MECA0525 : INTRODUCTION TO TIRE MECHANICS III:

Mathematical modelling of tire response curves

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Layout

- Introduction
- Tyre construction
- Classification: size, load and velocity indices
- Adhesion mechanisms
- Rolling resistance
- Generation of longitudinal forces
  - Brush model
  - Tractive and braking forces
  - Longitudinal slip ratio
  - Tractive and braking force curves

Layout (2)

- Lateral forces
  - Gough experiment
  - Lateral force as a function of the side slip angle
  - Cornering coefficient
  - Cornering stiffness
- Self aligning torque and pneumatic trail
- Camber thrust
  - Definition and mechanism
  - Camber coefficient
- Combined operations
  - Sakai experiment
  - Friction ellipse

- Modelling: Pacejka magic formula

Layout

- Pacejka magic formula
- Treatment of experimental data

References

- W. Milliken & D. Milliken. « Race Car Vehicle Dynamics », 1995, Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).

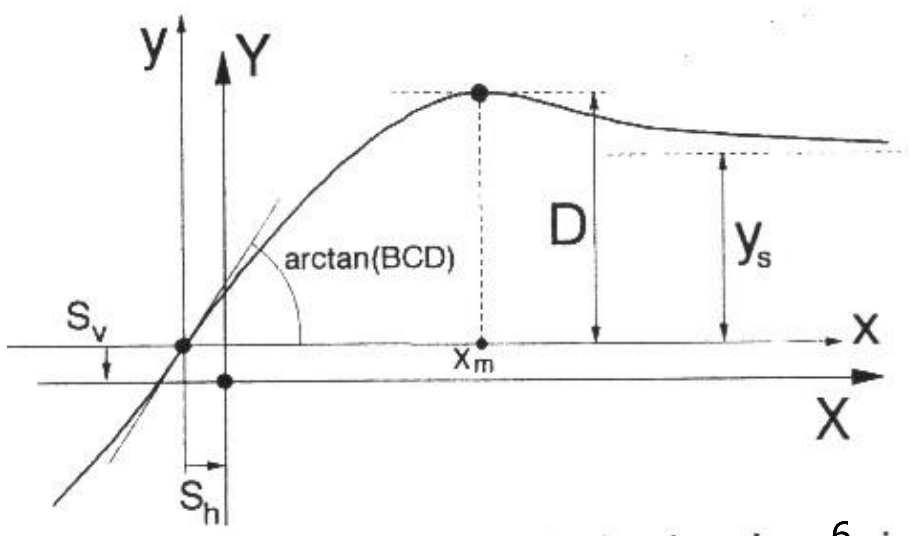
Pacejka magic formula

- For simple cases of pure (lateral) side slip and longitudinal slip, the magic formula by Pacejka can be used to describe the evolution curves of F_y , M_z and F_x as a function of the side slip angle α or the longitudinal slip rate κ.

$$y(x) = D \sin\left[C \arctan\{Bx - E\,(B\,x - \arctan(Bx))\}\right]$$

with

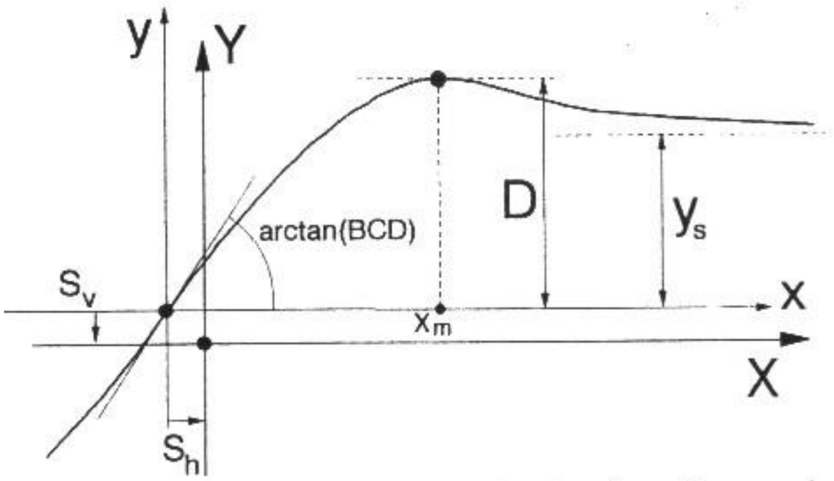
$$\begin{aligned} Y(X) &= y(x) + S_v \\ x &= X + S_h \end{aligned}$$



Pacejka magic formula

- One can provide the following interpretation of the coefficients of the magic formula

$$y(x) \;=\; D \,\sin\left[C\;\arctan\{Bx \;-\; E\;(B\,x - \arctan(Bx)\,)\,\}\,\right]$$



- BCD, the slope in zero

$$\left.\frac{d}{dx}y(x)\right|_{x=0} \;=\; BCD$$

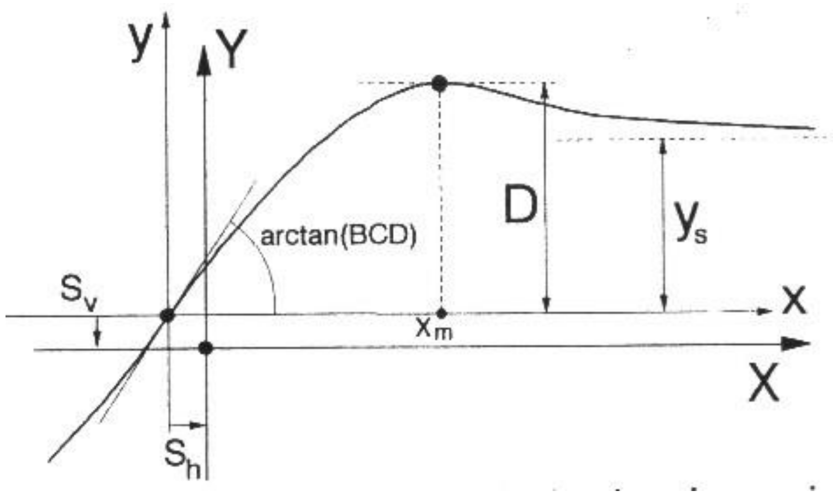
- D The maximum of the curve

$$x_m \;=\; \sup_x y(x) \quad \text{et} \quad D = \max_x y(x)$$

Pacejka magic formula

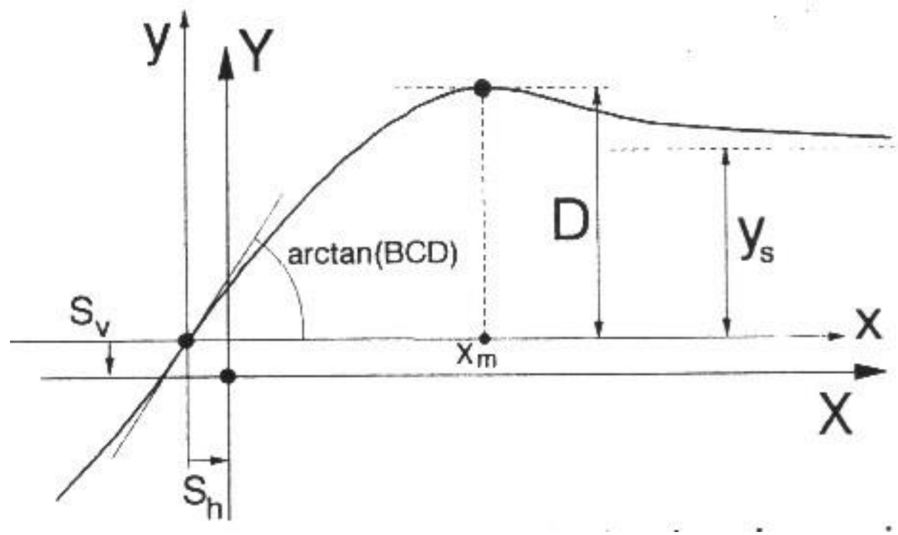
$$y(x) = D \sin\left[C \arctan\{Bx - E\,(B\,x - \arctan(Bx)\,)\,\}\right]$$

- The parameter C controls the bounds of the sinus argument. The parameter C rules the shape of the curve.
- Typical values of C:
  - C = 1.3 lateral force,
  - C = 2.4 self-aligning moment
  - C = 1.65 braking force



Pacejka magic formula

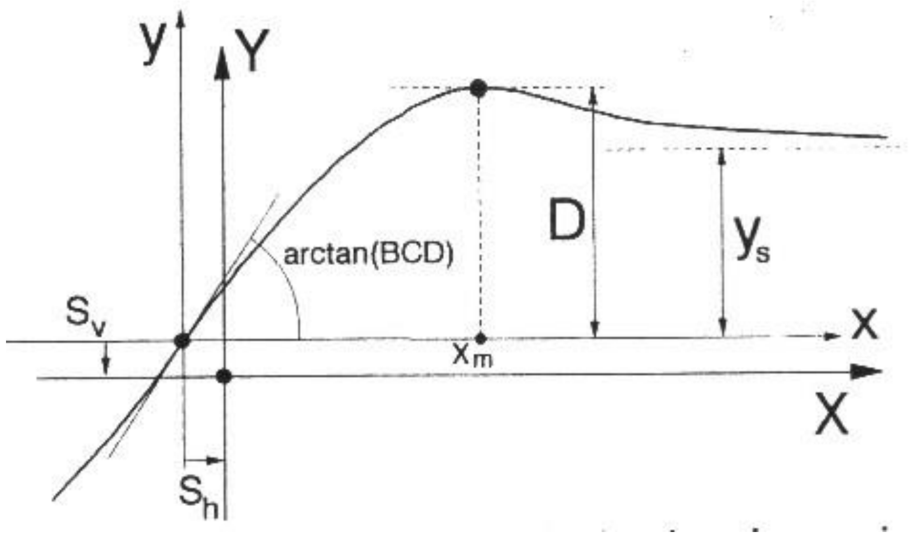
$$\frac{d}{dx}y(x)\bigg|_{x=0} = BCD$$



- The parameter B allows to adjust the slope at the origin, so that it is often denoted as the stiffness factor

Pacejka magic formula

$$E = \frac{Bx_m - \tan(\frac{\pi}{2C})}{Bx_m - \arctan(Bx_m)}$$



- The last paramter E enable to control the position of the maximum slipage x_m (if there is maximum in the curve)

Treatment of experimental data

- The key issue: to work with non dimensional numbers!
  - La lateral (cornering)
    $$\overline{F} = \frac{F_y}{\mu_y Z}$$
  - The self-aligning moment
    $$\overline{M_z} = \frac{M_z}{T_z \mu_y Z}$$
  - The overturning moment
    $$\overline{M_x} = \frac{M_x}{P_x \mu_y Z}$$
  - The tractive / braking force
    $$\overline{F}_x = \frac{F_x}{\mu_x Z}$$

Treatment of experimental data

- The side slip angle

$$\overline{\alpha} = \frac{C \tan \alpha}{\mu_y Z}$$

- The camber angle

$$\overline{\gamma} = \frac{G \sin \gamma}{\mu_y Z}$$

- The longitudinal slip ratio

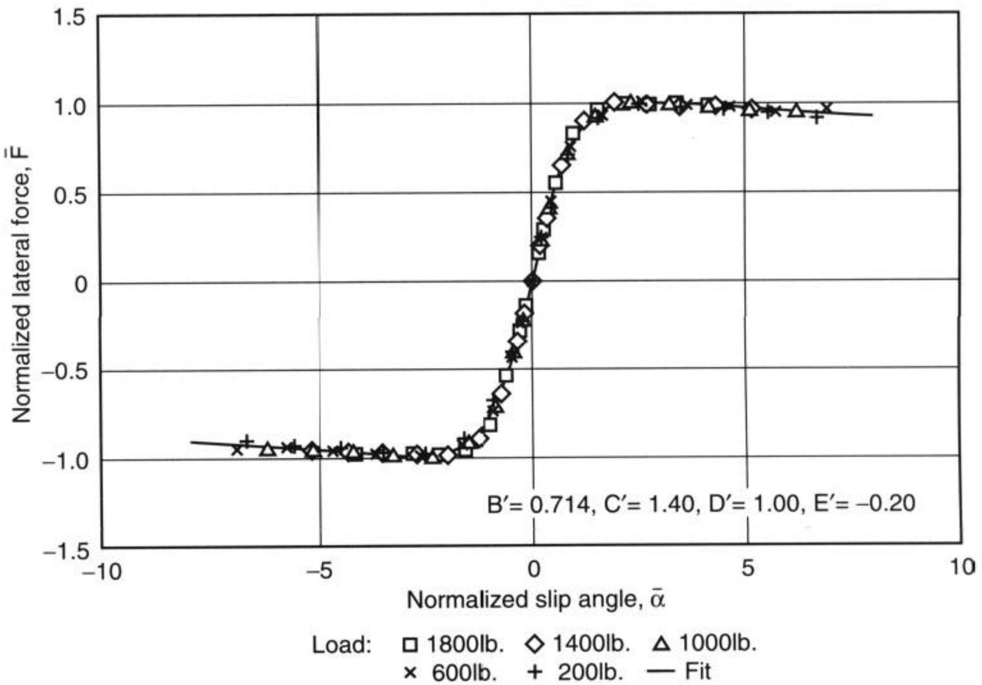
$$\overline{S} = \frac{k_x SR}{\mu_x Z}$$

$$SR = \frac{\Omega R_0 - V \cos \alpha}{V \cos \alpha}$$

Treatment of experimental data

EXAMPLE:
Tire P 195/70 R 14

Normalized lateral forces vs
Normalized side slip angle
Milliken Fig 14.1

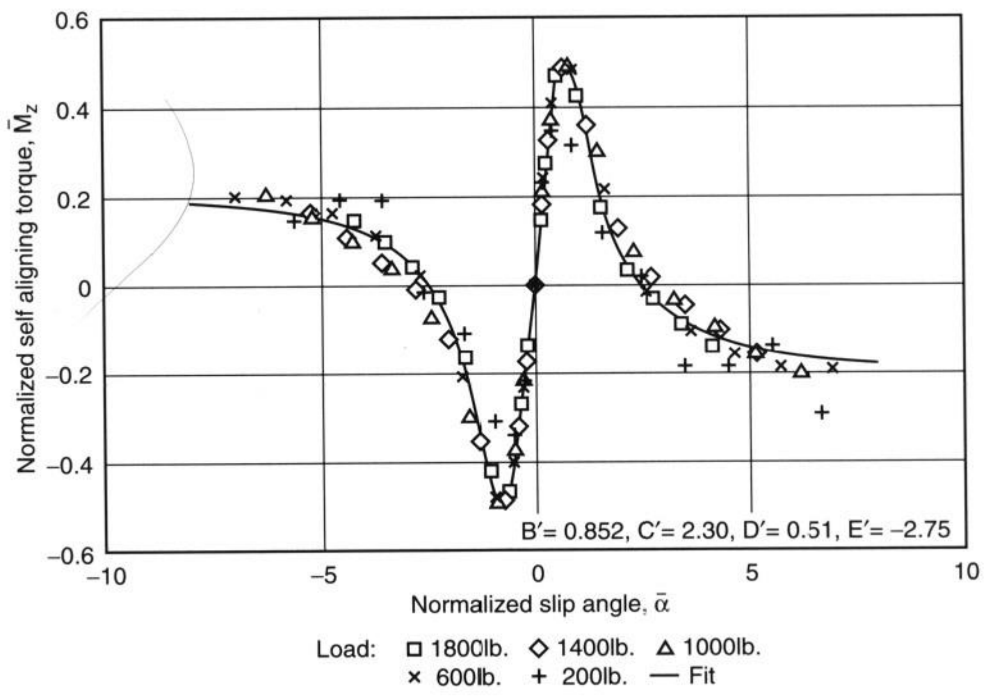


$$\bar{F} = D' \sin\theta$$
$$\theta = C' \arctan(B'\phi)$$
$$\phi = (1 - E')\,\bar{\alpha} + (E'/B')\,\arctan(B'\,\bar{\alpha})$$

Treatment of experimental data

EXAMPLE:
Tire P 195/70 R 14

Self-aligning Moment
vs
Normalized side slip angle
Milliken Fig 14.2

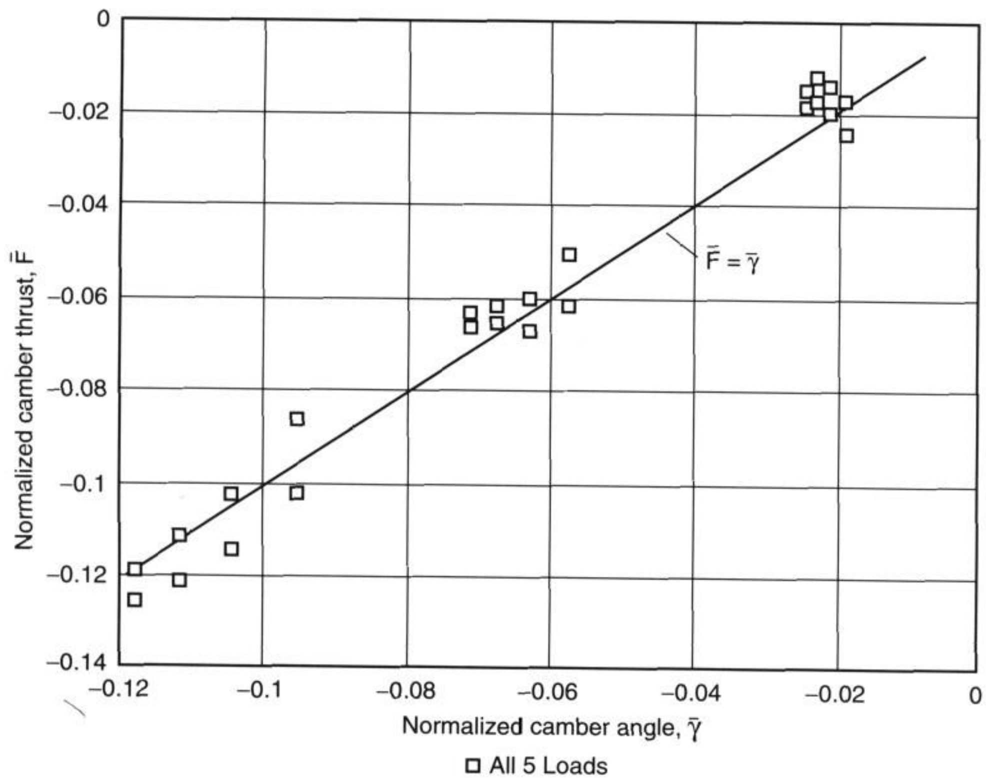


$$
\begin{aligned}
\bar{F} &= D' \sin\theta \\
\theta &= C' \arctan(B'\phi) \\
\phi &= (1 - E')\,\bar{\alpha} \;+\; (E'/B')\ \arctan(B'\,\bar{\alpha})
\end{aligned}
$$

Treatment of experimental data

EXAMPLE:
Tire P 195/70 R 14

Normalized Camber Thrust
vs
Normalized camber angle
Milliken Fig 14.3



Treatment of experimental data

EXAMPLE:
Tire P 195/70 R 14

Normalized Tractive Force
vs
Normalized longitudinal slip ratio
Milliken Fig 14.4