MECA0525 : Vehicle dynamics

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Lesson 1:
Steady State Cornering

Bibliography

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- G. Genta. «Motor vehicle dynamics: Modelling and Simulation ». Series on Advances in Mathematics for Applied Sciences - Vol. 43. World Scientific. 1997.

INTRODUCTION TO HANDLING

Introduction to vehicle dynamics

- Introduction to vehicle handling
- Vehicle axes system

- Tire mechanics & cornering properties of tires
  - Terminology and axis system
  - Lateral forces and sideslip angles
  - Aligning moment

- Single track model
- Low speed cornering
  - Ackerman theory
  - Ackerman-Jeantaud theory

Introduction to vehicle dynamics

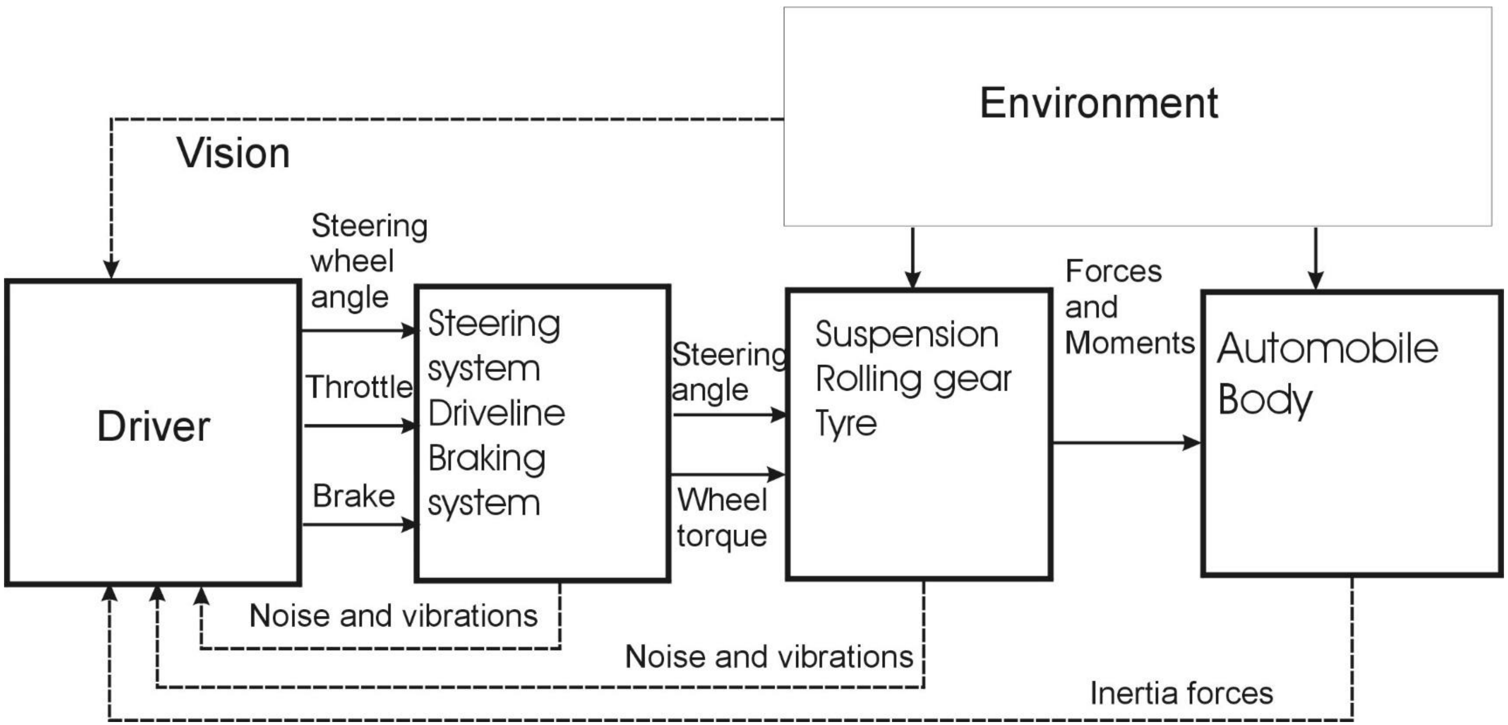
- High speed steady state cornering
  - Equilibrium equations of the vehicle
    - Gratzmüller equality
  - Compatibility equations
  - Steering angle as a function of the speed
  - Neutral, understeer and oversteer behaviour
  - Critical and characteristic speeds
  - Lateral acceleration gain and yaw speed gain
  - Drift angle of the vehicle
  - Static margin

- Exercises

Introduction

- In the past, but still nowadays, the understeer and oversteer character dominated the stability and controllability considerations
- This is an important factor, but it is not the sole one…
- In practice, one should consider the whole closed loop system vehicle + driver
  - Driver = intelligence
  - Vehicle = plant system creating the manoeuvring forces
- The behaviour of the closed-loop system is referred as the « *handling* », which can be roughly understood as the road holding

Introduction



Model of the system vehicle + driver

Introduction

- However because of the difficulty to characterize the driver, it is usual to study the vehicle alone as an open loop system.

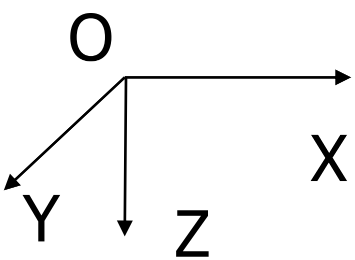
- Open loop refers to the vehicle responses with respect to specific steering inputs. It is more precisely defined as the ‘directional response’ behaviour.

- The most commonly used measure of open-loop response is the understeer gradient

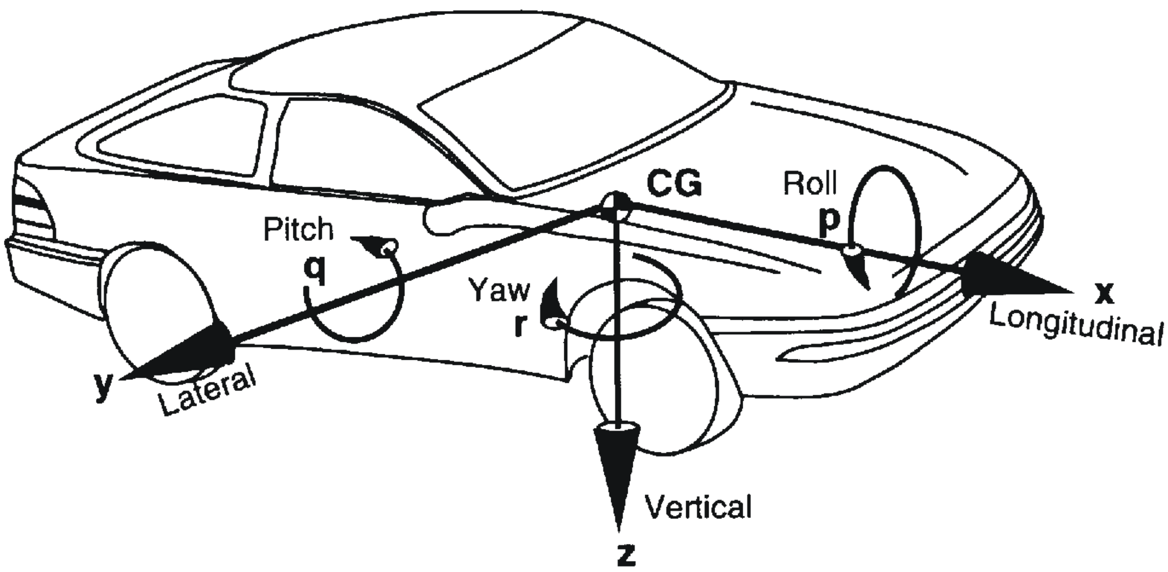
- The understeer gradient is a performance measure under steady-state conditions although it is also used to infer performance properties under non steady state conditions

AXES SYSTEM

Reference frames



Inertial coordinate system OXYZ



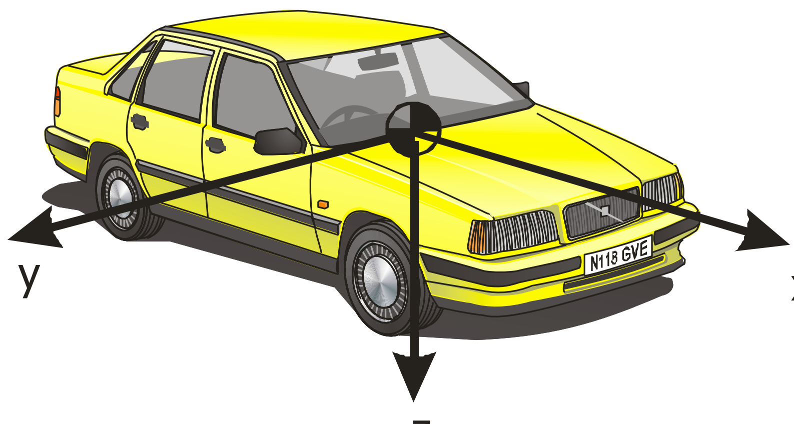
Local reference frame oxyz attached to the vehicle body - SAE (Gillespie, fig. 1.4)

Reference frames

- Inertial reference frame
  - X direction of initial displacement or reference direction
  - Y right side travel
  - Z towards downward vertical direction

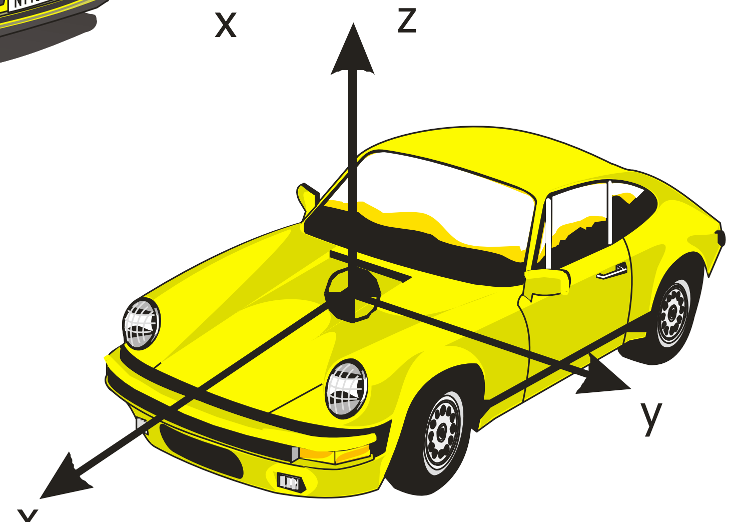
- Vehicle reference frame (SAE):
  - x along motion direction and vehicle symmetry plane
  - z pointing towards the center of the earth
  - y in the lateral direction on the right-hand side of the driver towards the downward vertical direction
  - o, origin at the center of mass

Reference frames



Système SAE

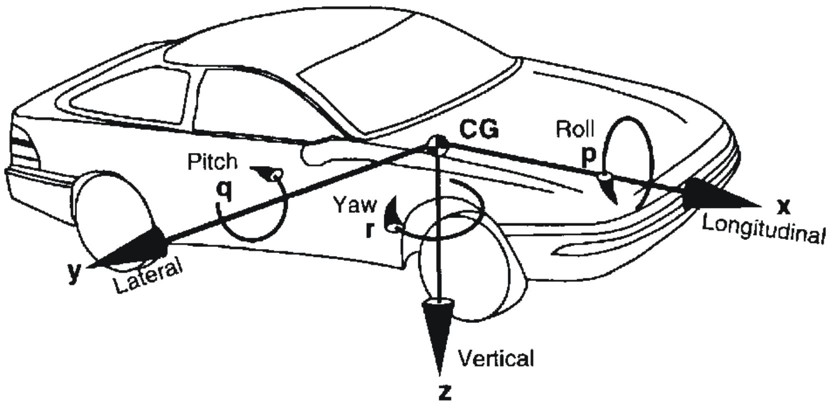
Comparison of conventions of SAE and ISO/DIN reference frames



Système ISO

Local velocity vectors

- Vehicle motion is often studied in car-body local systems
  - u : forward speed (+ if in front)
  - v : side speed (+ to the right)
  - w : vertical speed (+ downward)
  - p : rotation speed about x axis (roll speed) or ω_x.
  - q : rotation speed about y (pitch) or ω_y.
  - r : rotation speed about z (yaw) or ω_z.



Forces

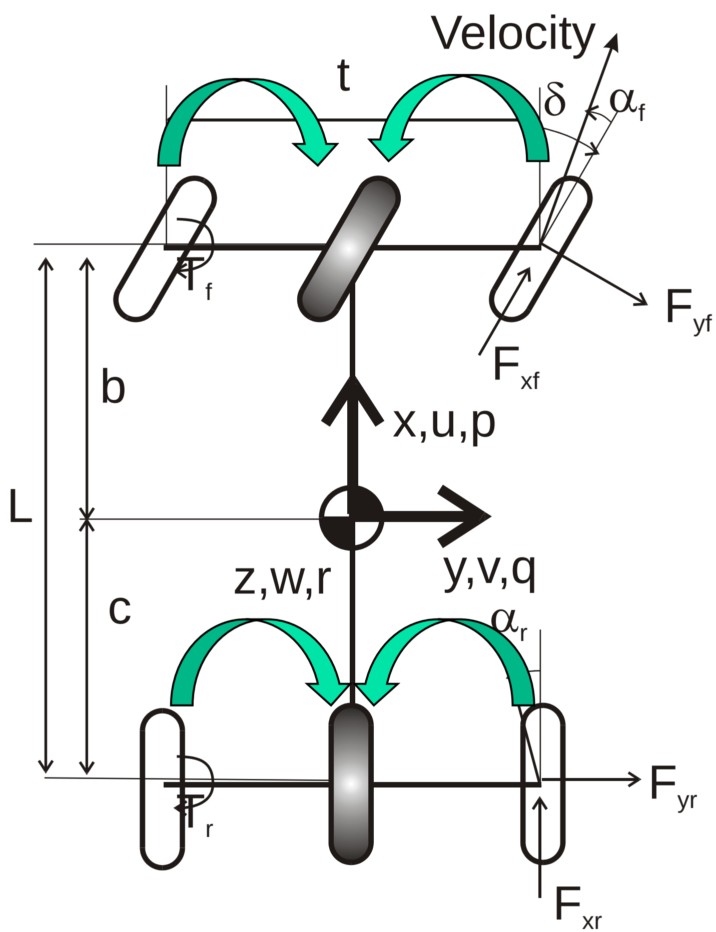
- *Forces and moments are accounted positively when acting onto the vehicle and in the positive direction with respect to the considered frame*

- Corollary
  - A positive F_x force propels the vehicle forward
  - The reaction force R_z of the ground onto the wheels is accounted negatively.

- Because of the inconveniency of this definition, the SAEJ670e « Vehicle Dynamics Terminology » names as normal force a force acting downward while vertical forces are referring to upward forces

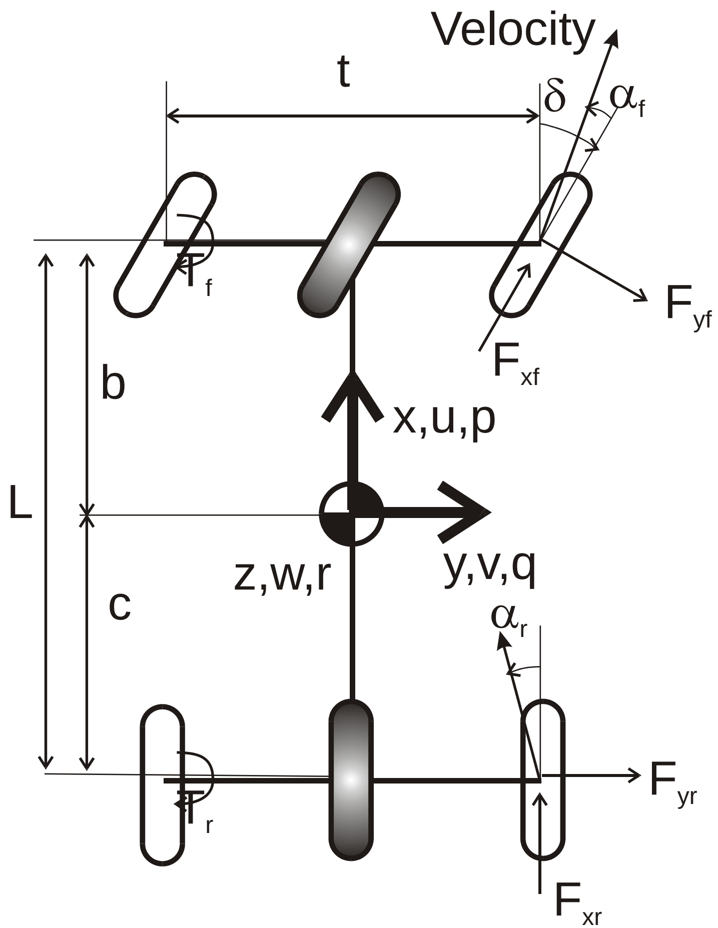
VEHICLE MODELING

The bicycle model



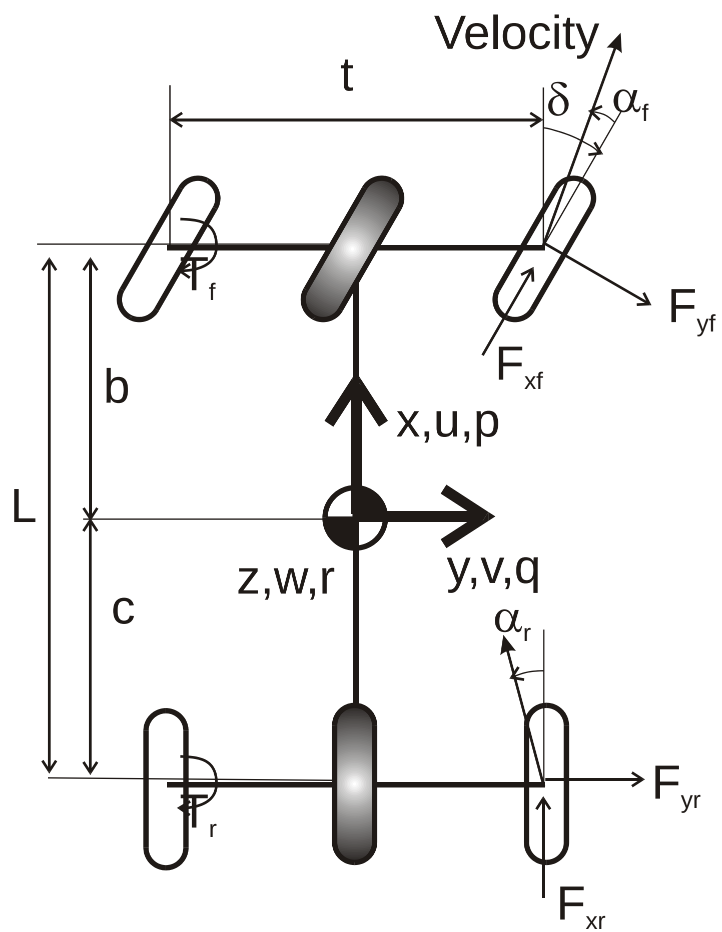
- When the behaviours of the left and right hand wheels are not that much different, one can model the vehicle as a single track vehicle known as the bicycle model or single track model.
- The bicycle model proved to be able to account for numerous properties of the dynamic and stability behaviour of vehicle under various conditions.

The bicycle model



- Geometrical data:
  - Wheel base: L
  - Distance from front (resp. rear) axle to CG: b (resp. c)
  - Track: t
- Tire variables
  - Sideslip angles of the front and rear tires: α_f and α_r
  - Steering angle (of front wheels) δ
  - Lateral forces developed under front and rear wheels respectively: F_{yf} and F_{yr}.
  - Longitudinal forces developed under front and rear wheel respectively: F_{xf} and F_{xr}.

The bicycle model



- Assumptions of the bicycle model
  - Negligible lateral load transfer
  - Negligible longitudinal load transfer
  - Negligible roll and pitch motion
  - The tires remain in linear regime
  - Constant forward velocity V
  - Aerodynamics effects are negligible
  - Control in position (whatever should be the control forces that are required)
  - No compliance effect of the suspensions and of the body

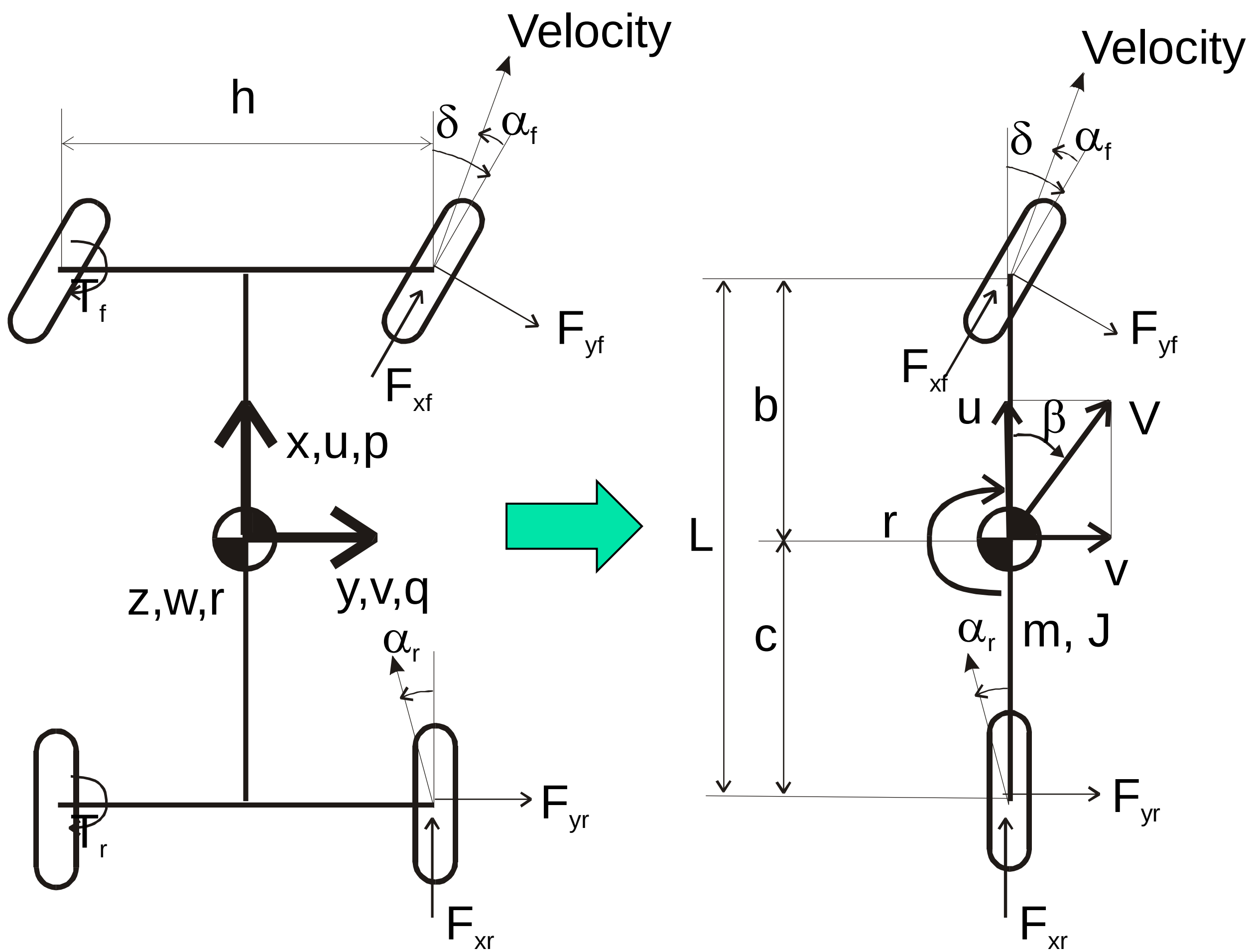
The bicycle model

- Remarks on the meaning of the assumptions
  - Linear regime is valid if lateral acceleration<0.4 g
    - Linear behaviour of the tire
    - Roll behaviour is negligible
    - Lateral load transfer is negligible
  - Small steering and slip angles, etc.
  - Smooth ground to neglect the suspension motion
  - Position control of the command : one can exert a given value of the input variable (e.g. steering system) independently of the control forces
  - The sole input considered here is the steering, but one could also add the braking and the acceleration pedal.

The bicycle model

- Assumptions :
  - Fixed: u = V = constant
  - No vertical motion: w=0
  - No roll p=0
  - No pitch q=0
- Bicycle model = 2 dof model :
  - $r=\omega_z$, yaw speed
  - v, lateral velocity or β, side slip of the vehicle $\tan\beta = \frac{v}{u} \simeq \frac{v}{V}$
- Vehicle parameters:
  - m, mass,
  - J_{zz} inertia about z axis
  - L, b, c wheel base and position of the CG

The bicycle model

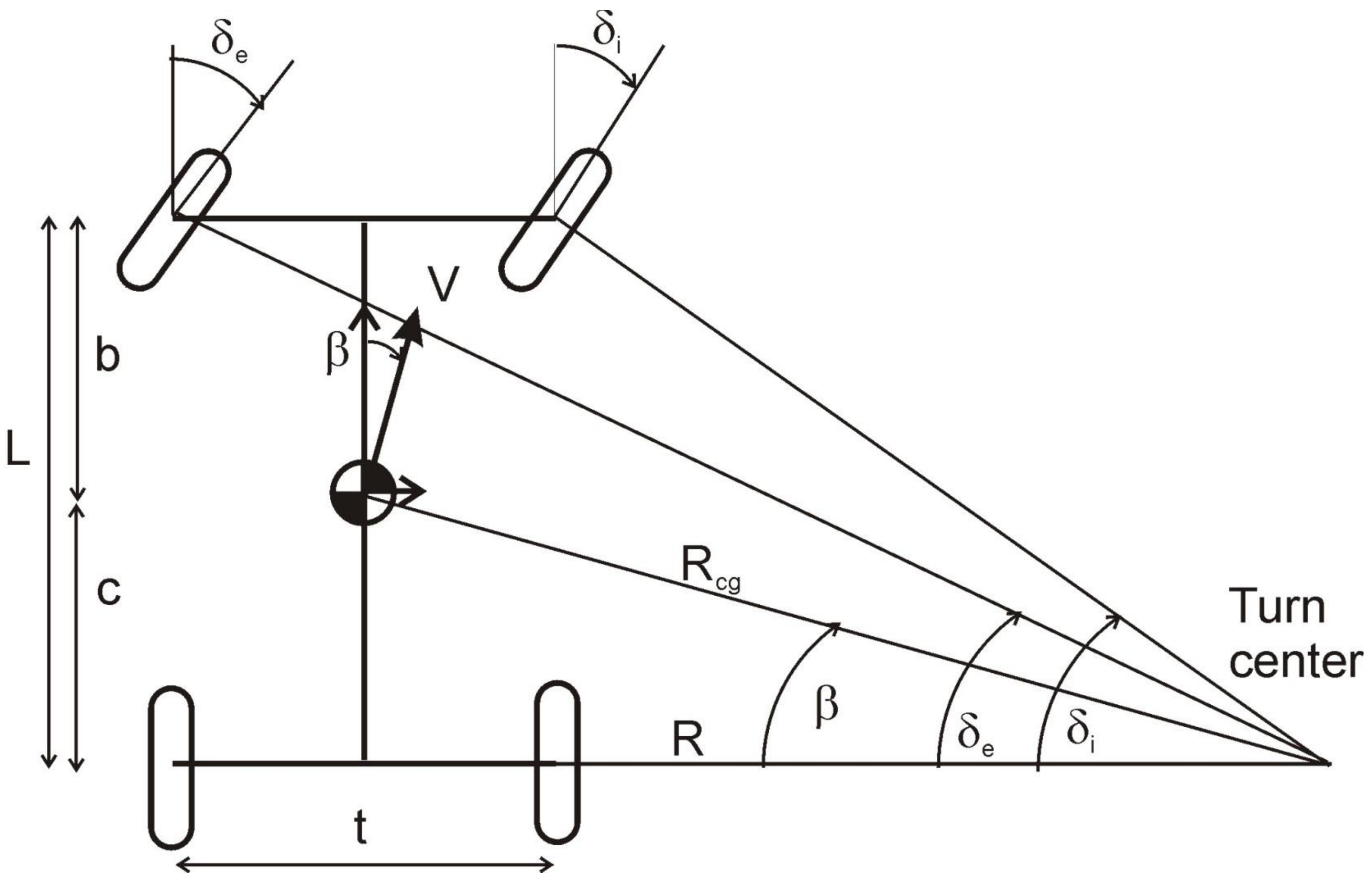


LOW SPEED TURNING

Low speed turning

- At low speed (parking manoeuvre for instance), the centrifugal accelerations are negligible and the tires have not to develop any lateral forces
- The turning is ruled by kinematics, that is the rolling motion of tires without (lateral) friction and without slip
- If the wheels experience no slippage, the instantaneous centres of rotation of the four wheels are coincident.
- The CIR is located on the perpendicular lines to the tire plan from the contact point
- In order that no tire experiences some scrub, the four perpendicular lines have to pass through the same point, that is the centre of the turn.

Ackerman-Jeantaud theory



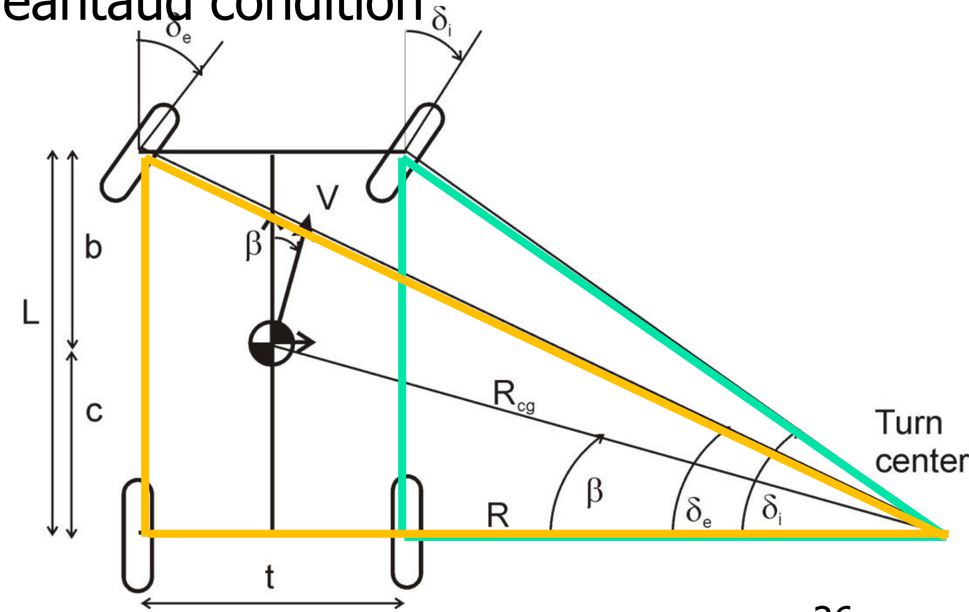
Ackerman-Jeantaud condition

- One can see that

$$\begin{aligned} \tan\delta_i &= L/(R - t/2) \\ \tan\delta_e &= L/(R + t/2) \end{aligned}$$

- This gives the Ackerman Jeantaud condition

$$\boxed{\cot\delta_e - \cot\delta_i = \frac{t}{L}}$$



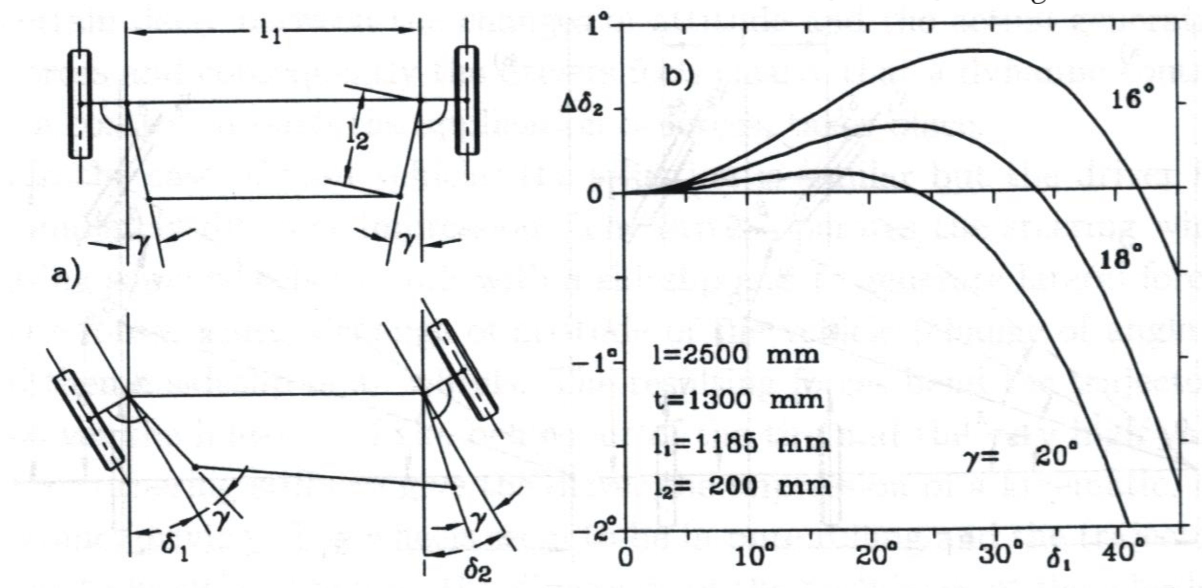
- Corollary

$$\delta_e \leq \delta_i$$

Ackerman-Jeantaud condition

- The Jeantaud condition is not always verified by the steering mechanisms in practice, as the four bar linkage mechanism

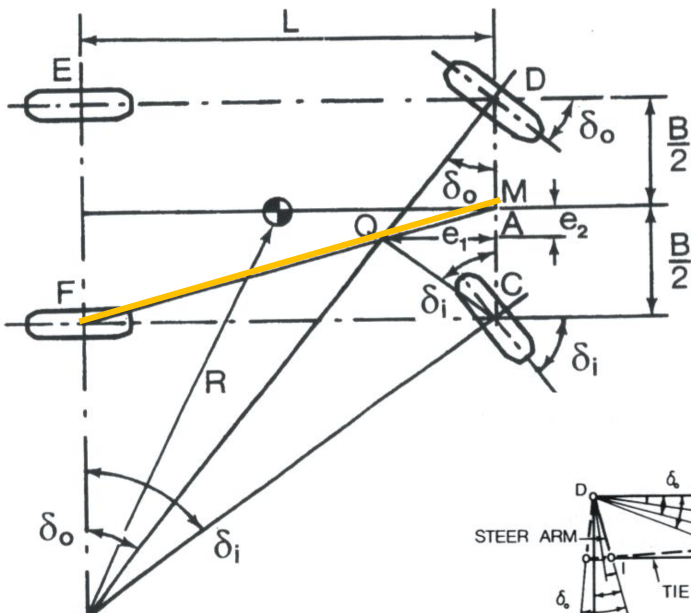
$$\Delta\delta_2 = \Delta\delta_e = \delta_e - \delta_e^c$$



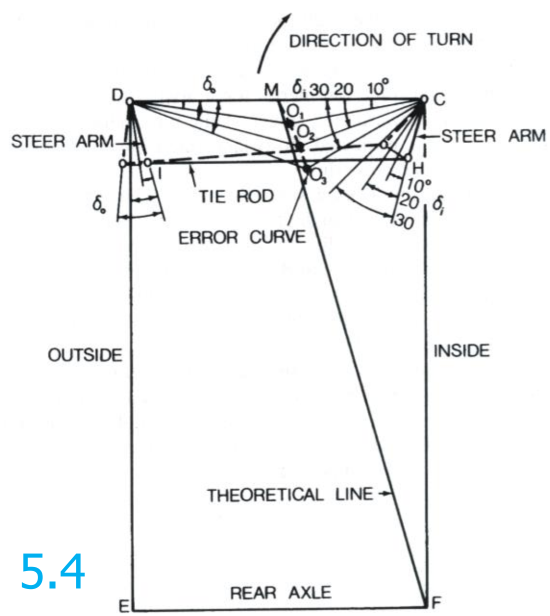
Genta Fig. 5.2

$$\sin(\gamma - \delta_e) + \sin(\gamma + \delta_i) = \sqrt{\left(\frac{l_1}{l_2} - 2\sin\gamma\right)^2 - (\cos(\gamma - \delta_e) - \cos(\gamma + \delta_i))^2}$$

Jeantaud condition



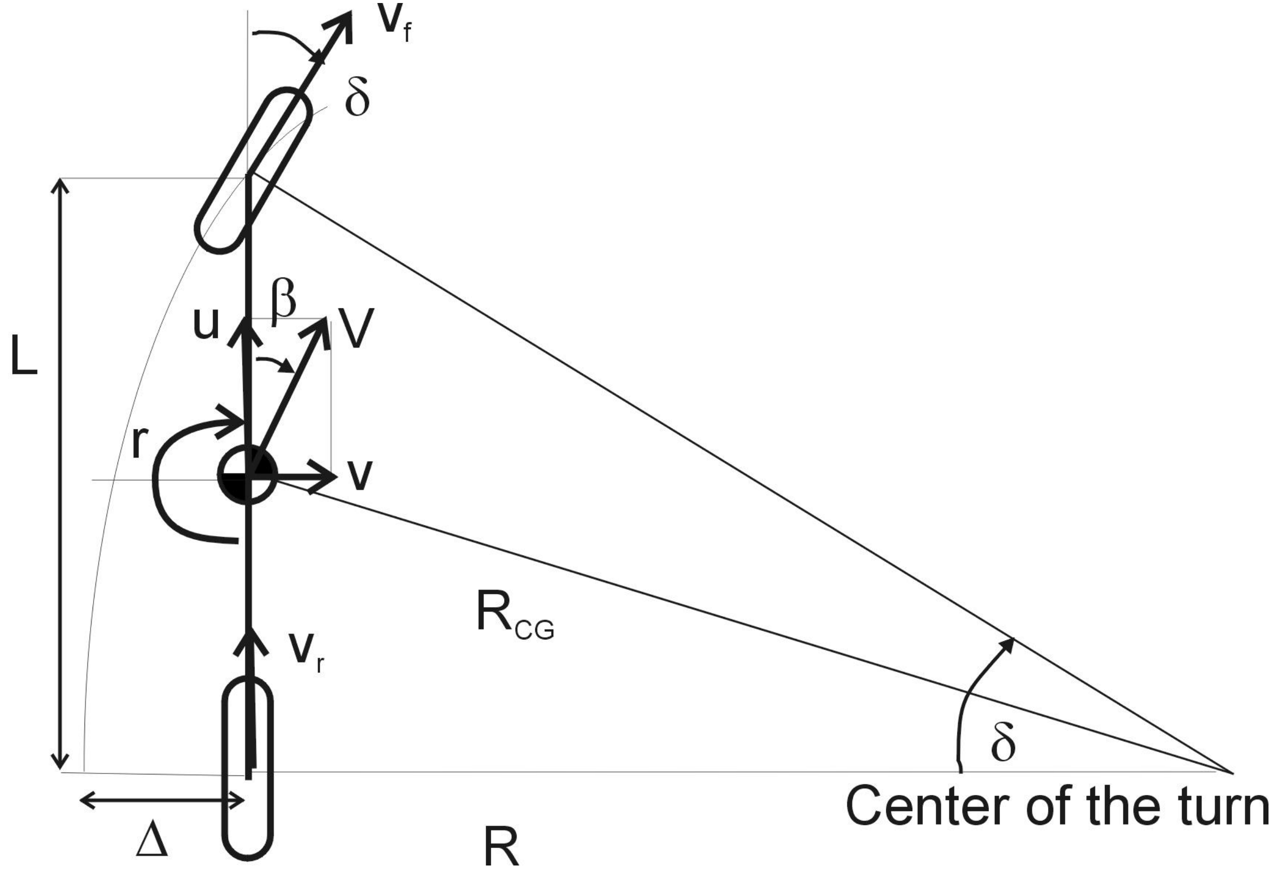
Wong Fig. 5.2



Wong Fig. 5.4

- The Jeantaud condition can be determined graphically, but the former drawing is very badly conditioned for a good precision
- Actually, one resorts to an alternative approach based on the following property
- Point Q belongs to the line MF when the Jeantaud condition is fulfilled
- The distance from Q to the line MF is a measure of the error from Jeantaud condition

Ackerman theory

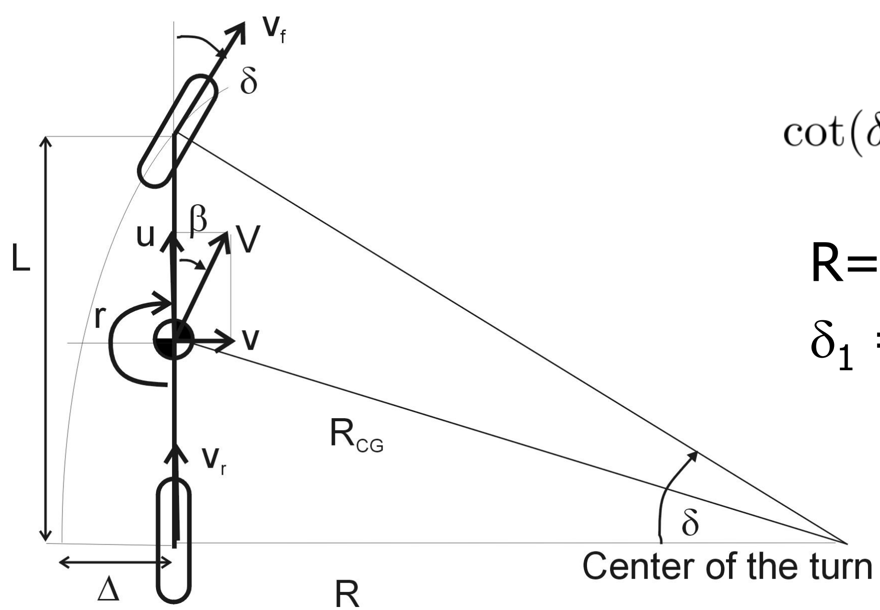


Ackerman theory

- The steering angle of the front wheels

$$\tan\delta = \frac{L}{R}$$

- The relation between the Ackerman steering angle δ and the Jeantaud steering angles δ_1 and δ_2



$$\cot(\delta) = \frac{R}{L} = \frac{\cot\delta_e + \cot\delta_i}{2}$$

R=10 m, L= 2500 mm, t=1300 mm

δ_1 = 15.090° δ_2= 13.305°

δ = 14.142°

$(\delta_1+\delta_2)/2$=14.197°

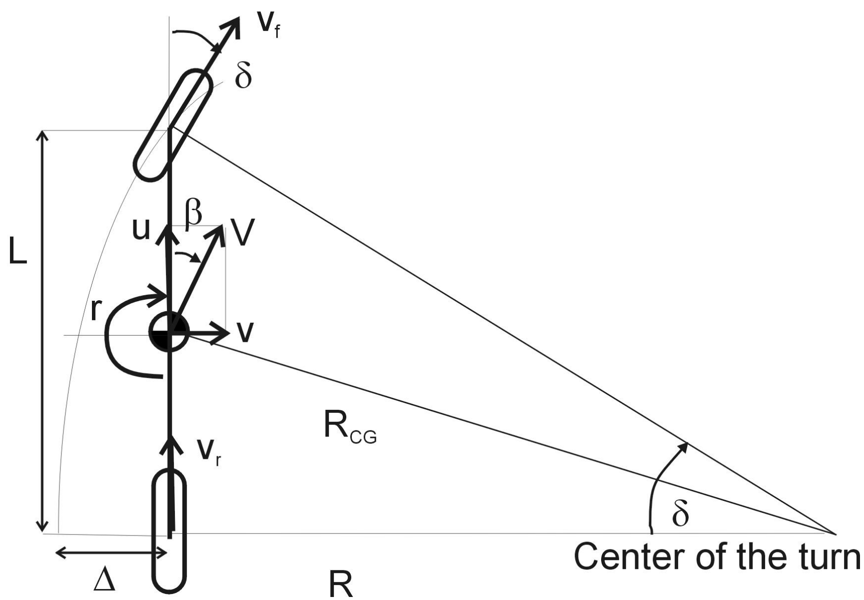
Ackerman theory

- Curvature radius at the centre of mass

$$R_{CG} = \sqrt{c^2 + R^2}$$

$$R_{CG} = R\sqrt{1 + c^2/R^2} \simeq R\left(1 + c^2/(2R^2)\right) \approx R \quad c \ll R \text{ and } L \ll R$$

- Relation between the curvature and the steering angle



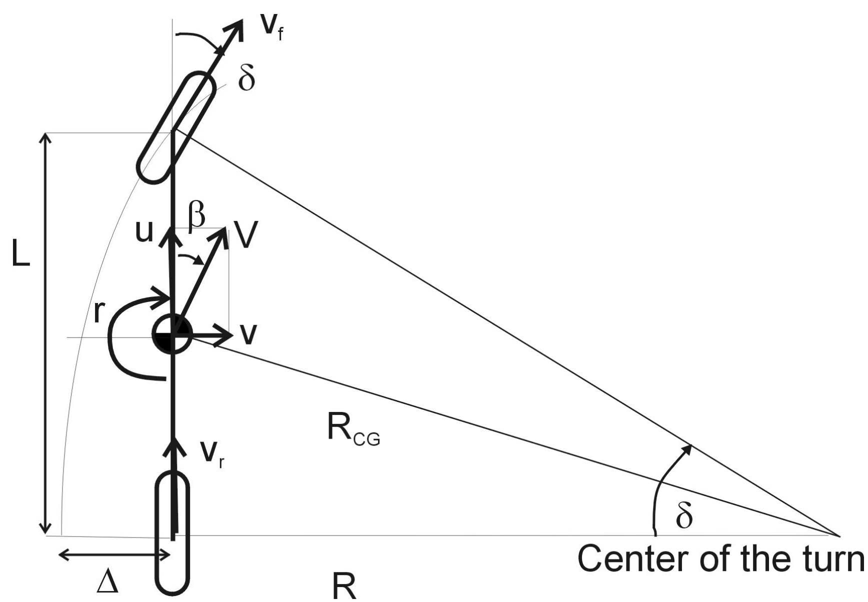
$$R_{CG} \approx R \approx L\,\cot\delta \approx \frac{L}{\delta}$$

$$\boxed{\frac{1/R}{\delta} = \frac{1}{L}}$$

Ackerman theory

- Side slip β at the centre of mass

$$\begin{aligned}\beta &= \arctan(\frac{c}{R}) \\ &= \arcsin(\frac{c}{R_{CG}}) = \arcsin(\frac{c}{\sqrt{R^2+c^2}})\end{aligned}$$



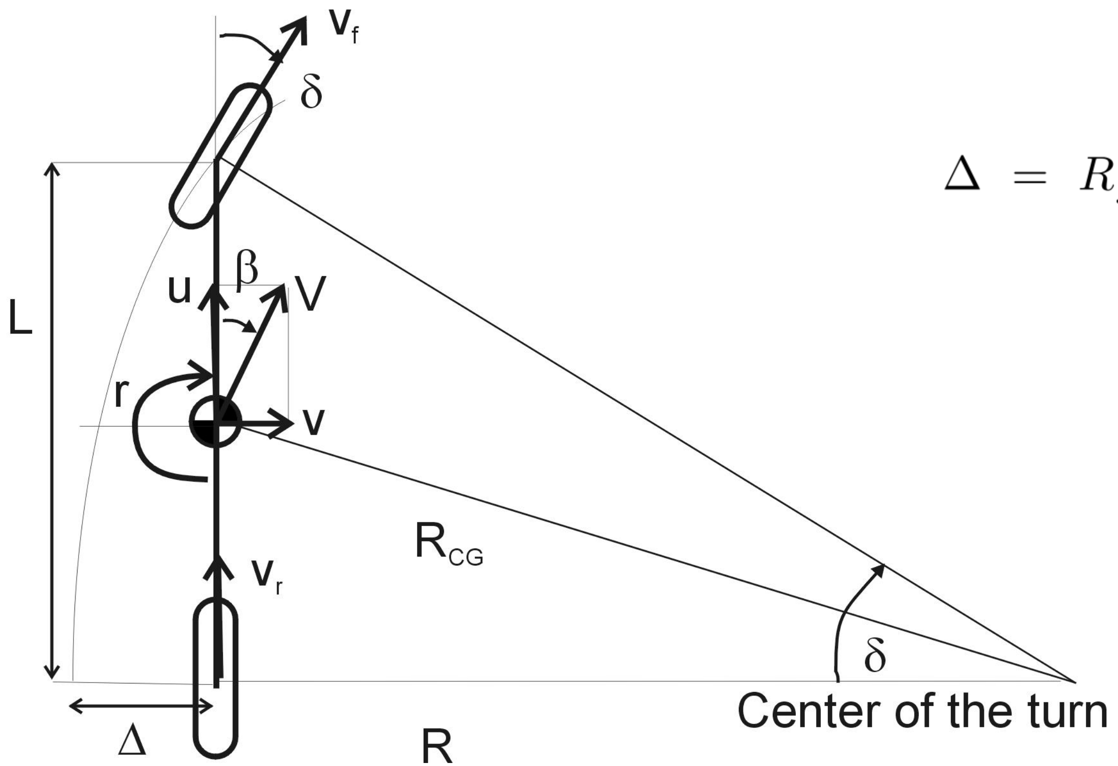
$$\tan\delta = \frac{L}{R} \quad \Rightarrow \quad \delta \simeq \frac{L}{R}$$

$$\tan\beta = \frac{c}{R} \quad \Rightarrow \quad \beta \simeq \frac{c}{R}$$

$$\boxed{\frac{\beta}{\delta} = \frac{c}{L}}$$

Ackerman theory

- The off-tracking of the rear wheel set



$$R_f = \sqrt{R^2 + L^2}$$

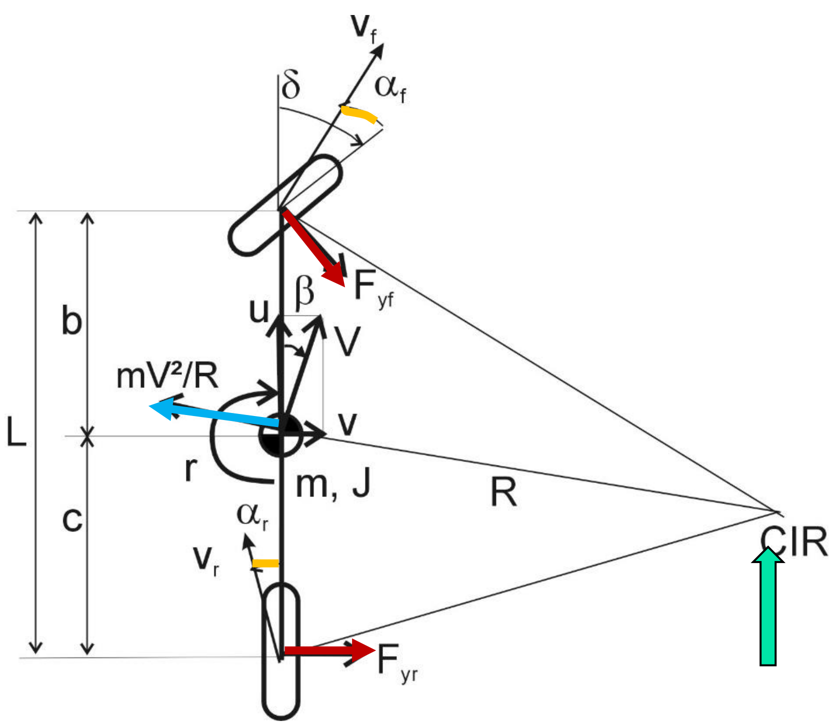
$$\Delta = R_f - R = R(\sqrt{1 + L^2/R^2} - 1)$$

$$\sqrt{1+x} \approx 1 + x/2 + \ldots$$

$$\boxed{\Delta = \frac{L^2}{2R}}$$

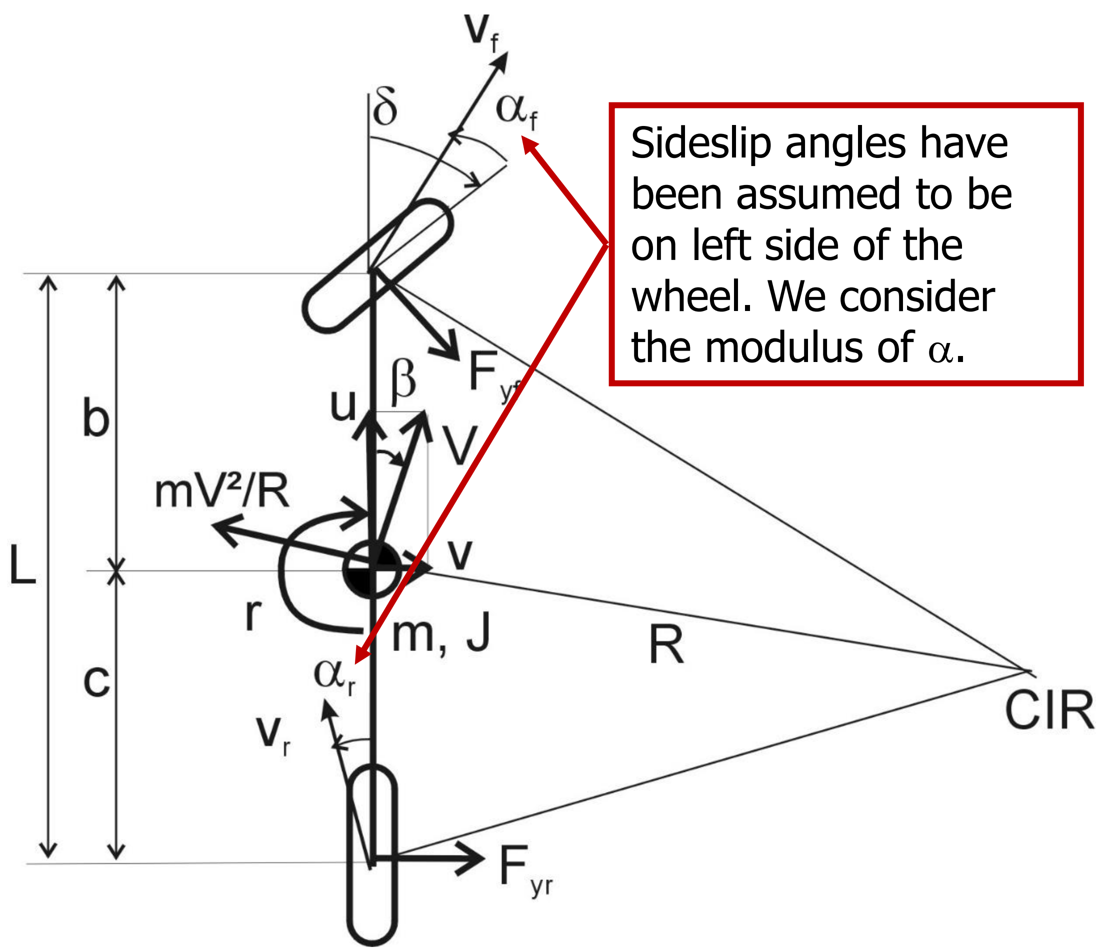
HIGH SPEED STEADY STATE CORNERING

High speed steady state cornering

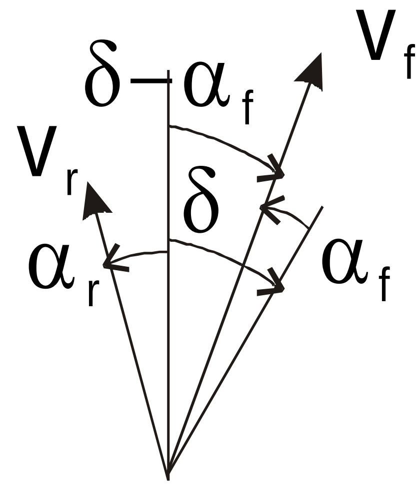


- At high speed, the tires have to develop lateral forces to sustain the lateral accelerations.
- The tire can develop forces if and only if they are subject to a side slip angle.
- Because of the kinematics of the motion, the IC is located at the intersection of the normal lines to the local velocity vectors under the tires.
- The IC, which was located at the rear axel for low-speed turn, is now moving to a point in front.

High speed steady state cornering



Sideslip angles have been assumed to be on left side of the wheel. We consider the modulus of α.

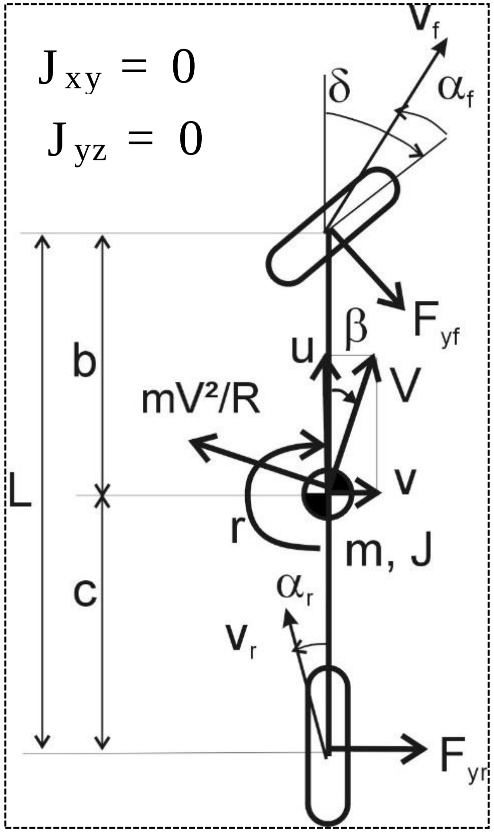


Dynamics equations of the vehicle motion

- Newton-Euler equilibrium equation in the non inertial reference frame of the vehicle body

$$\sum \vec{F} = m[\frac{d\vec{V}}{dt} + \vec{\omega} \times \vec{V}]$$

$$\sum \vec{M} = \frac{d}{dt}(J\vec{\omega}) + \vec{\omega} \times (J\vec{\omega})$$



- Model with 2 dof β & r

$$\vec{V} = [u\ v\ 0]^T \qquad \vec{\omega} = [0\ 0\ r]^T = [0\ 0\ \omega_z]^T$$

- Equilibrium equations in F_y and M_z :

$$F_y = m\,(\dot{v} + r\,u)$$

$$N = J_{zz}\,\dot{r}$$

- Operating forces
  - Tyre forces
  - Aerodynamic forces (can be neglected here)

Dynamics equations of the vehicle motion

- Newton-Euler equilibrium equation <u>in the non inertial reference frame of the vehicle body</u>

$$\sum \vec{F} = m[\frac{d\vec{V}}{dt} + \vec{\omega} \times \vec{V}]$$

$$\sum \vec{M} = \frac{d}{dt}(J\vec{\omega}) + \vec{\omega} \times (J\vec{\omega})$$

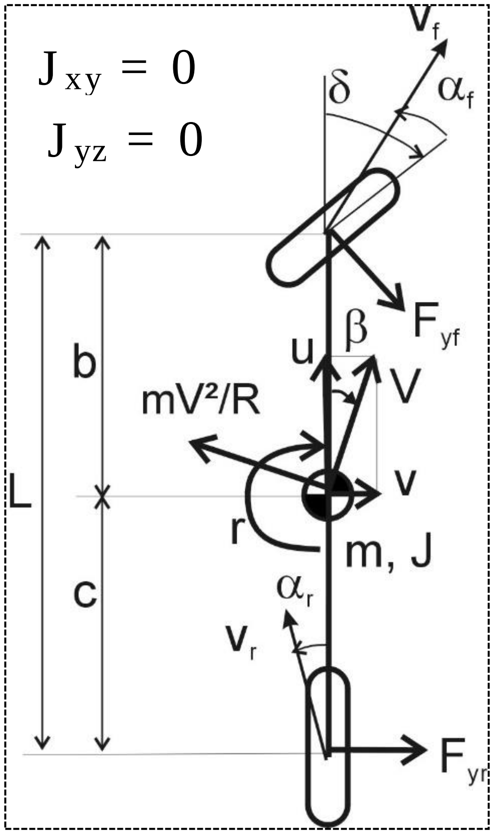
- Model with 2 dof β & r

$$\vec{V} = [u\ v\ 0]^T \quad \vec{\omega} = [0\ 0\ r]^T \quad \tan\beta \simeq \beta \simeq \frac{v}{V}$$

$$\vec{V} = u\vec{e}_x + v\vec{e}_y \quad \vec{\omega} = \omega_z \vec{e}_z$$

- Inertia tensor

$$J = \begin{pmatrix} J_{xx} & 0 & J_{xz} \\ 0 & J_{yy} & 0 \\ J_{xz} & 0 & J_{zz} \end{pmatrix}$$



Dynamics equations of the vehicle motion

- It comes

$$\vec{\omega} \times \vec{v} = \omega_z \vec{e}_z \times (u\vec{e}_x + v\vec{e}_y) = \omega_z u\, \vec{e}_y - \omega_z v\, \vec{e}_x$$

$$J\vec{\omega} = [J_{xy}\omega_z \;\; 0 \;\; J_{zz}\omega_z]^T$$

$$\vec{\omega} \times J\vec{\omega} = \omega_z \vec{e}_z \times (\omega_z J_{xz}\vec{e}_x + \omega_z J_{zz}\vec{e}_z) = \omega_z^2 J_{xz}\vec{e}_y$$

- And finally

$$\sum F_x = m(\dot{u} - \omega_z v) \qquad \sum M_x = 0$$

$$\sum F_y = m(\dot{v} + \omega_z u) \qquad \sum M_y = J_{xy}\omega_z^2$$

$$\sum F_z = 0 \qquad \sum M_z = J_{zz}\dot{\omega}_z$$

- The only nontrivial equations are

$$\sum F_y = m(\dot{v} + \omega_z u) \qquad \sum M_z = J_{zz}\dot{\omega}_z$$

Dynamics equations of the vehicle motion

- Circular motion
  - ω_z: rotation speed about vertical axis
  - V tangent velocity
  - R radius of the turn

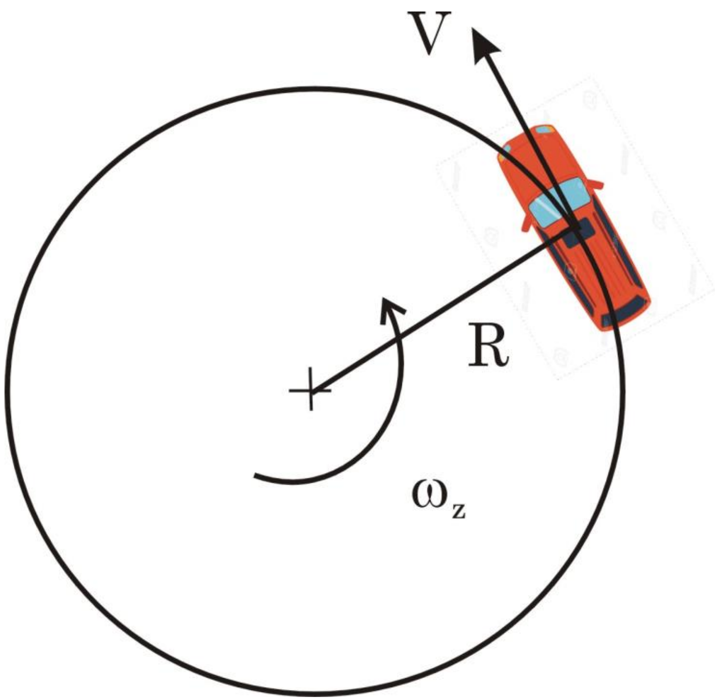
$$V = \omega_z R \qquad r = \omega_z = \frac{V}{R}$$

- Steady state

$$\dot{v} = 0 \qquad \dot{\omega}_z = 0$$

- Equation of motion

$$\sum F_y = m(\dot{v} + \omega_z u) = m(0 + \omega_z V)$$
$$= m\,\omega_z^2 R = m\frac{V^2}{R}$$
$$\sum M_z = J_{zz}\dot{\omega}_z = 0$$

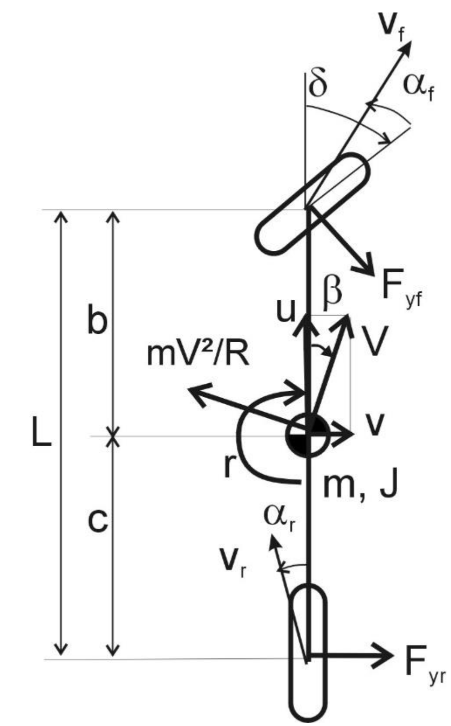


Equilibrium equations of the vehicle

- Equilibrium equations in lateral direction and rotation about z axis

$$F_{yf}\cos\delta \;+\; F_{yr} = m\frac{V^2}{R}\,cos\beta \qquad F_{yf} \;+\; F_{yr} = m\frac{V^2}{R}$$

$$F_{yf}\cos\delta\, b \;-\; F_{yr}\, c \;= 0 \qquad F_{yf}\, b \;-\; F_{yr}\, c \;= 0$$



$$\beta \ll 1 \quad \delta \ll 1 \qquad \cos\beta \simeq 1 \quad \cos\delta \simeq 1$$

- Solutions

$$F_{yf} \;=\; \frac{c}{L}\; m\frac{V^2}{R}$$

$$F_{yr} \;=\; \frac{b}{L}\; m\frac{V^2}{R}$$

The lateral forces are in the same ratio as the vertical forces under the wheel sets

Equilibrium equations of the vehicle

- Solving

$$F_{yf} + F_{yr} = m\frac{V^2}{R}$$
$$F_{yf}\, b - F_{yr}\, c = 0$$

- Can be made by using Cramer's rule

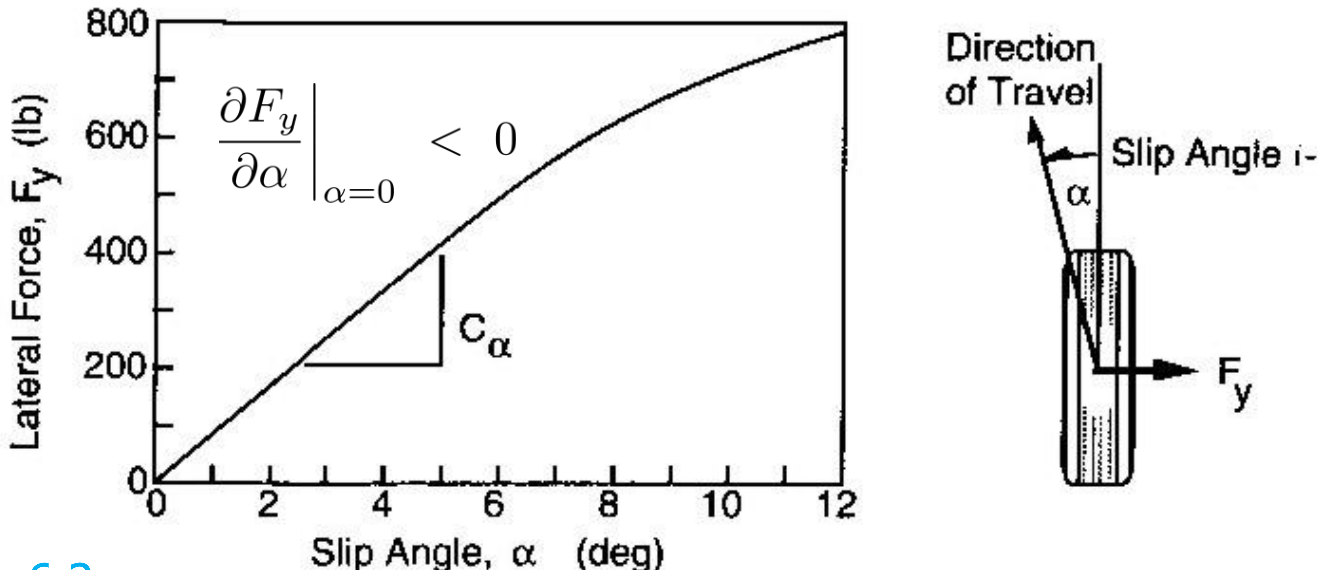
$$F_{yf} = \frac{\begin{vmatrix} mV^2/R & 1 \\ 0 & -c \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ b & -c \end{vmatrix}} = \frac{mV^2/R\,(-c)}{-c-b} = \frac{c}{L}\, m\, \frac{V^2}{R}$$

$$F_{yr} = \frac{\begin{vmatrix} 1 & mV^2/R \\ b & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ b & -c \end{vmatrix}} = \frac{-mV^2/R\, b}{-c-b} = \frac{b}{L}\, m\, \frac{V^2}{R}$$

Behaviour equations of the tires

- Cornering force for small slip angles

$$F_y = C_\alpha \, \alpha \qquad C_\alpha = -\left.\frac{\partial F_y}{\partial \alpha}\right|_{\alpha=0} > 0 \qquad C_\alpha = \sum_{i \in \text{axle}} C_{\alpha i}$$



Gillespie, Fig. 6.2

Gratzmüller equality

- Using the equilibrium and the behaviour condition, one gets

$$
\begin{aligned}
F_{yf} &= C_{\alpha f}\,\alpha_f = \frac{c}{L}\,m\frac{V^2}{R} \\
F_{yr} &= C_{\alpha r}\,\alpha_r = \frac{b}{L}\,m\frac{V^2}{R}
\end{aligned}
$$

- One yields the Gratzmüller equality

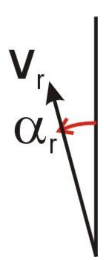
$$
\boxed{\frac{\alpha_f}{\alpha_r} = \frac{c\,C_{\alpha r}}{b\,C_{\alpha f}}}
$$

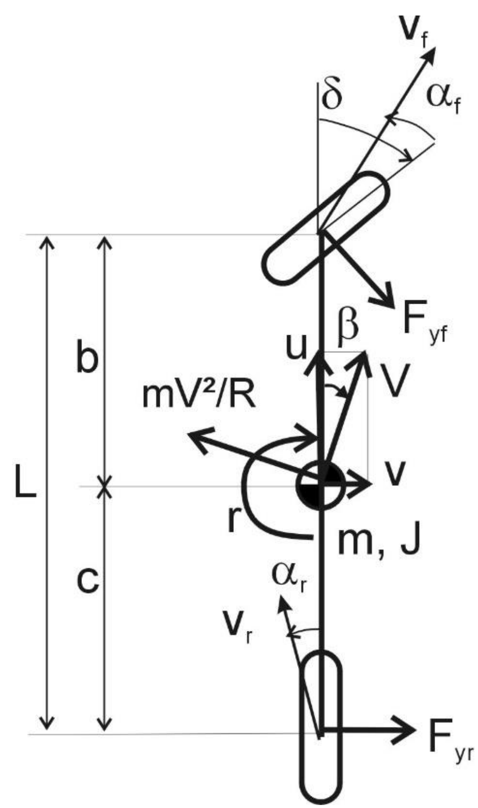
Compatibility equations

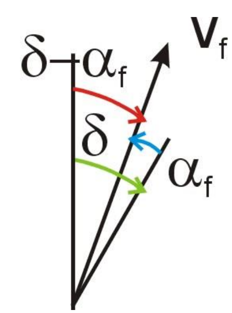
- Compatibility equation consists in evaluating the side slip angles in terms of the velocities

Because of assumption α_r<0!

$$\tan \alpha_r = \frac{-v_r}{u_r}$$







$$\tan(\delta - \alpha_f) = \frac{v_f}{u_f}$$

Because of steering action δ!

Compatibility equations

- Evaluation of velocities under front and rear axles thanks to the Poisson transport equation $\vec{v}_P = \vec{v}_{CG} + \vec{\omega} \times \vec{r}_{P/CG}$

$$\vec{v}_{CG} = u\,\vec{e}_x + v\,\vec{e}_y$$

$$\vec{\omega} = r\,\vec{e}_z$$

$$\vec{r}_f = b\,\vec{e}_x$$

$$\vec{\omega} \times \vec{r}_f = b\,r\,\vec{e}_y$$

$$u_f = u \simeq V$$

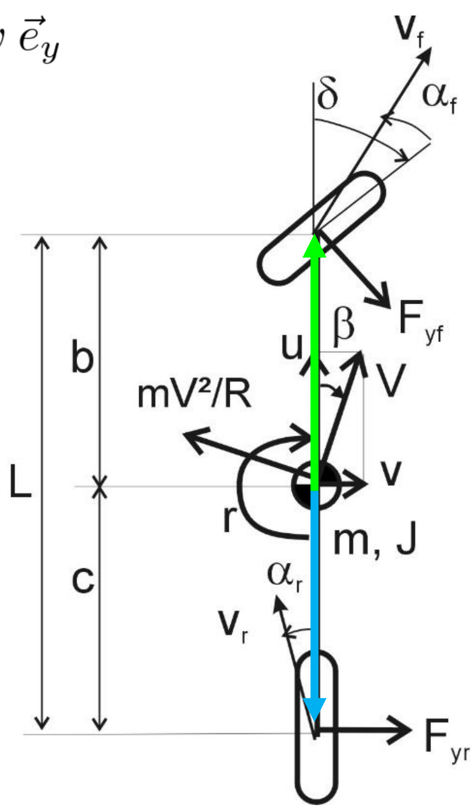
$$v_f = v + b\,r$$

$$\vec{r}_r = -c\,\vec{e}_x$$

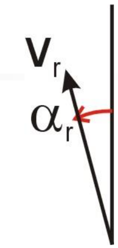
$$\vec{\omega} \times \vec{r}_r = -c\,r\,\vec{e}_y$$

$$u_r = u \simeq V$$

$$v_r = v - c\,r$$



Compatibility equations



- The velocity under the rear wheels are given by

$$u_r = u \simeq V$$

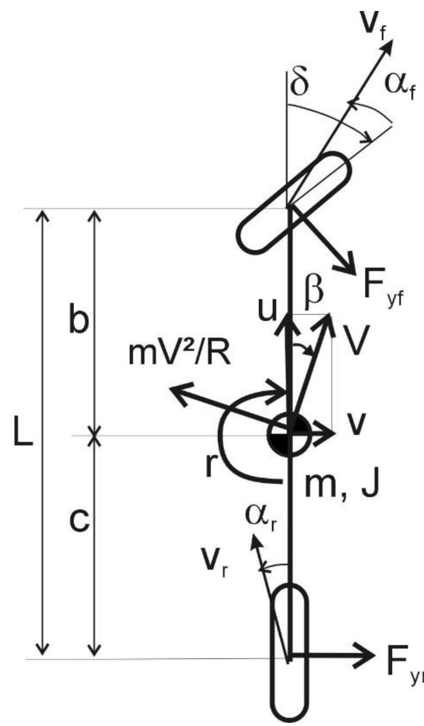
$$v_r = v - c\,r$$

- The compatibility of the velocities yields the slip angle under the rear wheels

$$\tan \alpha_r = \frac{-v_r}{u_r} = \frac{-v + c\,r}{V}$$

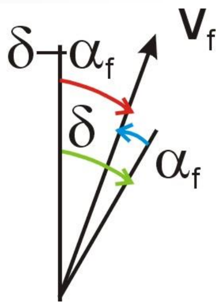
$$V = r\,R$$

$$\boxed{\alpha_r = -\beta + \frac{c}{R}}$$



Compatibility equations

- The velocity under the front wheels are given by



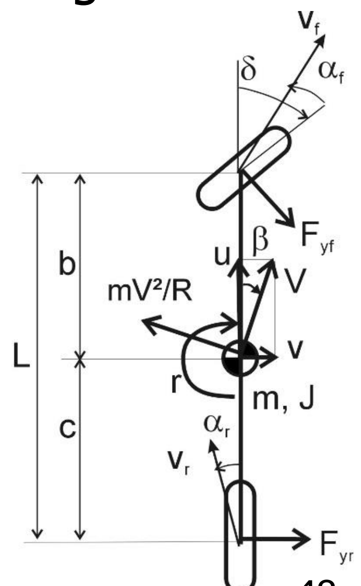
$$u_f = u \simeq V$$
$$v_f = v + b\,r$$

- The compatibility of the velocities yields the slip angle under the front wheels

$$\tan(\delta - \alpha_f) = \frac{v_f}{u_f} = \frac{v + b\,r}{V}$$

$$V = r\,R$$

$$\boxed{\delta - \alpha_f = \beta + \frac{b}{R}}$$



Steering angle

- Steering angle as a function of the slip angles under front and rear wheels

$$\delta - \alpha_f = \beta + \frac{b}{R}$$

$$\alpha_r = -\beta + \frac{c}{R}$$

$$\delta - \alpha_f + \alpha_r = 0 \; + \; \frac{b+c}{R}$$

- This gives relation between the steering angles and the

$$\boxed{\delta \; = \; \frac{L}{R} + \underbrace{\alpha_f - \alpha_r}}$$

Ackerman angle

Correction due to side slip

Steering angle

- Steering angle as a function of the slip angles under front and rear wheels

$$\delta = \frac{L}{R} + \alpha_f - \alpha_r$$

- Let's insert the expression of the side slip angles in terms of lateral forces and cornering stiffness

$$\alpha_f = \frac{F_{yf}}{C_{\alpha f}} \qquad \alpha_r = \frac{F_{yr}}{C_{\alpha r}}$$

$$\alpha_f = \frac{m\frac{V^2}{R}\frac{c}{L}}{C_{\alpha f}} = \frac{m\,c/L}{C_{\alpha f}}\,\frac{V^2}{R} \qquad \alpha_r = \frac{m\frac{V^2}{R}\frac{b}{L}}{C_{\alpha r}} = \frac{m\,b/L}{C_{\alpha r}}\,\frac{V^2}{R}$$

Steering angle

- The expression of the steering angle as a function of the slip angles under front and rear wheels

$$\delta = \frac{L}{R} + \alpha_f - \alpha_r$$

- Inserting the values of the side slip angles as a function of the velocity and the cornering stiffness of the wheels sets yields

$$\delta = \frac{L}{R} + \left(\frac{m\,c}{C_{\alpha f}\,L} - \frac{m\,b}{C_{\alpha r}\,L}\right)\frac{V^2}{R}$$

- Or

$$\delta = \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)\frac{V^2}{gR}$$

  - with

$$W_f = mg\,\frac{c}{L} \qquad W_r = mg\,\frac{b}{L}$$

Understeer gradient

- The steering angle is expressed in terms of the centrifugal acceleration

$$\delta = \frac{L}{R} + \left(\frac{m\,c}{C_{\alpha f}\,L} - \frac{m\,b}{C_{\alpha r}\,L}\right)\frac{V^2}{R}$$

- So

$$\boxed{\delta = \frac{L}{R} + K\,\frac{V^2}{R}}$$

$$\boxed{\delta = \frac{L}{R} + K'\,\frac{V^2}{g\,R}}$$

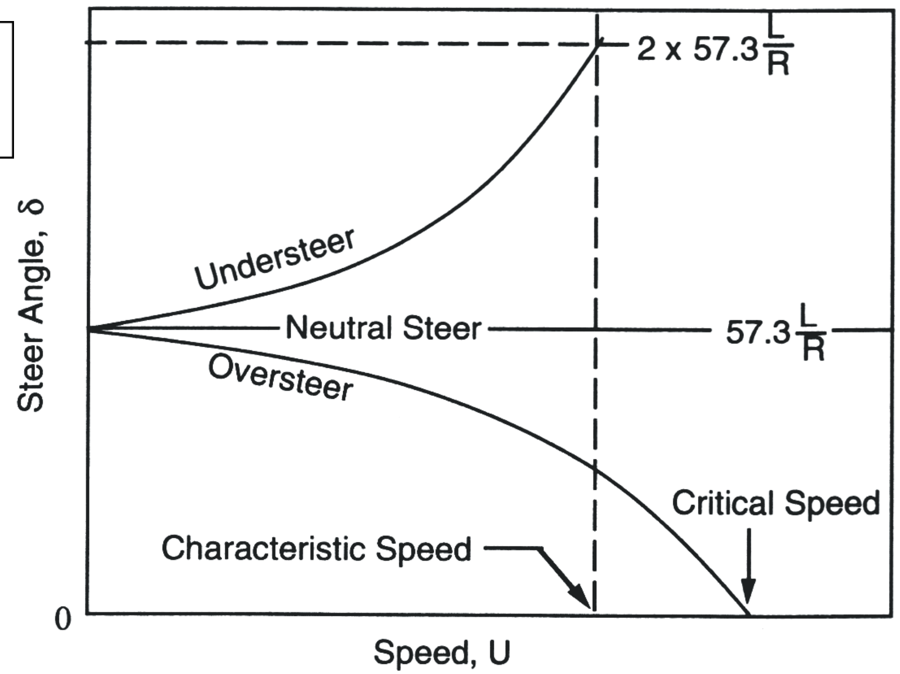
- With the understeer gradient K of the vehicle

$$\boxed{K = \frac{m\,c}{C_{\alpha f}\,L} - \frac{m\,b}{C_{\alpha r}\,L}}$$

$$\boxed{K' = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}}$$

Steering angle as a function of V

$$\delta = \frac{L}{R} + K\frac{V^2}{R}$$



Gillespie. Fig. 6.5 Modification of the steering angle as a function of the speed

Neutralsteer, understeer and oversteer vehicles

$$K = \frac{m\,c}{C_{\alpha f}\,L} - \frac{m\,b}{C_{\alpha r}\,L} = \frac{m}{L}\,\frac{c\,C_{\alpha r} - b\,C_{\alpha f}}{C_{\alpha f} C_{\alpha r}}$$

- If K=0, the vehicle is said to be *neutralsteer*:

  $$K = 0 \quad \Leftrightarrow \quad c\,C_{\alpha r} = b\,C_{\alpha f} \qquad \alpha_f = \alpha_r$$

  The front and rear wheels sets have the same directional ability

- If K>0, the vehicle is *understeer* :

  $$K > 0 \quad \Leftrightarrow \quad c\,C_{\alpha r} > b\,C_{\alpha f} \qquad \alpha_f > \alpha_r$$

  Larger directional factor of the rear wheels

- If K<0, the vehicle is *oversteer*:

  $$K < 0 \quad \Leftrightarrow \quad c\,C_{\alpha r} < b\,C_{\alpha f} \qquad \alpha_f < \alpha_r$$

  Larger directional factor of the front wheels

Characteristic and critical speeds

- For an understeer vehicle, the understeer level may be quantified by a parameter known as the characteristic speed. It is the speed that requires a steering angle that is twice the Ackerman angle (turn at V=0)

$$\delta = 2L/R \qquad V_{\text{characteristic}} = \sqrt{\frac{L}{K}}$$

- For an oversteer vehicle, there is a critical speed above which the vehicle will be unstable

$$\delta = 0 \qquad V_{\text{critical}} = \sqrt{\frac{L}{|K|}}$$

$$\delta = \frac{L}{R} + K\frac{V^2}{R} = 0 \quad \Leftrightarrow \quad V^2 = -\frac{L}{K}$$

Lateral acceleration and yaw speed gains

- Lateral acceleration gain $a_y = V^2/R$

$$\delta = \frac{L}{R} + K a_y$$

$$\frac{a_y}{\delta} = \frac{\frac{V^2}{L}}{1 + \frac{KV^2}{L}}$$

- Yaw speed gain $r = \frac{V}{R}$

$$\frac{r}{\delta} = \frac{\frac{V}{L}}{1 + \frac{KV^2}{L}}$$

Lateral acceleration gain

- Purpose of the steering system is to produce lateral acceleration

$$\delta = \frac{L}{R} + K a_y \Longrightarrow \frac{a_y}{\delta} = \frac{\frac{V^2}{L}}{1 + \frac{KV^2}{L}}$$

- For neutral steer, K=0 and the lateral acceleration gain is increasing constantly with the square of the speed : V^2/L
- For understeer vehicle, K>0, the denominator >1 and the lateral acceleration is reduced with growing speed
- For oversteer vehicle, K<0, the denominator is < 1 and becomes zero for the critical speed, which means that any perturbation produces an infinite lateral acceleration

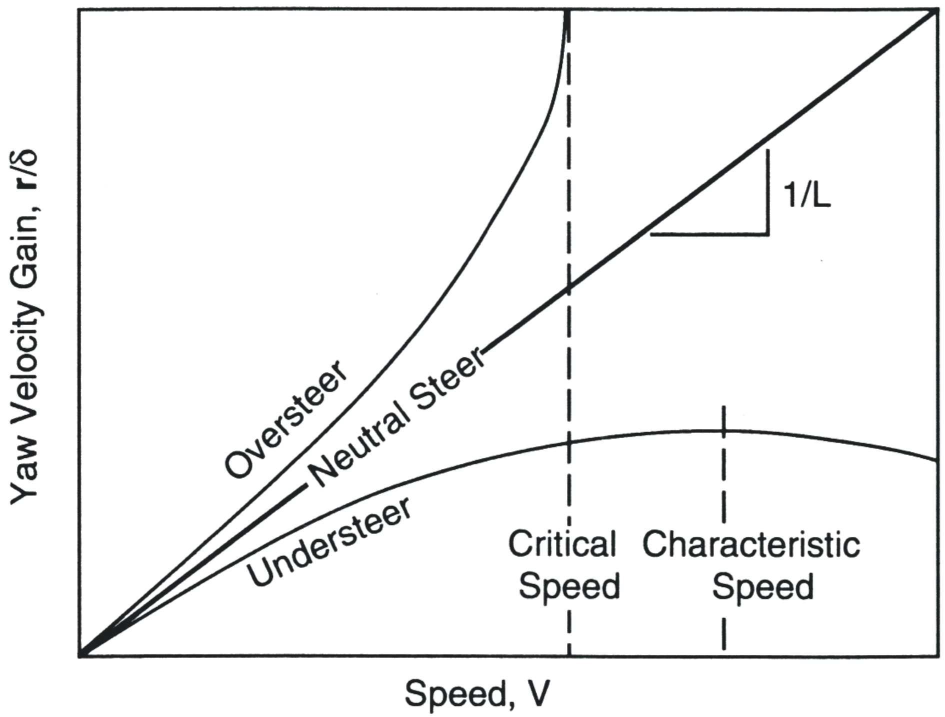
Yaw velocity gain

- The second raison for steering is to change the heading angle by developing a yaw velocity

$$r = \frac{V}{R} \quad \Longrightarrow \quad \frac{r}{\delta} = \frac{\frac{V}{L}}{1 + \frac{KV^2}{L}}$$

- For neutral vehicles, the yaw velocity is proportional to the steering angle and increases with the speed (slope 1/L)
- For understeer vehicles, the yaw gain angle is lower than proportional. It is maximum for the characteristic speed.
- For oversteer vehicles, the yaw rate becomes infinite for the critical speed and the vehicles becomes uncontrollable at critical speed.

Yaw velocity gain



$$\frac{r}{\delta} = \frac{\frac{V}{L}}{1 + \frac{KV^2}{L}}$$

Gillespie. Fig. 6.6 Yaw rate as a function of the steering angle

Sideslip angle at centre of mass

- Definition (reminder)

$$\beta = \frac{v_{cg}}{u_{cg}}$$

- Value

$$\alpha_r = -\beta + \frac{c}{R} \quad \longrightarrow \quad \beta = \frac{c\,r}{V} - \alpha_r = \delta - \alpha_f - \frac{b\,r}{V}$$

- Value as a function of the speed V

$$\beta = \frac{c}{R} - \frac{W_r}{C_{\alpha r}}\,\frac{V^2}{gR}$$

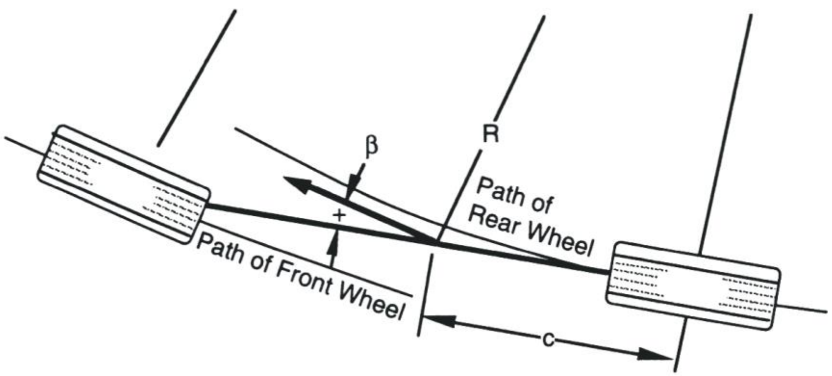
- Becomes zero for the speed

$$V_{\beta=0} = \sqrt{cg\frac{C_{\alpha r}}{W_r}} = \sqrt{\frac{c\,L\,C_{\alpha r}}{b\,m}}$$

independent of R !

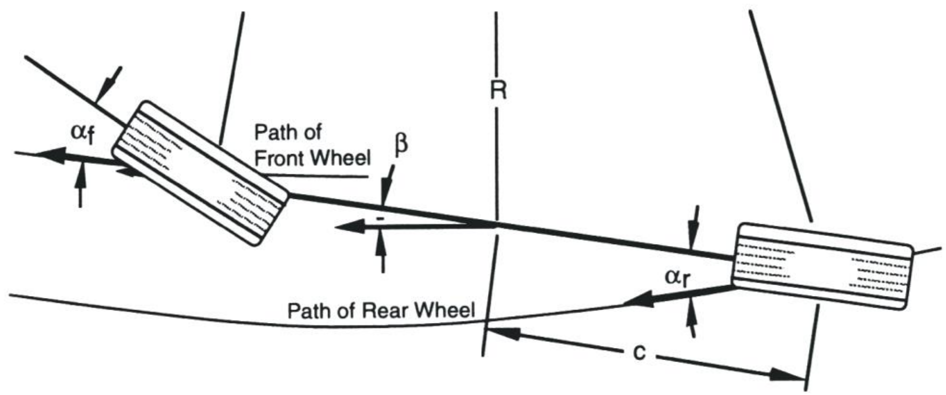
Sideslip angle

$\beta > 0$



Gillespie. Fig. 6.7 Sideslip angle for a **low speed** turn

$\beta < 0$

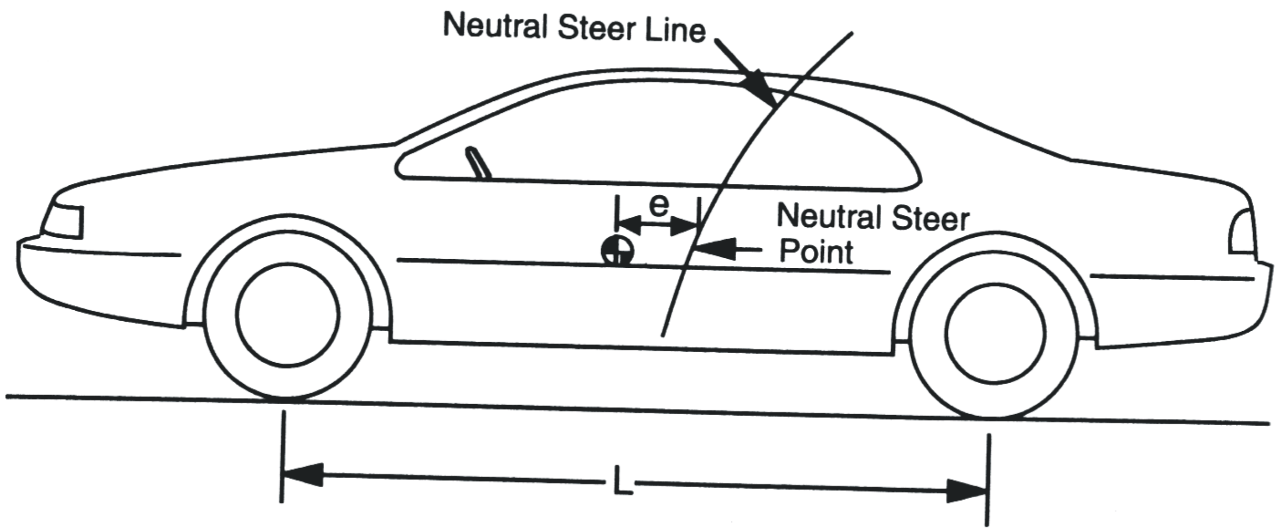


Gillespie. Fig. 6.8 Sideslip angle for a **high speed** turn

This is true whatever the vehicle is understeer or oversteer

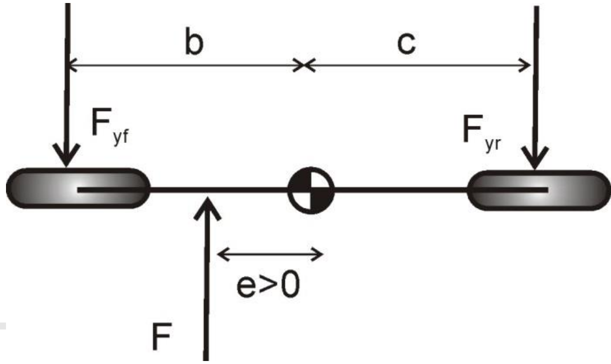
Static margin

- The static margin provides another (equivalent) measure of the steady-state behaviour



Gillespie. Fig 6.9 Neutral steer line
e>0 if it is located in front of the vehicle centre of gravity

Static margin



- Suppose the vehicle is in straight line motion (δ=0)
- Let a perturbation force F applied at a distance e from the CG (e>0 if in front of the CG)
- Let's write the equilibrium

$$\begin{aligned} F_{yf} + F_{yr} &= F \\ F_{yf}\, b - F_{yr}\, c &= F\, e \end{aligned} \quad \Longrightarrow \quad F_{yf}(b-e) - F_{yr}(c+e) = 0$$

- The static margin is the point such that the perturbation lateral forces F do not produce any steady-state yaw velocity
- That is:

$$\begin{aligned} r &= 0 \\ \delta &= 0 \end{aligned} \quad \Longrightarrow \quad R = \infty \quad \Longrightarrow \quad \delta = \frac{L}{R} + \alpha_f - \alpha_r \quad \Leftrightarrow \quad \alpha_f = \alpha_r$$

Static margin

- It comes

$$C_{\alpha f}(b-e) - C_{\alpha r}(c+e) = 0$$

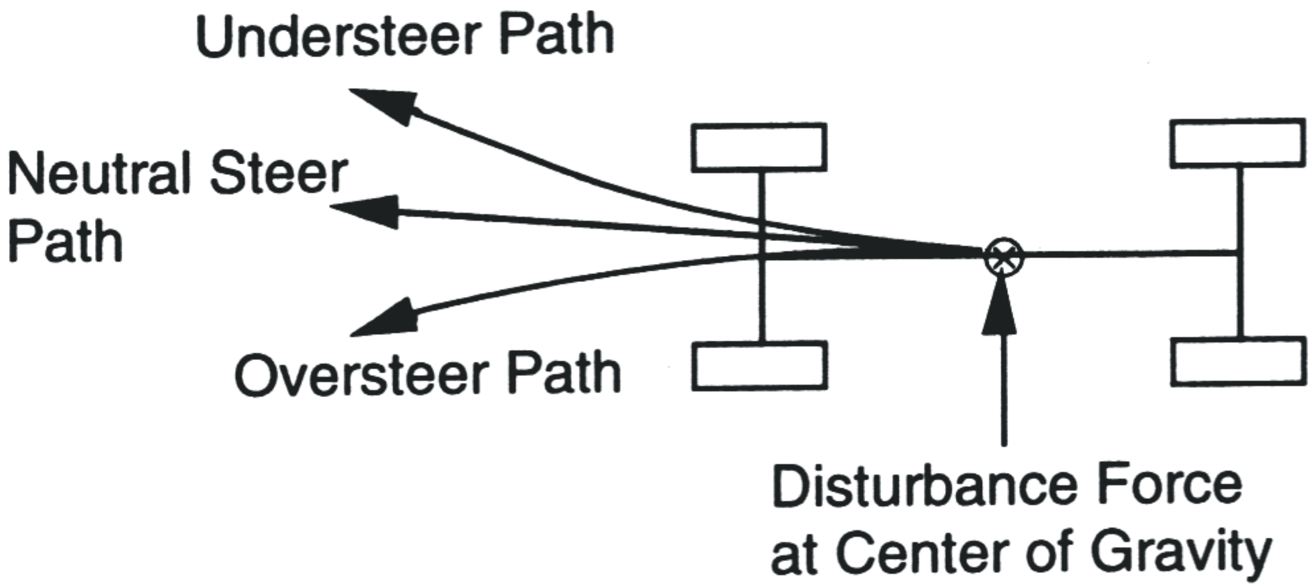
- So the static margin writes

$$\boxed{e = \frac{b\,C_{\alpha_f} - c\,C_{\alpha_r}}{C_{\alpha_f} + C_{\alpha_r}}}$$

- A vehicle is
  - Neutral steer if e = 0
  - Under steer (K>0) if e<0 (behind the CG)
  - Over steer (K<0) if e>0 (in front of the CG)

- Remember that $K = \frac{m}{L}\left[\frac{c\,C_{\alpha r} - b\,C_{\alpha f}}{C_{\alpha f}\,C_{\alpha r}}\right]$

Static margin



Gillespie. Fig. 6.10 Maurice Olley's definition of understeer and over steer