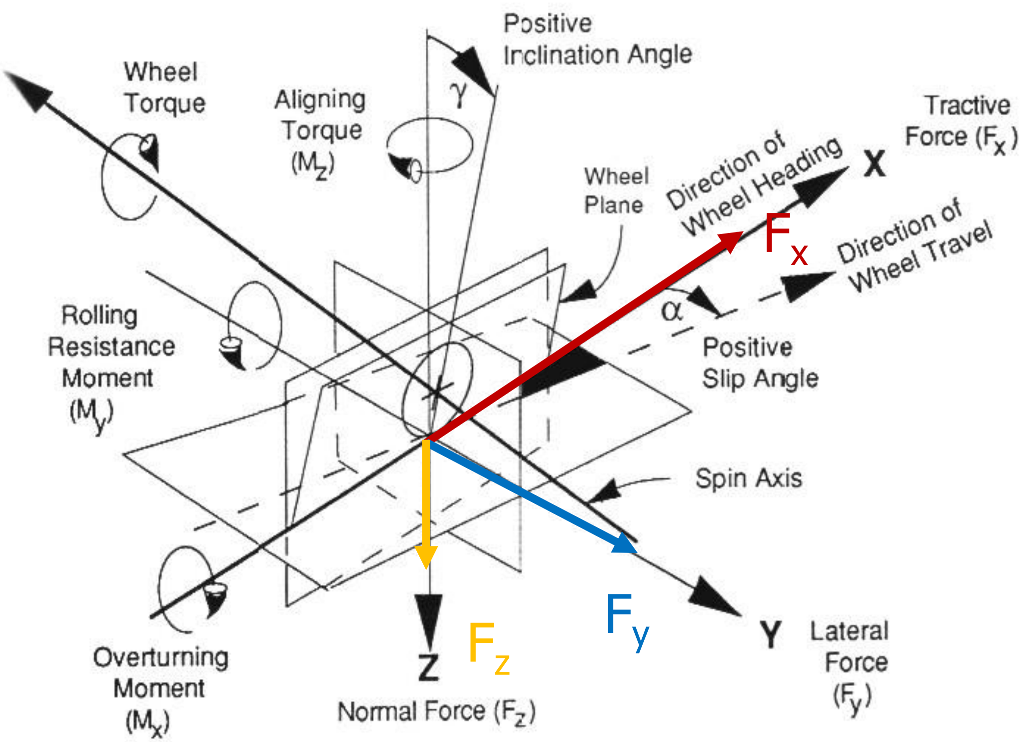
Forces

- *Forces and moments are accounted positively when acting onto the vehicle and in the positive direction with respect to the considered frame*

- Corollary
  - A positive F_x force propels the vehicle forward
  - The reaction force R_z of the ground onto the wheels is accounted negatively.

- Because of the inconveniency of this definition, the SAEJ670e « Vehicle Dynamics Terminology » denotes by normal force a force acting downward while vertical forces are referring to upward forces

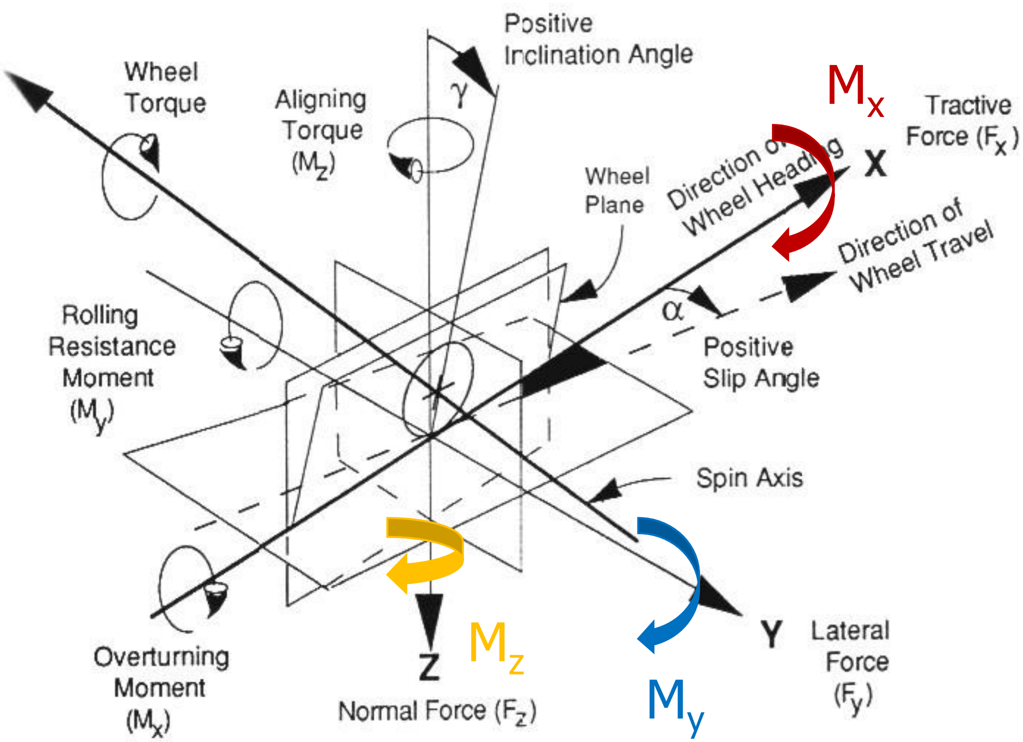
Terminology and axis system (SAE)



Gillespie Fig. 10.3

- Longitudinal Force F_x : component of the force acting on the tire by the road in the plane of the road and parallel to the intersection of the wheel plane with the road.
- Lateral Force F_y : component of the force acting on the tire by the road in the plane of the road and normal to the intersection of the wheel plane with the road plane.
- Normal Force F_z : component of the force acting on the tire by the road which is normal to the plane of the road.

Terminology and axis system (SAE)



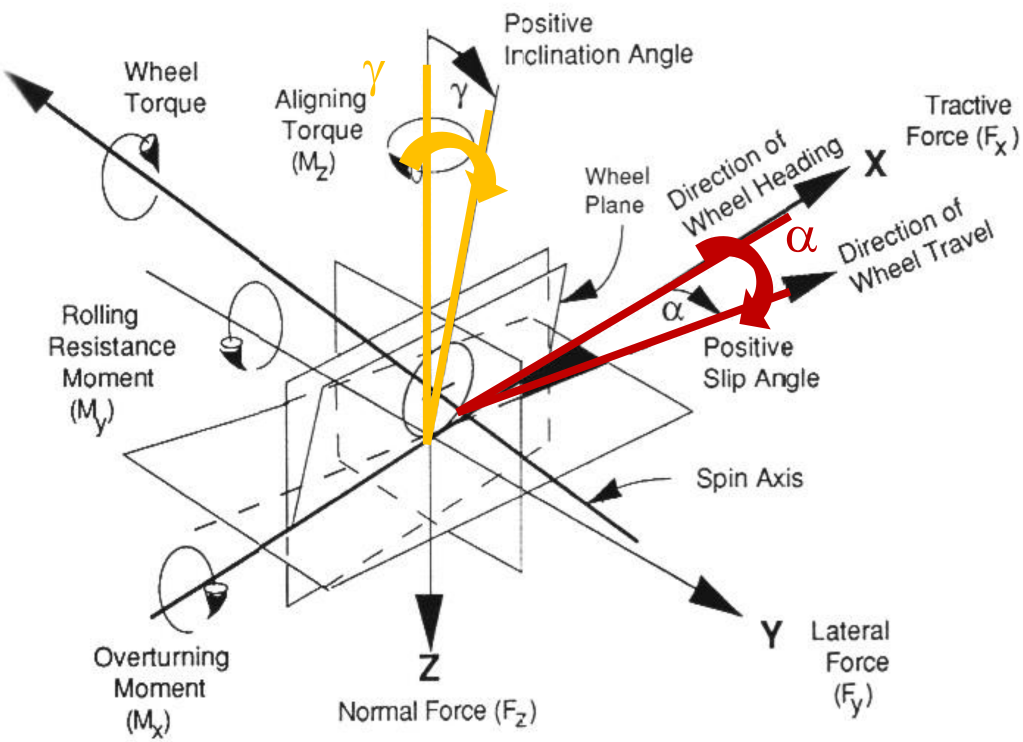
Gillespie Fig. 10.3

- Overturning Moment M_x : moment acting on the tire by the road in the plane of the road and parallel to the intersection of the wheel plane with the road plane.

  Rolling Resistance Moment M_y : moment acting on the tire by the road in the plane of the road and tending to rotate the wheel about the y axis.

  Aligning Moment M_z : moment acting on the tire by the road which is normal to the road plane.

Terminology and axis system (SAE)



Gillespie Fig. 10.3

- <u>Slip Angle (α):</u> angle between the direction of the wheel heading and the direction of travel.

  A positive sideslip corresponds to a tire moving to the right while rolling in the forward direction.

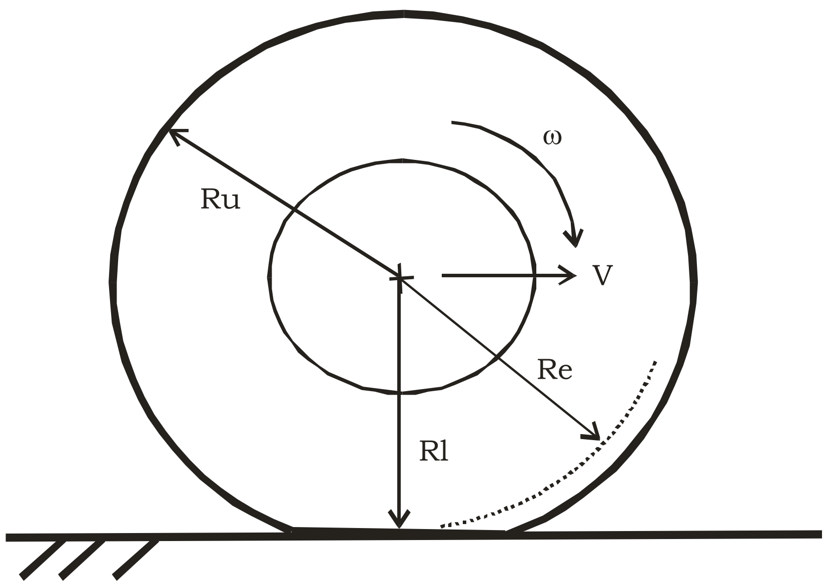
- <u>Camber Angle (γ):</u> angle between the wheel plane and the vertical direction.

  A positive camber corresponds to the top of the tire leaned outward from the vehicle.

RADIUS OF THE TYRE

- The radius definition is important for the formulation of the longitudinal slip ratio
- One can see:
  - The unloaded tire radius R_u: this is the rigid tire without deformation, inflated at standard pressure
  - The loaded radius R_l: is measured by the distance between the wheel rotation center and the central contact patch point
  - The effective rolling radius R_e: it is the radius calculated to the constant ratio between the velocity and the rotation speed

$$R_e = \frac{V}{\omega}$$



RADIUS OF THE TYRE

- Relation between the three radii

- In the flat part of the contact patch

$$L/2 = \Delta x = R_u \, \sin \Delta\phi$$

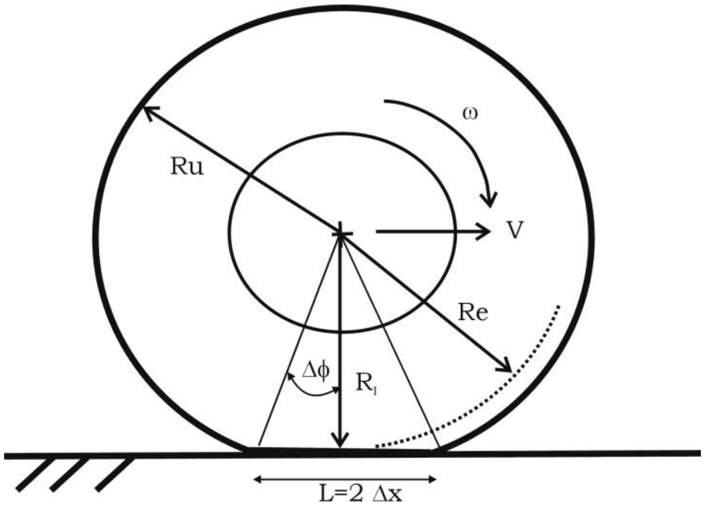
- The kinematic relationship of the virtual rigid tyre rolling with slippage

$$\Delta x = R_e \, \Delta\phi$$

- It comes

$$\Delta x = R_e \, \Delta\phi = R_u \, \sin \Delta\phi$$

$$R_e = R_u \, \frac{\sin \Delta\phi}{\Delta\phi}$$



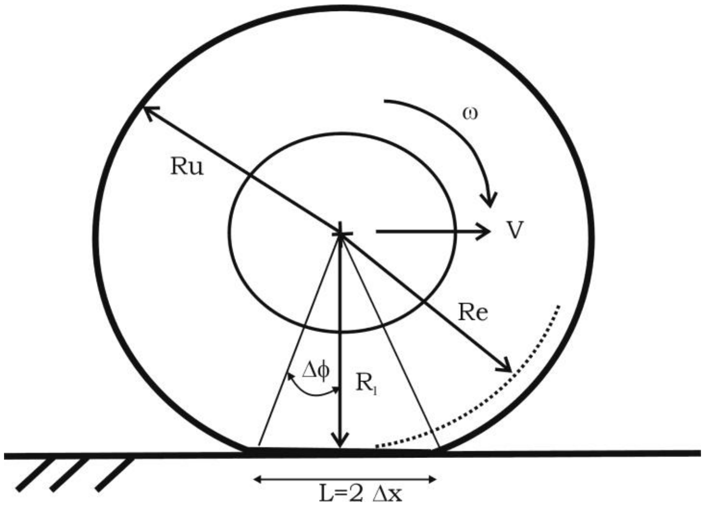
RADIUS OF THE TYRE

- If we expand the sinus $\Delta\phi$

$$R_e = R_u \frac{\sin \Delta\phi}{\Delta\phi} \simeq R_u \frac{\Delta\phi - \frac{1}{6}\Delta\phi^3}{\Delta\phi} = R_u \left(1 - \frac{\Delta\phi^2}{6}\right)$$

- Let's now calculate $\Delta\phi$ in terms of R_l:

$$\cos \Delta\phi = \frac{R_l}{R_u}$$



- Again using Taylor expansion

$$\cos \Delta\phi = 1 - \frac{1}{2}\Delta\phi^2$$

- It comes

$$\Delta\phi^2 = 2(1 - \cos \Delta\phi) = 2(1 - \frac{R_l}{R_u})$$

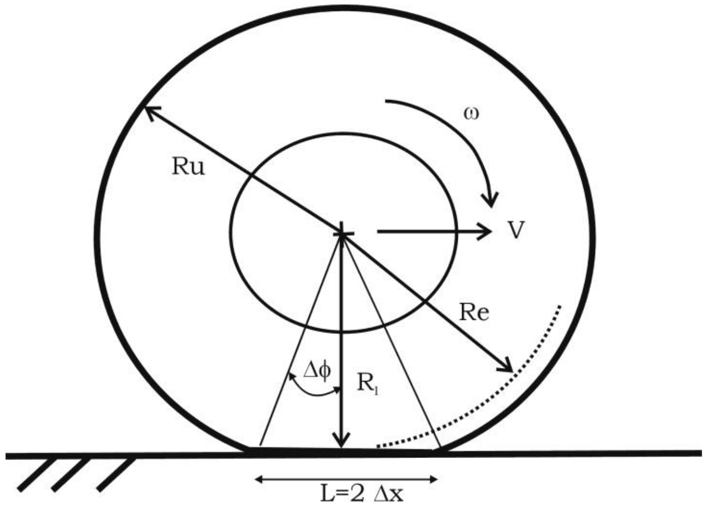
RADIUS OF THE TYRE

- Finally we have

$$R_e = R_u \left(1 - \frac{\Delta\phi^2}{6}\right) \qquad \Delta\phi^2 = 2(1 - \frac{R_l}{R_u})$$

- So

$$\begin{aligned} R_e &= R_u \left(1 - \frac{\Delta\phi^2}{6}\right) \\ &= R_u \left(1 - \frac{2}{6}(1 - \frac{R_l}{R_u})\right) \\ &= R_u \left(\frac{2}{3} + \frac{1}{3}\frac{R_l}{R_u}\right) \end{aligned}$$



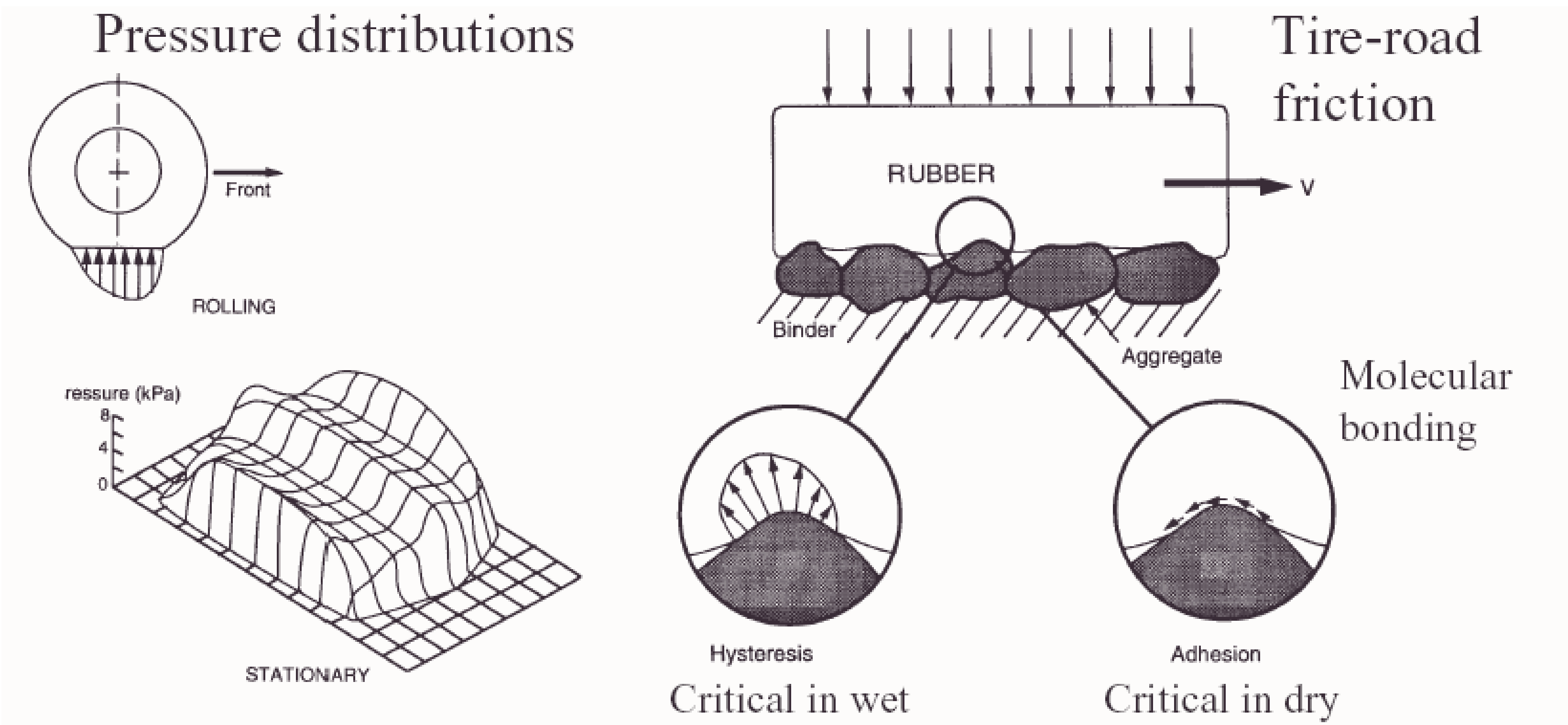
- Order of magnitude:

$$R_e \simeq 0,98\, R_u$$

Tyre Force Generation Mechanisms

Tyre force generation mechanisms

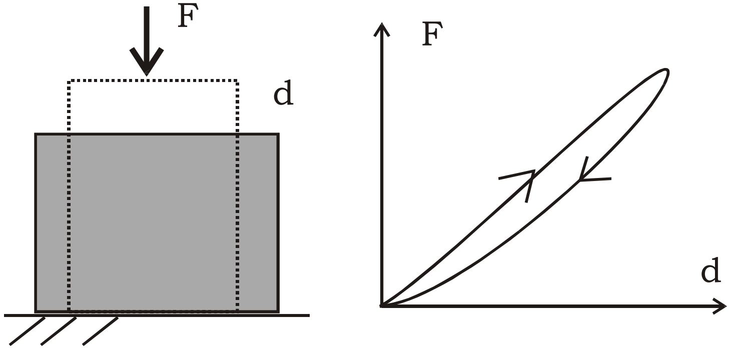
Mechanism of force generation at the ground–tire interaction



Gillespie. Fig 10.4 et Fig10.5

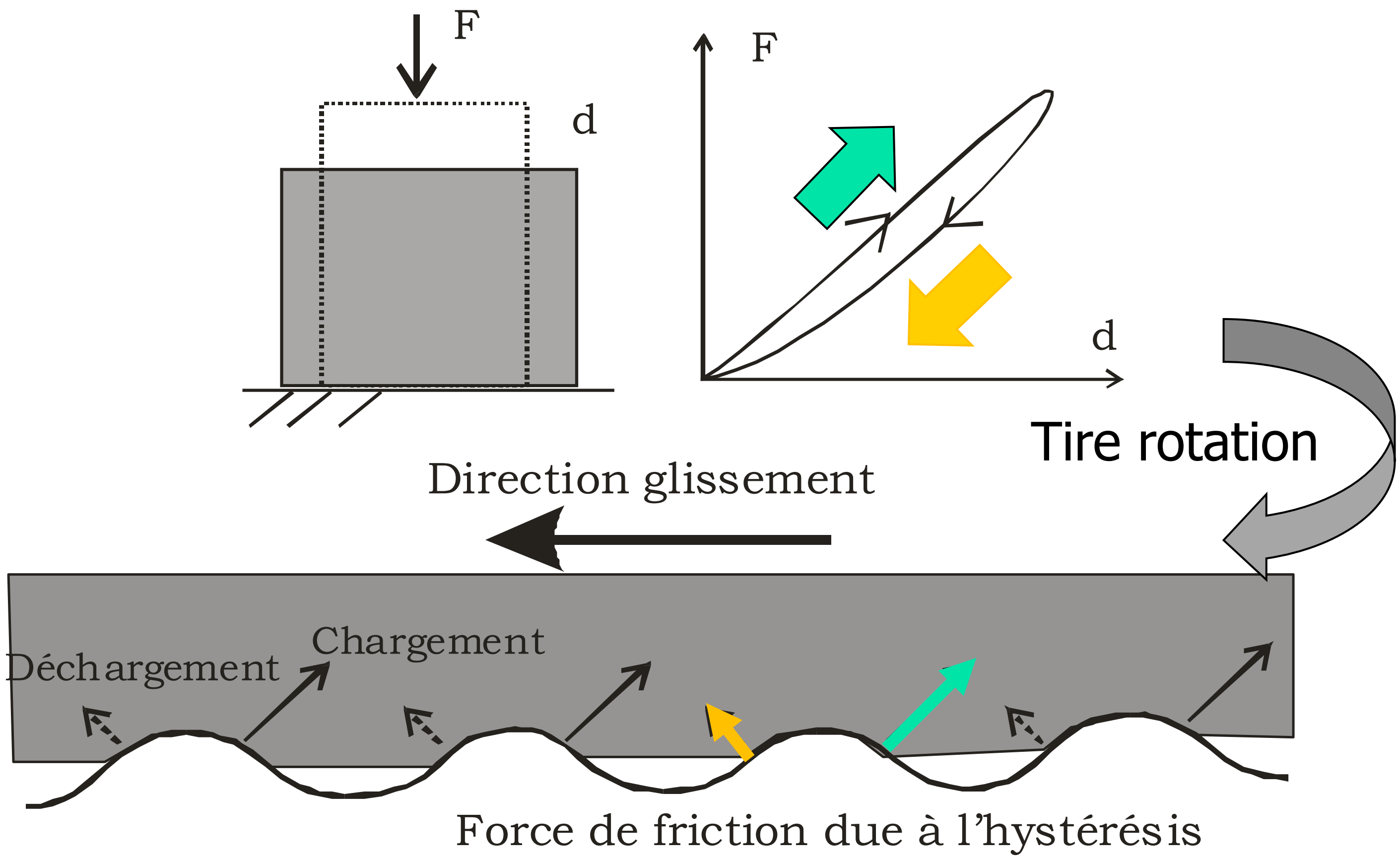
Tyre force generation mechanisms

- **Hysteresis mechanism**
  - Viscoelastic material is characterized (among others) by a lower unloading forces compared to the load that is necessary during the load increase
  - Even if the surface is lubricated, rubber allows to develop an adherence force because the rubber will exert a different pressure on the front faces of road asperities where load is increasing while pressure is lower on rear faces where load is decreasing.



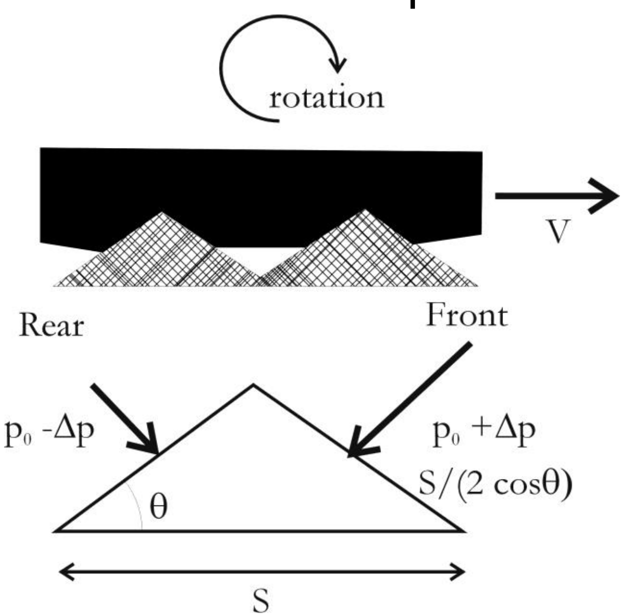
Tyre force generation mechanisms

- **Hysteresis mechanisms**



Tyre force generation mechanisms

- **Corollary of hysteresis mechanisms**
  - The hysteresis mechanism is weakly impacted by the presence of water. So rain tyres can make use of high hysteresis materials in the tread.
  - The friction coefficient does not depend on the scale of the road asperities but mostly on the roughness and on their angles.



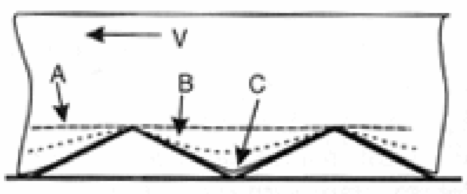
$$S' = \frac{S}{2\ \cos\theta}$$

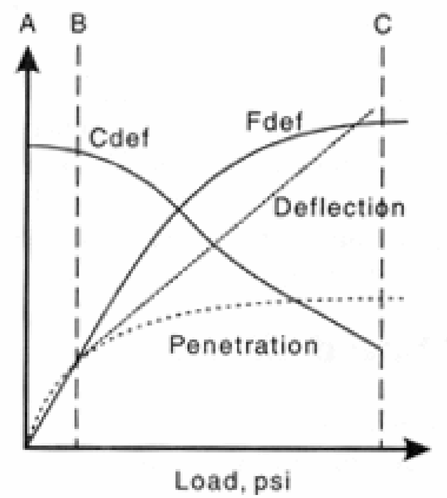
$$\begin{aligned}\sum F_x &= -(p_0 - \Delta p)S' \sin\theta + (p_0 + \Delta p)S' \sin\theta \\ &= 2\,\Delta p\, S'\ \sin\theta = 2\Delta p\,\frac{S}{2\ \cos\theta}\ \sin\theta \\ &= \Delta p\, S\ \tan\theta\end{aligned}$$

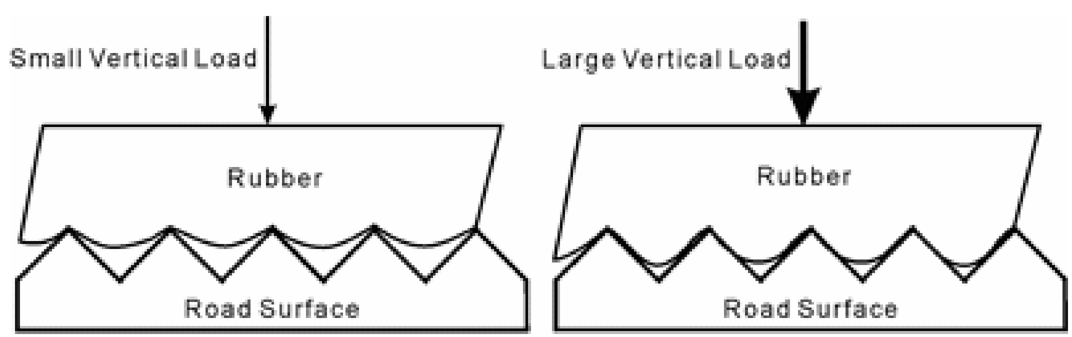
$$\begin{aligned}\sum F_z &= (p_0 - \Delta p)S' \cos\theta + (p_0 + \Delta p)S' \cos\theta \\ &= 2\, p_0\, S'\ \cos\theta = p_0\, S\end{aligned}$$

Tyre force generation mechanisms

- **Corollary of hysteresis mechanisms**
  - Conversely to many pairs of materials, the rolling adhesion coefficient of rubber is dependent of the contact surface, because of its viscoelastic nature.
  - Increasing vertical load does not increase the contact surface in the same ratio. Contact surface growths with a lower rate than linear.
  - Thus friction coefficient $\mu=F_T/F_N$ decreases with the pressure force. Typically the adhesion coefficient is reduced like power –0.15 of the mean pressure.

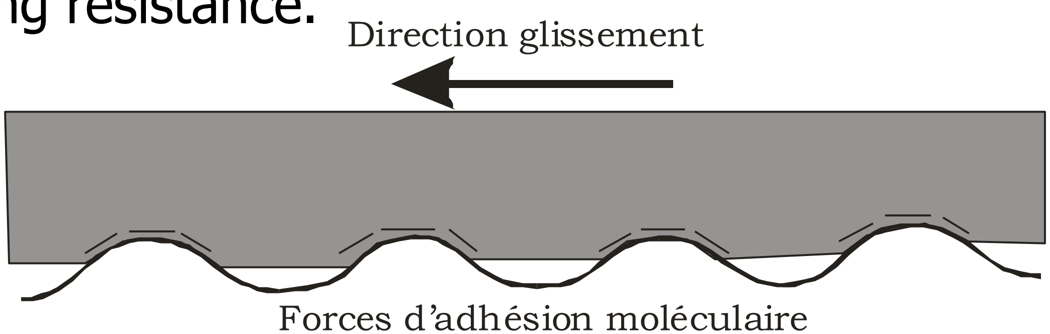






Tyre force generation mechanisms

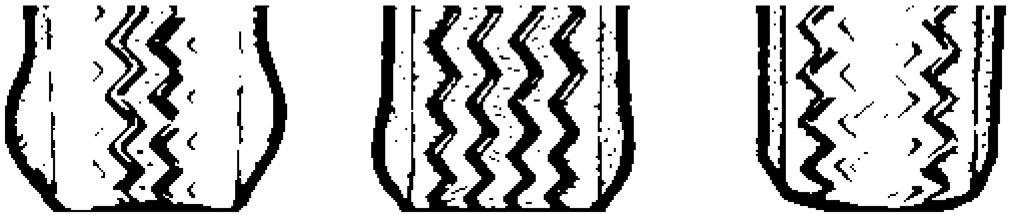
- **Adhesion forces at the interface:**
  - Rubber is able to build intermolecular bondings (Van der Waals forces) between rubber and road surface molecules
  - The phenomenon is mainly present on dry road surface and it is greatly reduced on wet roads because of the polar property of water molecules. It comes that adherence forces drop in rain conditions and they are cut by a factor two.
  - Energy liberated by adhesion bonding is not important but breaking them at the end of the contact patch also require some additional energy input that increases a little bit the rolling resistance.

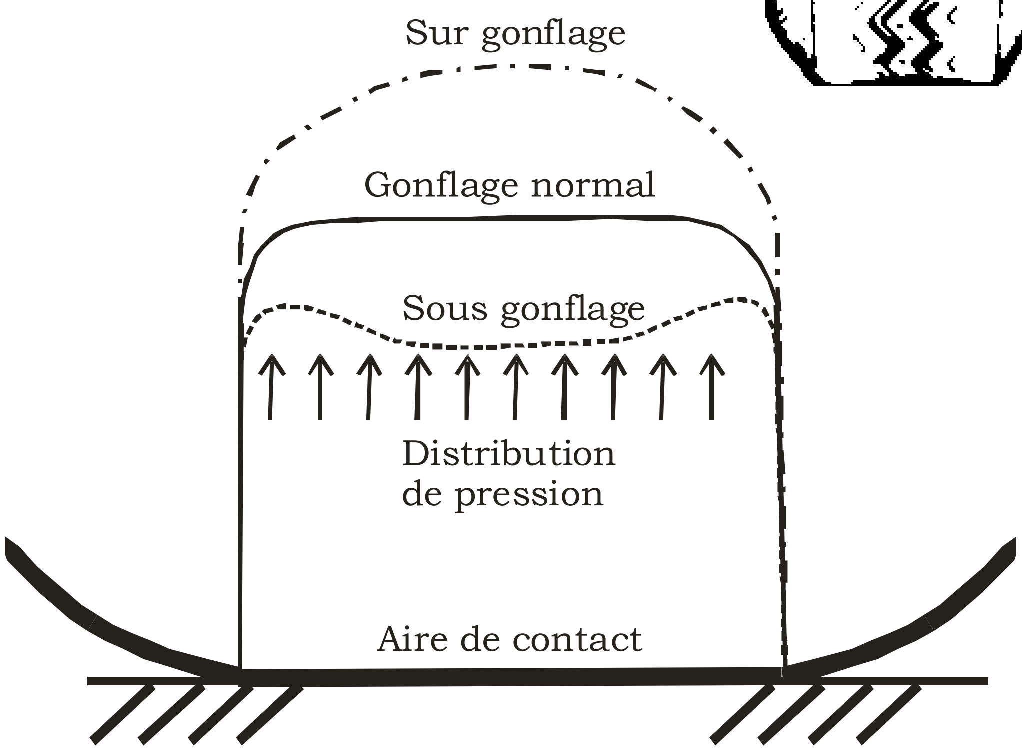


Tyre force generation mechanisms

- **Sliding dry friction:**
  - When local dry friction coefficient is exceeded, rubber experiences slippage and dry friction occurs in a limited part of the contact patch.
  - This mechanism is not massively present and does not provide the main contribution to the force generation.
  - The two first mechanisms are function of a moderate relative slippage at the tire ground interface.

Distribution of contact pressure





- Adherence generation mechanism is sensitive to the contact pressure, especially to a uniform contact pressure distribution.
- A proper inflation of the tire allows to produce a uniform pressure distribution all over the contact patch.

Tire rolling resistance

Rolling resistance forces

- Under free rolling conditions, it is necessary to apply a torque to maintain the motion and counteract the rolling resistance moment.

- The rolling resistance covers a large number of phenomena of different natures:
  - The energy dissipation in the tire due to the hysteresis of the material due to alternate motion in the sidewalls and in the tread blocks
  - Air drag inside and outside the tire
  - The scrubbing of the tire on the ground
  - The friction in the driveline
  - The dissipation of energy in the shock absorber
  - The misalignment of the tires, the longitudinal and lateral slip
  - The deformation of the road surface

Rolling resistance forces

- Experiments show that generally, the global rolling resistance force F_{RR} can be, with a very good agreement, modelled by a linear function of the vertical force F_z applied onto the tire

$$F_{RR} = f_{RR} F_z = f_{RR} mg \cos\theta$$

The coefficient f_{RR} is the rolling resistance coefficient

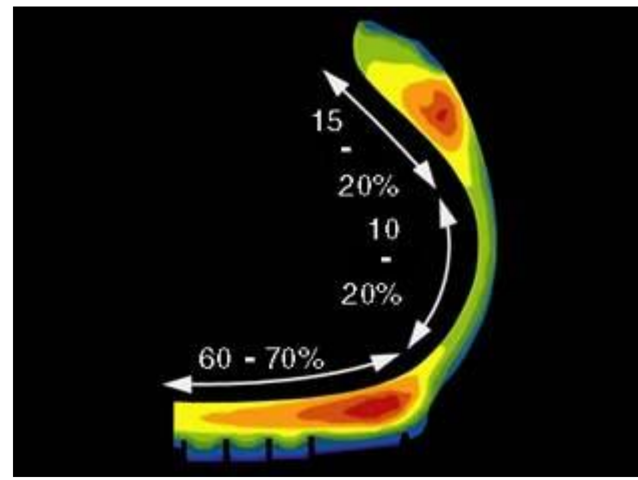
- **The rolling resistance coefficient** f_{RR}, ratio between the rolling resistance force and the normal force encompasses the complicated and interdependent physical properties of the tire and the ground.

Rolling resistance forces

- 1st cause: hysteresis of the tire materials (viscoelastic rubber) because of cyclic deformation loading

- Other sources:
  - Frictions during slippage
  - Air ventilation inside and outside

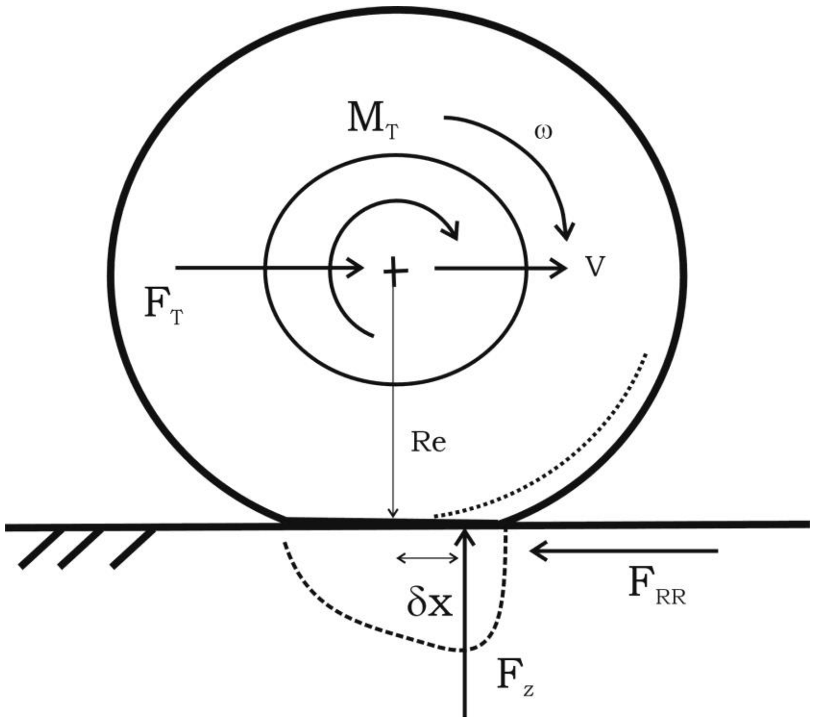
- Example: truck tire at 130 km/h
  - 90-95 % = hysteresis
  - 2-10 % friction
  - 1.5 – 3.5 % aerodynamic dissipation



Energy dissipation in the rolling tyre

Rolling resistance forces

- The resulting contact force is located in front of the theoretical contact point.
- The pressure distribution gives rise to a rolling resistance moment that is statically equivalent to a resistance force in the contact patch



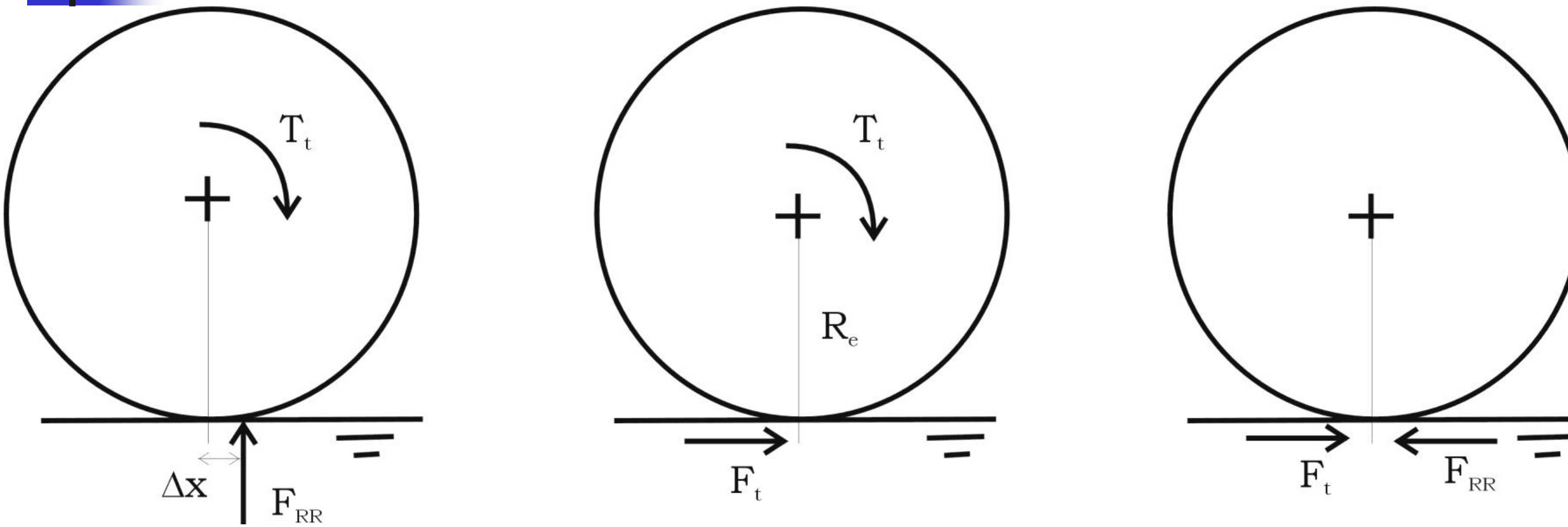
$$M_t = F_z \Delta x + M_{fr} \simeq F_z \Delta x$$

$$M_t = F_t\, R_e \;=\; F_{RR}\, R_e$$

$$F_{RR} = \frac{F_z \Delta x + M_{fr}}{R_e} \simeq \frac{F_z \Delta x}{R_e}$$

$$F_{RR} \simeq \frac{\Delta x}{R_e} F_z = f_{RR}\, F_z$$

Rolling resistance forces

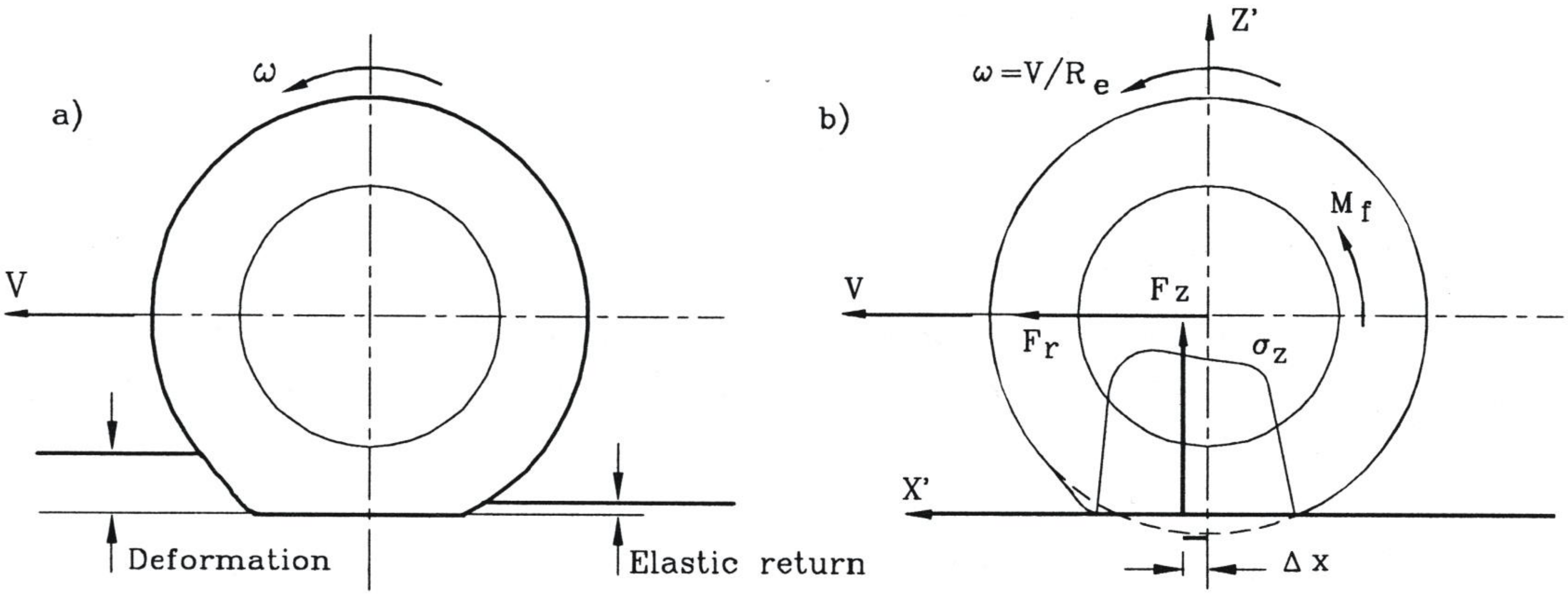


$$J\dot{\omega} = T_t - F_z\,\Delta x = 0 \qquad J\dot{\omega} = T_t - F_t\,R_e = 0 \qquad m\dot{V} = F_t - F_{RR} = 0$$

$$\begin{aligned} T_t &= F_t\,R_e \\ T_t &= F_{RR}\,R_e \\ F_z\,\Delta x &= = F_{RR}\,R_e \end{aligned} \quad \Longleftrightarrow \quad F_{RR} = \frac{\Delta x}{R_e}\,F_z = f_{RR}\,F_z$$

Rolling resistance forces

- Rolling resistance is higher on **soft grounds** because of the additional deformation work of the soil



Genta Fig 2.7 : Origin of the rolling resistance

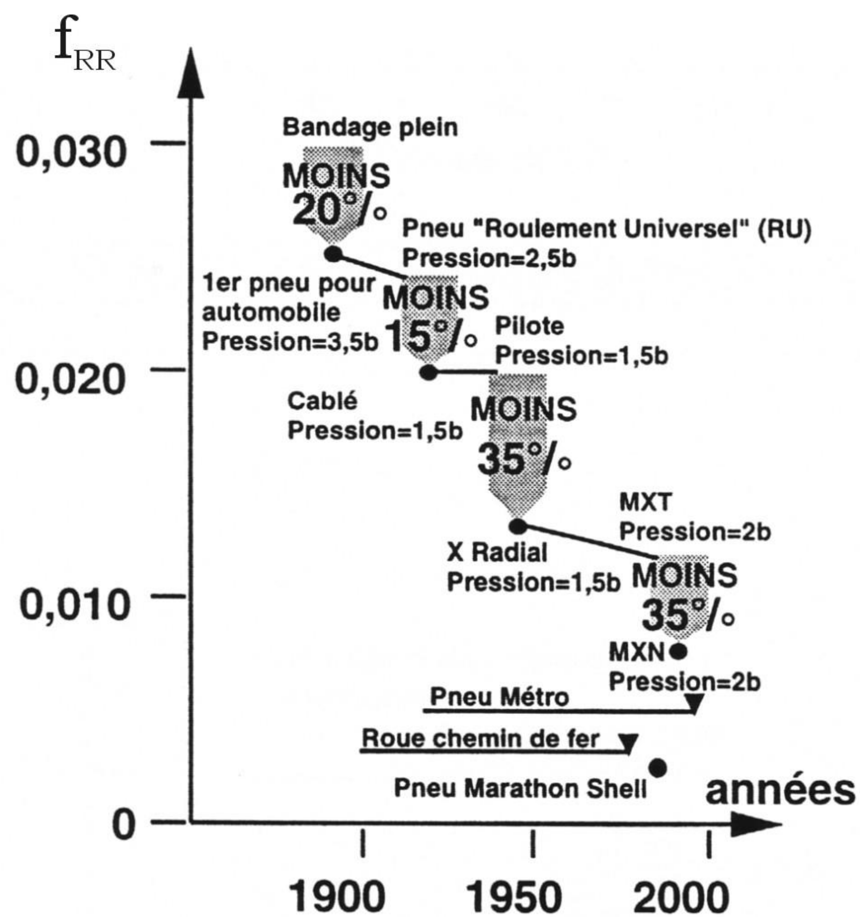
Rolling resistance forces

- Hysteresis generates heat that :
  - Increases the tyre temperature and its internal pressure and so reduces the tyre deformation in the contact patch
  - Increases wear of the tyre
  - Reduces the bending fatigue life time

- The rolling resistance is influenced by the **tyre structure**:
  - The rolling resistance of bias tires is higher than radial tires
  - Tires with low max speed has also a lower rolling resistance at low velocities

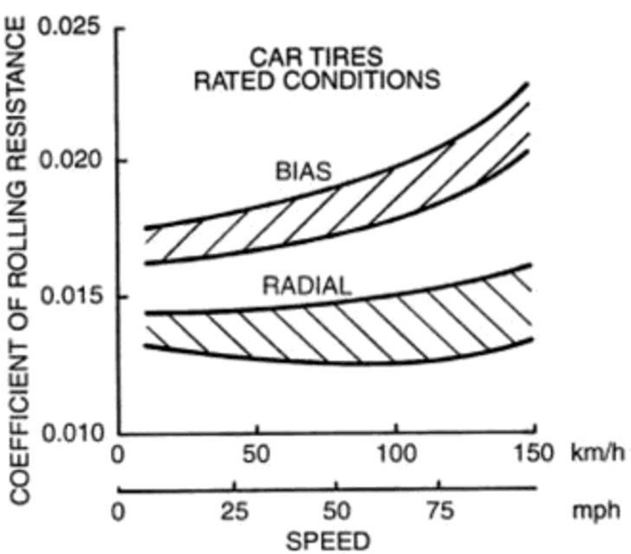
Influence of tire construction

- Tyre construction has been improved along the years
- Following several technological evolutions (inflated tire, radial structure, silicate compounds…)
- For the benefit of safety and energy consumption

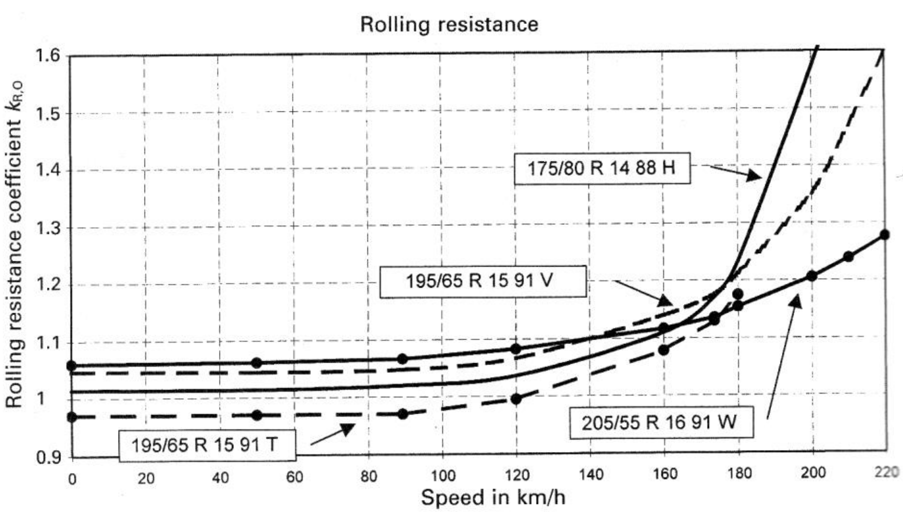


Influence of tire construction

- Radial tires have a lower rolling resistance
  - Bias ply tires ~ 0,015 to 0,020
  - Radial tires ~ 0,010 to 0,013
- Lower speed index tires have a lower RR at low speed but experience a sharp RR increase after critical speed



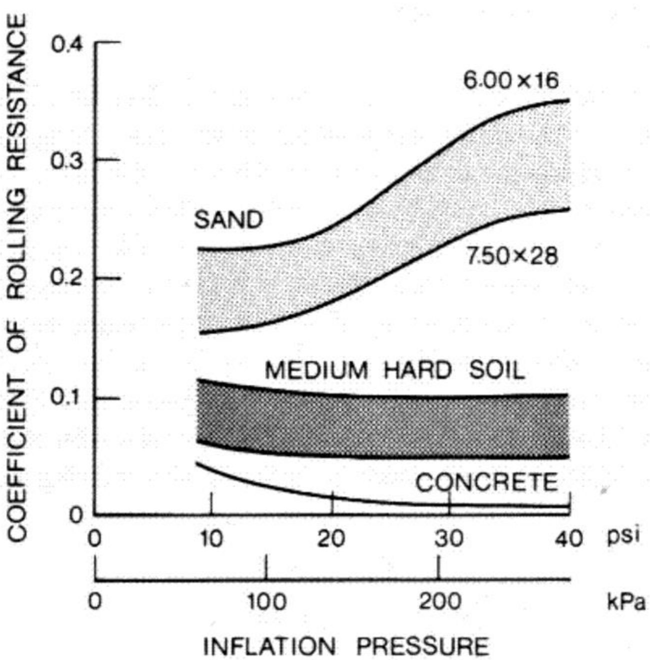
Wong Fig 1.3



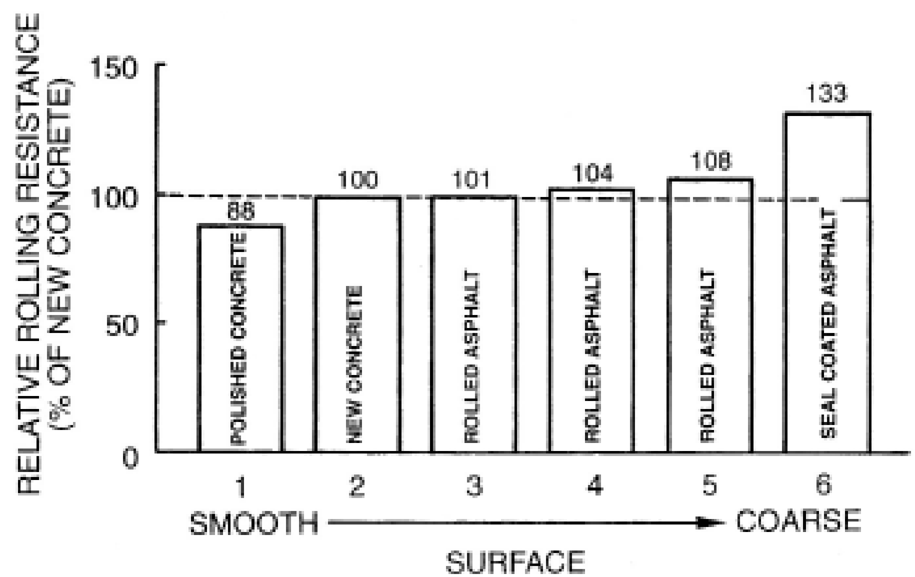
Reimpel et al. Fig. 2.31

Influence of ground nature

- Increase of RR on deformable ground
- Road roughness and degradation increase the RR



Wong Fig 1.8

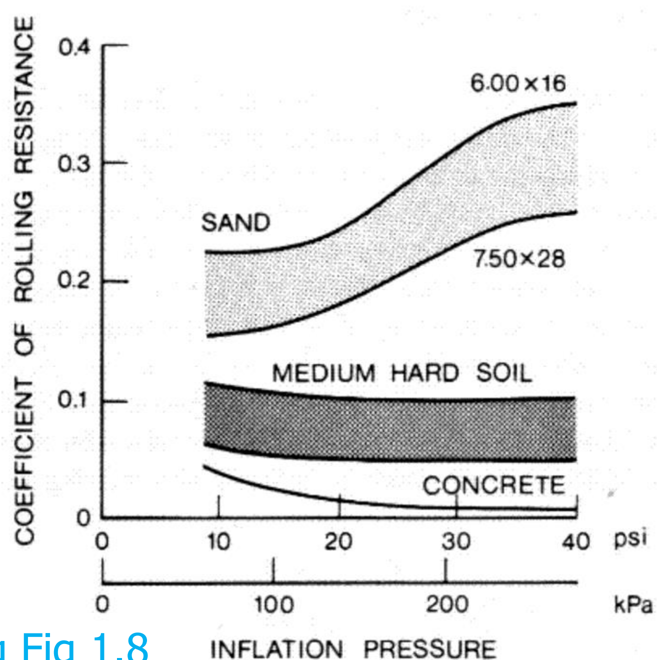


Rolling resistance forces

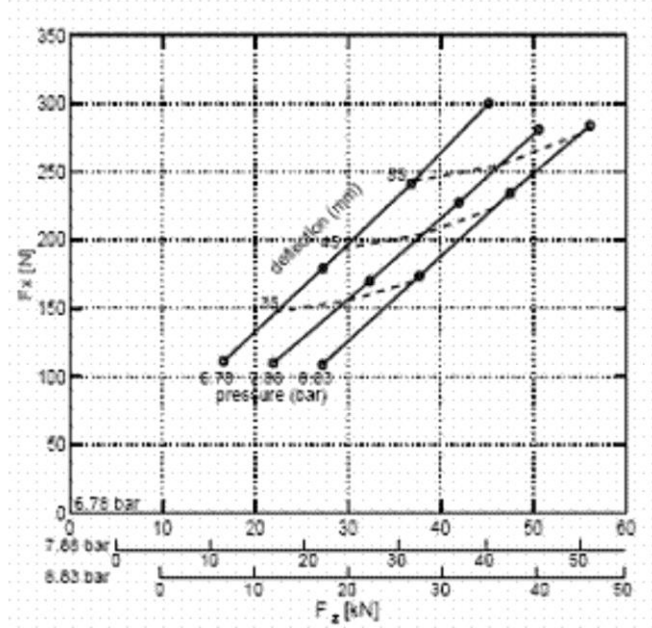
- The **operating conditions** mainly:
  - The inflating pressure: the rolling resistance is reduced for a higher inflation pressure
  - Vertical load also increases rolling resistance
  - Temperature elevation reduces the energy dissipation and reduces the rolling resistance
  - The vehicle speed: one observes a slight increase of F_{RR} with v at low speed. A sharp and dramatic increase of F_{RR} after a critical speed because of the development of high-energy standing waves
  - The longitudinal and lateral slip: the rolling resistance increases as the square of the side slip.

Influence of inflation pressure and vertical load

- Influence of inflation pression is positive on RR reduction for hard soils and negative for soft grounds



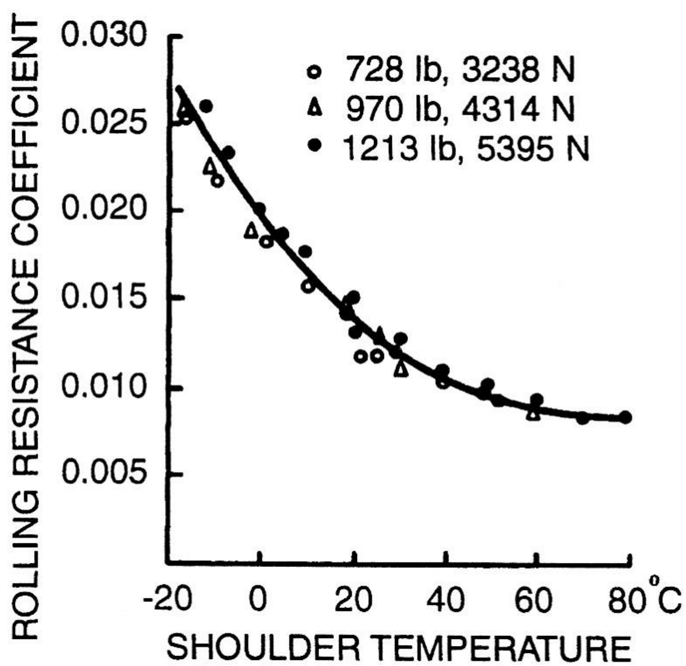
Wong Fig 1.8



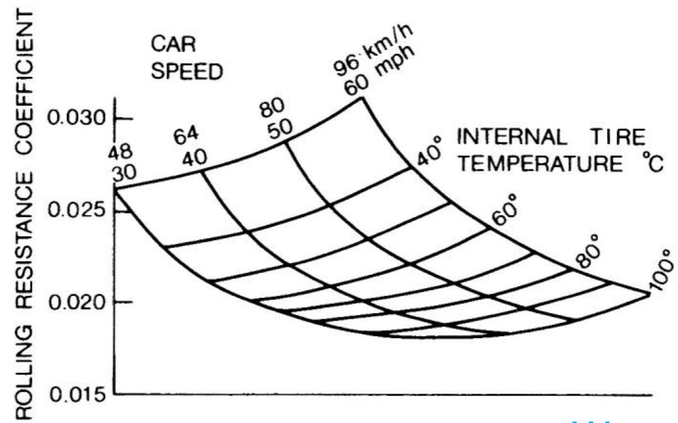
- RR increases nearly linearly with respect to the vertical load increase

Influence of temperature

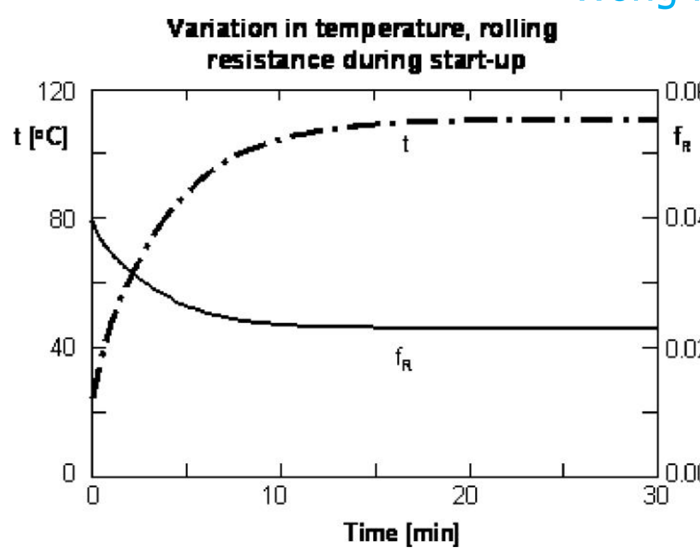
- Shoulder temperature decreases the RR



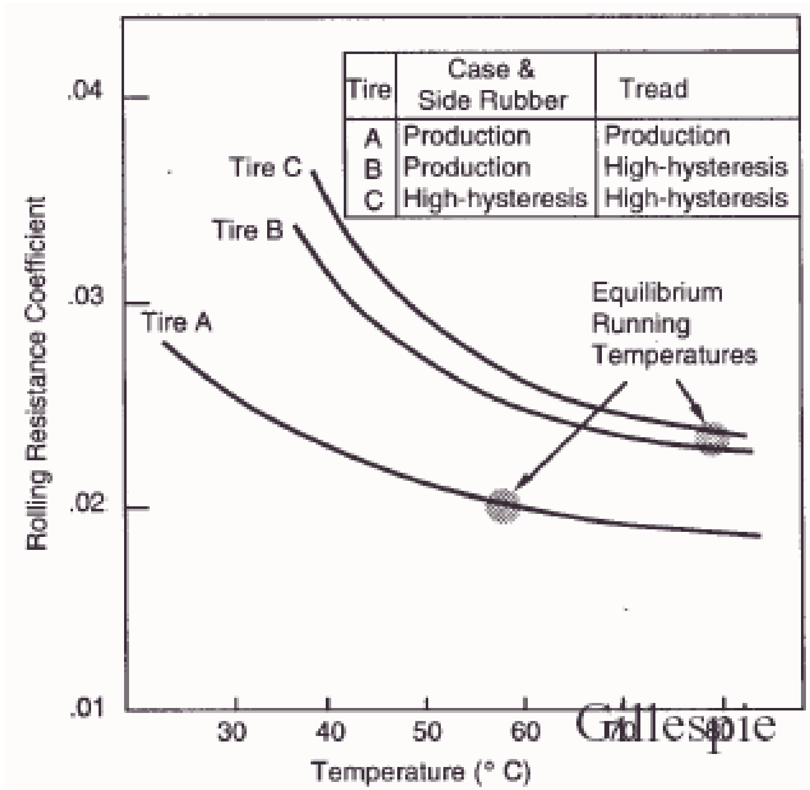
Wong Fig 1.12



Wong Fig 1.11



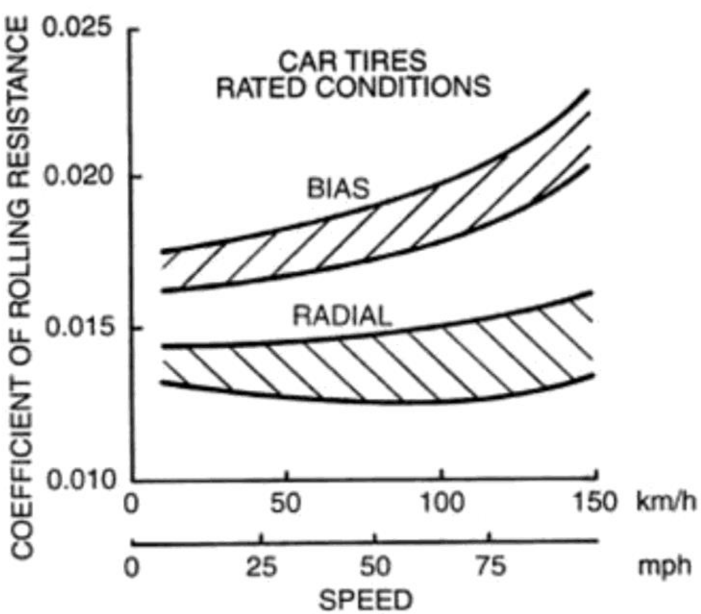
Influence of temperature and rubber quality



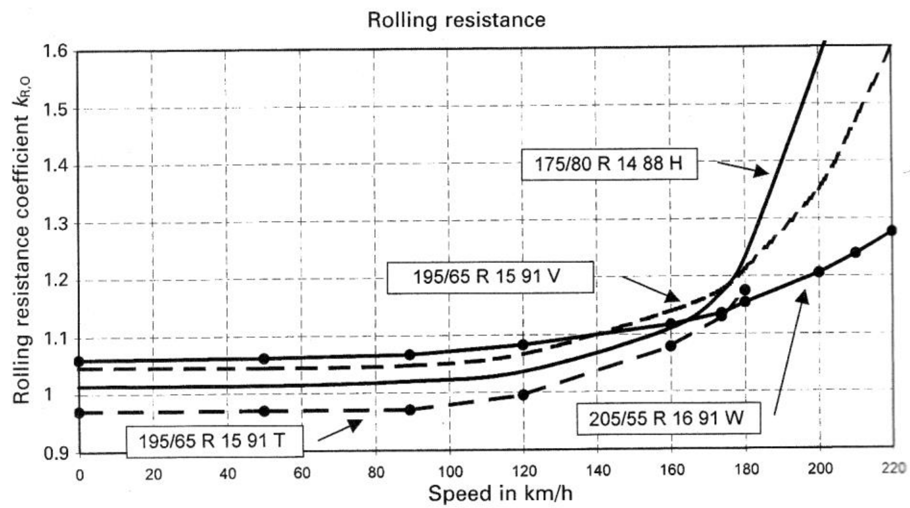
Gillespie Fig. 4.32 : Influence of the rubber quality, dissipation rate and working temperature

Influence of velocity

- Velocity influence



Wong Fig 1.3

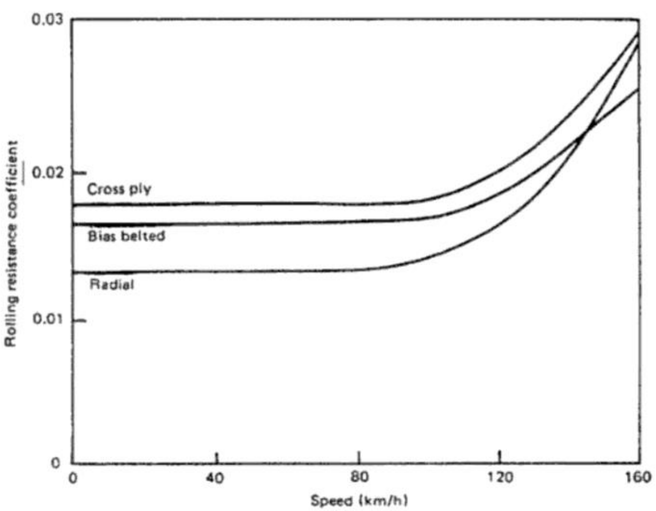


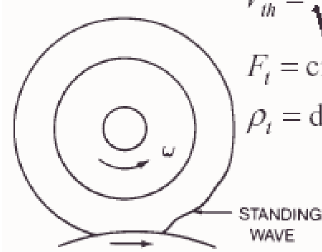
Reimpel et al. Fig. 2.31

- Sharp increase of RR after 80-100 km/h
- Increase like power 4 of velocity

Rolling Resistance forces

- At high speed, after a certain critical speed that depends on the prestress tension and the tread mass density, one observes stationary vibration waves developing in the tread.
- These ones lead to a large energy dissipation and a rapid growth of the rolling resistance.
- The tread wear also increases abruptly



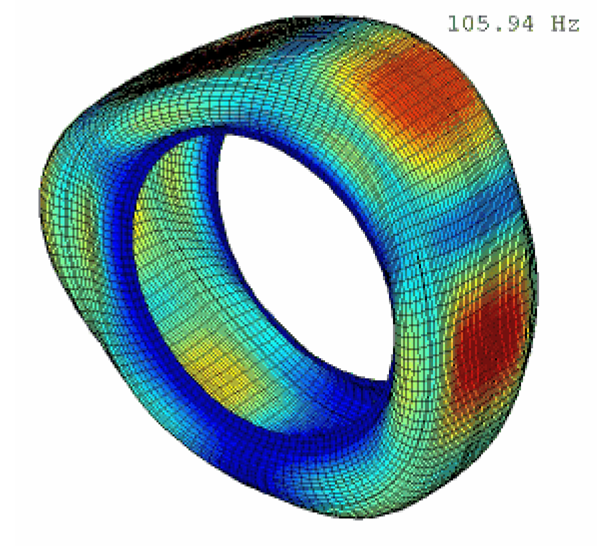


$$V_{th} = \sqrt{F_t / \rho_t}$$

F_t = circumfrential tension

ρ_t = density per unit area

Wong Fig 1.10



Influence of speed

- Shape of RR coefficient vs speed

$$f_{RR} = f_0 + \sum_{k=1}^{N} f_k V^k$$

- Best fit (V in km/h)

$$f_{RR} = f_0 + f_1 \frac{V}{100} + f_4 \left(\frac{V}{100}\right)^4$$

- Rolling resistance approximation recommended by SAE for road resistance expression

$$f_{RR} \simeq f_0 + f_2 V^2$$

Influence of speed

- Wong's expression of rolling resistance coefficient
  - For passenger car radial tires

$$f_{RR} = 0.0136 \; + \; 0.40\, 10^{-7}\, V^2$$

  - For passenger car diagonal tires

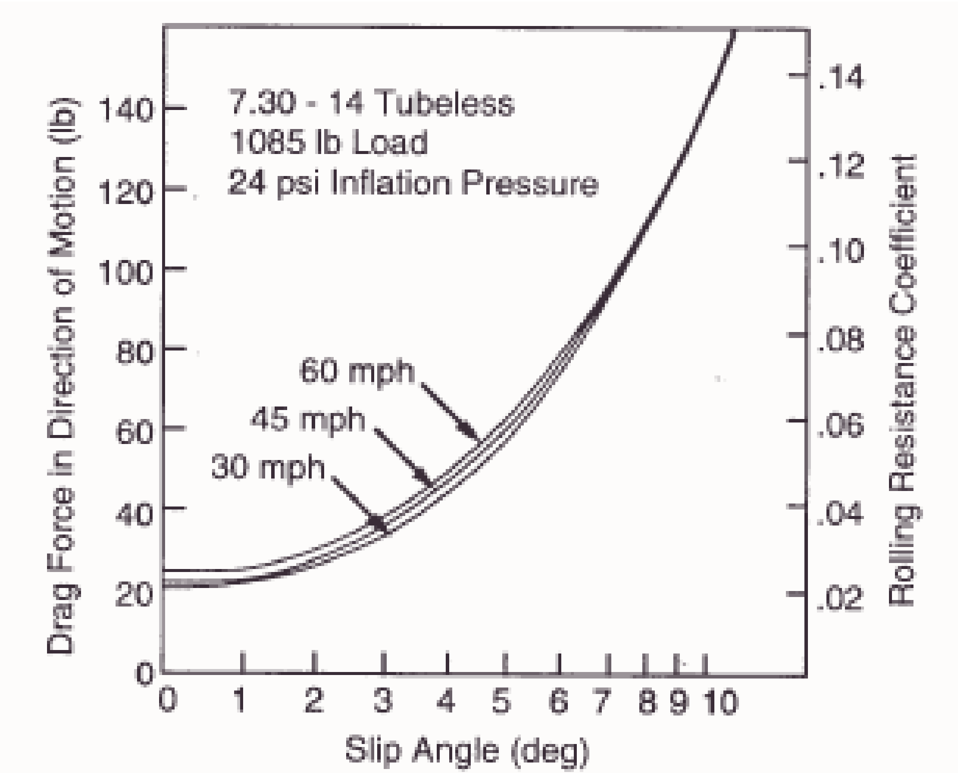
$$f_{RR} = 0.0169 \; + \; 0.19\, 10^{-6}\, V^2$$

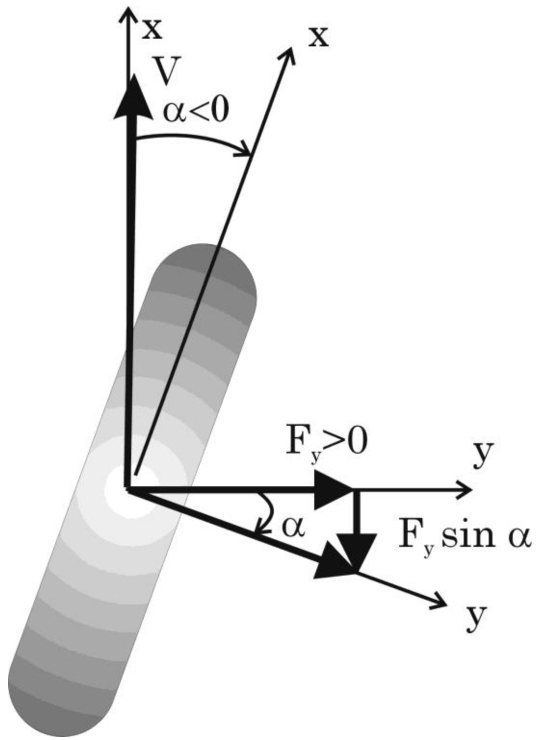
V in km/h

  - For truck radial tires

$$f_{RR} = 0.006 \; + \; 0.23\, 10^{-6}\, V^2$$

Influence of lateral forces





Gillespie Fig. 4.33 : Influence of side slip angle and cornering forces

Influence of lateral forces

- For a given side slip angle

$$F_y \simeq C_\alpha \, \alpha$$

- Component in longitudinal direction of vehicle → resistance

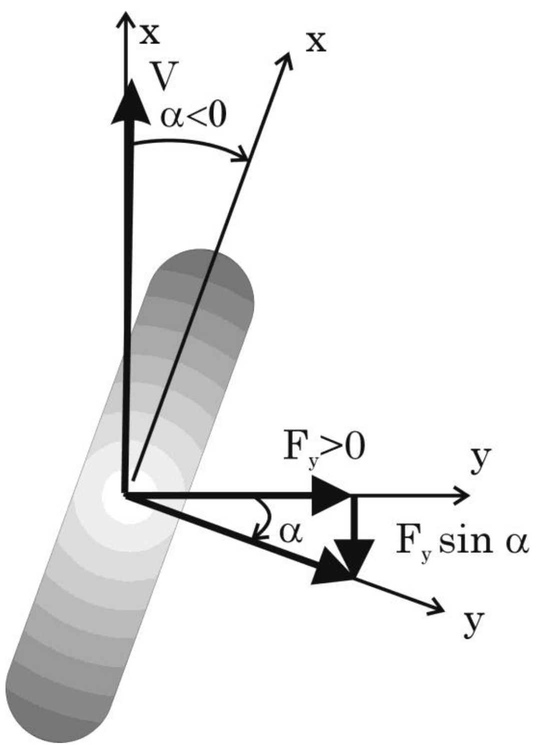
$$F_{RES} = F_y \, \sin\alpha \simeq C_\alpha \, \alpha^2$$

- Influence of 1° of side slip

$$CC_\alpha \simeq 10 \text{ N/N/rad} \qquad 1^\circ = 1.7452\,10^{-2} \text{ rad}$$

$$F_{RES} = 10.0\,(1.7452\,10^{-2})^2\,F_z = 0.003\,F_z$$

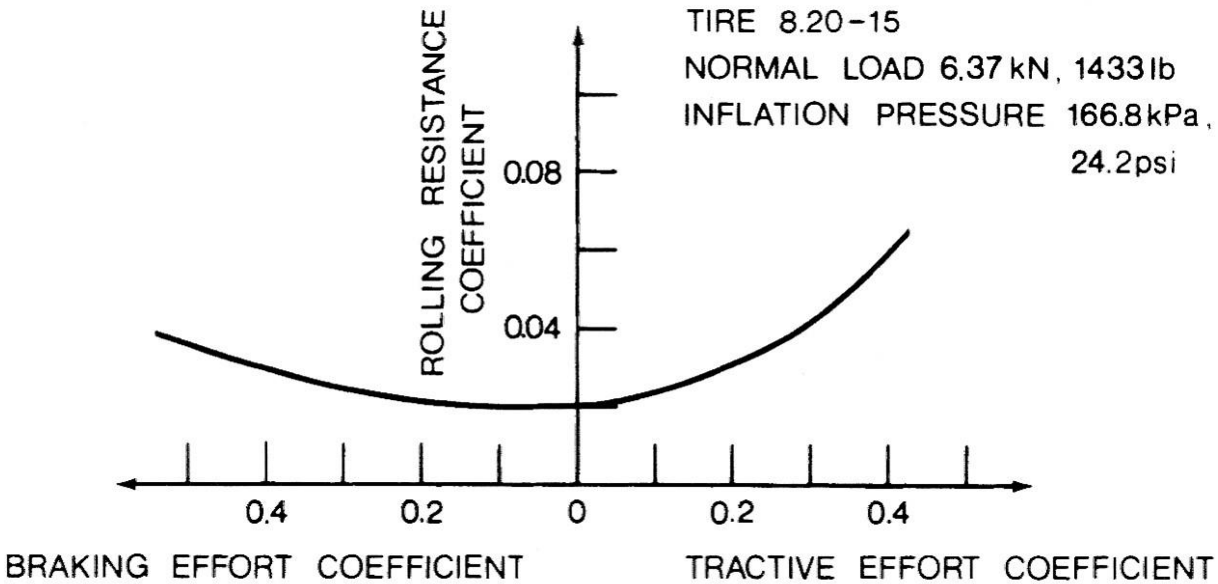
$$F_{RR} = f_{RR}\,F_z \sim 0.010\,F_z$$



$$F_{RES}(\alpha = 1^\circ) = 0.003 F_Z$$

Influence of longitudinal forces

- Longitudinal forces increase rolling resistance



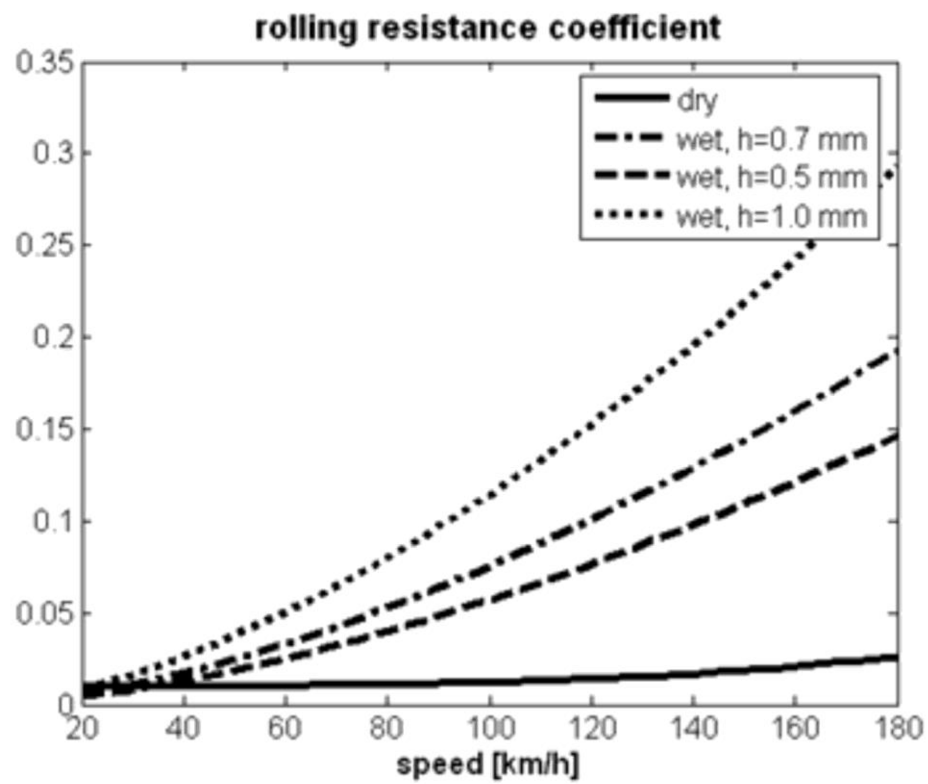
Wong Fig 1.14

Influence of presence of water

- RR drastically increases on wet and snow roads

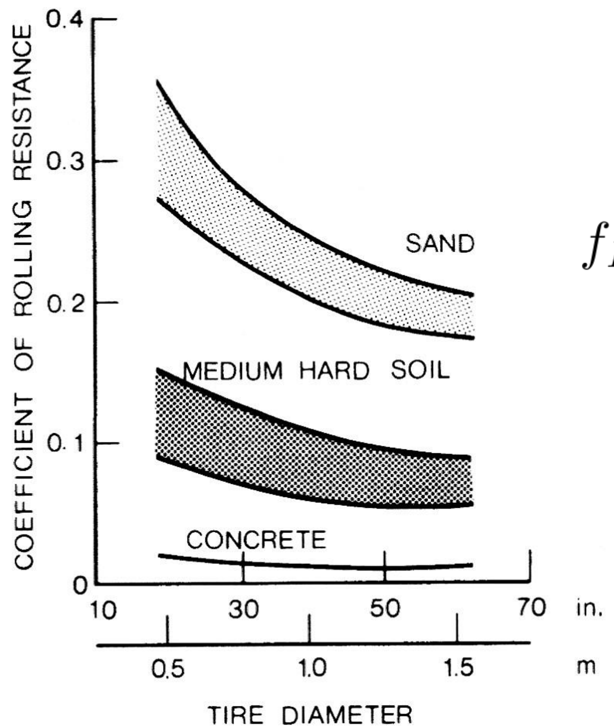
$$F_{Water} \simeq A\, w\, V^n \quad n > 1$$

  - n~1.6 if h< 0.5 mm
  - n~2.2 if h> 2.0 mm
  - w tire width
  - V velocity
  - A coefficient



Influence of tyre size on RR

- RR is reduced when increasing the tyre size (radius) R_u and reducing its aspect ratio h_t/w_t.



$$f_{RR} = C\,\frac{F_z}{2\,R_u}\sqrt{\frac{h_t}{w_t}}$$

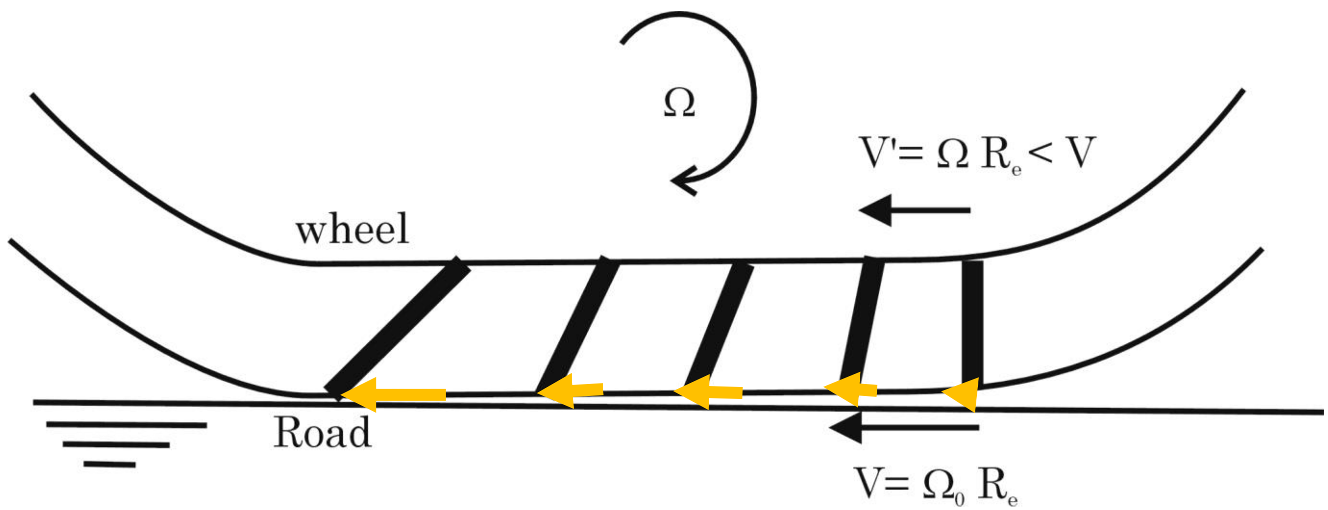
Wong Fig 1.13

Longitudinal forces

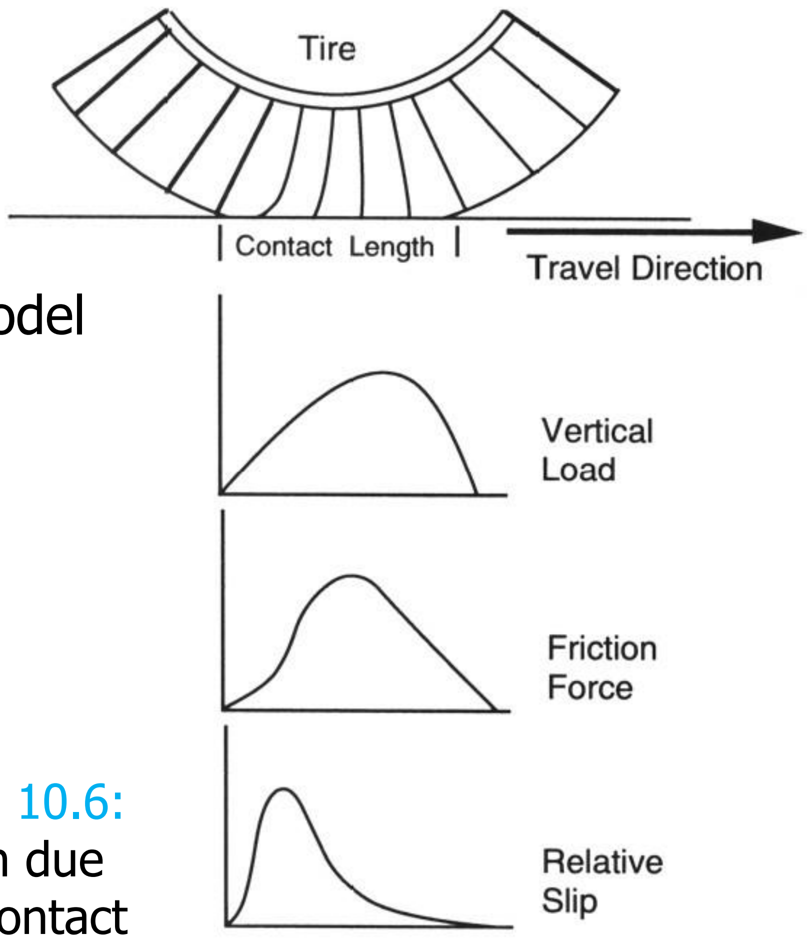
Origine of longitudinal forces

- Longitudinal forces are much beyond what can be expected from the pure dry friction of the rubber
- Large road adhesion coefficients are due to the tire elastic deformation of the tire tread and side wall

  ➔ Brush model



Longitudinal force generation in tires



"Brush" model

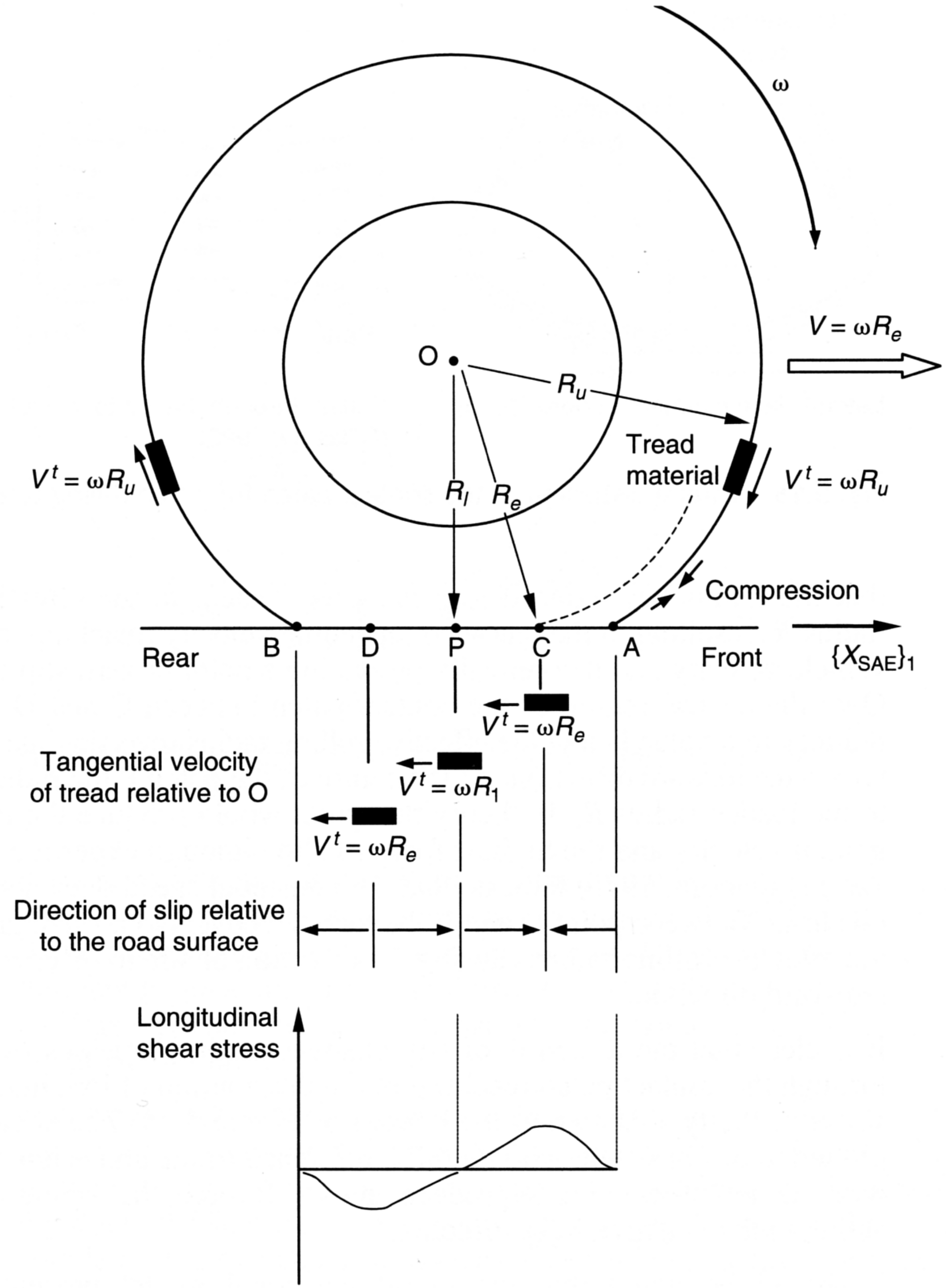
Gillespie Fig 10.6: Deformation due braking in contact patch

Gillespie Fig. 10.6

- When braking, the tire rotation speed is lower than the equivalent ground velocity in the contact patch
- Tread and side walls radial fibers are in shear because of the local friction between road and tire
- Slip occurs only at the end of the contact patch
- The resultant of the shear forces in the contact patch gives rise to the longitudinal force developed by the tire

Free wheel rolling

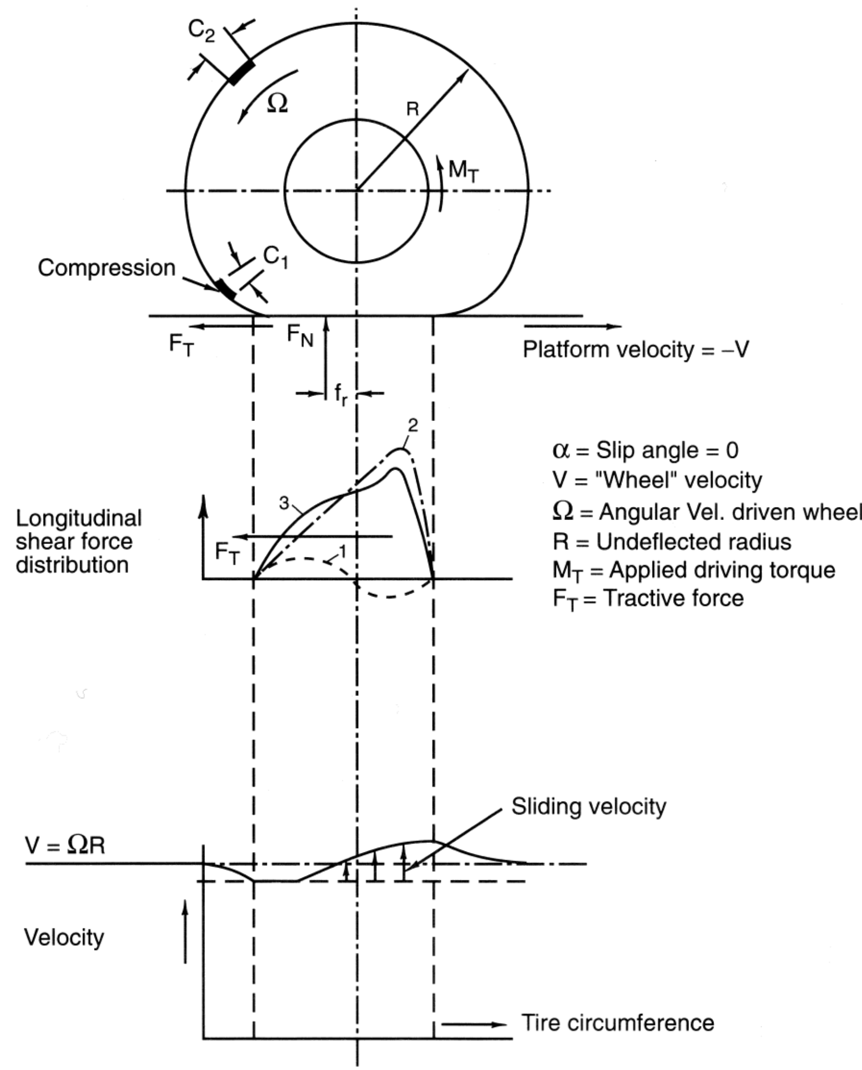
- If the tyre is freely rolling, one observes the development of shear stresses due to the enforced tire radius reduction when tread elements are pushed into the contact patch
- For a given rolling rotation speed Ω_0, the tangential speed is dropping to a minimum before increasing again
- This leads to a shear stress distribution that is alternated



Tractive force

Longitudinal force

- Shear stress distribution (3) in the contact patch is the superposition of two effects:
  - The <u>alternated shear stress</u> distribution due to the radius reduction as in free rolling conditions (1)
  - A <u>triangular shape distribution</u> related to the brush model with increasing shear from the beginning to the end of the contact zone (2)
- The shear stress (exerted by the road onto the tire rubber) is acting frontward and tread elements are bended forward (2)
- Local shear loads are linearly increasing



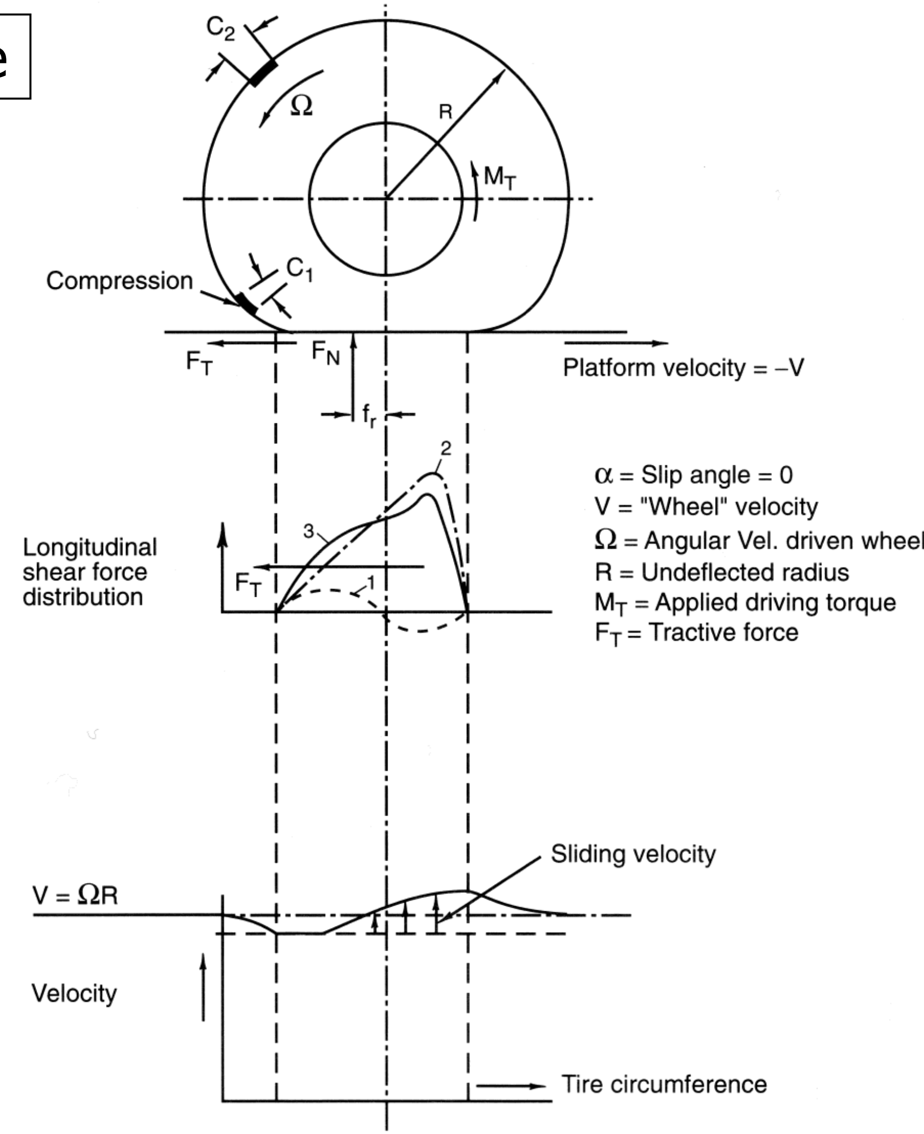
Distribution of forces and sliding velocity, over the contact length of a tire under the action of a driving torque M_T.

Milliken. Fig. 2.14

Tractive force

Longitudinal force

- Tractive torque ➔ tractive force deforming the contact patch frontward, setting the tread elements in compression state in front of the contact patch
- The shear force drops at the end of the contact patch because the contact pressure is vanishing and shear forces are locally exceeding the local dry friction capability. There is a local slippage between the tire and the road in the rear part of the contact patch.



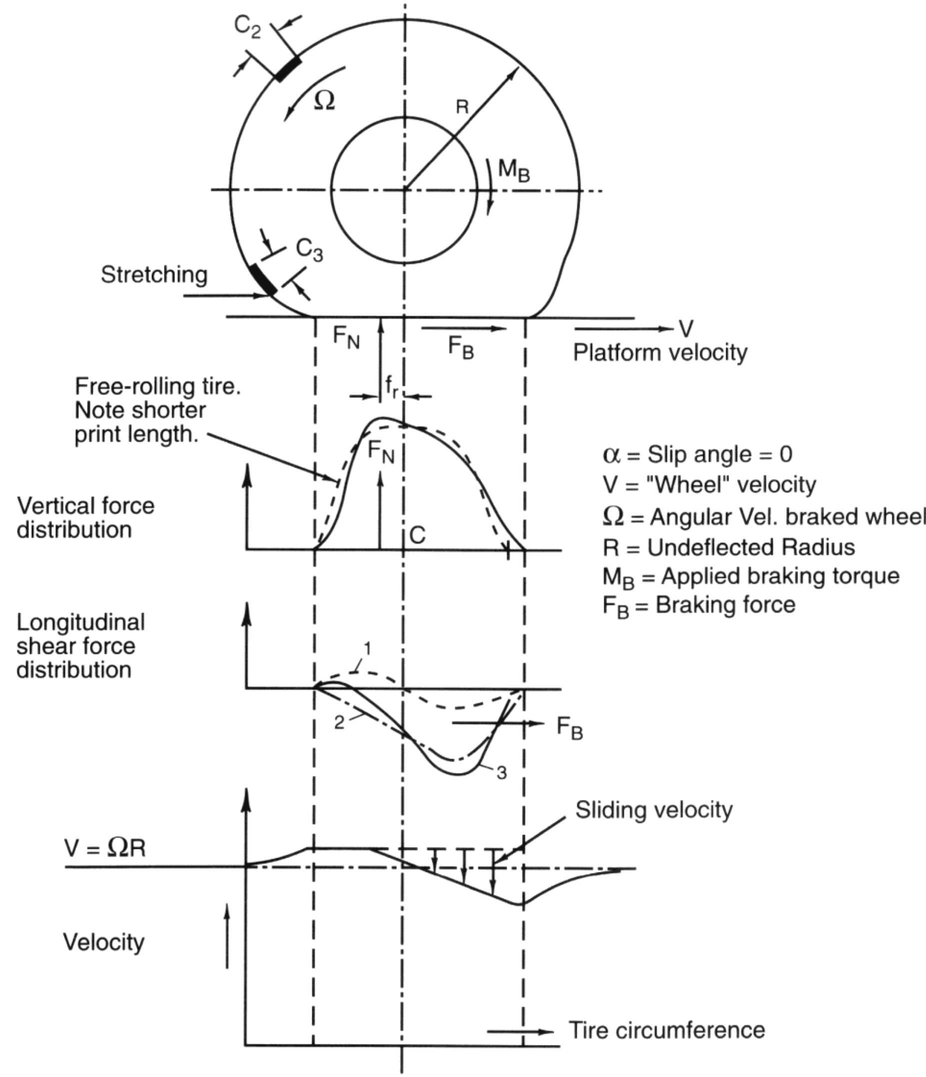
Distribution of forces and sliding velocity, over the contact length of a tire under the action of a driving torque M_T.

Milliken. Fig. 2.14

Braking force

Longitudinal force

- A braking torque yields a net force pointing to the back of the tire. Contact patch is forced to deform rearward. Tread elements are compressed at the rear of the contact patch while front ones are stretched.
- For moderate braking, the elements in contact with the road are stretched.
- The shear stresses are linearly growing
- When the pressure is fading in the rear of the patch, there is a limited local slip between tire elements and road surface.



Distribution of forces and sliding velocity over the contact length of a tire under the action of a braking torque M_B.

Milliken. Fig. 2.15

Longitudinal force generation in tires

- Longitudinal slip (Slip ratio)

$$SR = \frac{\Omega - \Omega_0}{\Omega_0} = \frac{\Omega}{\Omega_0} - 1$$

If R_e is the effective rolling radius of the tire

$$\Omega_0 = \frac{V}{R_e}$$

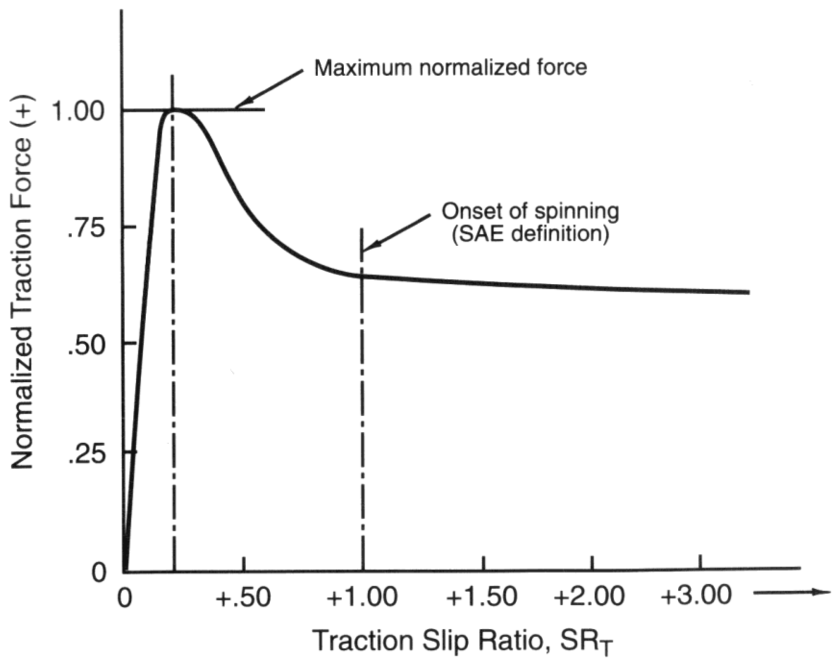
$$SR = \frac{\Omega\, R_e}{V} - 1$$

$$V = \Omega_0\, R_e$$

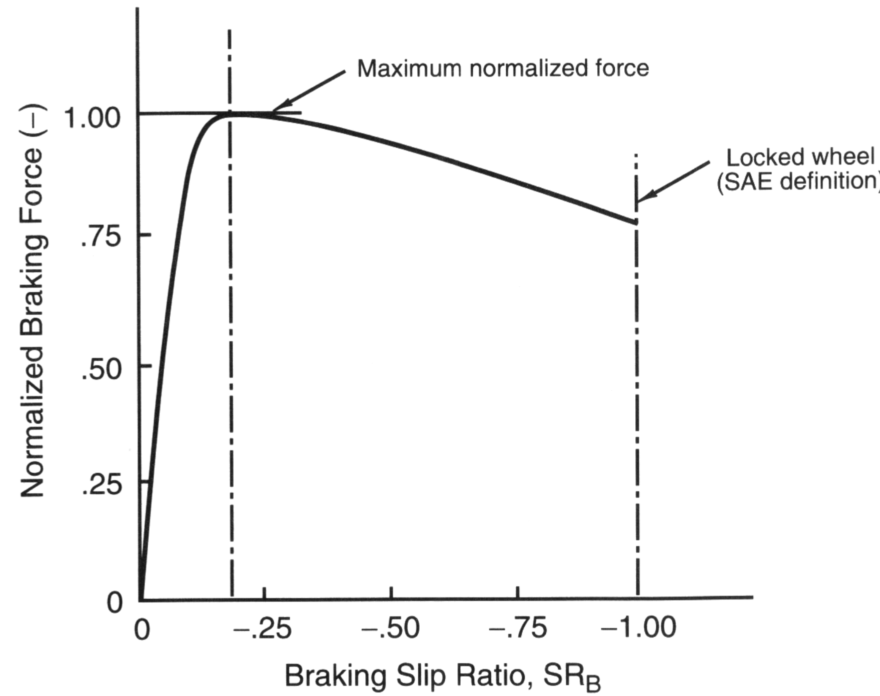
  - Free rolling ($\Omega = \Omega_0$): SR = 0
  - Wheel blocked in braking ($\Omega = 0$): SR = -1
  - Spinning ($\Omega = 2\,\Omega_0$): SR = +1
  - Free slipping ($\Omega_0 \rightarrow 0$): : SR $\rightarrow \infty$

- The tractive/braking forces can be plotted in terms of the slip ratio

Longitudinal force



Milliken. Fig. 2.16



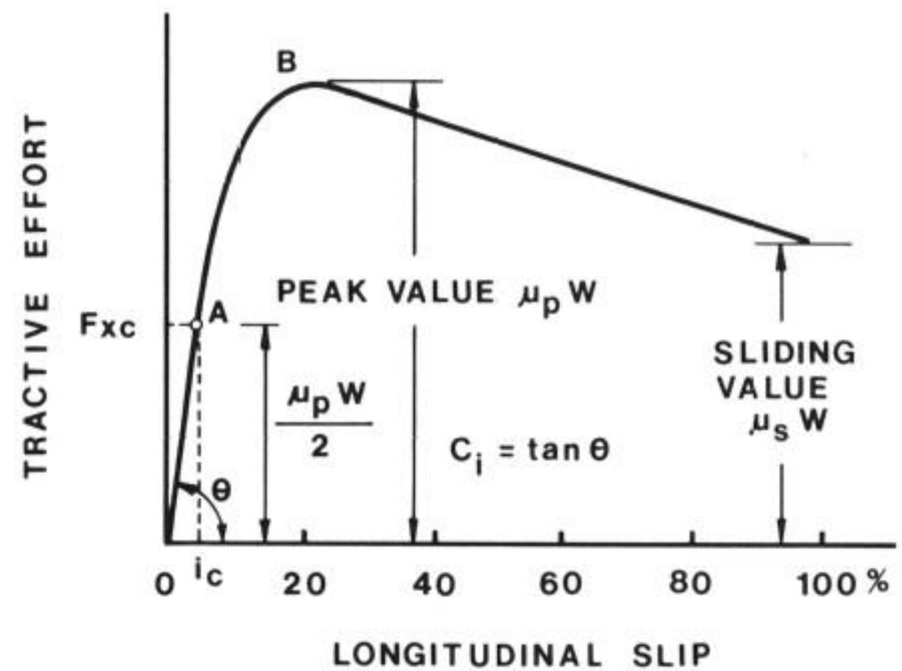
Milliken. Fig. 2.17

Passenger tire with bias ply carcass

Longitudinal force

- Peak value μ_p: maximum tractive force arises for a moderate slip ratio
- Sliding value μ_s: tractive force for a tire totally blocked
- Longitudinal stiffness C_i: rate of tractive force per unit of side slip around zero side slip

$$\left.\frac{\partial F_x}{\partial SR}\right|_{SR=0} = C_s$$



Wong Fig 1.16

Coefficient of longitudinal friction

- One can define a coefficient of longitudinal friction:

$$\mu_x = \frac{F_x}{F_z}$$

- The first part of the curve can also be linearized (λ=SR<10%)

$$F_x \simeq \frac{\partial F_x}{\partial \lambda}\lambda = C_\lambda \, \lambda$$

- Peak value reached for moderate slip ratio λ= SR~15% to 20%

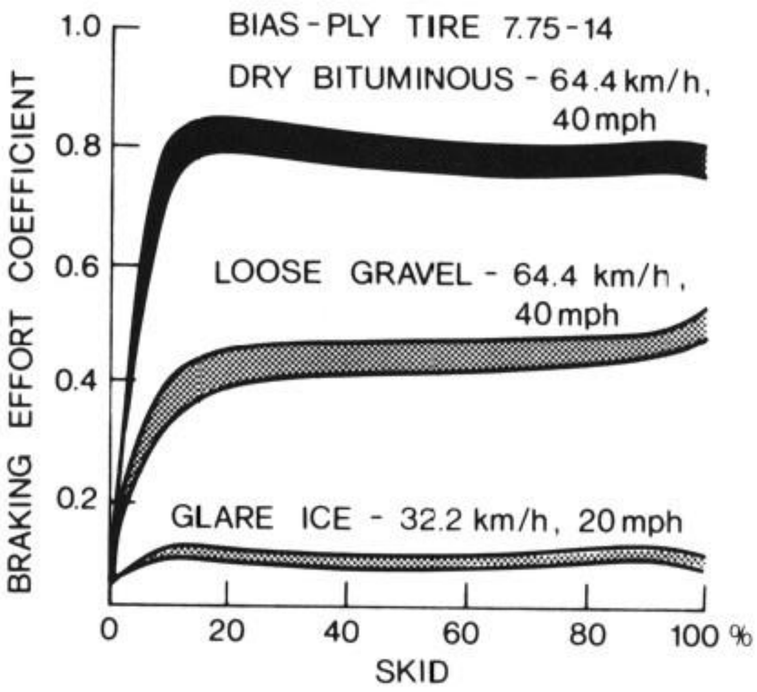
$$\max_\lambda F_x = \mu_P F_z$$

- For λ= SR=100%, the tyre is sliding as a whole

$$F_x(\lambda = 100\%) = \mu_s F_z$$

Longitudinal force

- Strong influence of the operating conditions:
  - Humidity
  - Nature of the road
  - Level of water
  - Ice, snow…

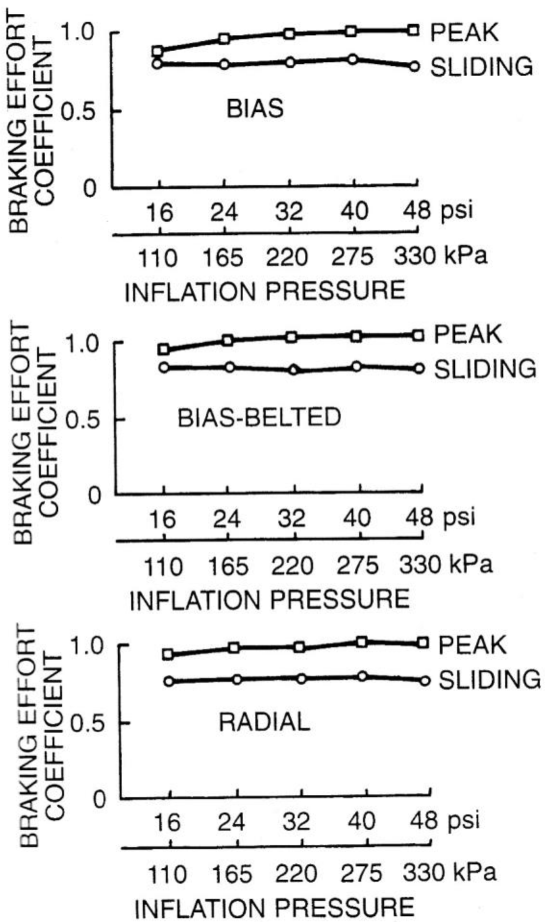


Wong Fig 1.18

Typical values of peak and sliding longitudinal coefficients

Surface	Peak Value μ_P	Sliding Value μ_S
Asphalt and concrete (dry)	0.8-0.9	0.75
Asphalt (wet)	0.5-0.7	0.45-0.6
Concrete (wet)	0.8	0.7
Gravel	0.6	0.55
Earth road (dry)	0.68	0.65
Earth road (wet)	0.55	0.4-0.5
Snow (hard-packed)	0.2	0.15
Ice	0.1	0.07

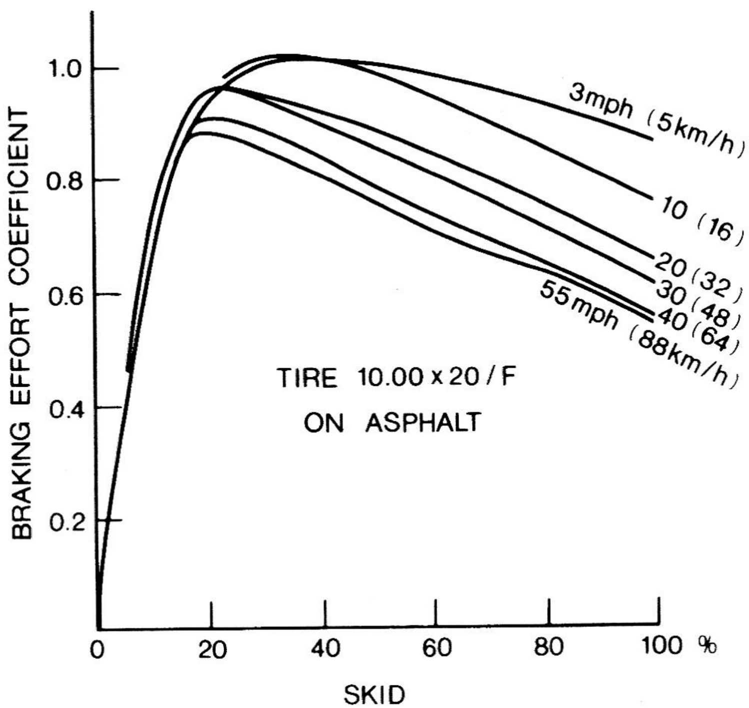
Influence of tyre construction and inflating pressure



Wong Fig 1.19

- Tyre construction has a moderate influence on the peak and sliding values of longitudinal friction coefficients
- Inflation pression has a weak influence on the peak and sliding values of the longitudinal friction coefficients

Influence of velocity and vertical load on the longitudinal friction coefficients

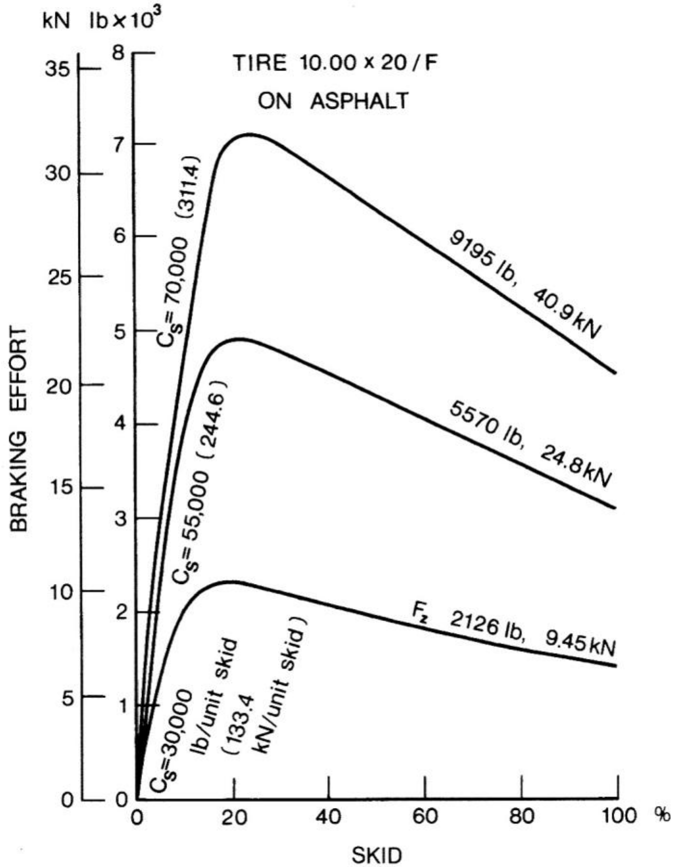


Wong Fig 1.20

- The effect of speed on braking / tractive force generation is quite big.
- Increasing speed reduces the peak value μ_P as well as the sliding value of the friction coefficient μ_S.
- The longitudinal stiffness coefficient C_s is not affected by the speed.

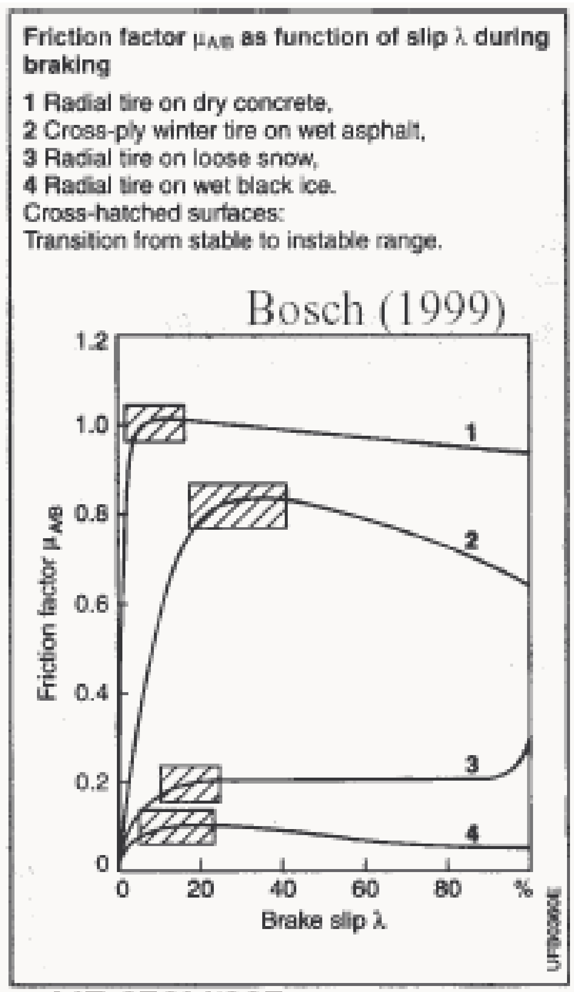
Influence of velocity and vertical load on longitudinal friction coefficients

- The effect of vertical load F_z on braking / tractive force generation is illustrated with a bias ply truck tire on dry asphalt.
- The longitudinal stiffness coefficient C_s increases noticeably with the increase of the vertical load F_z.
- For a given inflation pressure, larger vertical load leads to longer contact patches, which is favorable to develop more braking/tractive forces.



Wong Fig 1.21

Longitudinal force



Variation of longitudinal adhesion coefficient with operating conditions

Coefficient of friction μ_{HF} between tire and road surface

Vehicle speed km/h	Tire condition	Dry road surface μ_{HF}	Wet road surface (water-depth 0.2 mm) μ_{HF}	Heavy rain (water-depth 1 mm) μ_{HF}	Puddles (water-depth 2 mm) μ_{HF}	Surface ice (black ice) μ_{HF}
50	new	0.85	0.65	0.55	0.5	0.1 and lower
	worn out	1	0.5	0.4	0.25	
90	new	0.8	0.6	0.3	0.05	
	worn out	0.95	0.2	0.1	0.0	
130	new	0.75	0.55	0.2	0	
	worn out	0.9	0.2	0.1	0	

Definition of longitudinal sliding rate

- Definition by SAE (SAE J670):

$$S = (\Omega R_e / V \cos\alpha) - 1$$

  - R_e effective rolling radius in free rolling conditions (zero slip angle)

- Definition by Calspan TIRF (for testing machines):

$$SR = (\Omega R_l / V \cos\alpha) - 1$$

  - R_l loaded radius (measured between the belt and the ground)

Definition of longitudinal sliding rate

- Pacejka:
  - Practical slip quantity :

$$K_x = (\Omega R_e / V \cos\alpha) - 1$$

  - Independent slip quantity :

$$\sigma_x = (V \cos\alpha / \Omega R_e) - 1$$

- Sakaï (JSAE):
  - Tractive

$$S_t = (V \cos\alpha / \Omega R_e) - 1$$

  - Braking

$$S_b = (\Omega R_e / V \cos\alpha) - 1$$