MECA0525 : INTRODUCTION TO TIRE MECHANICS II:
Lateral Forces and Combined Operations

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Layout

- Introduction
- Tyre construction
- Classification: size, load and velocity indices
- Adhesion mechanisms
- Rolling resistance
- Generation of longitudinal forces
  - Brush model
  - Tractive and braking forces
  - Longitudinal slip ratio
  - Tractive and braking force curves

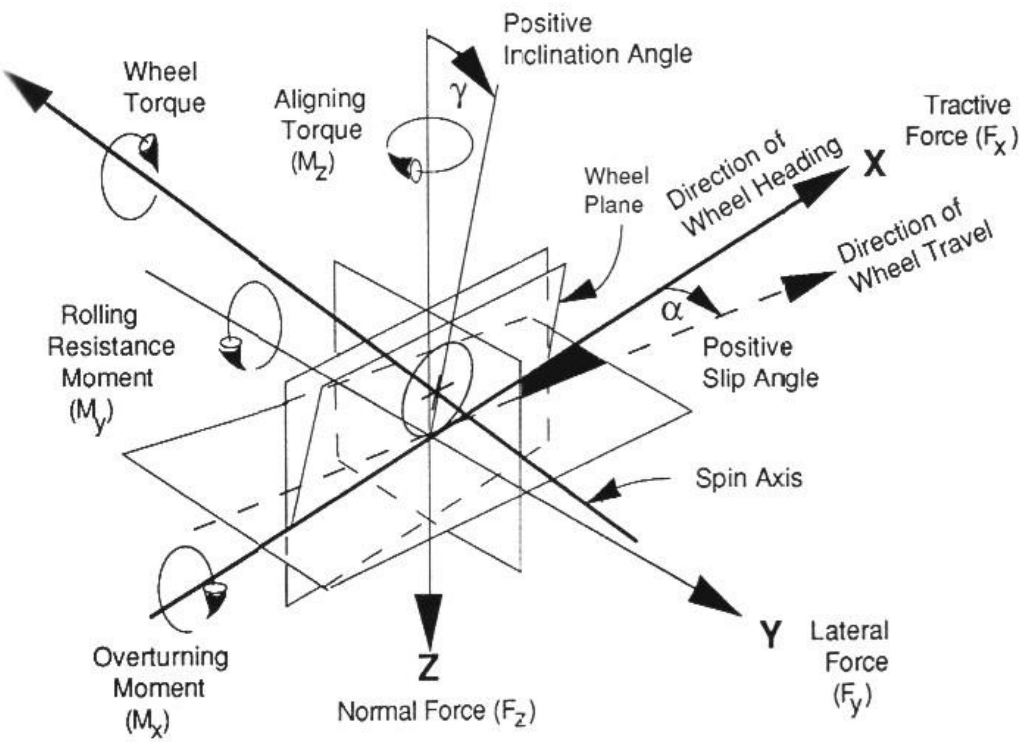
Layout (2)

- Lateral forces
  - Gough experiment
  - Lateral force as a function of the side slip angle
  - Cornering coefficient
  - Cornering stiffness
- Self aligning torque and pneumatic trail
- Camber thrust
  - Definition and mechanism
  - Camber coefficient
- Combined operations
  - Sakai experiment
  - Friction ellipse

- Modelling: Pacejka magic formula

AXES SYSTEM AND TERMINOLOGY

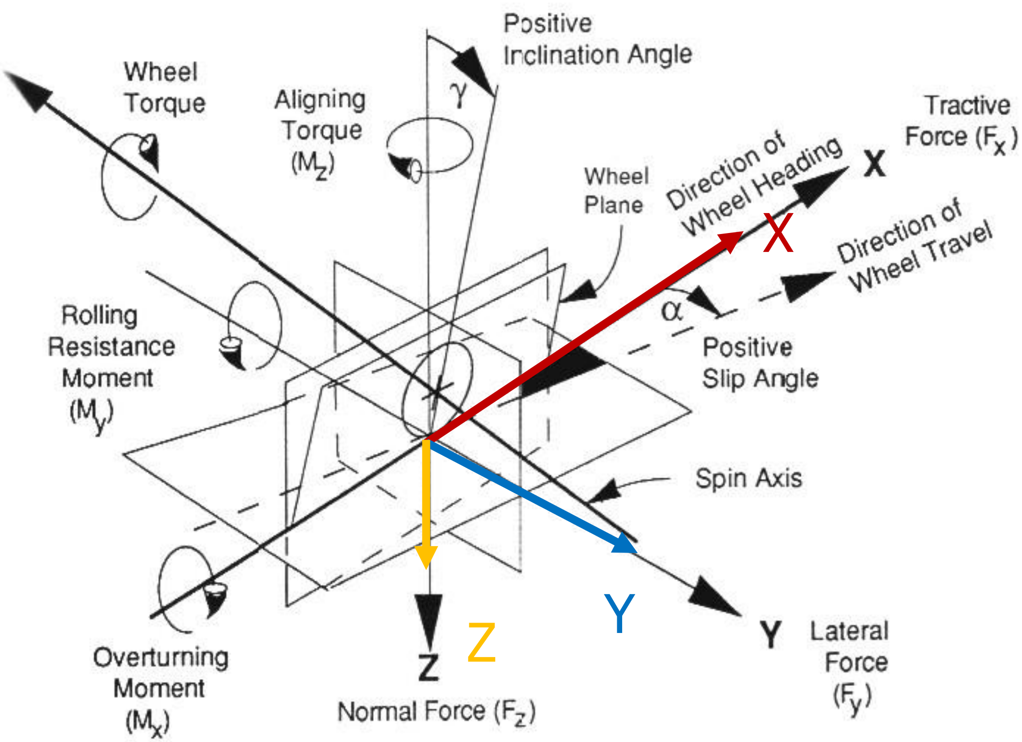
Terminology and axis system (SAE)



Gillespie Fig. 10.3

- <u>Wheel plane:</u> central plane of the tire normal to the axis of rotation
- <u>Wheel centre:</u> intersection of the spin axis with the wheel plane
- <u>Centre of Tire Contact:</u> intersection of the wheel plane and the projection of the spin axis onto the road plane
- <u>Loaded radius:</u> distance from the centre of tire contact to the wheel centre in the wheel plane

Terminology and axis system (SAE)



Gillespie Fig. 10.3

- <u>X-axis :</u> is the intersection of the wheel plane and the road plane, with a positive direction pointing into forward direction.
- <u>Z-axis:</u> is perpendicular to the road plane with a positive direction downward.
- <u>Y-axis:</u> is in the road plane; it is perpendicular to both X-axis and Z-axis and chosen to make an orthogonal right hand reference system. Y-axis points to the right hand side of the driver.
- <u>Origin O:</u> is located in the wheel centre

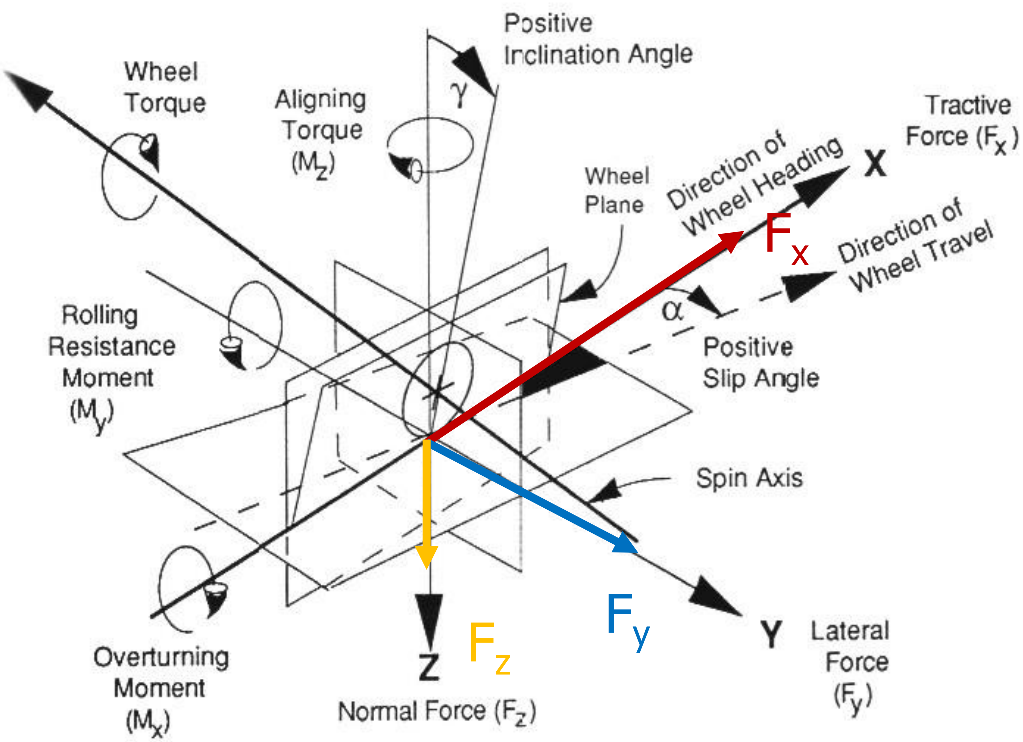
Forces

- *Forces and moments are accounted positively when acting onto the vehicle and in the positive direction with respect to the considered frame*

- Corollary
  - A positive F_x force propels the vehicle forward
  - The reaction force R_z of the ground onto the wheels is accounted negatively.

- Because of the inconveniency of this definition, the SAEJ670e « Vehicle Dynamics Terminology » denotes by normal force a force acting downward while vertical forces are referring to upward forces

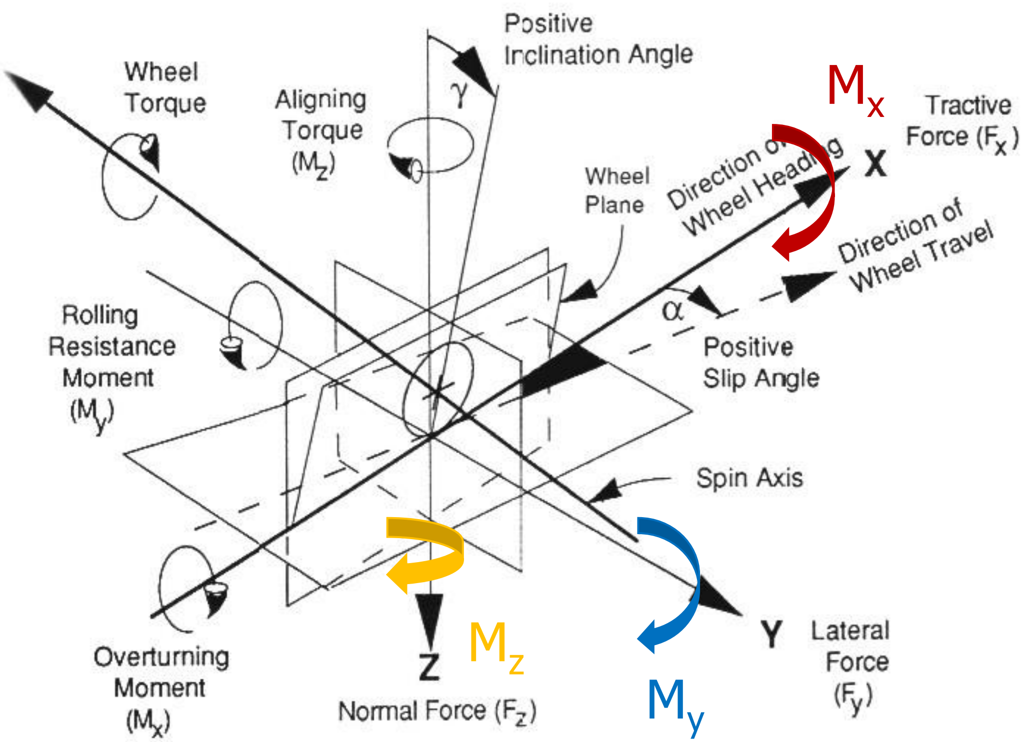
Terminology and axis system (SAE)



Gillespie Fig. 10.3

- Longitudinal Force F_x : component of the force acting on the tire by the road in the plane of the road and parallel to the intersection of the wheel plane with the road
- Lateral Force F_y : component of the force acting on the tire by the road in the plane of the road and normal to the intersection of the wheel plane with the road plane
- Normal Force F_z : component of the force acting on the tire by the road which is normal to the plane of the road

Terminology and axis system (SAE)



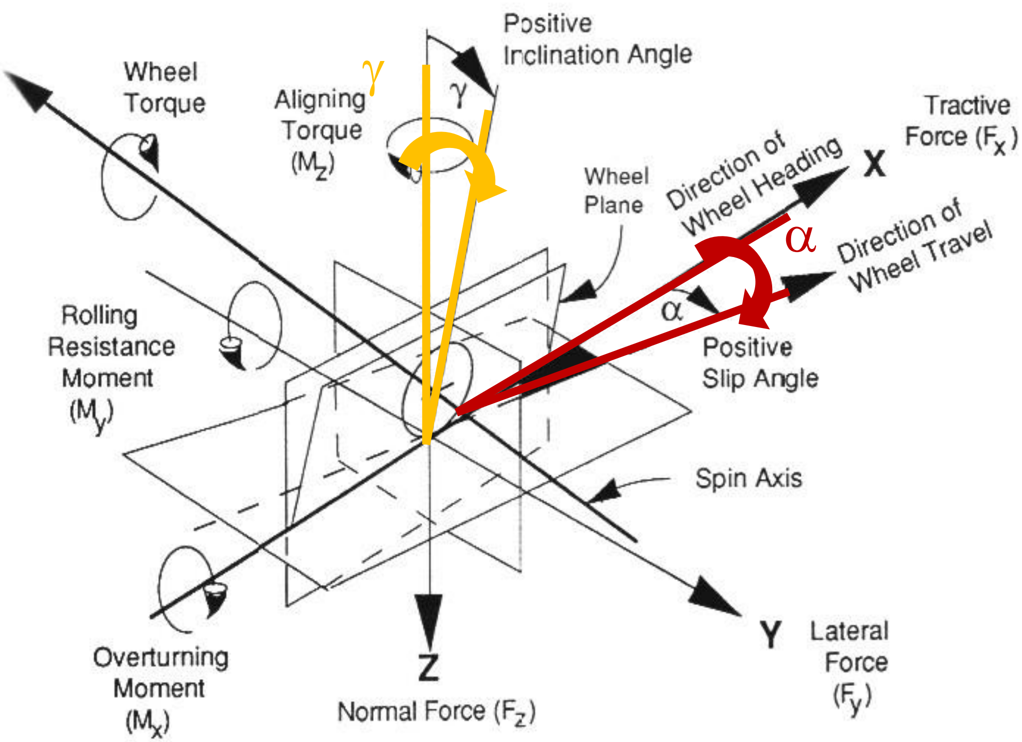
Gillespie Fig. 10.3

- Overturning Moment M_x : moment acting on the tire by the road in the plane of the road and parallel to the intersection of the wheel plane with the road plane

  Rolling Resistance Moment M_y : moment acting on the tire by the road in the plane of the road and tending to rotate the wheel about the y axis

  Aligning Moment M_z : moment acting on the tire by the road which is normal to the road plane

Terminology and axis system (SAE)



Gillespie Fig. 10.3

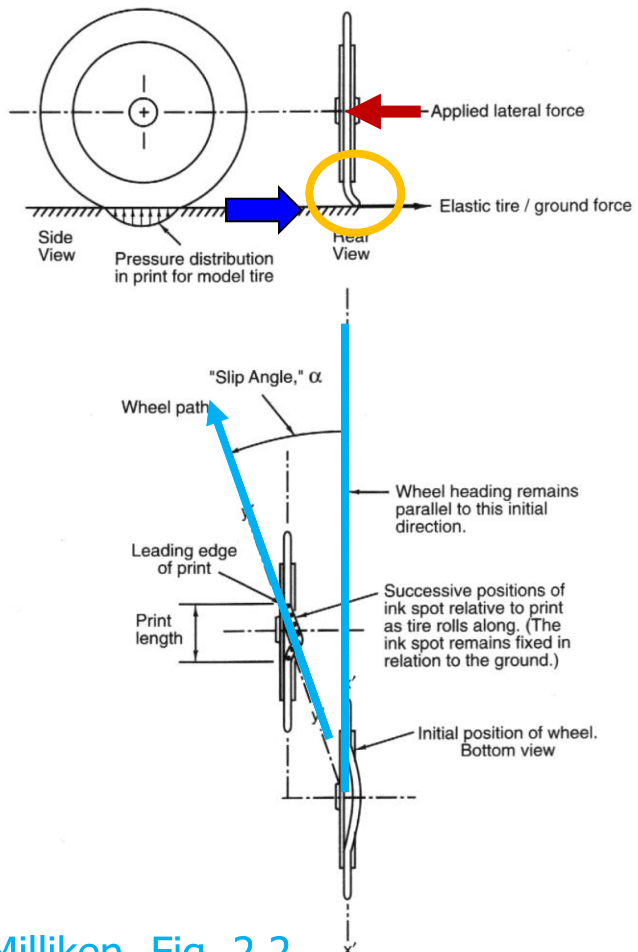
- Slip Angle (α): angle between the direction of the wheel heading and the direction of travel.

  A positive sideslip corresponds to a tire moving to the right while rolling in the forward direction
- Camber Angle (γ): angle between the wheel plane and the vertical direction.

  A positive camber corresponds to the top of the tire leaned outward from the vehicle

TIRE MECHANICS:
CORNERING PROPERTIES

LATERAL FORCES

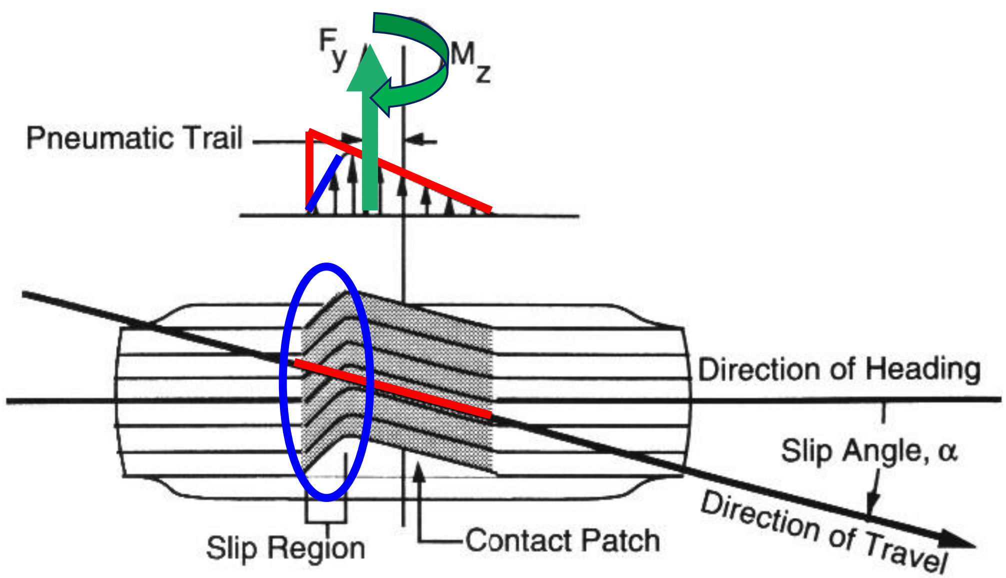


Milliken, Fig. 2.2

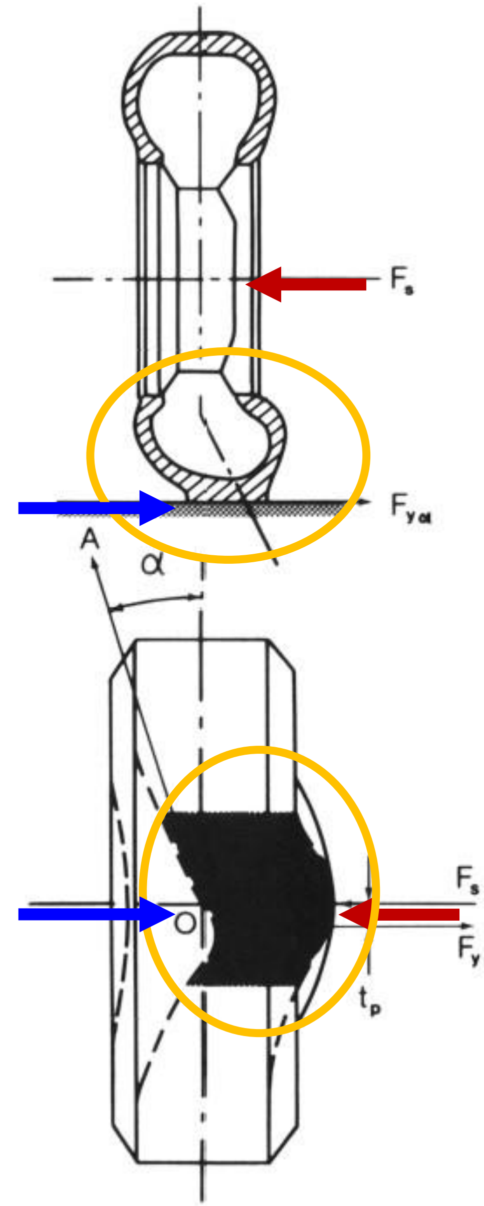
- When a lateral force is applied to the tire, one observes that the contact patch is deformed and the tire develops a lateral force that is opposed to the applied force.
- When rolling, applying a lateral force, the tire develops a (reaction) lateral force opposed to the applied force and the tire moves forward with an angle α with respect to the heading direction, called slip angle.
- The relation between the lateral force that is developed and the sideslip angle is a fundamental matter for the study the vehicle dynamics and its stability

ORIGIN OF LATERAL FORCES

- The origin of the lateral force generation is the carcass deformation under the applied forces

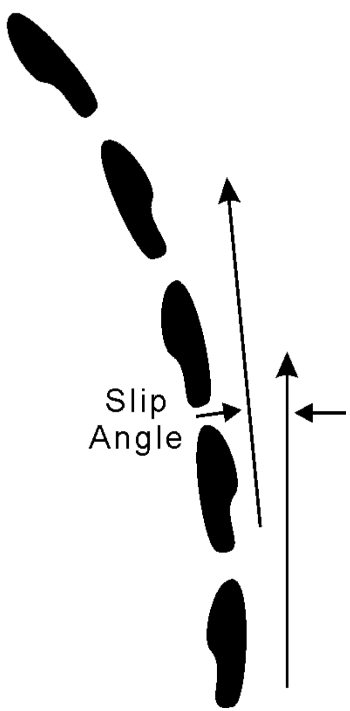


Gillespie, Fig 10.10



Wong Fig 1.22

LATERAL FORCES

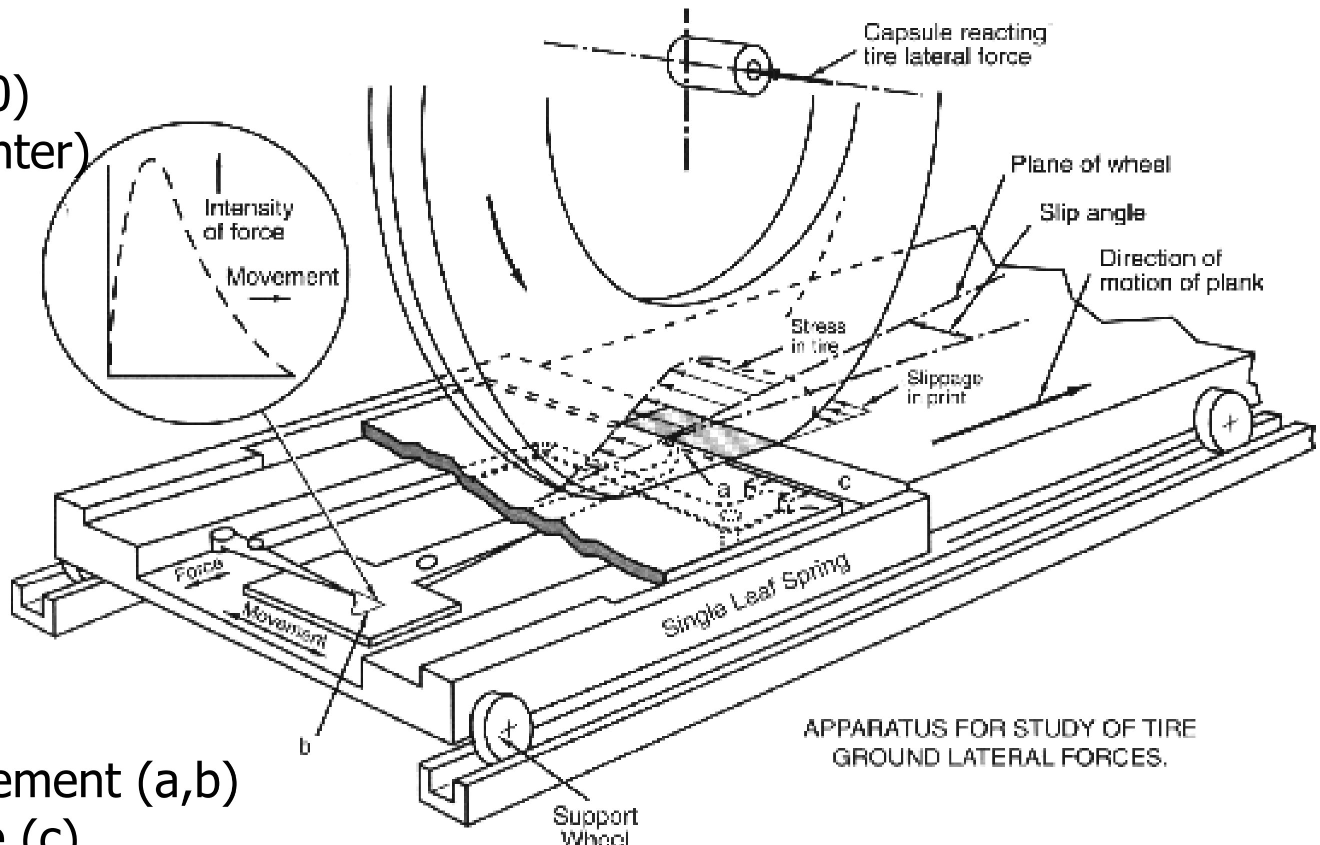


Analogy with walking on a snow slope

- The name of sideslip is misleading since there is no global slip of the tire with respect to the ground (except in a very limited part at the back of the contact patch).
- The sideslip of the tire is due to the flexible character of the rubber of the tire that allows keeping a heading direction while having a lateral motion
- The lateral force may be a cause or a consequence of the sideslip.
  - Lateral forces (gust) → sideslip → reaction forces under the tire
  - Steering the wheel → sideslip → lateral forces to turn

LATERAL FORCES

Gough machine (1950)
(Dunlop Research Center)



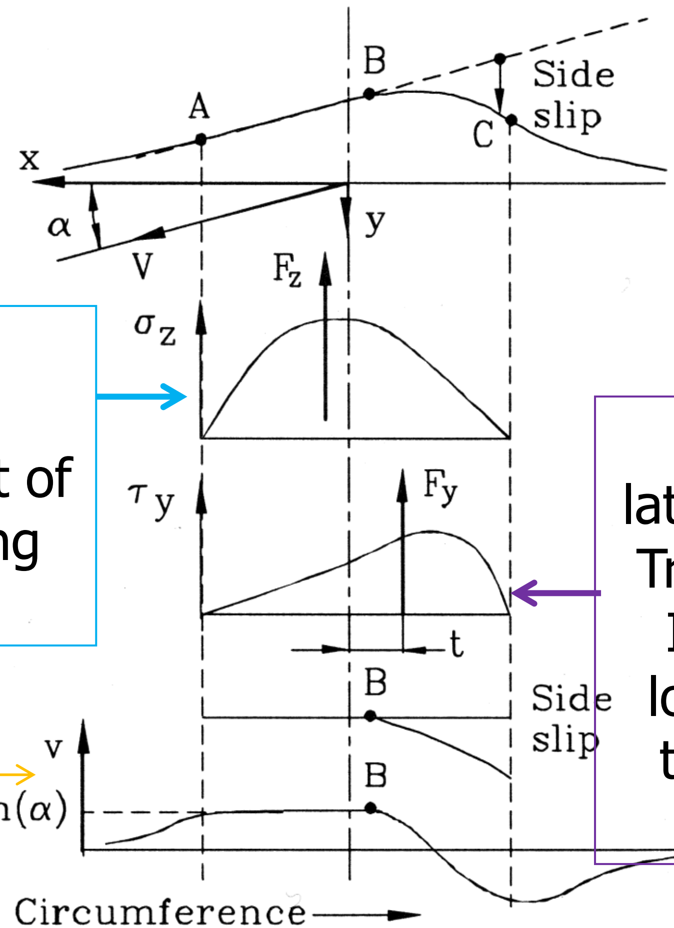
Records:

- the lateral displacement (a,b)
- the force of a slice (c)
- the reaction force on the tire groove

Milliken, Fig. 2.4

LATERAL FORCES

Side displacements



Distribution of contact pressure σ_z: centre of pressure is in the front part of the contact patch ➔ rolling resistance

Distribution of τ_{zy} = lateral shear of the tire: Triangular distribution. Its resultant force is located in the back of the contact patch ➔ aligning moment

Lateral speed
A→B : V sinα
B→C: local slip

Genta Fig 2.24

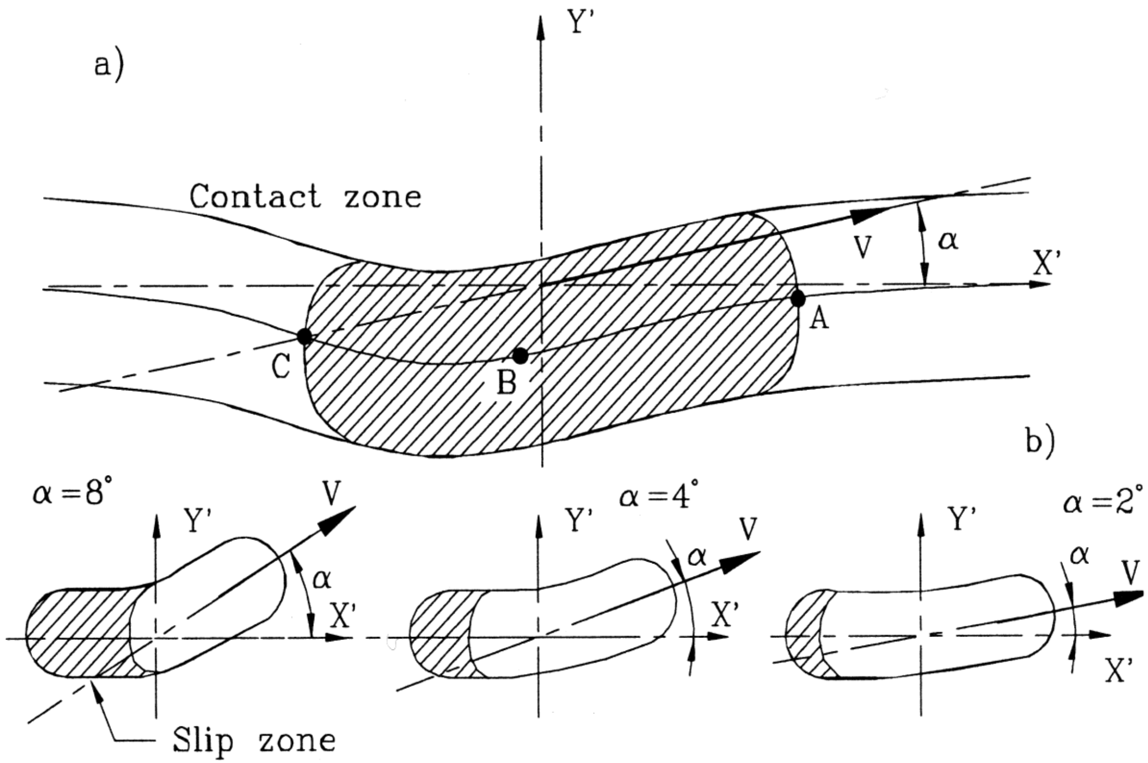
LATERAL FORCES

- From experimental observations (for instance on Gough machine), it comes that:
  - The resultant of the lateral forces F_y is located in the rear part of the pneumatic the contact patch centre point.
  - The distance between the resultant force position and the centre of the contact pact is the pneumatic trail t
  - The lateral forces produces also an aligning moment

$$M_z = F_y\, t$$

  - The local slippage of the tire on the road is limited to a small zone at the back of the contact patch. Its span depends on the sideslip angle.
  - The characteristics of the lateral forces are related, in the small sideslip angles range, to the lateral displacements due to the rolling process but they are mostly independent of the speed.

LATERAL FORCES



Genta Fig 2.23 : Contact zone in presence of sideslip
a/ Contact zone and trajectory of a point in the tread
b/ Contact zone and slippage zone for different sideslip angles

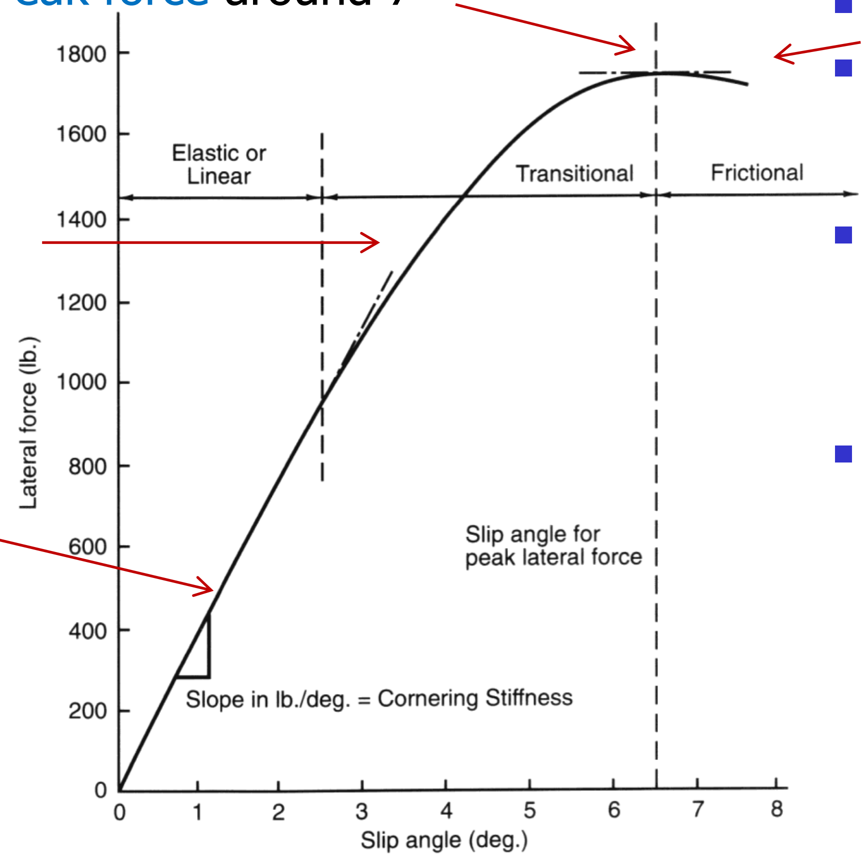
LATERAL FORCES

- Three parts in the lateral force F_y curve in terms of side slip angle: linear, transition, dry friction

- Peak force around 7°

- Transition zone: end of linear regime (>3°) to the peak value of the curve (<7°)

- Linear part: for small slip angles (<3°)

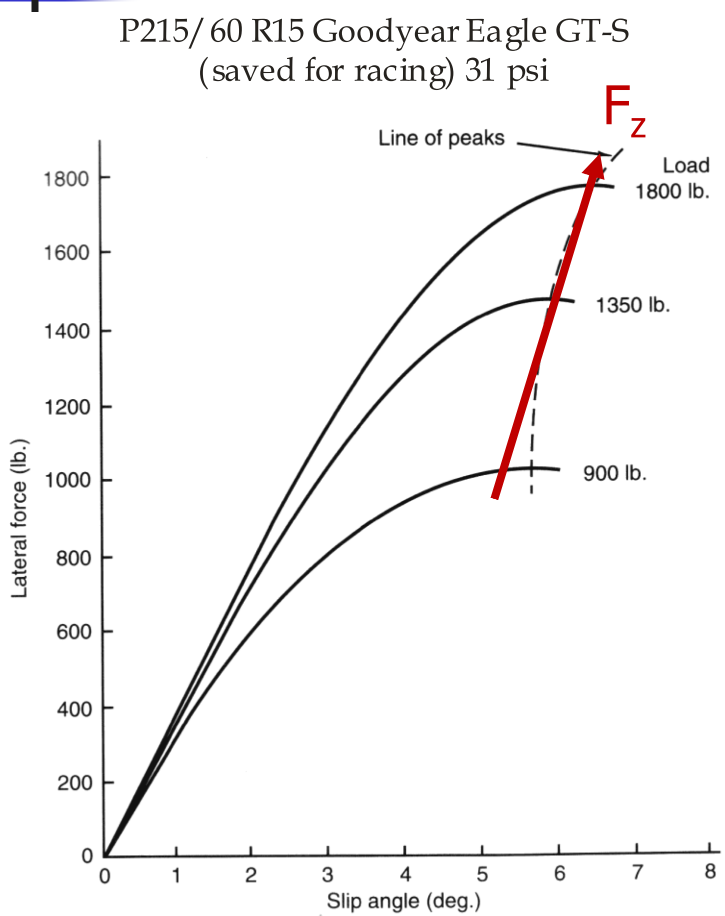


Milliken. Fig. 2.7

- Dry friction part (>7°)
- After the peak, the curve can drop or stay more or less constant
- On dry road, the max value is reduced and the drop is more pronounced
- At the max value and beyond, most of the contact patch is experiencing sliding and the lateral force is due to dry friction

P215/60 R15 GoodYear Eagle GT-S (shaved for racing) @31 psi. Vertical load 1800 lb

LATERAL FORCES



Milliken. Fig. 2.8

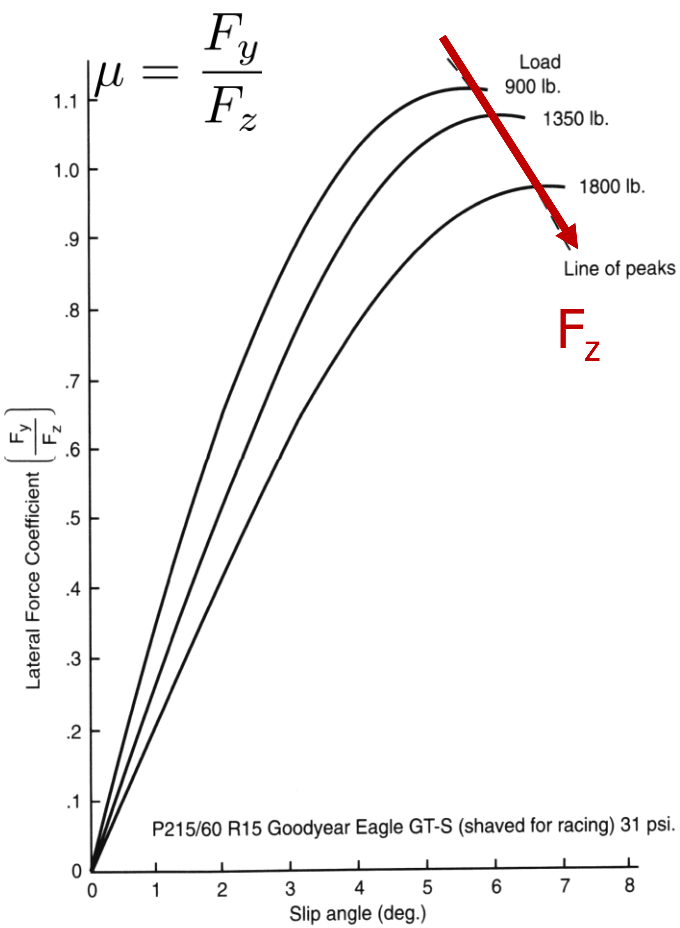
- Increasing vertical load F_z ➔ Increasing the lateral force generated F_y

LATERAL FORCES

- One introduces the friction coefficient as the ratio of the lateral force and of the applied vertical load

$$\mu_y = \frac{F_y}{F_z}$$

- Increasing vertical load F_z ➔ Reducing μ_y = the lateral force generated F_y per unit of vertical load F_z
- The peak value of the lateral friction coefficient is reduced with the vertical load: this is the load sensitivity phenomenon.



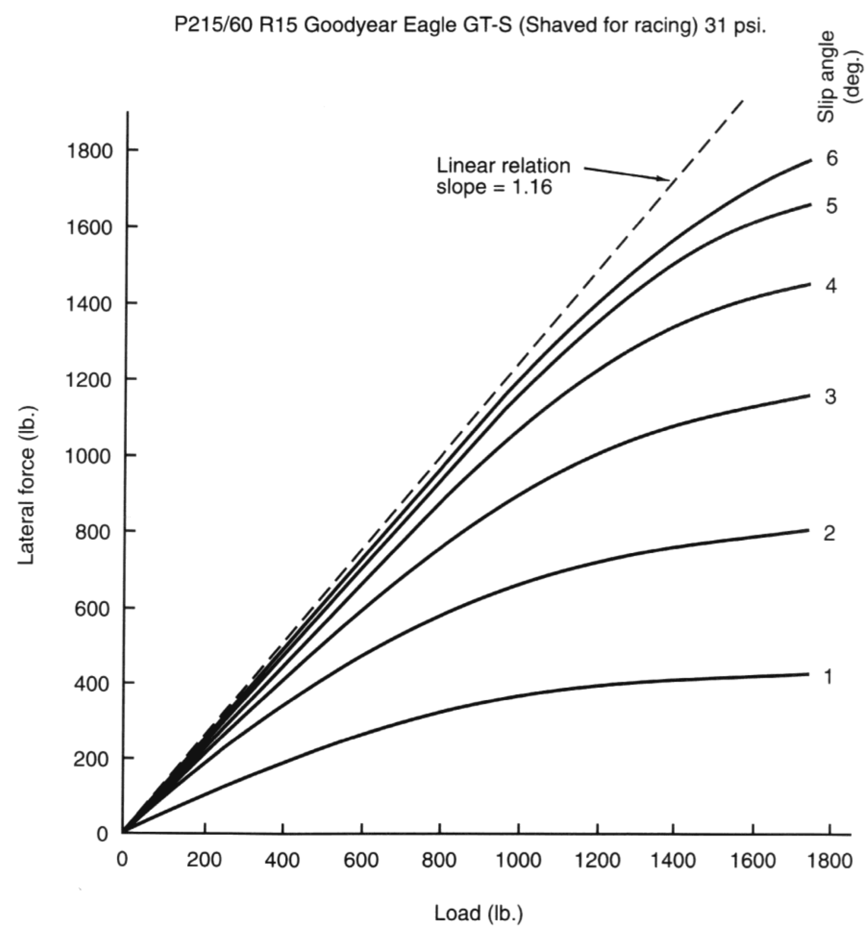
Milliken. Fig. 2.9

LATERAL FORCES

- The load sensitivity phenomenon is better illustrated when plotting the lateral generated load F_y with respect to vertical load F_z for a fixed value à the side slip angle a:

- The modification of the lateral force with the vertical load can be important with old bias tires.

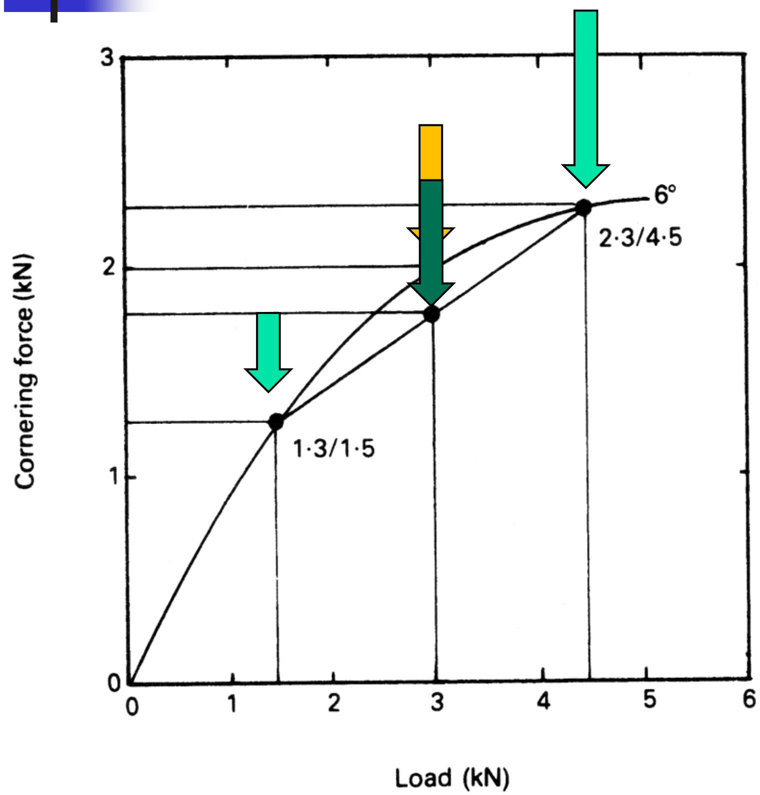
- This affects the total lateral force capability of the wheel axle when experiencing lateral load transfer.

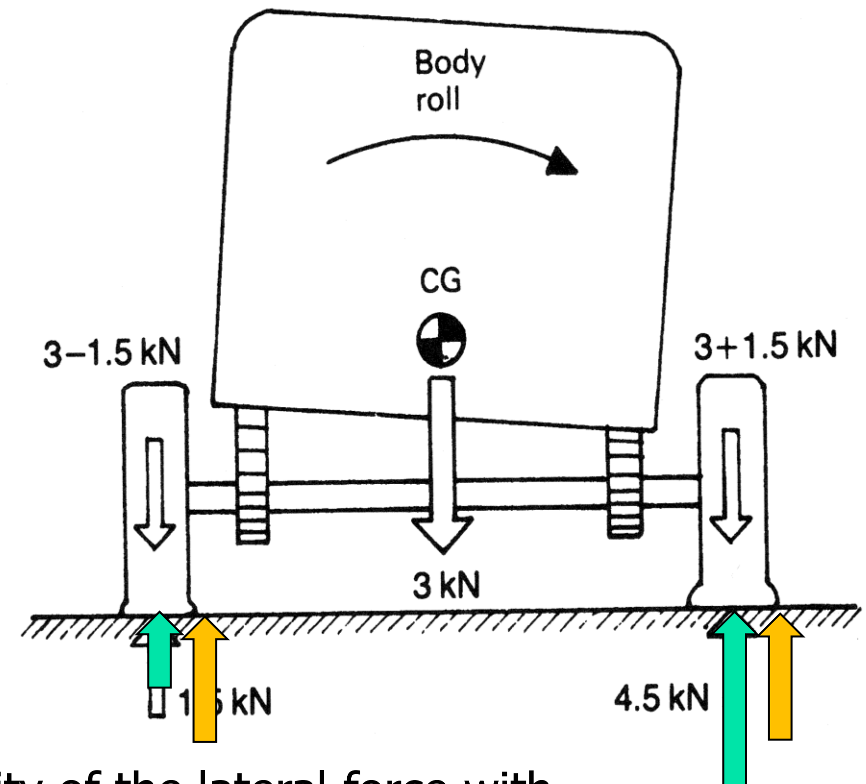


Milliken. Fig. 2.10

Load sensitivity

- What is gained on the loaded external tire never compensates the loss by the inner unloaded.
- It comes a net loss of lateral force generated by the axle.





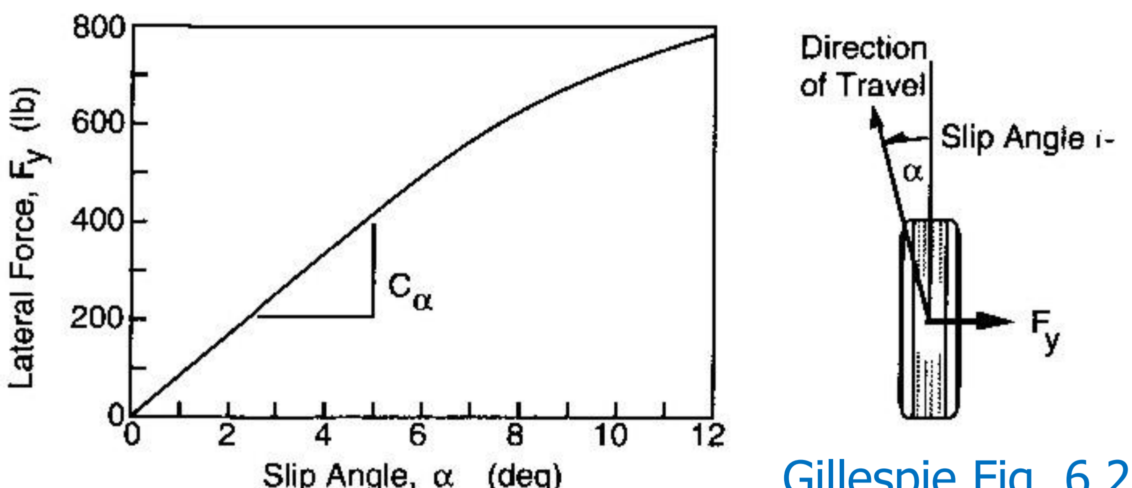
Heisler: Fig 8.42 et 8.43 : Sensitivity of the lateral force with respect to the load transfer

Cornering stiffness

- In its linear part (small sideslip angles) the lateral force curve can be approximated by its Taylor first order expansion :

$$\boxed{F_y = -C_\alpha\, \alpha}$$

  - C_α is called as the cornering stiffness.
  - The cornering stiffness is negative



Gillespie Fig. 6.2

$$F_y(\alpha) = F_y(0) + \frac{\partial F_y}{\partial \alpha}(\alpha - 0)$$

$$= \frac{\partial F_y}{\partial \alpha}\, \alpha$$

$$C_\alpha = \left.\frac{\partial F_y}{\partial \alpha}\right|_{\alpha=0}$$

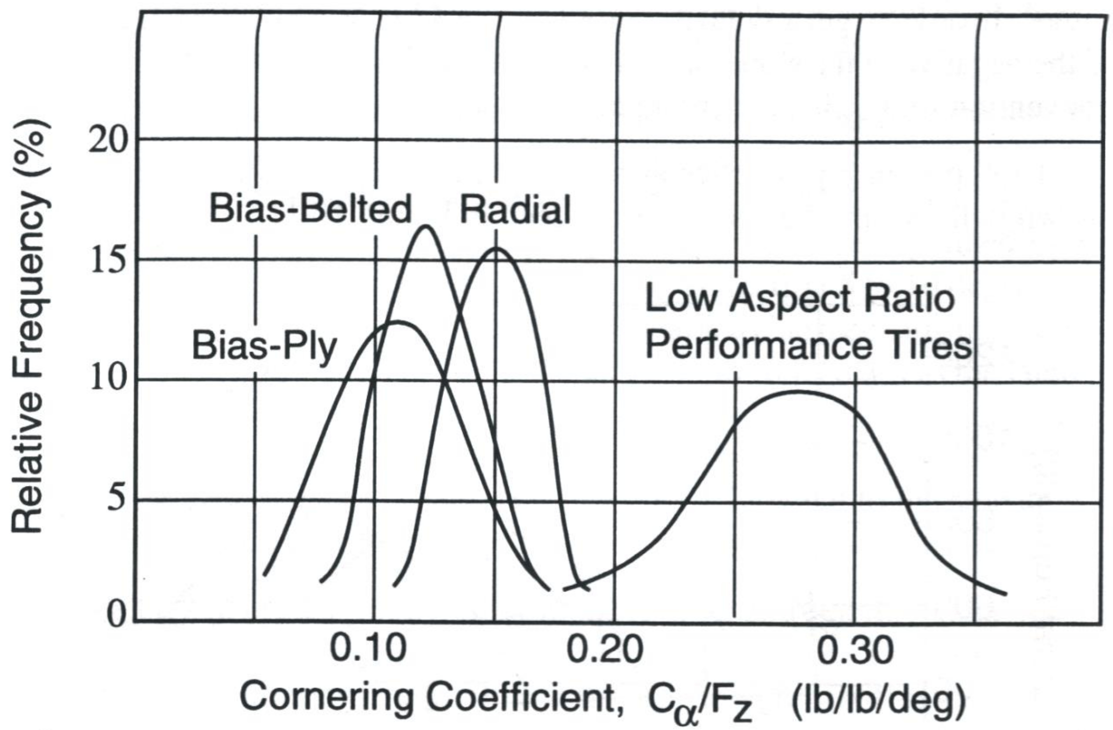
Cornering stiffness

- The cornering stiffness depends on several parameters:
  - The construction of the tire, its sizes and width,
  - The inflating pressure
  - The vertical load
- The order of magnitude of the cornering stiffness is about 50000 N/rad ~ 875 N/deg
- As the lateral force is sensitive to the vertical load, one generally prefers to use the cornering coefficient that is defined as the ratio of the cornering stiffness by the vertical load:

$$\boxed{CC_\alpha = C_\alpha / F_z}$$

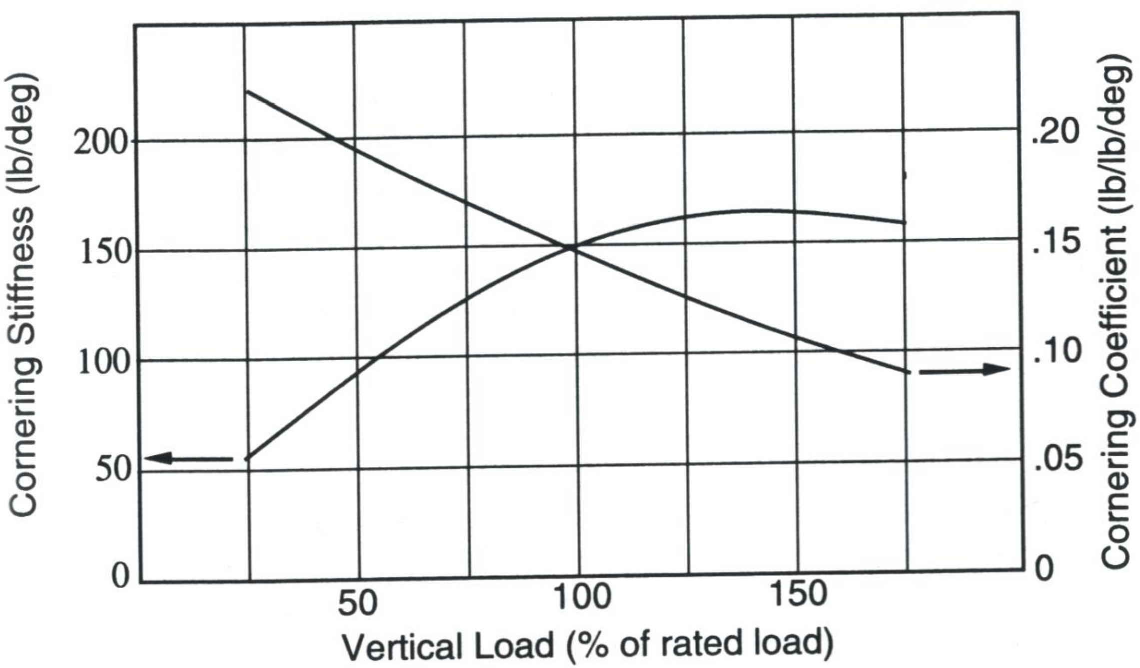
- The order of magnitude of CC_α is in the range 0.1 to 0.2 N/N degre^{-1}

Cornering stiffness



Gillespie: Fig 10.14 : Cornering coefficient for different populations of tires

Cornering stiffness



Gillespie: Fig 10.15 : Load sensitivity of the cornering stiffness and cornering coefficient

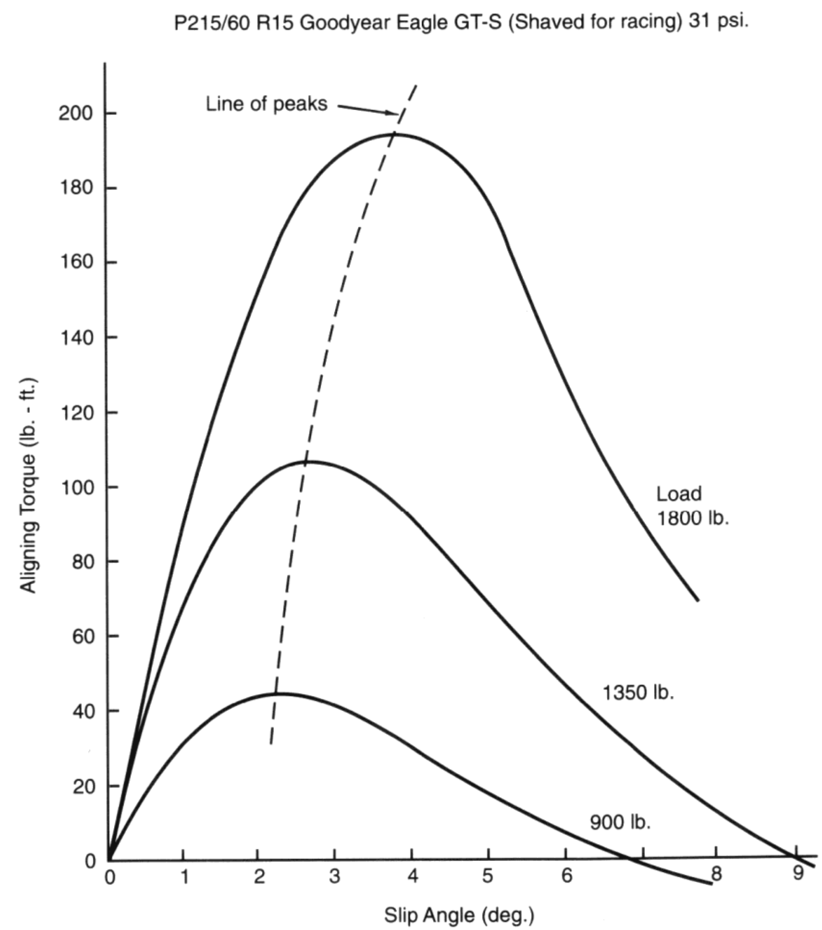
ALIGNING MOMENT AND PNEUMATIC TRAIL

- The aligning moment reflects the tendency of the tire to rotate about its vertical axis to self align with the motion direction
- For small and medium sideslip angles, the tire tends to align itself with the velocity vector direction. i.e. to reduce its sideslip
- Origin of the aligning moment:
  - Triangular distribution of the local shear forces in the contact patch with a resultant located behind the contact centre
- The pneumatic trail is the distance between the lateral force resultant and the contact centre
  - Trail = Aligning moment / Lateral force

Aligning moment and pneumatic trail

- Linear part - small sideslip angles (<3°):
  - The largest shear stresses in the aft work to reduce the sideslip

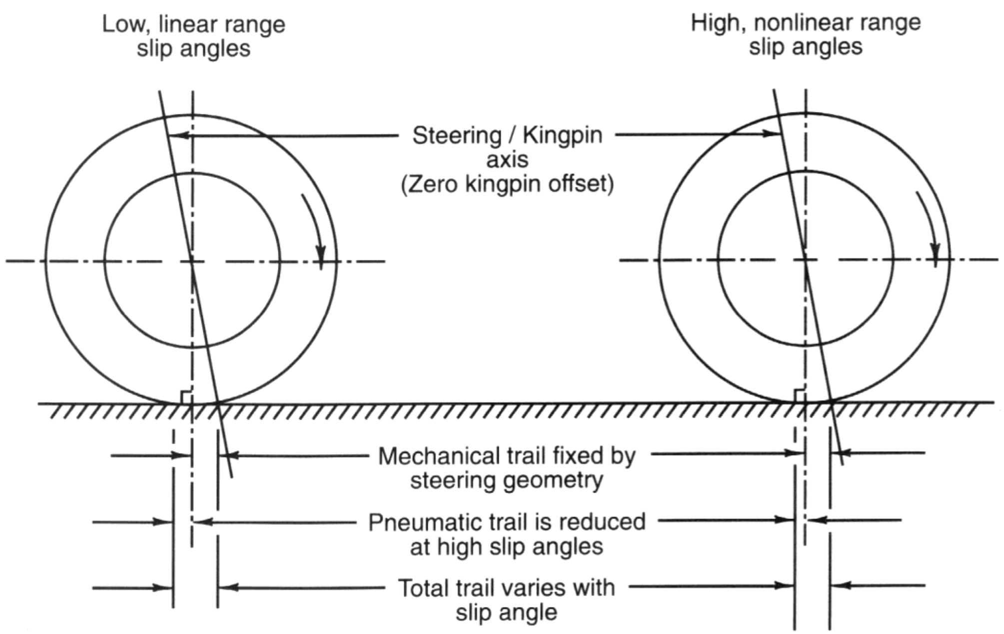
- Non-linear part – medium and large sideslip angles
  - Maximum of the curves about 3 to 4°
  - When the rear of the contact patch is invaded with local friction, the aligning moment is reducing
  - At the maximum of the lateral force, the aligning moment drops to zero and may even become negative for larger sideslip angles α > 7° à 10°



Milliken. Fig. 2.11

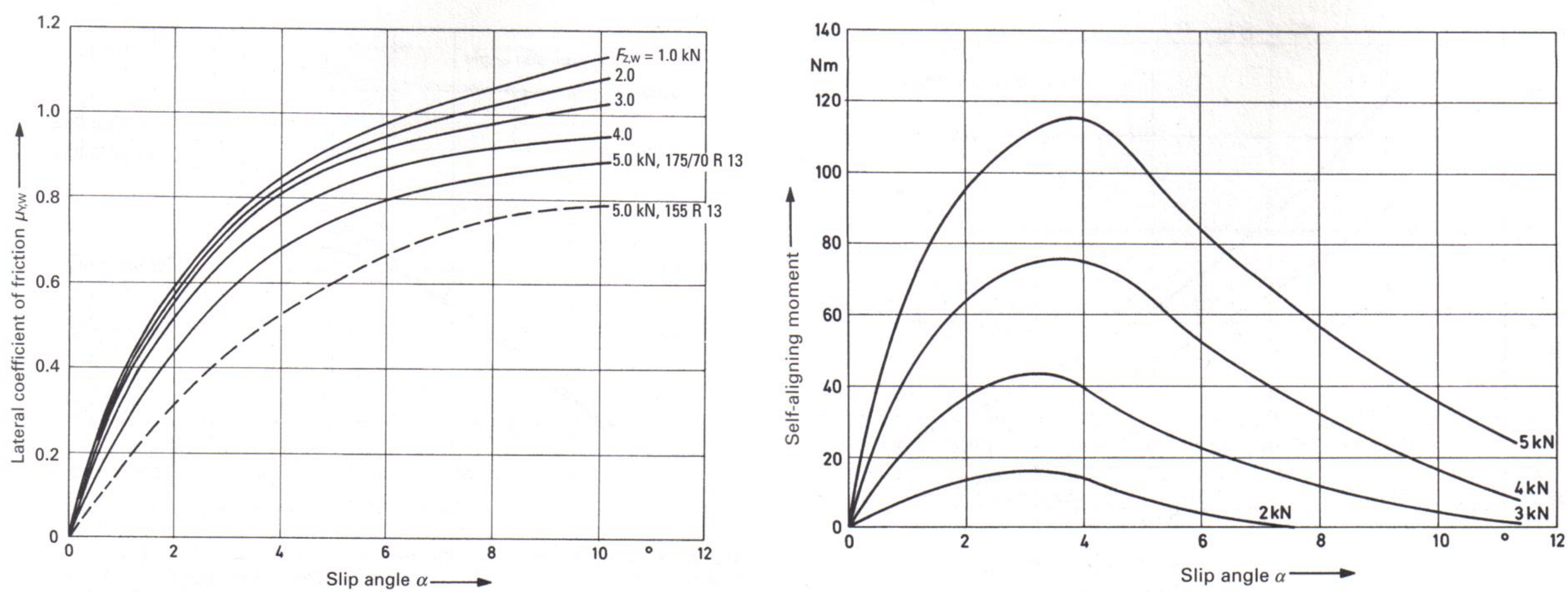
Aligning moment and pneumatic trail

- The aligning moment can also be reinforced with a mechanical trail coming from the wheel and the steering geometry
- Optimum combination of both trails :
  - For small mechanical trail, the vehicle can loose its aligning torque capability with the saturation of lateral force
  - For large mechanical trail, one looses the feeling of the maximum of the lateral force curve



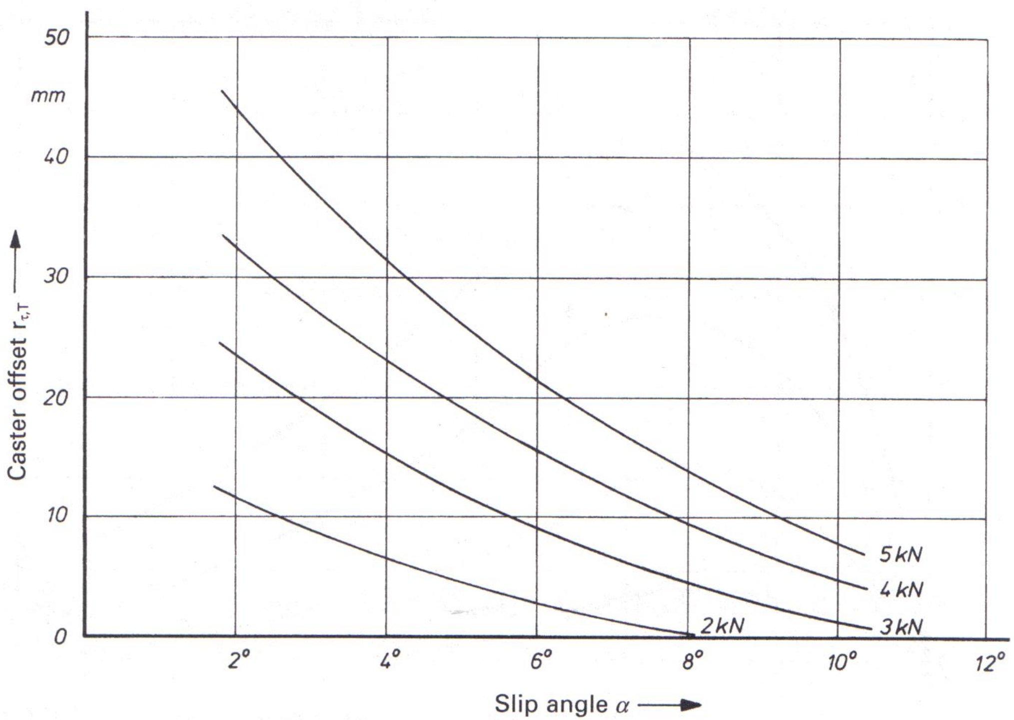
Milliken. Fig. 2.12

Aligning moment and pneumatic trail



Reimpell et al. (2001) Fig 2.45 & Fig 2.49 Lateral force and aligning moment for a tire 175/70 R 13 82S

Aligning moment and pneumatic trail



Reimpell et al. (2001) Fig 2.50 Pneumatic trail for a tire 175/70 R 13 82S

- Trail = Aligning Moment / Lateral Force

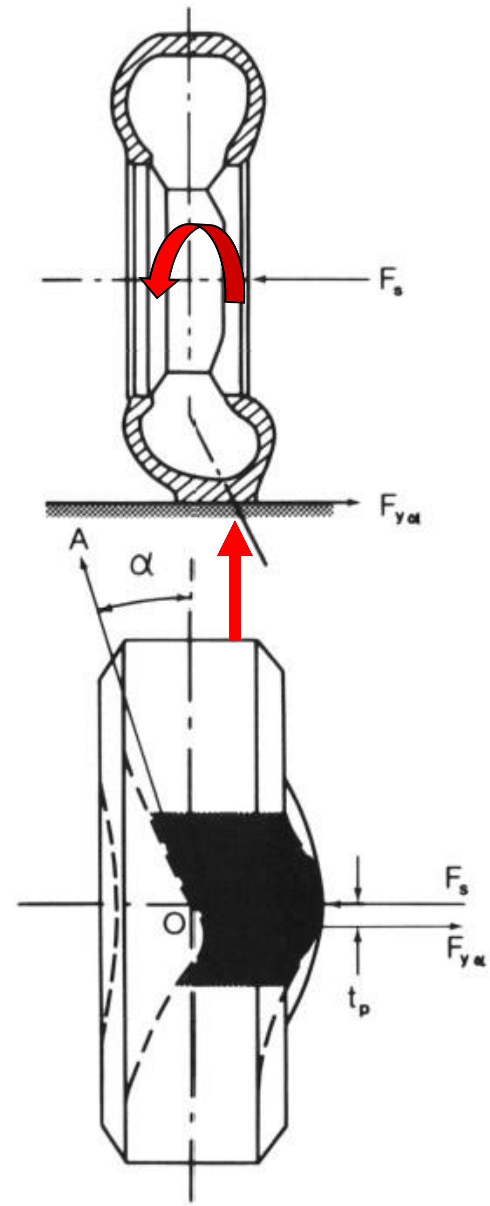
$$t = M_Z / F_y$$

- Larger lateral forces result in low aligning moment and a small pneumatic trail.
  - For small sideslip angles, only the profile is deformed, which gives rise to a resultant far in the back.
  - For important side slip angles, the sidewall works much more and the resultant F_y gets closer to the centre of the contact patch

OVERTURNING MOMENT

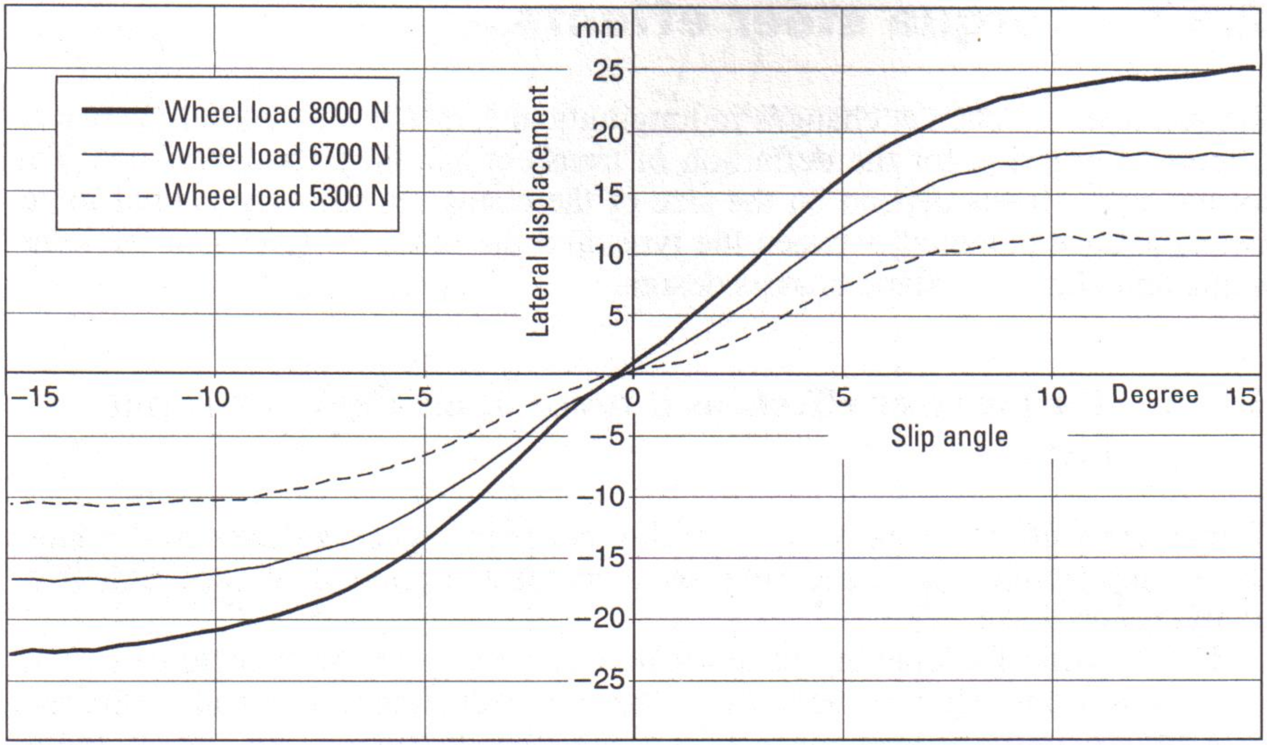
- The contact patch being deformed, it experiences a lateral displacement and the vertical resultant generates an overturning moment about the 'x' axis of the tyre.
- The phenomenon is dependent on :
  - Tyre size,
  - Lateral force magnitude
  - Presence of a camber angles
  - The tyre construction

- Surprisingly the low aspect ratio tyres are more subject to lateral displacements of the vertical resultant (cfr Reimpell)



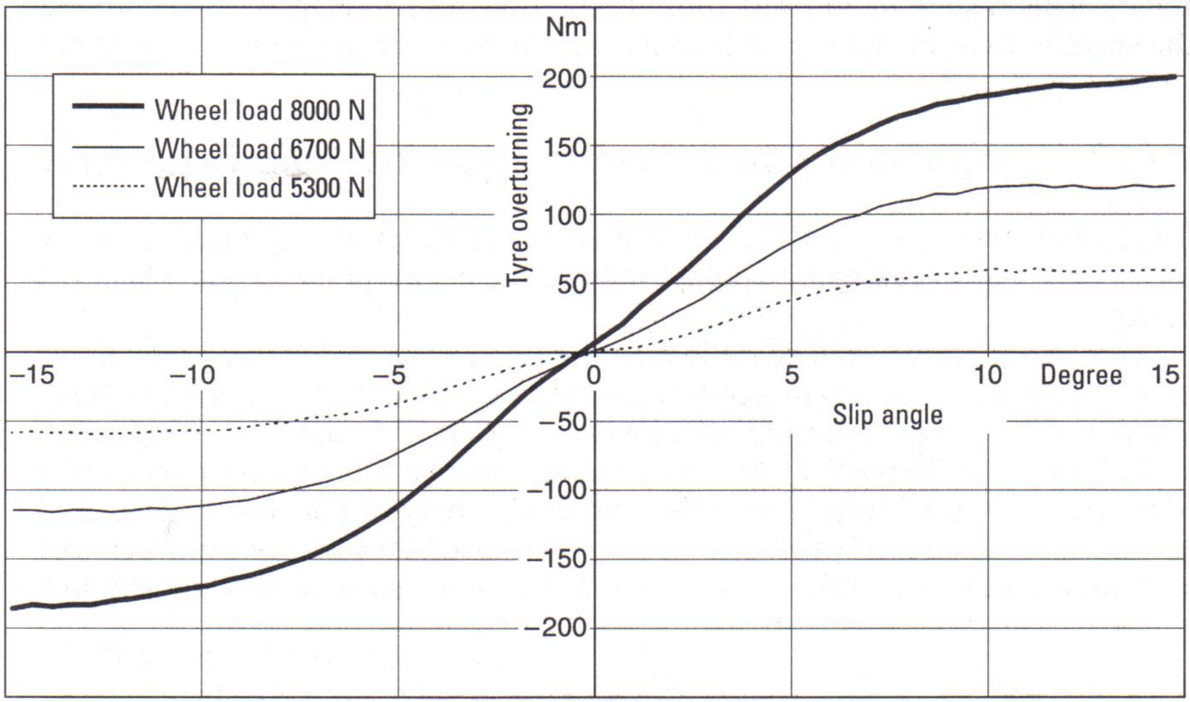
Wong Fig 1.22

OVERTURNING MOMENT



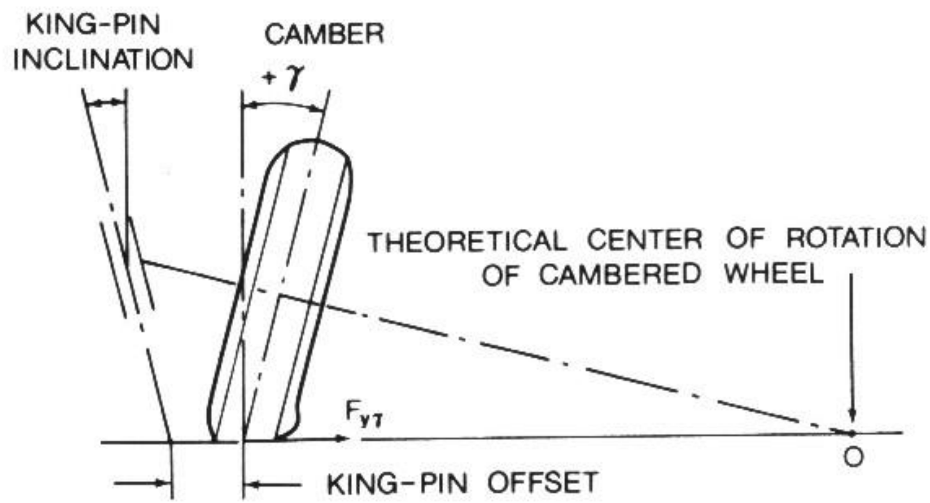
Reimpell Fig. 2.51: Lateral displacement of the contact patch centre as a function of the side slip angle and of the vertical load. Tire 205/65 R15 94 V ContiEcoContact CP

OVERTURNING MOMENT



Reimpell 2.52: Overturning moment as a function of the side slip angle and of the vertical load. Tire 205/65 R15 94 V ContiEcoContact CP

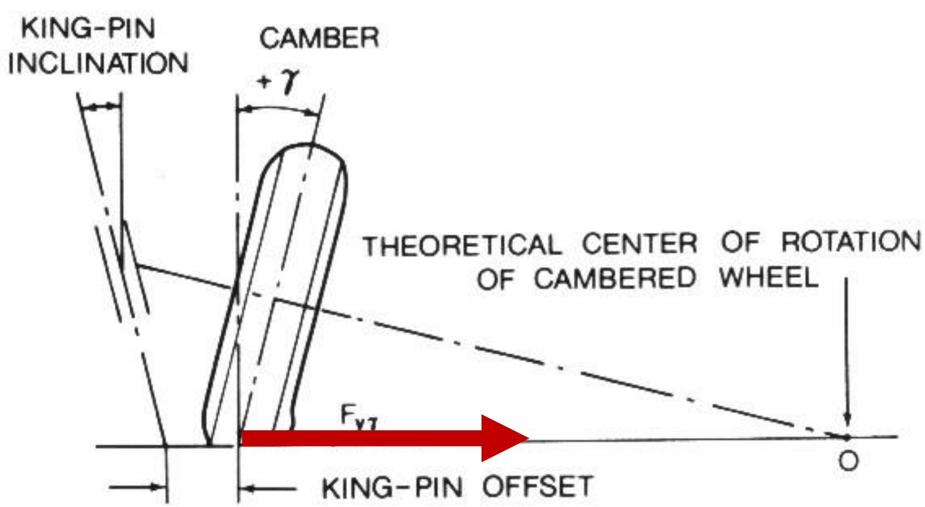
EFFECT OF CAMBER



Definition of camber
Wong Fig.1.40

Camber change

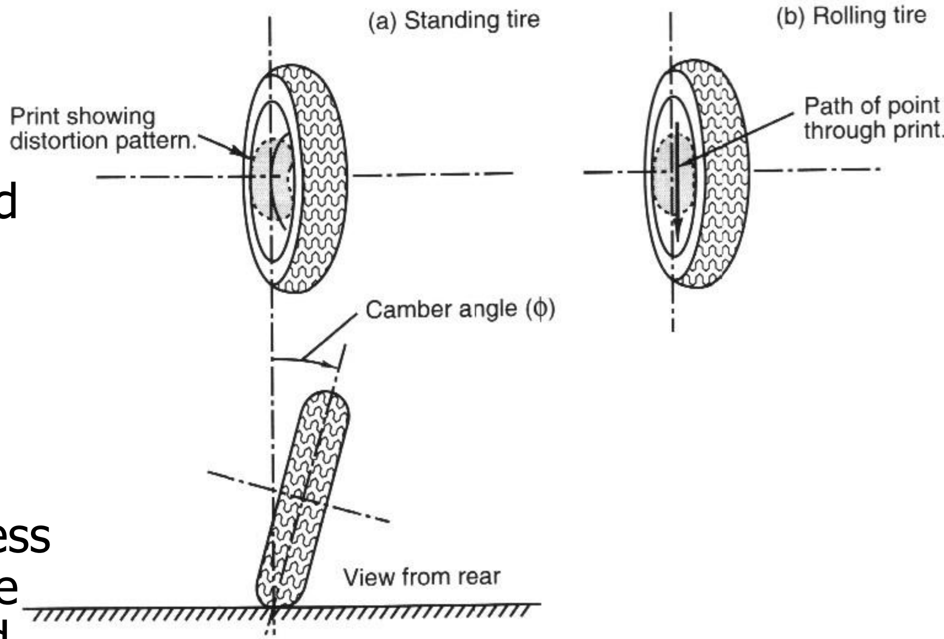
- The inclination of the wheel outward from the body is called the camber angle.
- Wheel camber will produce a lateral force known as camber thrust.
- The camber thrust is oriented toward the intersection of the wheel rotation axis with the ground plane.



Wong Fig.1.33

Camber thrust

- Origin of the camber thrust:
  - The contact patch is deformed and takes a banana shape
  - The tyre particles would like to follow a circular trajectory which creates a double shear stress distribution.
  - The net resultant of the shear stress distribution is oriented towards the centre of rotation of the cambered wheel



Milliken : Fig 2.23

Distorted contact patch because of camber

Camber thrust

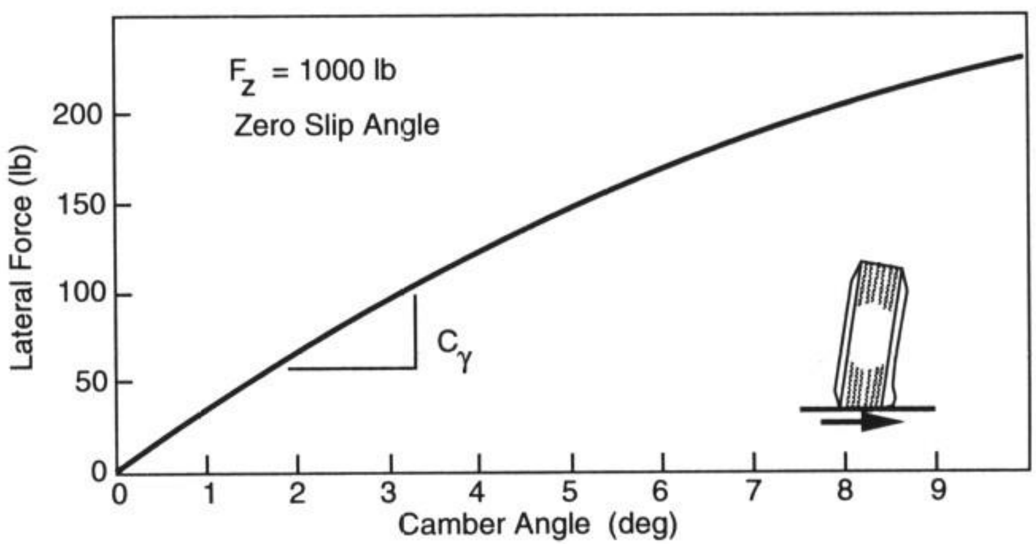
- The camber thrust is oriented towards the centre of rotation of the cambered wheel
- The tyre develops a lateral force even if there is no slip angle.
- Camber thrust and the side slip angle cornering forces due to slip are additive phenomena at least for small values
- The magnitude of the camber thrust depends on
  - The camber angle
  - The type of tyre, its construction characteristics
  - The shape of the tread
  - The inflating pressure (strongly)
  - The braking / tractive forces
  - The slip angle
- The camber thrust is little sensitive to load and speed

Camber thrust

- In its linear part, (small camber angles), the camber thrust angle can be approximated by the linear relation:

$$\boxed{F_\gamma = C_\gamma\,\gamma} \qquad C_\gamma = \left.\frac{\partial F_y}{\partial \gamma}\right|_{\gamma=0} > 0$$

- C_γ is the camber stiffness



Gillespie Fig. 6.14

Camber thrust

- The camber thrust produces much smaller lateral forces than an equivalent slip angle.
    - About 4 to 6 degrees of camber produce the same lateral force as 1 degree of side slip for a bias tyre.
    - For radial tyres, as much as 10 to 15 degrees are necessary for 1 degree of slip
    - This can be understood because the side slip produces a much bigger deformation of the tyre tread. For radial tyre, the sidewall are even softer and the belts are very rigid.

- For passenger car, the camber thrust is fading out for camber thrust over 5 to 10 degrees
- For motorbikes tyres with round profiles and bias structures, the camber thrust can be generated up to 50 degrees

Camber thrust coefficient

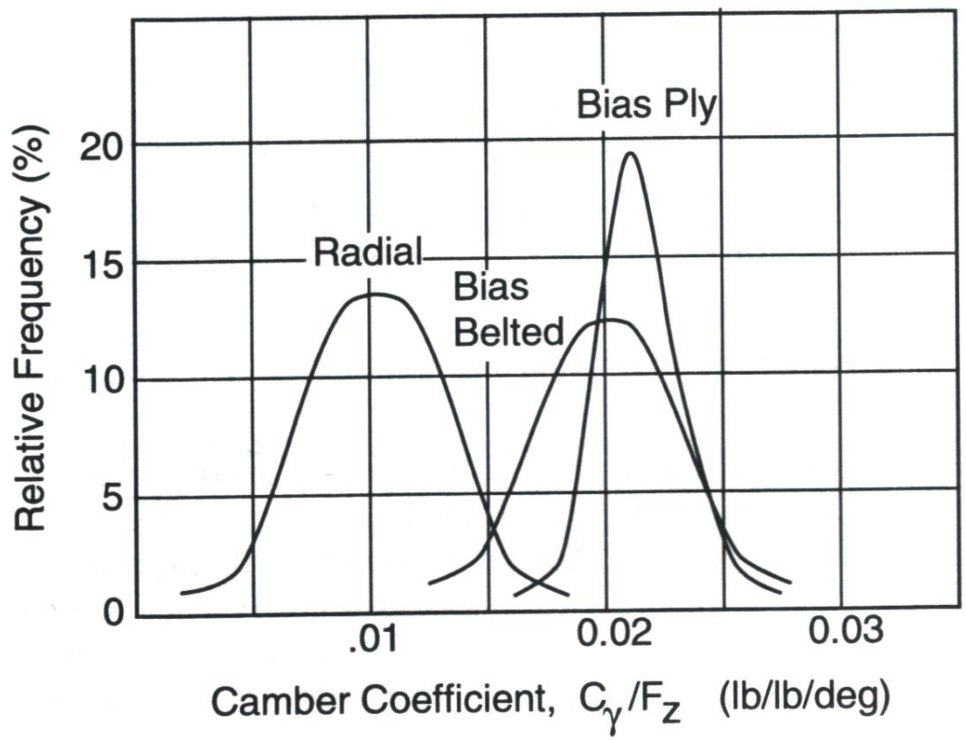
- As the lateral force is sensitive to the vertical load, one generally prefers to use the camber thrust coefficient that is defined as the ratio of the camber thrust stiffness by the vertical load:

$$CC_\gamma = C_\gamma / F_z$$

- The camber thrust coefficient depends heavily of the constructive properties of the tire.

- The order of magnitude of CC_γ is in the range 0.01 for radial tire and is about to 0.02 to 0.03 N/N degre^{-1} for bias ply tires

Camber stiffness



Gillespie: Fig 10.17 Camber stiffness for different population
Of radial, bias and bias belted tyres.

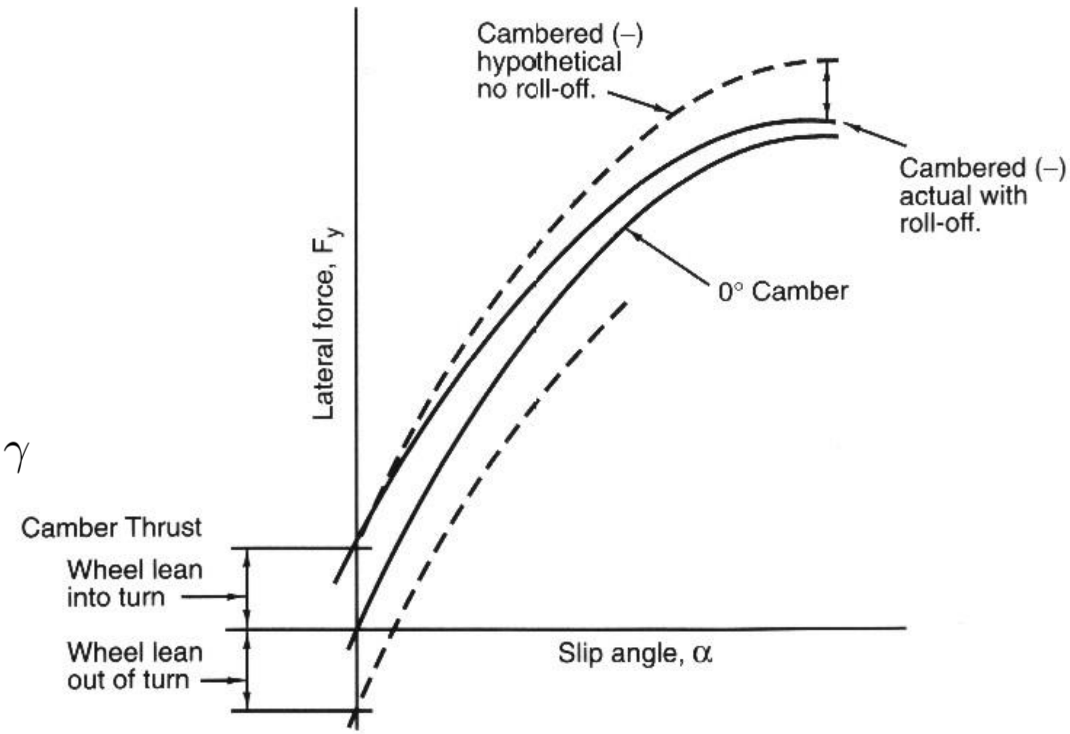
Camber thrust & side slip forces

- Camber thrust is additive to cornering forces from slip angle (at least for small angles).
- For small side slip and camber angles, one can expand the lateral force

$$F_y(\alpha,\gamma) \simeq \left.\frac{\partial F_y}{\partial \alpha}\right|_{\alpha=0} \alpha + \left.\frac{\partial F_y}{\partial \gamma}\right|_{\gamma=0} \gamma$$

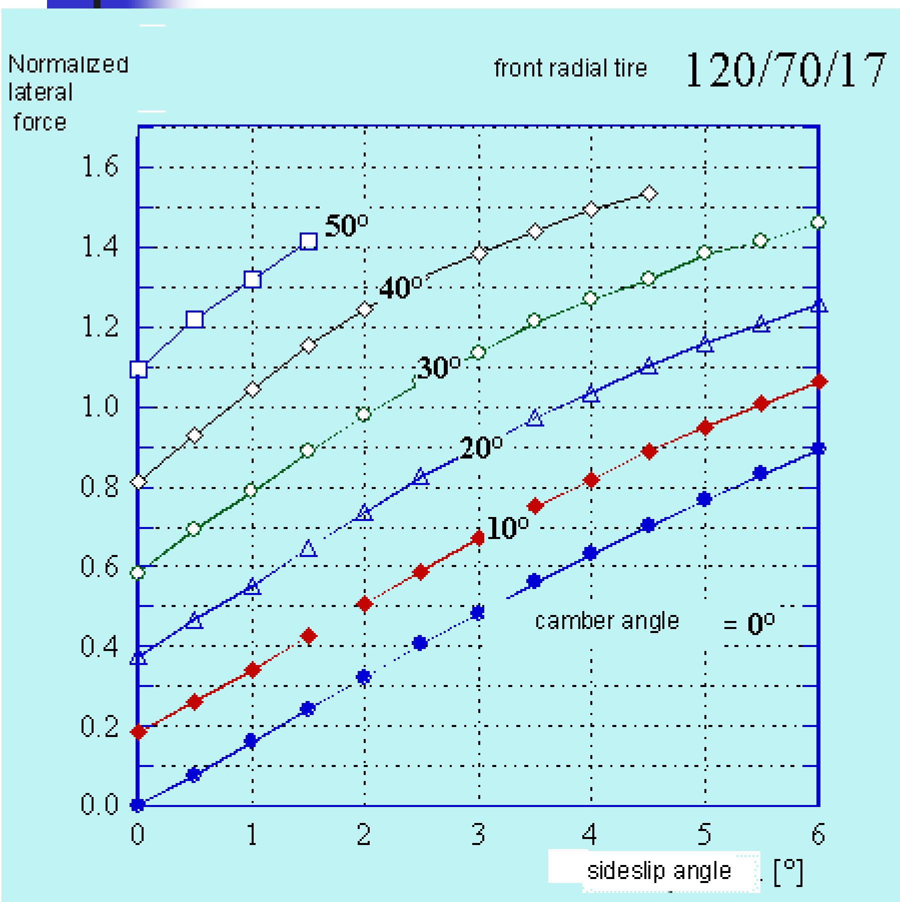
$$\boxed{F_\gamma = -|C_\alpha|\alpha + C_\gamma\,\gamma}$$

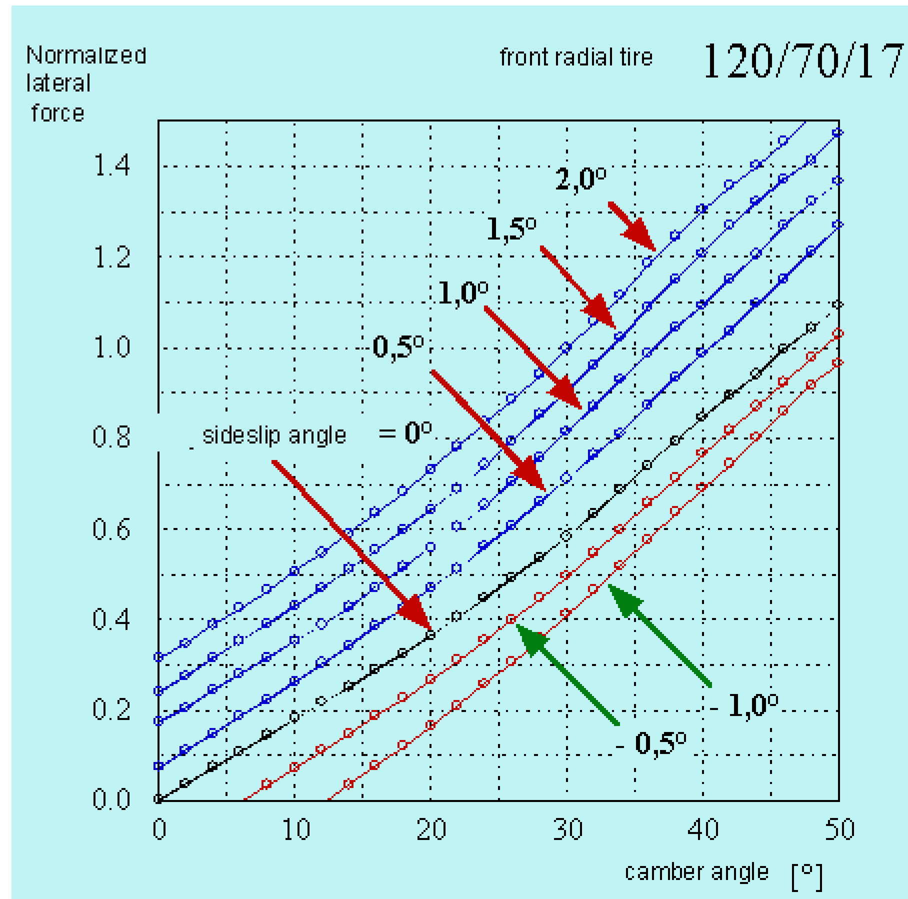
- For large angles, both phenomena interfere and a saturation is observed



Camber thrust and slip angles
Milliken Fig 2.24

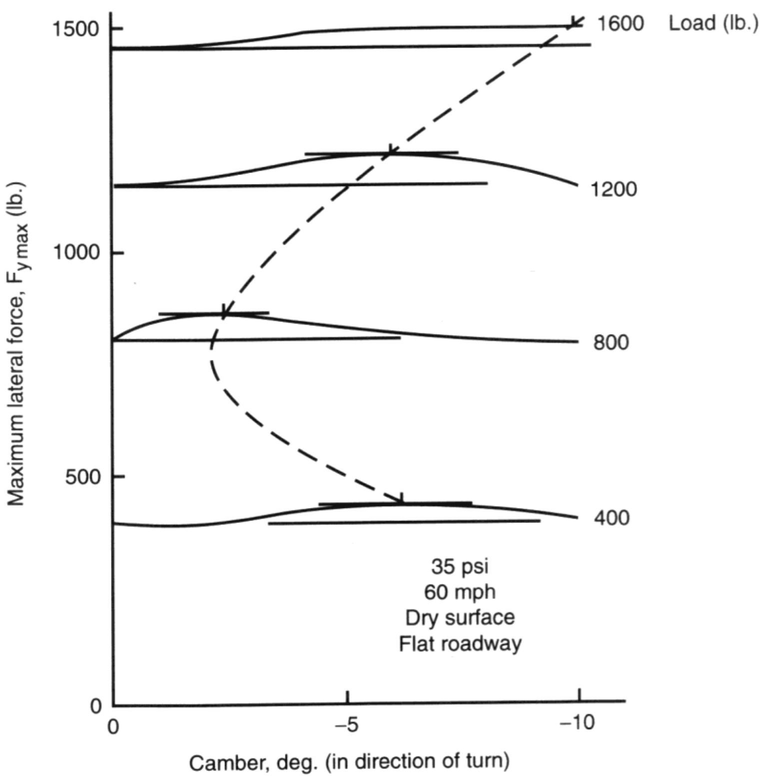
Camber thrust



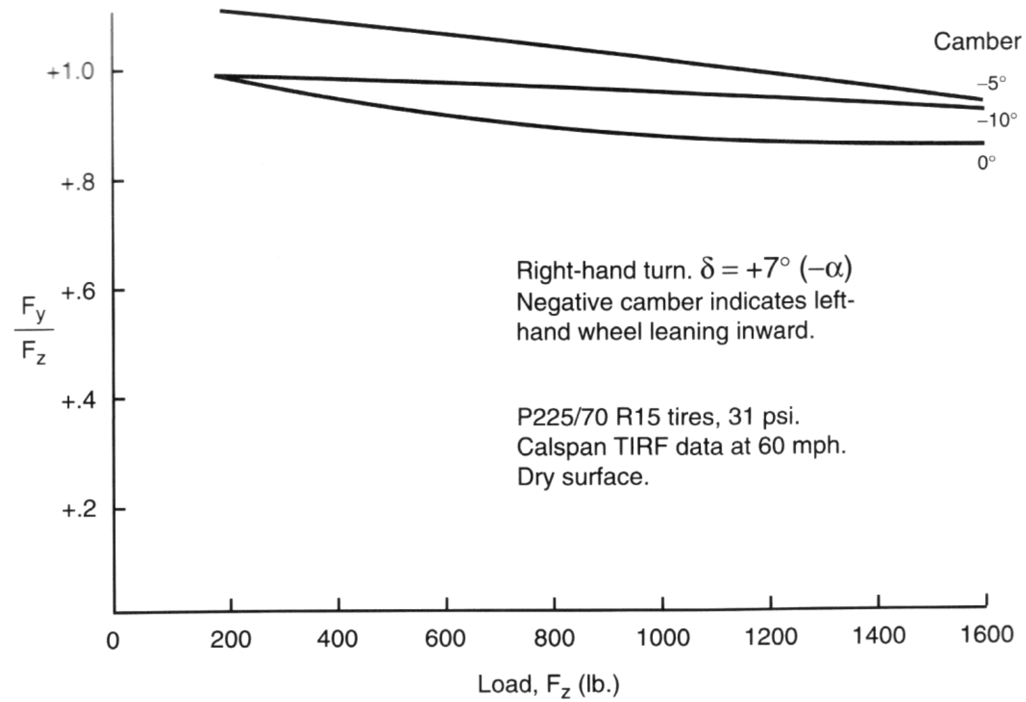


Camber thrust for a motorbike tyre

Influence of normal force on camber thrust



Milliken Fig 2.26 Influence of vertical load on optimized camber angle.



Milliken Fig 2.30 Influence of vertical force on lateral forces for different camber angles.

Aligning torque due to camber

- Generally, there is no significant effect of camber thrust onto aligning moment.
  - Because of the symmetric pattern of the shear forces
- If any, camber tends to destabilize the drift angle
- It always requires a mechanical trail

Combined operations

COMBINED OPERATIONS

- Combined Operations results from simultaneous effects of
  - Longitudinal forces (braking / tractive)

  AND
  - Lateral forces (cornering)

Definition of longitudinal sliding rate

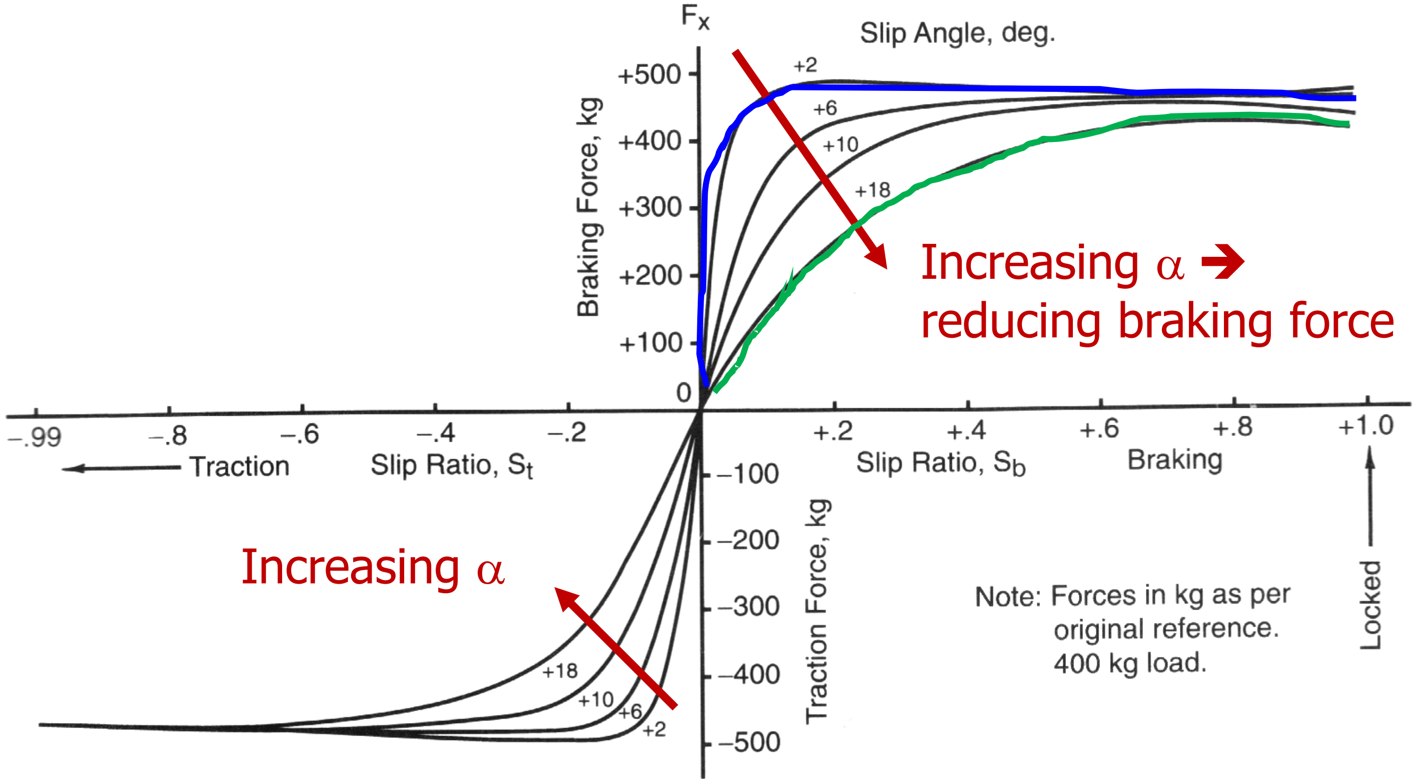
- Definition by SAE (SAE J670):
  - $S = (\Omega R_e / V \cos\alpha) - 1$
  - R_e effective rolling radius in free rolling conditions (zero slip angle)
- Definition by Calspan TIRF (for testing machines):
  - $SR = (\Omega R_l / V \cos\alpha) - 1$
  - R_l loaded radius (measured between the belt and the ground)
- Pacejka:
  - Practical slip quantity $K_x = (\Omega R_e / V \cos\alpha) - 1$
  - Independent slip quantity : $\sigma_x = (V \cos\alpha / \Omega R_e) - 1$
- Sakaï (JSAE):
  - Tractive $S_t = (V \cos\alpha / \Omega R_e) - 1$
  - Braking $S_b = (\Omega Re / V \cos\alpha) - 1$

Combined Operations

- Sakaï experiment
  - Japan Automotive Research Institute (JARI)
  - One of the few complete sets of results to be published in public domain
  - Small tires for passenger car with a moderate load of 400 kg (882 lb) and a vehicle speed of 20 km/h (12.4 mph)

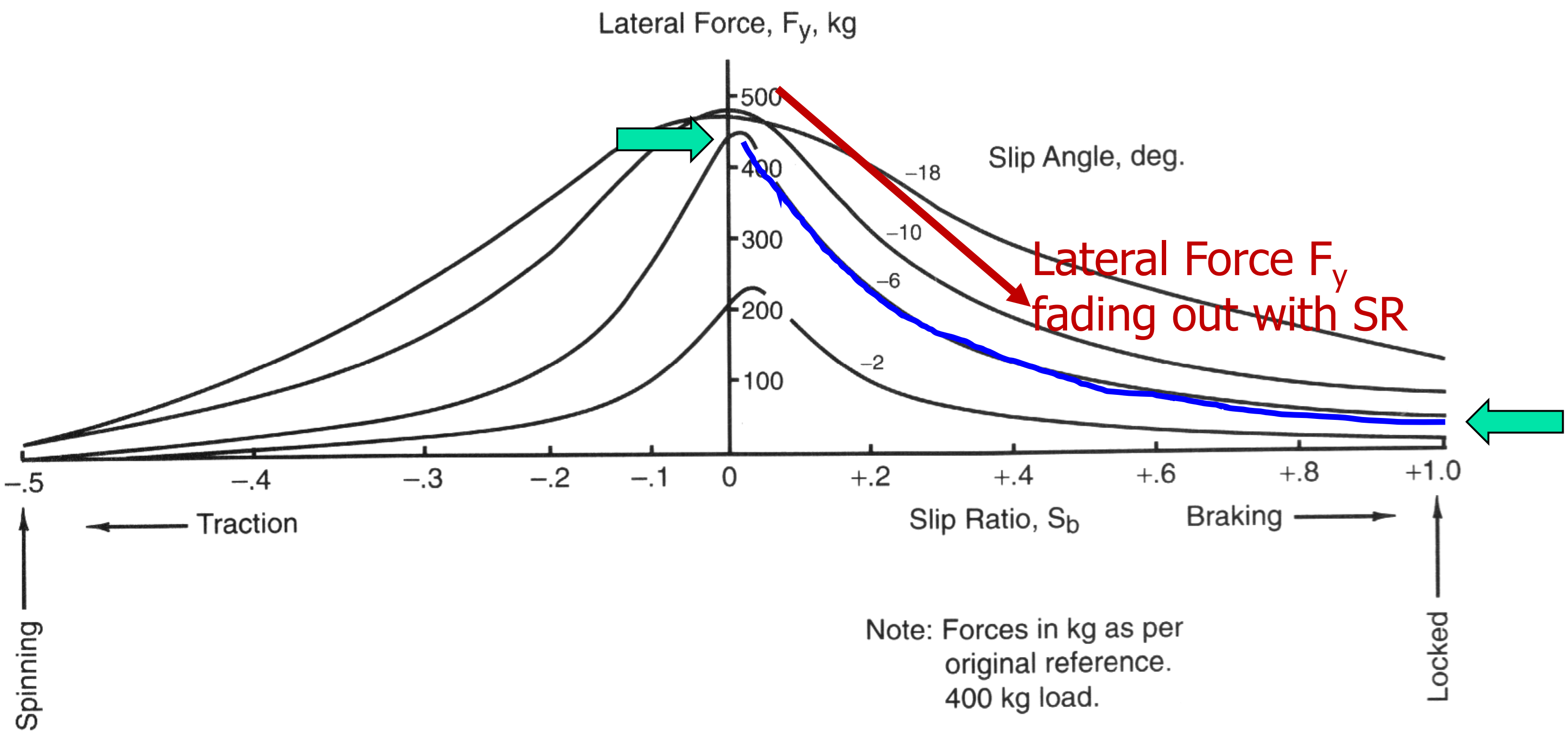
- Definition of longitudinal slide slip
  - Traction $S_t = (V \cos\alpha / \Omega R_e) - 1$
  - Braking $S_b = (\Omega Re / V \cos\alpha) - 1$
  - Free rolling $S_t = S_b = 0$
  - Wheel blocked while braking $S_b = -1$
  - Spinning wheel (acceleration) $S_t = -½$
  - Blocked wheel (acceleration) $S_t = -1$

Sakaï experiment



Milliken Fig 2.18 Braking/ Tractive forces vs sliding rate and side slip angle

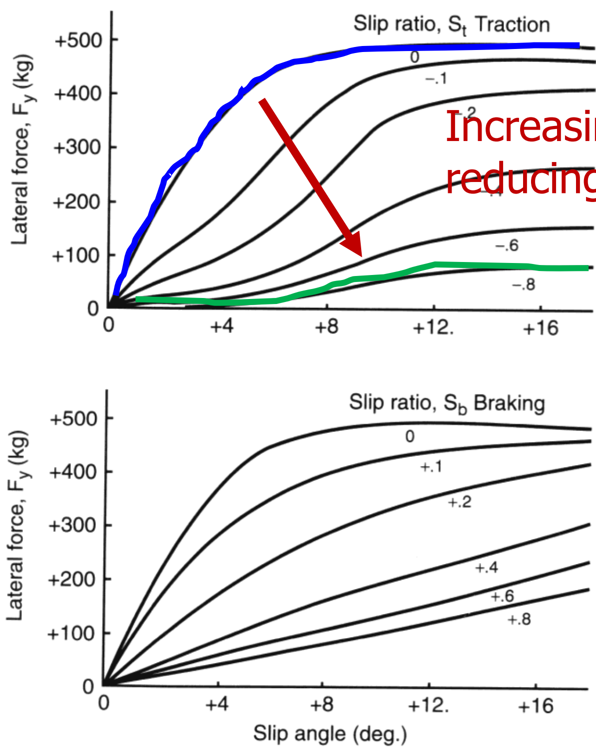
Sakaï experiment



Lateral forces vs side slip and longitudinal slip
From Milliken Fig 2.19

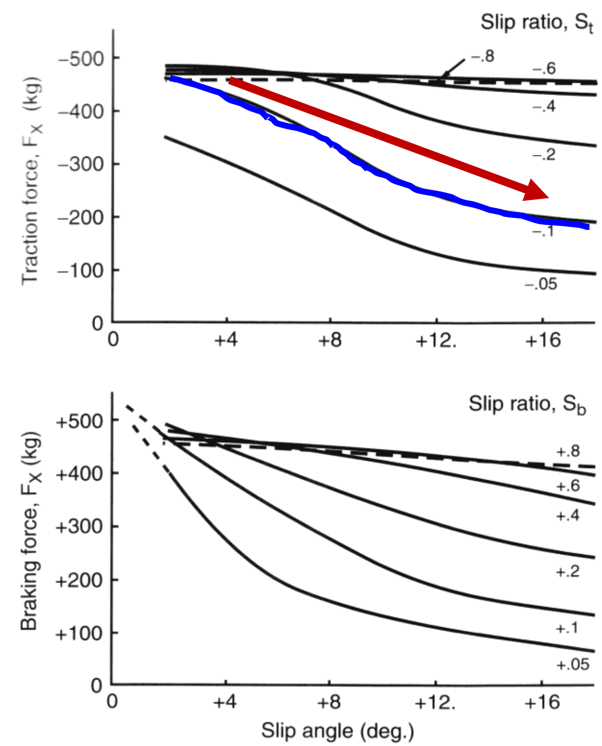
Sakaï experiment

Tractive Force F_x weakening with α



Notes: Force in kg as per original reference.
Lateral force is normal to wheel plane.
400 kg load.

Effect of longitudinal slip ratio upon lateral force curve. Milliken Fig 2.20



Notes: Longitudinal force in kg as per original reference.
400 kg load

Effect of side slip and longitudinal rate onto tractive and braking forces. Milliken Fig 2.21

Force magnitude v.s. effective sliding

- The tyre does feel only the sliding speed and the relative longitudinal slip ➔ reconciliation and unification of the concepts of longitudinal slip ratio and side slip
- The tyre does feel only the relative velocity between the ground and the tyre.
- Speed :

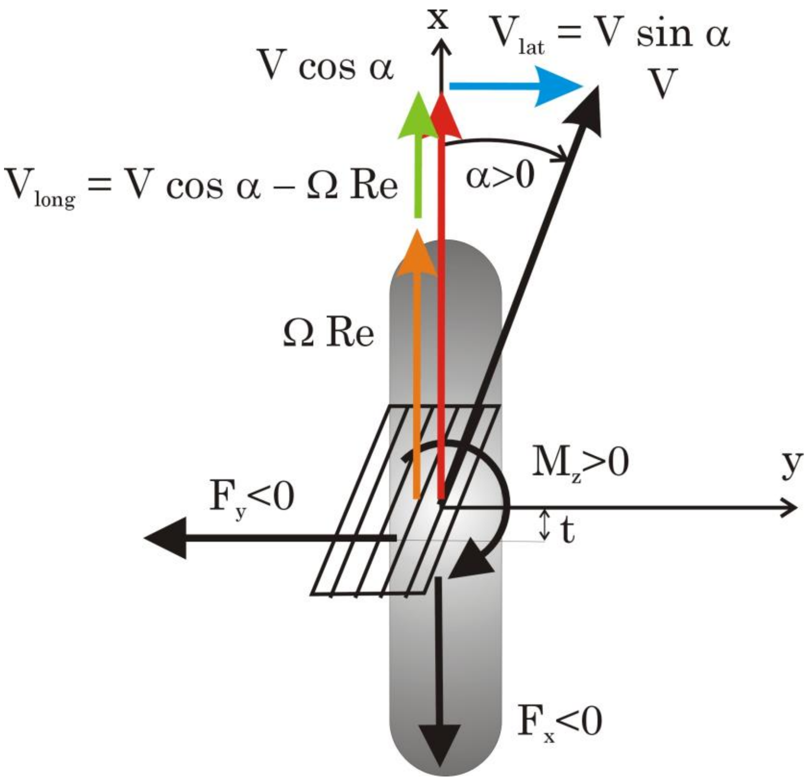
$$v_{lat} = V \sin\alpha$$

$$v_{long} = V\cos\alpha - \Omega R_e$$

$$v_{res} = \sqrt{v_{long}^2 + v_{lat}^2}$$

- Force :

$$F_{res} = \sqrt{F_x^2 + F_y^2}$$



Force magnitude v.s. effective sliding

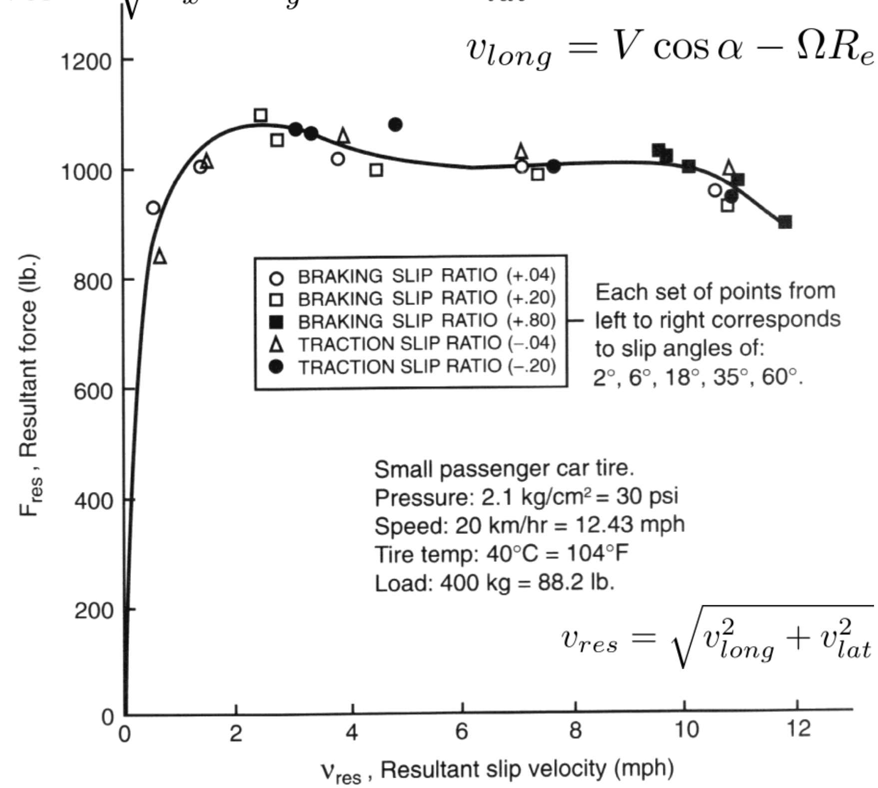
- Using the unified concept of relative velocity and the resulting generated force

- One gests a unique force slip velocity curve
- Conclusion: both longitudinal and cornering forces have a common origin: the deformation of the contact patch
- There is a strong interaction between longitudinal and lateral forces !
- This interaction is summarized by the concept of friction circle

$$F_{res} = \sqrt{F_x^2 + F_y^2}$$

$$v_{lat} = V \sin\alpha$$

$$v_{long} = V\cos\alpha - \Omega R_e$$



$$v_{res} = \sqrt{v_{long}^2 + v_{lat}^2}$$

Resultant force v.s. generalized sliding rate
Milliken Fig 2.22

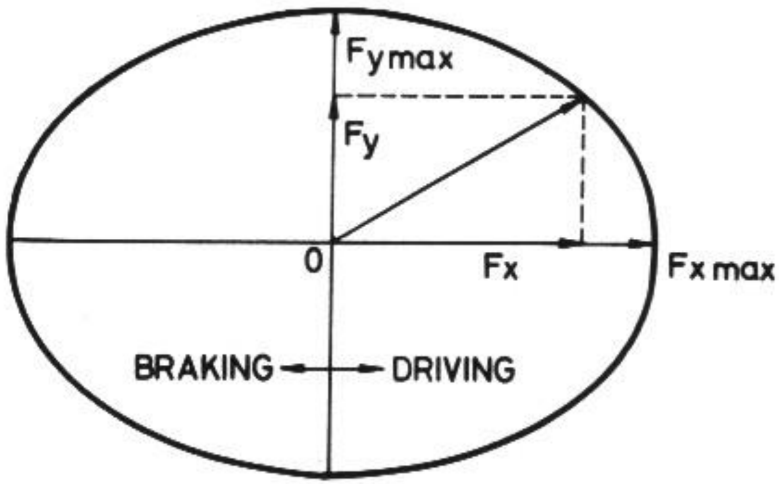
Friction circle (ellipsis)

- Goal : to combine in a single diagram all relevant information given in longitudinal and lateral forces as an <u>implicit function of the slip angles and longitudinal slip ratio</u>.

- Motivation: Visualization of the resulting force whose both longitudinal and lateral components are limited by the tire capability to generate adherence force.

Friction circle (ellipsis)

- The friction circle represents the limit that a tyre can ever develop under given operational conditions (charge, temperature, ground texture…)

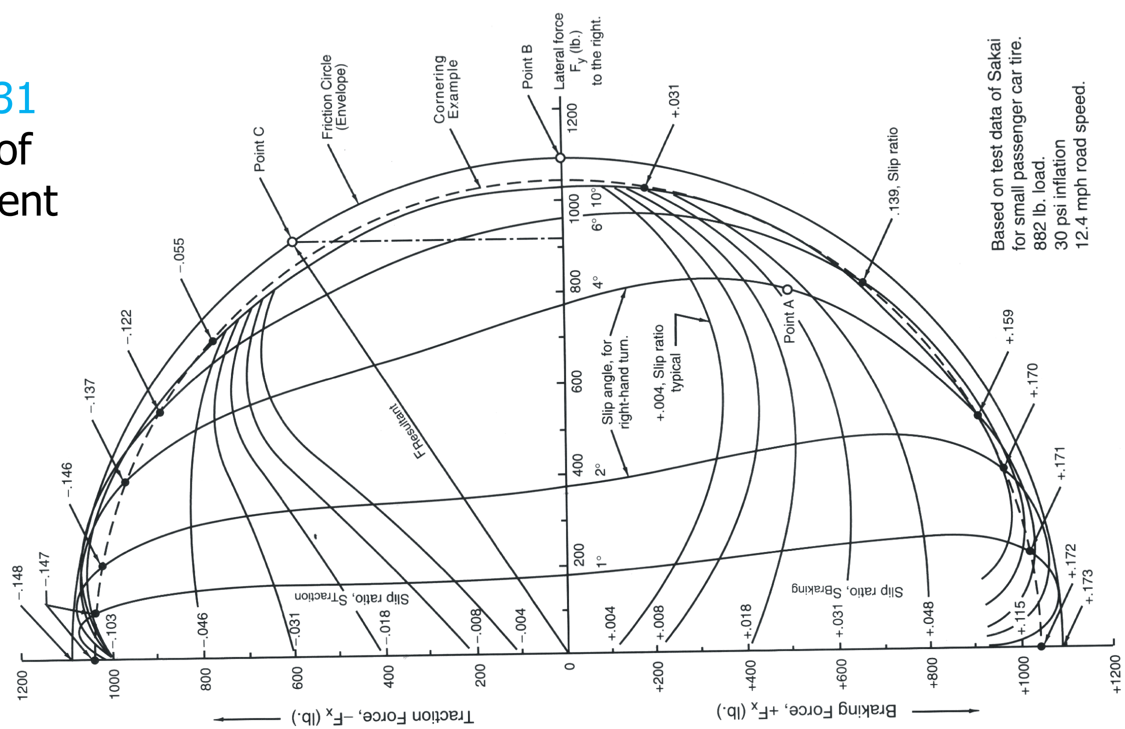


Concept of friction circle (ellipsis) for a given tyre
Wong Fig. 1.35

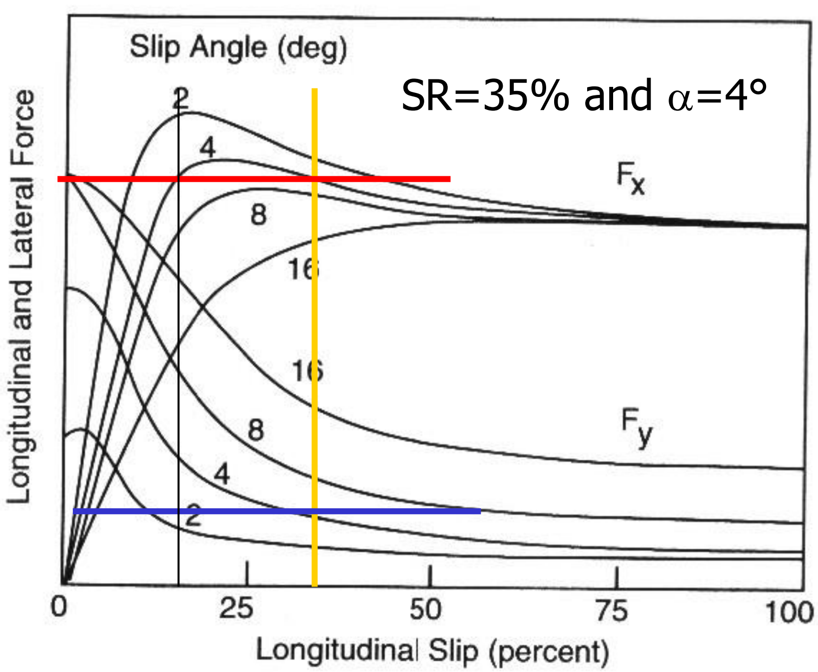
Friction circle

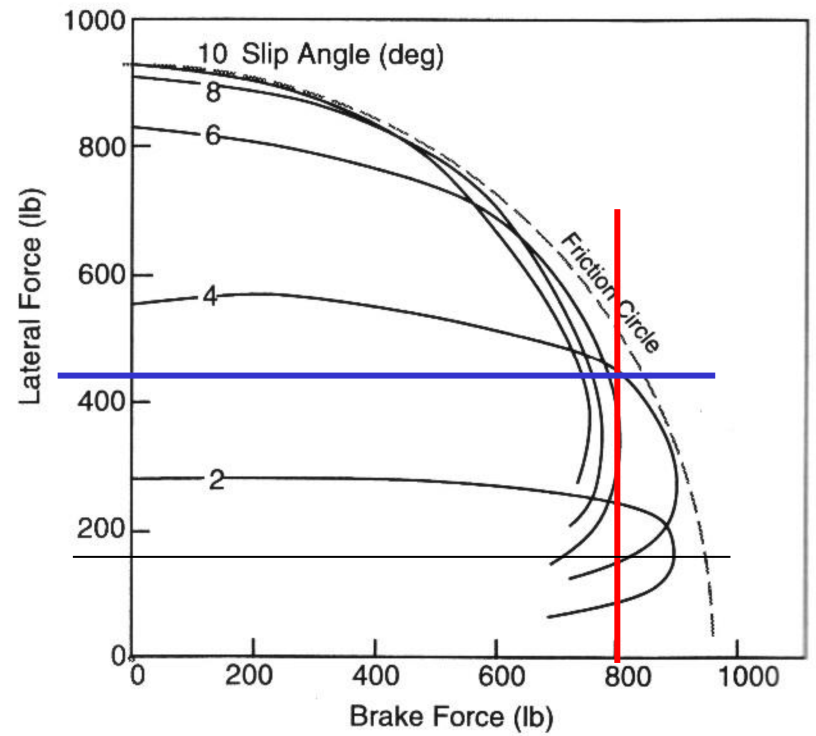
- Friction circle is built as the envelop of all feasible points (F_x, F_y) that can developed by the tyre under a set of longitudinal slip ratio SR=λ and side slip angle α.

Milliken Fig 2.31
Friction circle of
Sakaï experiment



Friction circle (ellipsis)





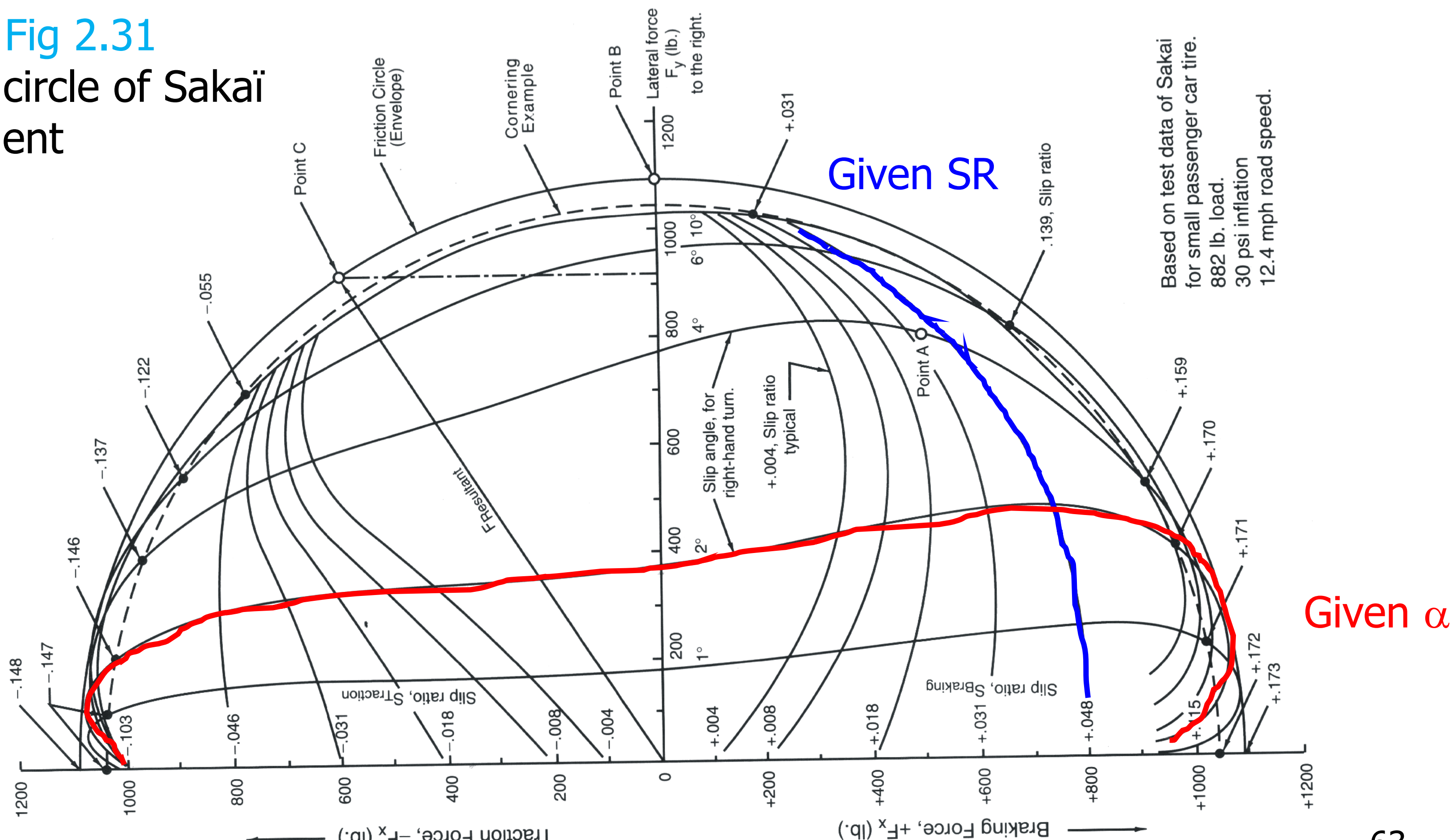
Braking forces and Lateral forces as a function of the sliding parameter from Gillespie Fig. 10.22

Lateral force as a function of the braking force and of the sliding rate from Gillespie Fig. 10.23

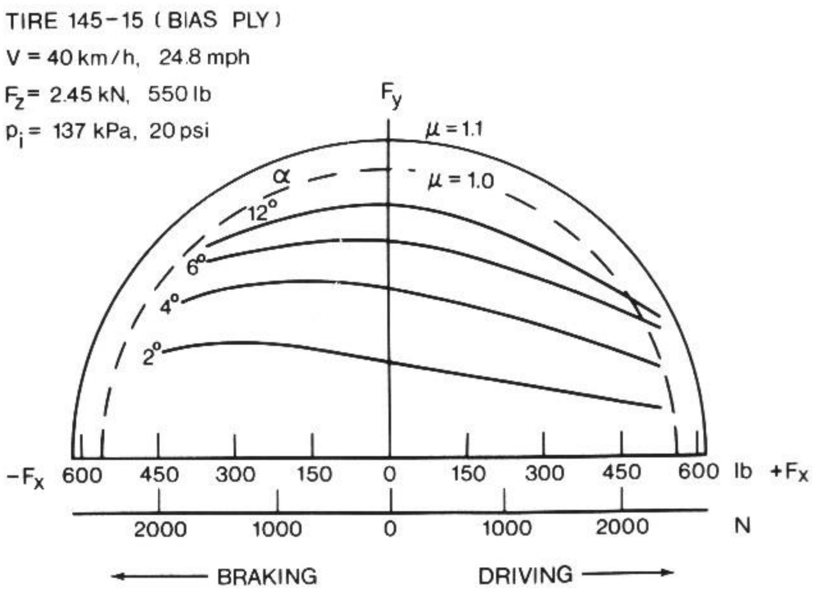
Friction circle (ellipsis)

Milliken Fig 2.31

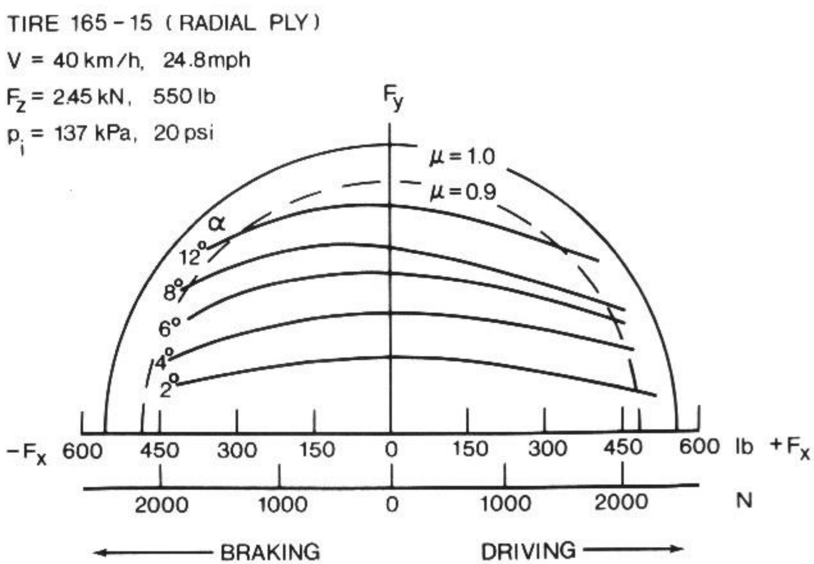
Friction circle of Sakaï experiment



Friction circle (ellipsis)

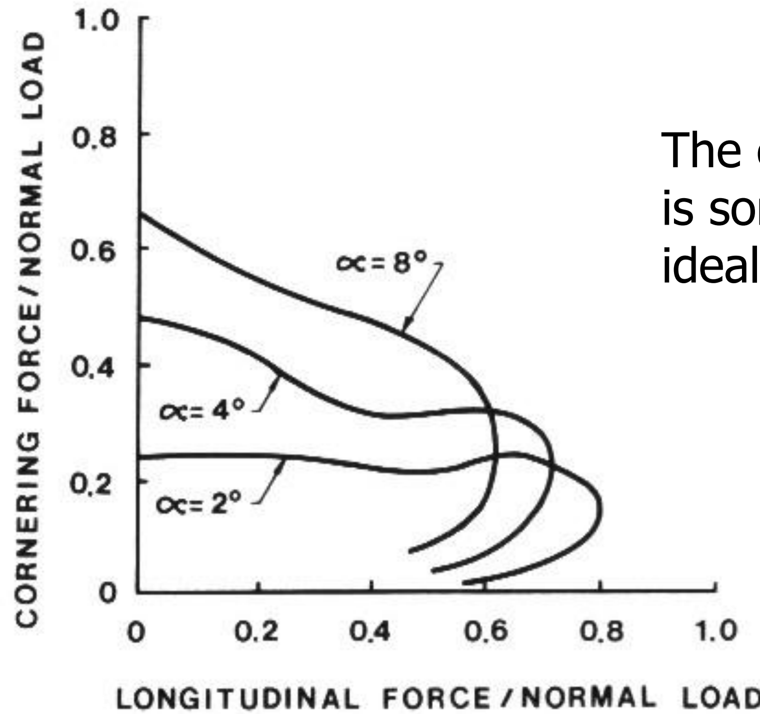


Friction circle for a diagonal tyre
Wong Fig. 1.33



Friction circle for radial tyres
Wong Fig. 1.33

Friction circle (ellipsis)



The concept of circle is sometimes is idealization!

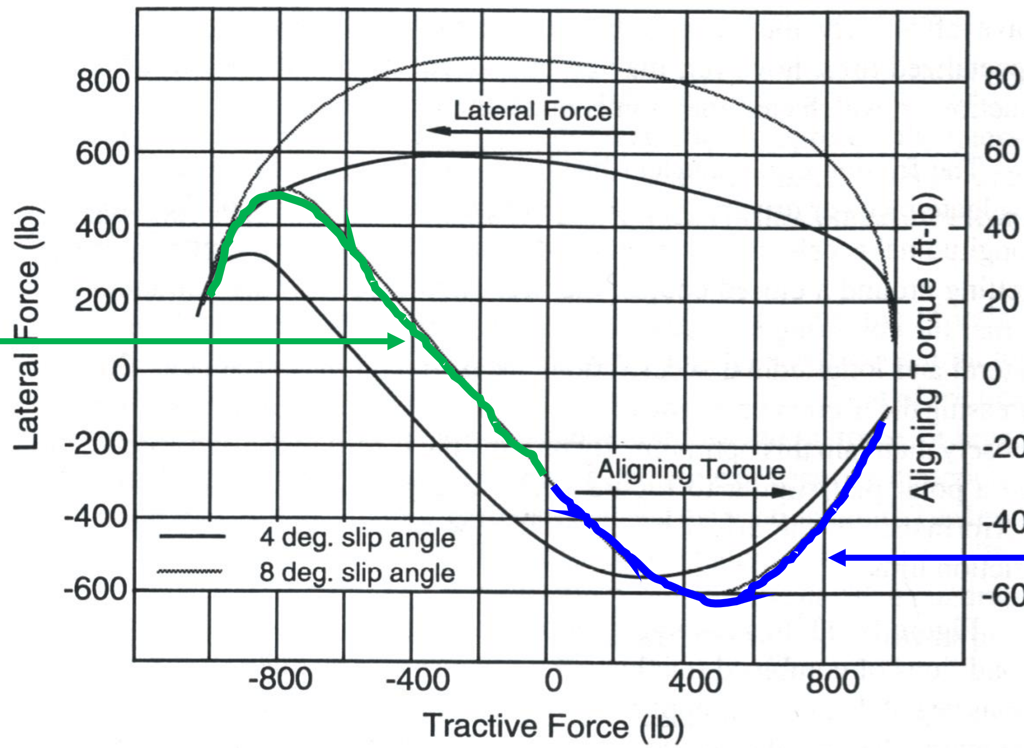
Friction circle for a truck tyre
Wong Fig. 1.34

Friction circle for Self Aligning Moment

- As the cornering force and the tractive/braking forces are coupled, it is the same for the self aligning moment.
- However, the coupling is here more complex because of the influence of the lateral and longitudinal displacement of the centre point of the tire contact patch.
- The effects superimpose to each other to explain the observed curves of the self aligning moment with respect to the tractive / braking forces.

Friction circle (ellipsis)

Under braking forces, the self aligning moment is reduced and becomes destabilizing before vanishing because of the overall friction

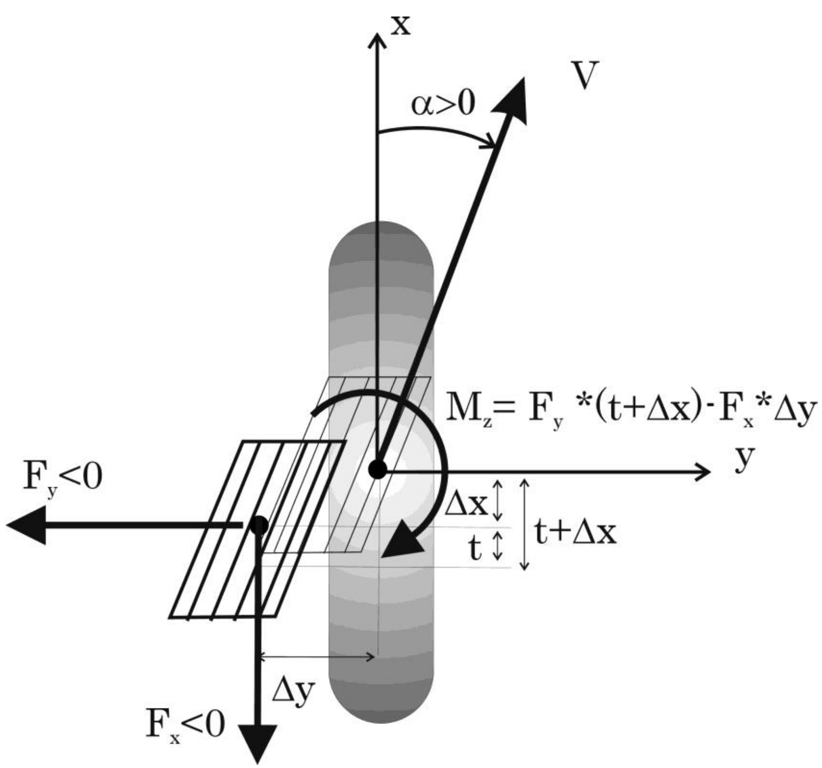


Tractive force reinforces first the self aligning before weakening it due to massive friction in the contact patch

Lateral forces and yaw moment in terms of the tractive / braking force.

Gillespie Fig. 10.24

Friction circle (ellipsis)

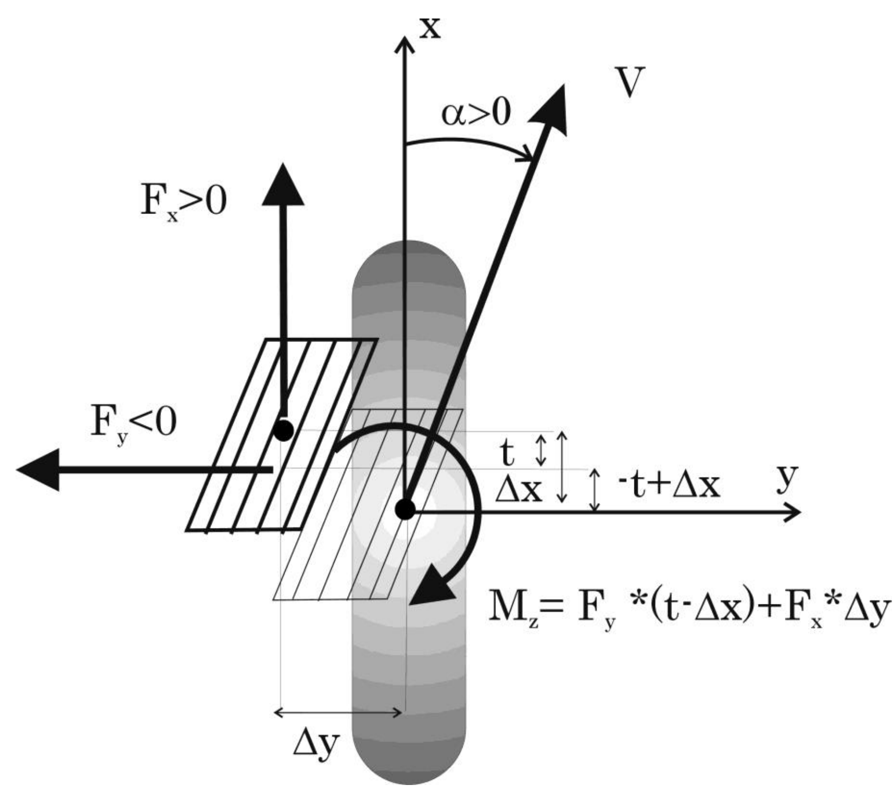


Lateral forces and yaw moment in terms of the braking force.

- When applying braking forces, the lateral displacement of the contact patch and the applied longitudinal force create an additional moment that acts against the self aligning moment related to the pneumatic trail.
- For large braking forces, the overall friction covers the contact patch, and the self aligning moment vanishes.

Friction circle (ellipsis)

- In presence of tractive forces, the lateral displacement of the contact patch leads to a moment that reinforces the self aligning moment.
- Massive friction accuring for large tractive forces gives rise to a uniform distribution and leading to a null yaw moment.



Lateral forces and yaw moment in terms of the tractive force.