MECA0525 : Vehicle dynamics

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Lesson 2:
Suspension effects on cornering

Bibliography

- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- J. Dixon. « Tires, Suspension, and Handling » 2nd edition. 1996, Society of Automotive Engineers (SAE)
- H. Heisler. Advanced Vehicle Technology. 2nd edition. Butterwoth Heinemann. 2002.
- W. Milliken & D. Milliken. « Race Car Vehicle Dynamics », 1995, Society of Automotive Engineers (SAE)
- J. Reimpel, H. Stoll, J. Betzler. The Automotive Chassis - Engineering Principles. Butterworth Heinemann. Oxford. 2001.
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).

Layout

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  - Roll moment distribution
  - Roll angle
- Understeer gradient due to lateral load transfer
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  - Understeer gradient due to load transfer

Layout

- Understeer gradient due to camber
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  - Roll steer
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  - Lateral force compliance
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- Aligning torque

INTRODUCTION

Suspension effects on cornering

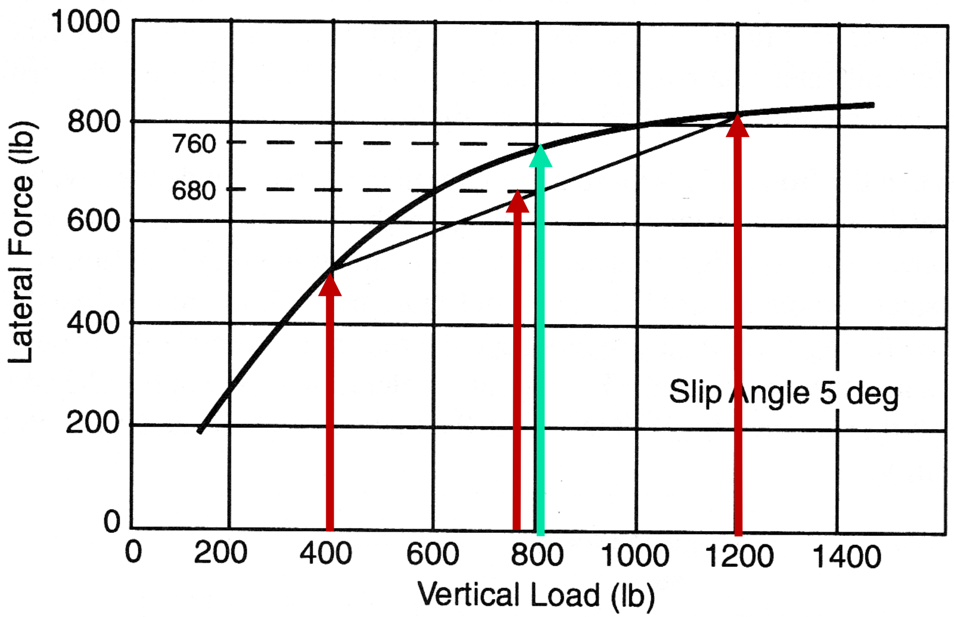
- So far our first theory has shown that the behaviour is dependent on the balance between the load/cornering coefficient W/C_α (called cornering compliance [deg/g]) of the front and rear axles

$$K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}$$

- Weight and cornering coefficients can be affected by suspension mechanics due to
  - Lateral load transfer
  - Change in camber angles
  - Roll steer
  - Lateral force compliance steer
  - Aligning torque
  - Effect of tractive force on cornering

Roll Moment distribution

- For virtually all pneumatic tires, the cornering forces are nonlinearly dependent on the vertical load (load sensitivity)



Straight line: F_y= 760 lb

Cornering: F_y=680 lb

Gillespie Fig. 6.11

Roll Moment distribution

- Lateral load transfer occurs in cornering because of the elevation of the CG with respect to the ground.
- Because of the load sensitivity of tyre cornering force, the lateral force developed by the axle is affected in cornering.
- Thus to sustain the lateral forces, the slip angle has to be increased.

$$\delta = \frac{L}{R} + \alpha_f - \alpha_r$$

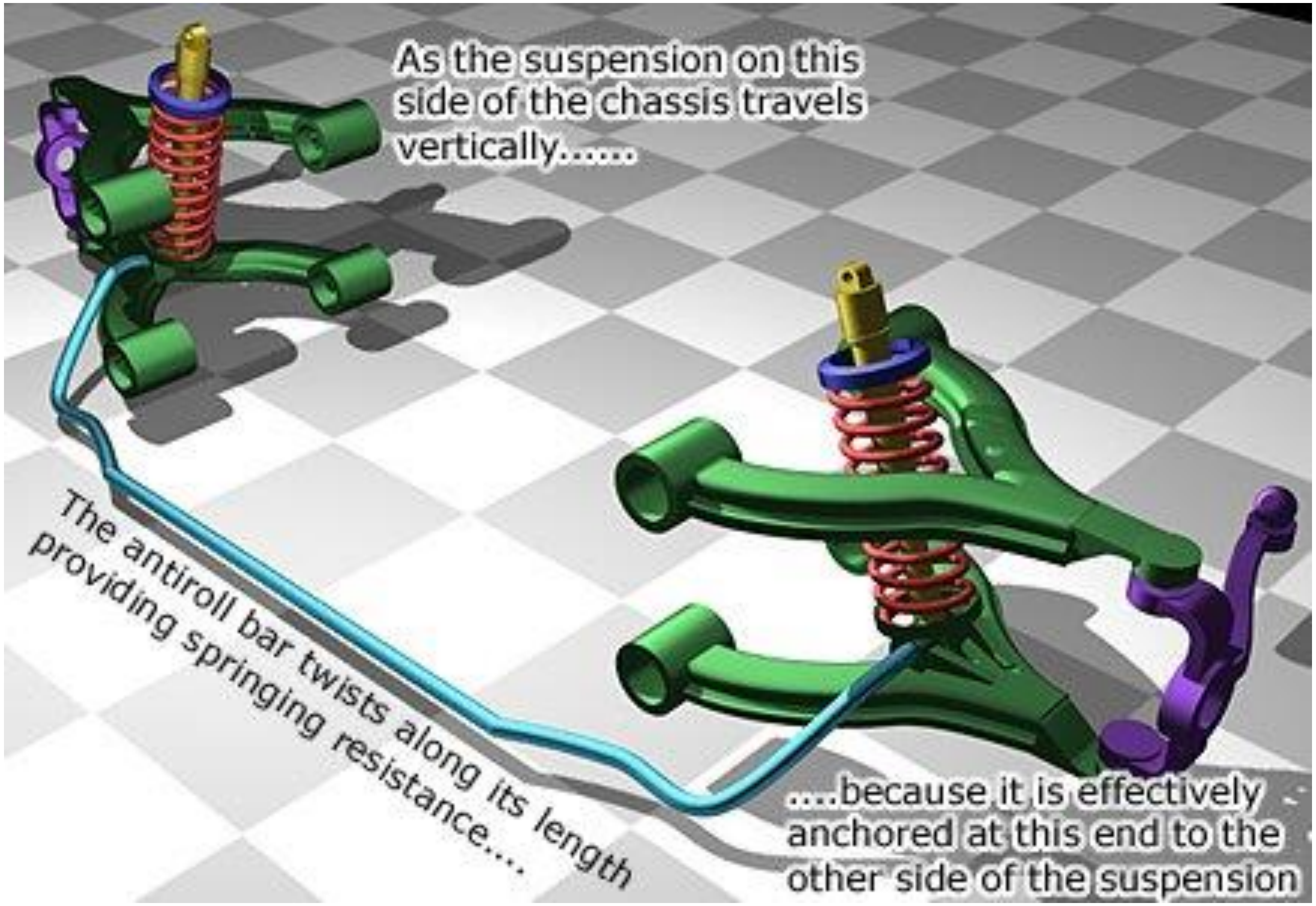
- For front tyres, greater slip angles lead to an understeer effect
- For rear tyres, greater slip angles lead to an oversteer behaviour

Roll Moment distribution

- The load transfer is in action on both front and rear axles. The relative importance of the load transfers will depend on the balance of the roll moments distributed over the front and the rear axles.
  - More roll on front axles contributes to understeer
  - More roll on rear axles contributes to oversteer

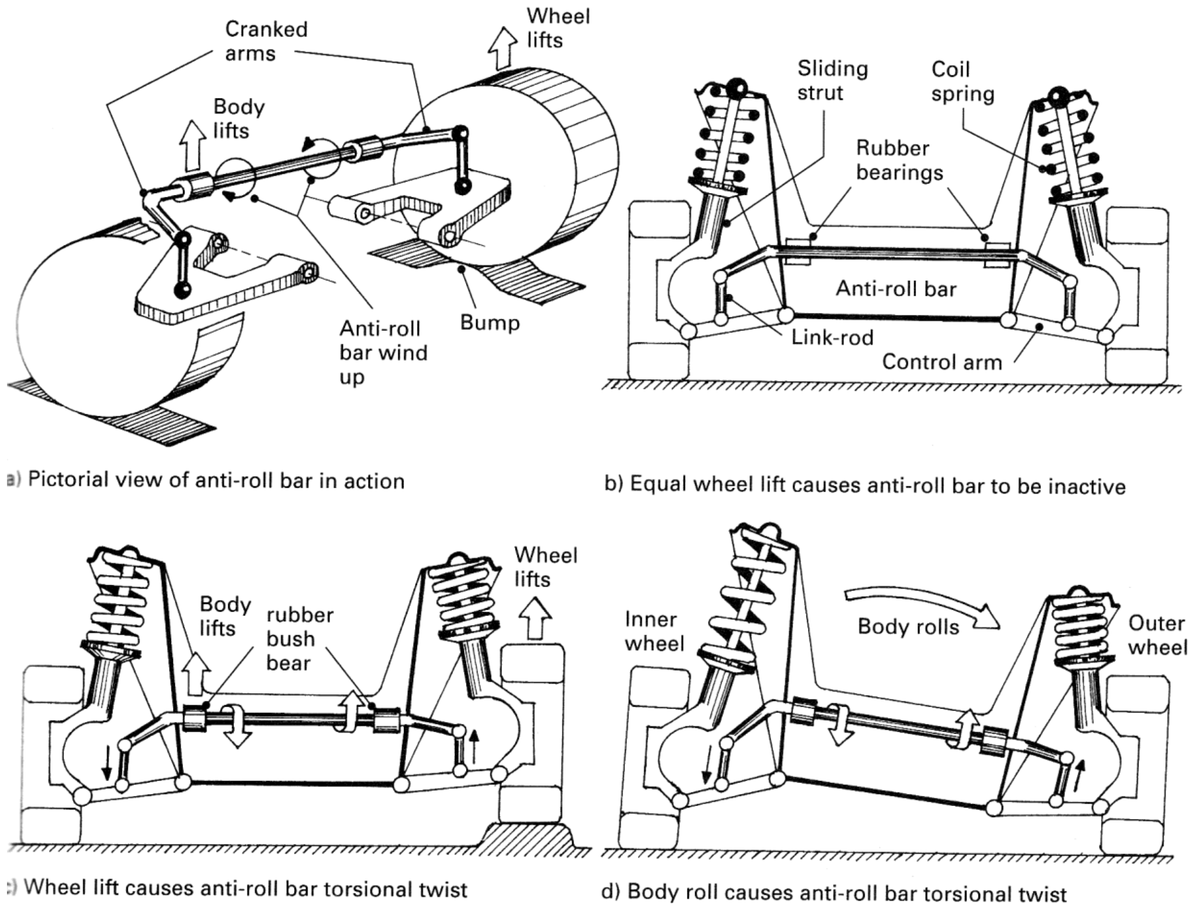
- Auxiliary roll stiffeners (stabilizer bars) alter the handling performance through the redistribution of the relative importance of the roll between the axles

Antiroll bars



Antiroll bars

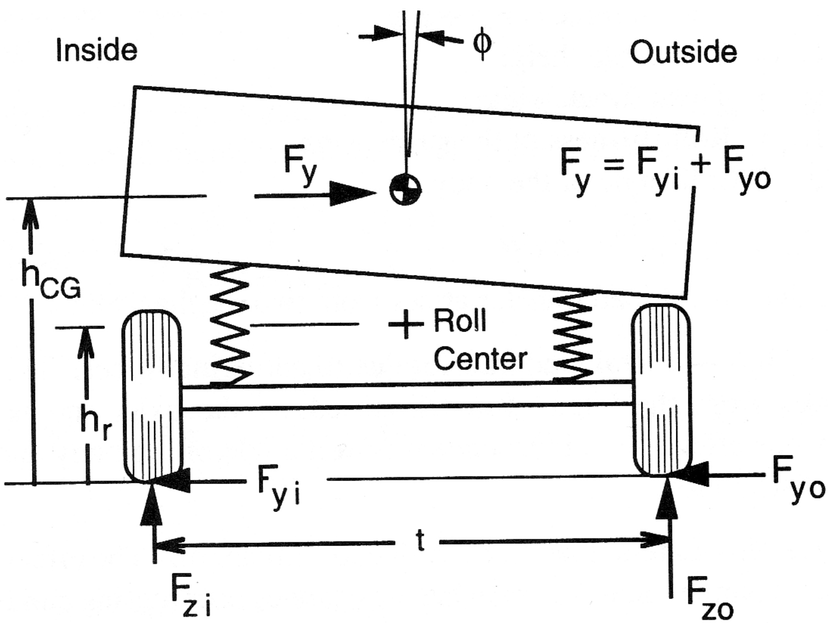
Heisler Vehicle
And engine
Technology
Fig 7.38



SUSPENSION CHARACTERISTICS

- Roll Stiffness
- Roll Center

Roll Stiffness of an Axle



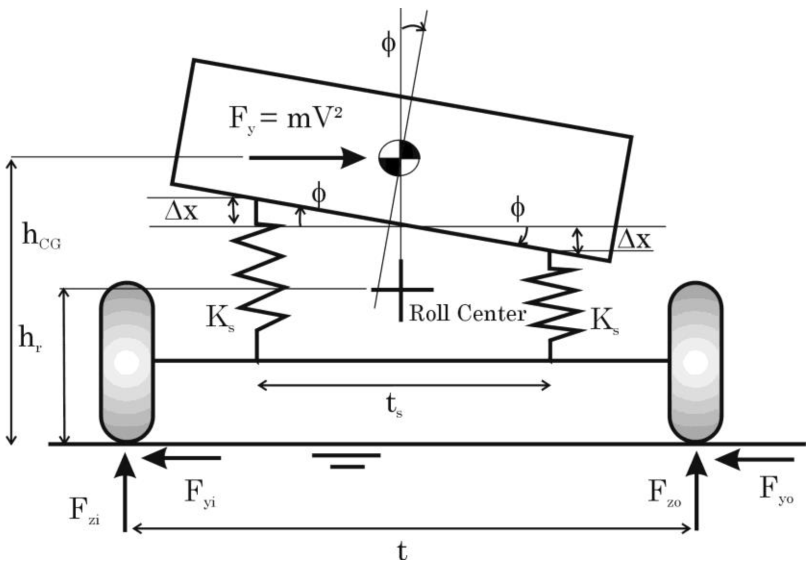
Gillespie Fig. 6.12

- The mechanics governing the roll moment
- All suspensions are functionally equivalent to two springs.
- The stiffness of the axle is

$$K_\phi = \frac{1}{2} K_s \, t_s^2$$

  - Springs of stiffness K_s
  - Separated by a distance t_s
  - K_ϕ roll stiffness of the suspension

Roll Stiffness of an Axle



$$\Delta x \simeq \phi \, \frac{t_s}{2}$$

- Equivalent roll stiffness of the two coil springs K_s separated by distance t_s:
- Energy accumulated by the two springs

$$U = 2 \times \left(\frac{1}{2} K_s \, \Delta x^2 \right)$$

$$= 2 \times \left(\frac{1}{2} K_s \, \phi^2 \, \frac{t_s^2}{4} \right)$$

$$U = \frac{1}{2} K_\phi \, \phi^2$$

- It comes

$$K_\phi = K_s \, \frac{t_s^2}{2}$$

Roll Center

- **Kinematic definition:** The kinematic roll center is the point in the transverse vertical plane of the suspension about which the body rolls in cornering relatively to the axle [Dixon]

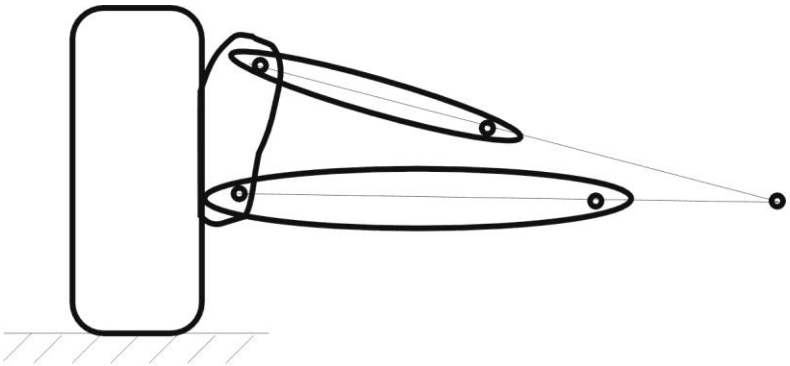
- The roll center is also the point around which the body rolls without any lateral movement at either of the wheel contact areas [Happian Smith]

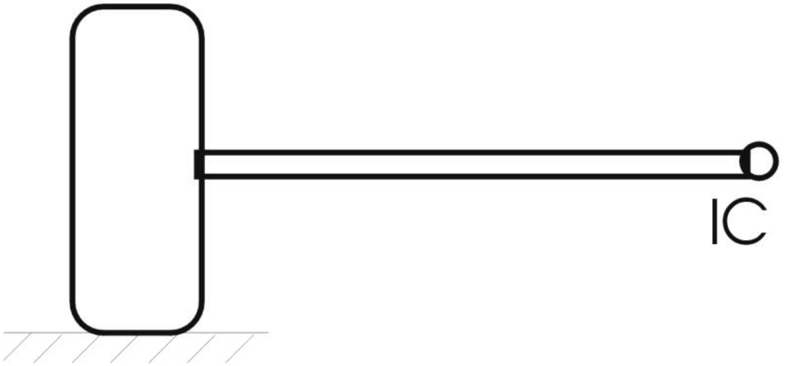
- **Static definition [SAE]:** the roll center (RC) is the point in the transverse vertical plane through any pair of wheel centers at which the lateral forces may be applied to the sprung mass without producing any suspension roll. [SAE definition]

- The roll center can also be regarded as the notional point at which the applied lateral (cornering) forces are reacted to the vehicle body.

Roll Center

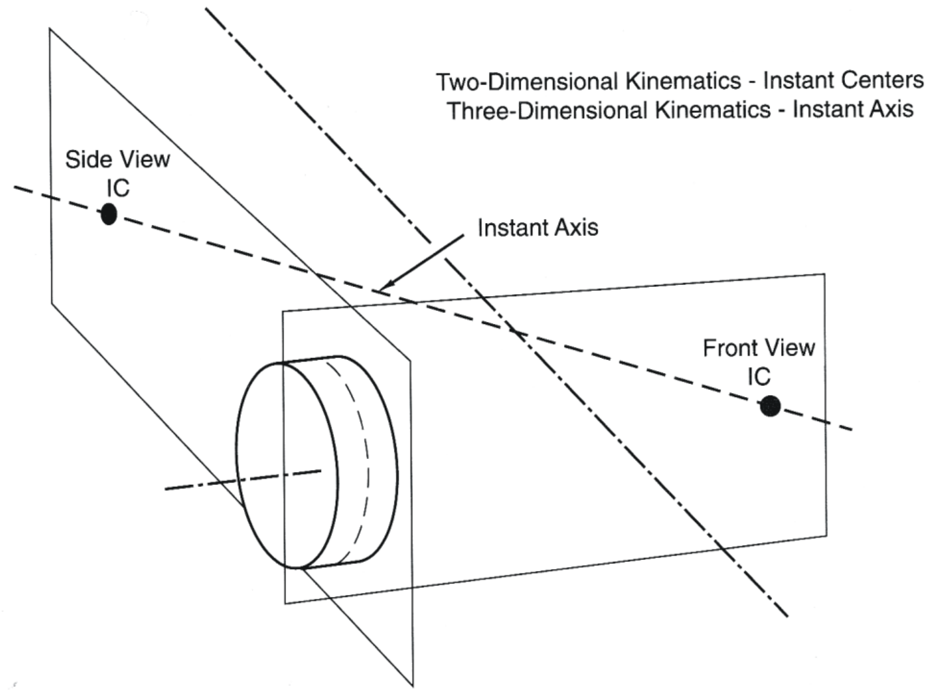
- The concept of instantaneous center of rotation (IC) is very useful to determine the kinematic parameters of a suspension mechanism
- At a given time t, the motion is equivalent to a pure rotation about a fictitious hinge located at the ICR
- One can replace the complex suspension mechanism by a rigid bar rotating about the IC
- When the mechanism undergoes large motions, the IC is also modified!





Roll Center

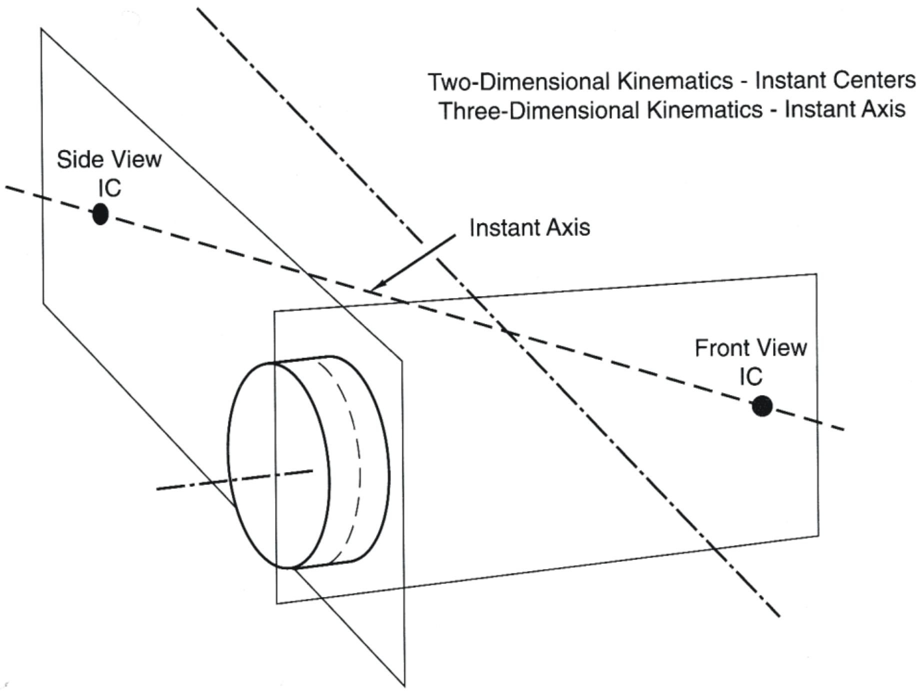
- The concept of instantaneous rotation center is basically two dimensional.
- In 3D the concept of instantaneous centre of rotation becomes an instantaneous axis of rotation.
- For 3D problems, one can be brought back to plane problems by using projections onto projection planes including the wheel center: front view and lateral view.



Milliken Fig 17.6 : CI in the front and lateral view planes

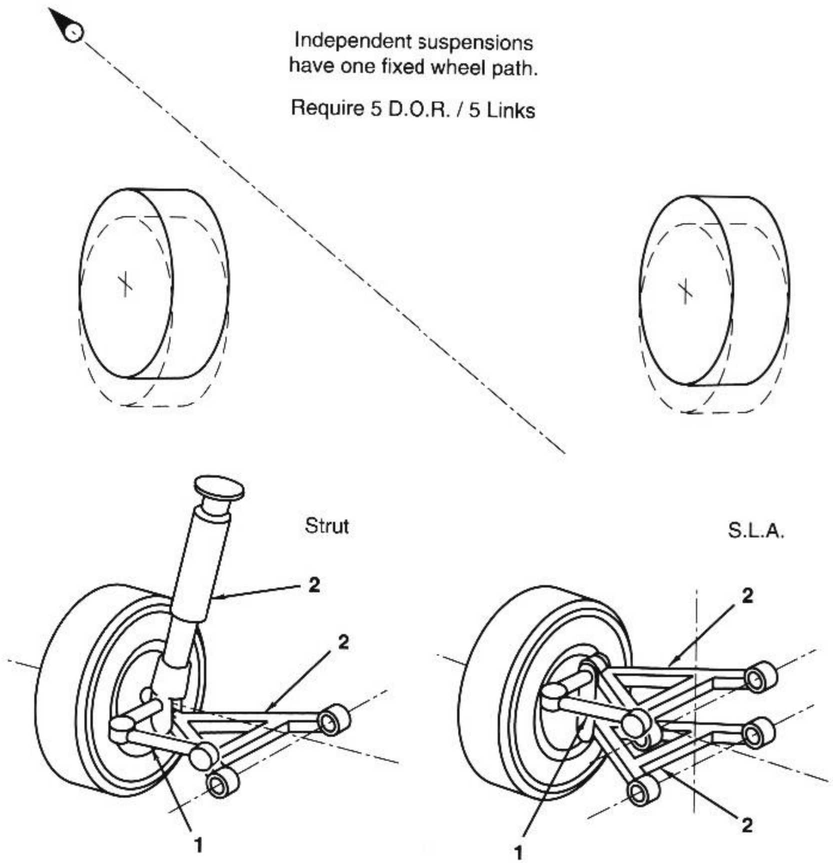
Roll Center

- The instantaneous axis of rotation is the axle about which the wheel rotates with respect to the body.
- Independent suspension mechanisms have <u>one instantaneous rotation axis</u>
- Rigid axle suspensions have <u>two instantaneous </u>rotation axes, one for the rebound and one for the roll degree of freedom.
- The IC axes are changing with the suspension travel.

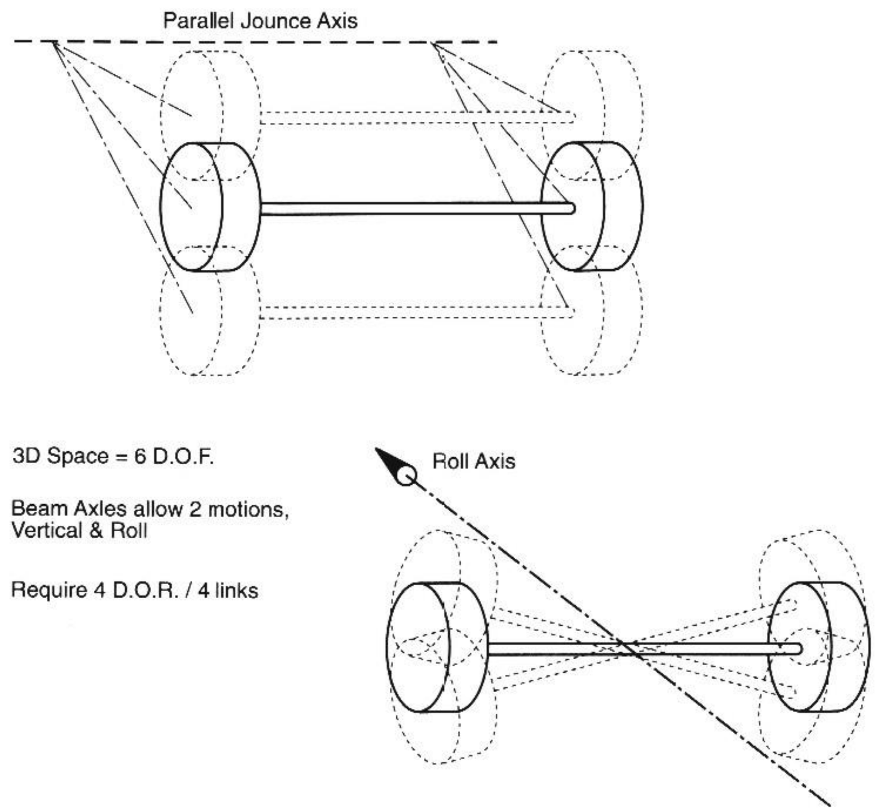


Milliken Fig 17.6 : CI in the front and lateral view planes

Suspension Mechanisms



Milliken : Fig 17.3



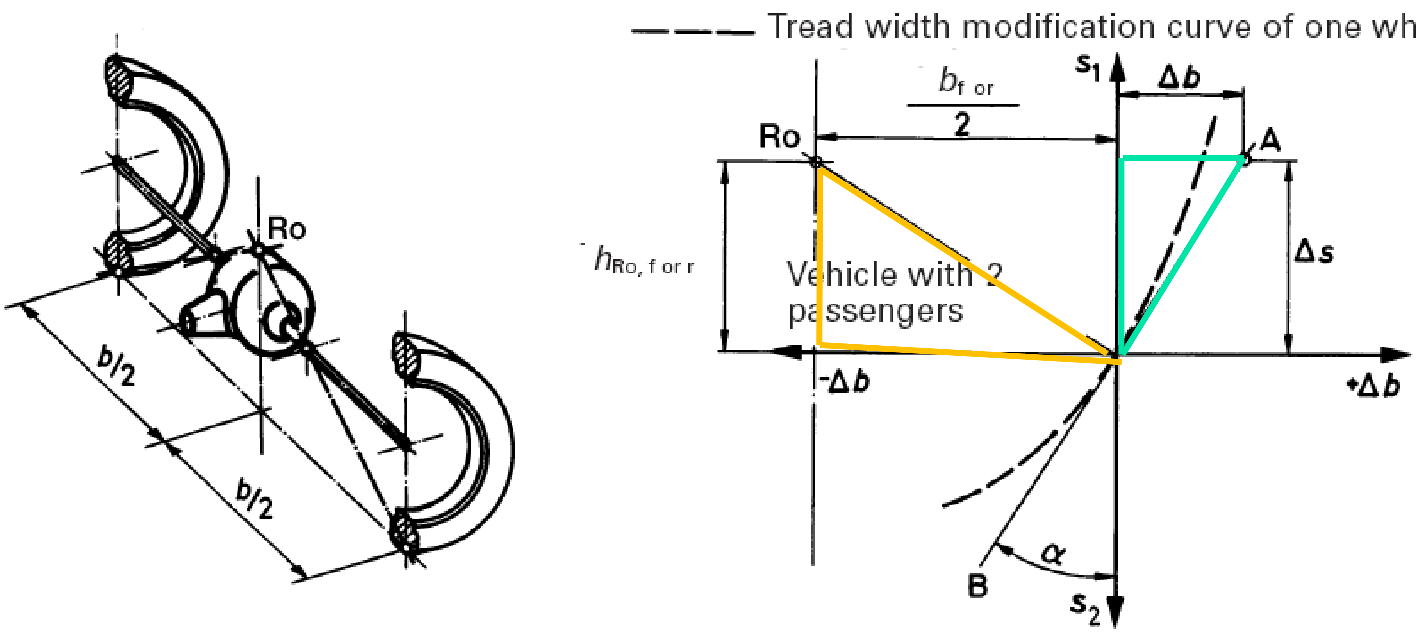
Milliken : Fig 17.4

Roll Center

- How can we calculate / determine the roll centre?
  - By using the methods of instantaneous rotation centers and making use of the Kennedy theorem
    - *The CIR of 3 bodies undergoing relative motions are aligned*
  - By using the principle of virtual work
  - By using the curve of half track modification with the wheel travel

Roll Center

- Determining the roll centre using the curve of half track modification with the wheel travel (Reimpel, p 165)



$$\tan\theta = \frac{\Delta b}{\Delta s}$$

$$h_{RoC} = \tan\theta\, b/2$$

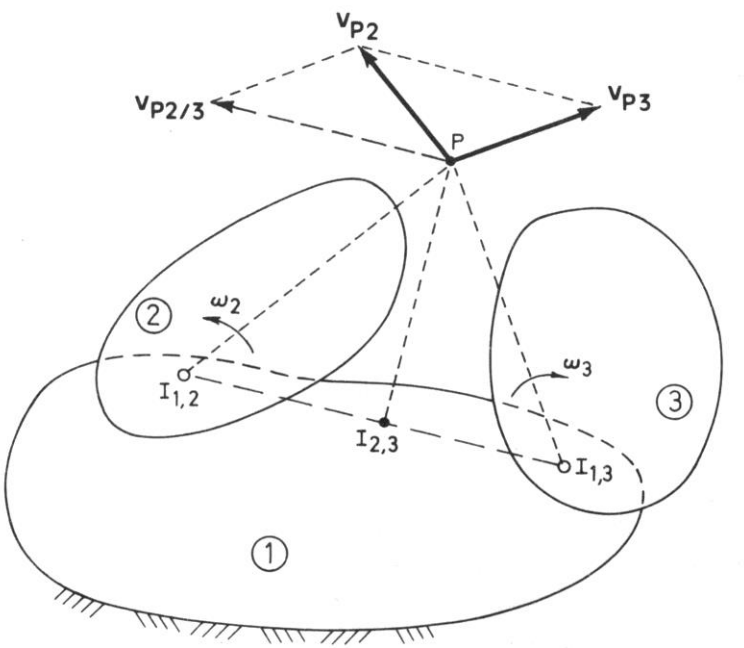
$$h_{Ro,f} = \frac{\Delta b}{\Delta s} \times \frac{b_f}{2}$$

$$h_{Ro,r} = \frac{\Delta b}{\Delta s} \times \frac{b_r}{2}$$

Fig. 3.22 The height $h_{Ro,f\ or\ r}$ of the body roll centre can be determined using a tangent from the measured track alteration curve in the respective load condition.

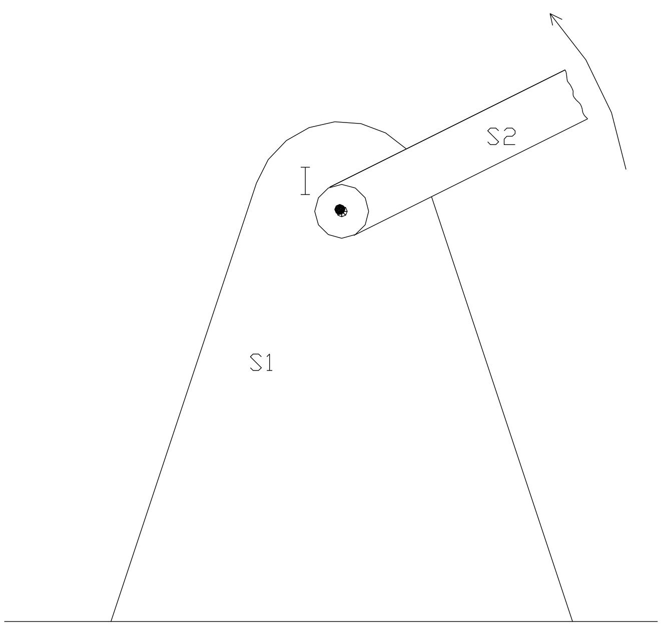
Roll Centre

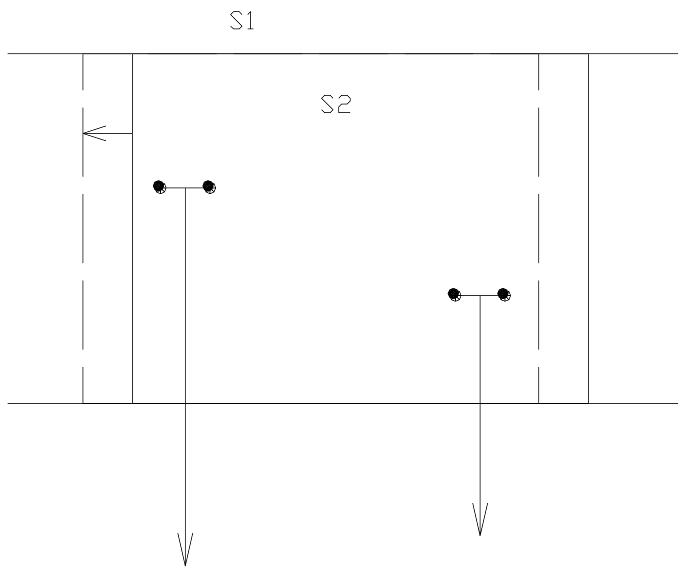
- Determination of the kinematic roll center using Kennedy theorem of the three instantaneous center of rotations: *When three bodies move relative to one another they have three instantaneous centers all of which lie in the same straight line*.



Roll Centre

Some basic ICs





Hinge: CIR is on the joint

Prismatic joint: CIR at infinity in the perpendicular direction to the hinge sliding direction

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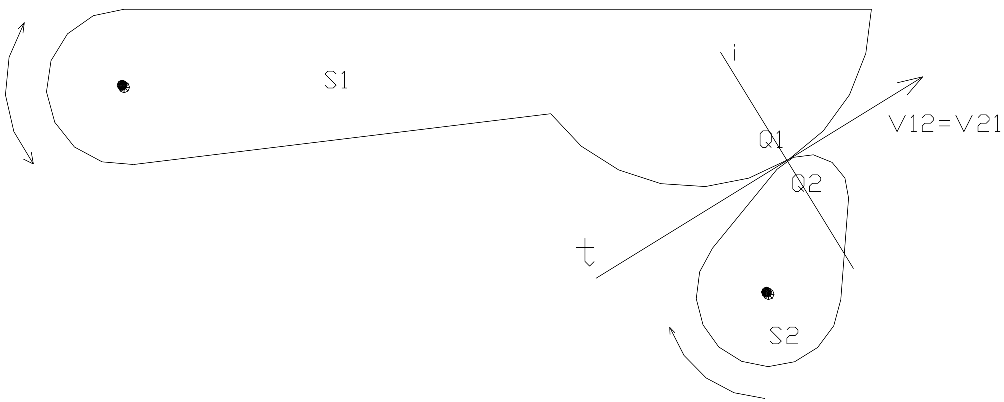
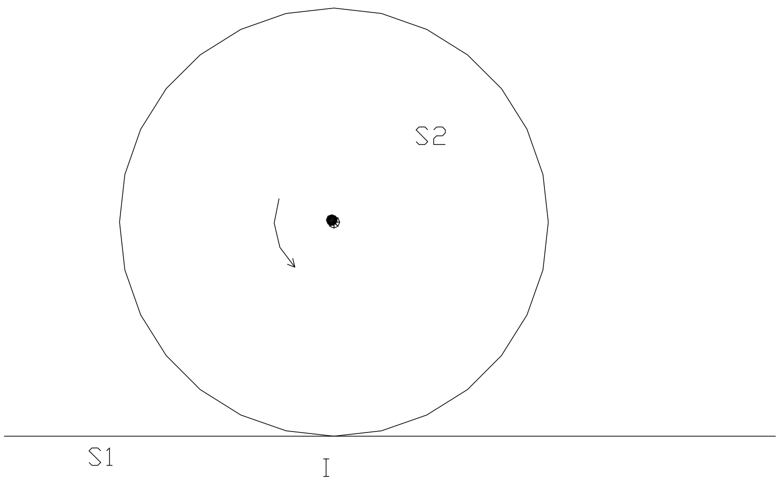
Roll Centre

Some basic ICs





Cam: CIR is located on the perpendicular line to the common tangent line

Wheel with non slip rotation: CIR is at the contact point

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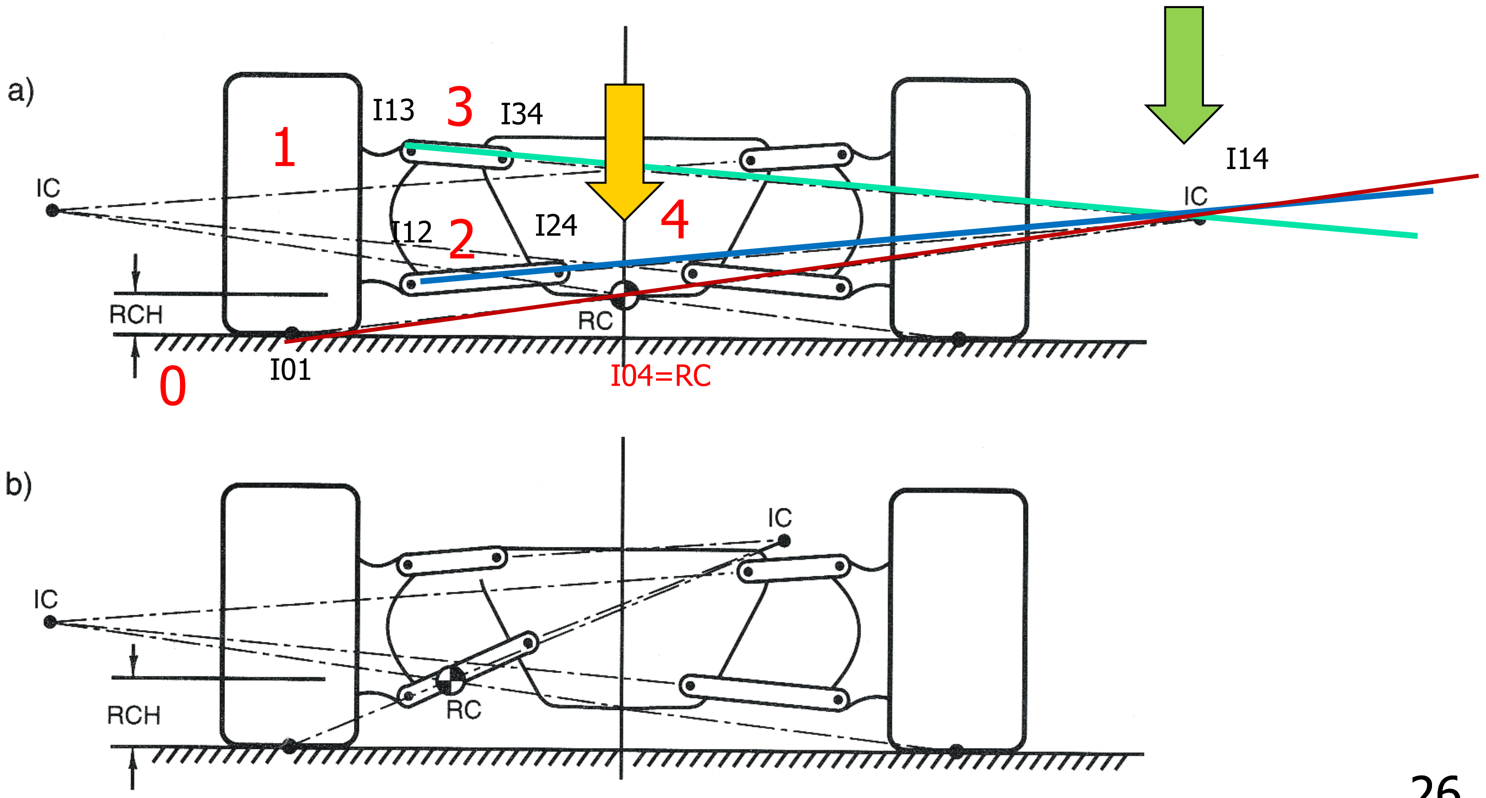
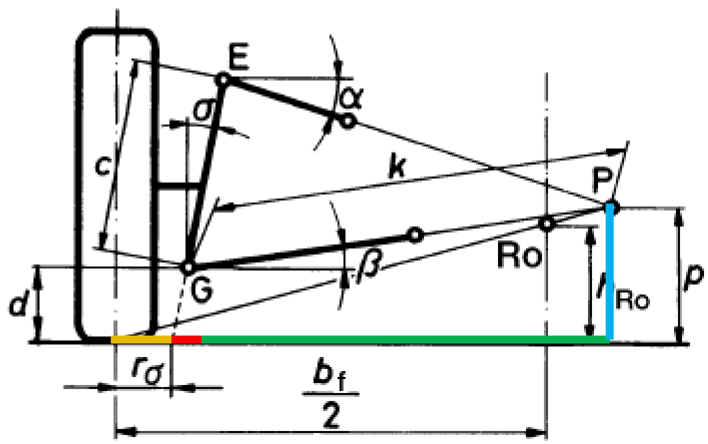
Roll Center

- Graphical determination of the roll center using the methods of instantaneous roll centers



Roll Center

- Graphical determination of the roll center for a double wishbone suspension



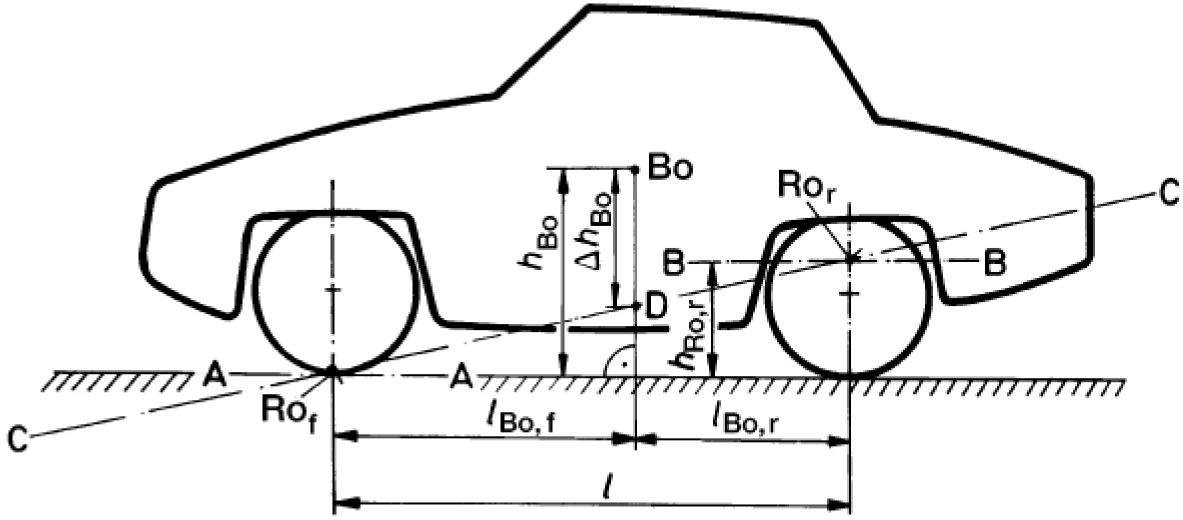
$$h_{Ro} = \frac{b_f}{2} \cdot \frac{p}{k \cos \beta + d \tan \sigma + r_\sigma}$$

$$p = k \sin \beta + d$$

$$k = c \, \frac{\sin(90^\circ + \sigma - \alpha)}{\sin(\alpha + \beta)}$$

Roll axis

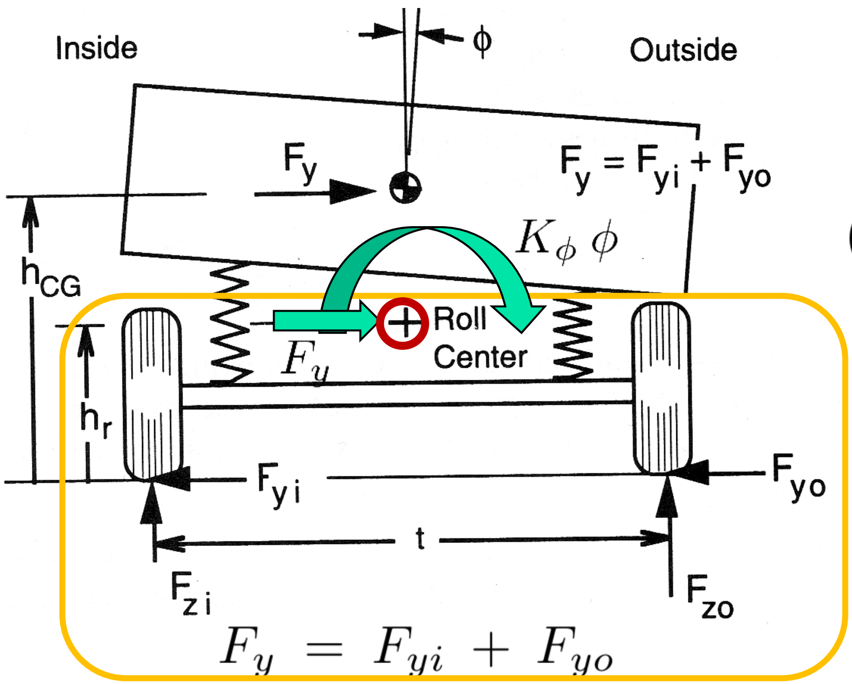
- In 3D, the roll center is replaced by the roll axis. Front and rear axles have different roll centers. Roll axis passes through the two roll centers.



LATERAL LOAD TRANSFER

- Load transfer in the axle
- Roll moment distribution
- Roll angle

Load transfer



- Equilibrium of loads applied to axle (free body diagram)

$$F_y = F_{yo} + F_{yi}$$

$$(F_{zo} - F_{zi})\frac{t}{2} = (F_{yi} + F_{yo})\,h_r + K_\phi\,\phi$$

  - t track width
  - h_r roll centre height
- Relation between the wheel loads, the lateral forces and the roll angle.

$$F_{zo} - F_{zi} = 2\,F_y\,\frac{h_r}{t} + 2\,K_\phi\frac{\phi}{t} = 2\,\Delta F_z$$

Load transfer

- Load transfer

$$2\,\Delta F_z = F_{zo} - F_{zi} = 2\,F_y\,\frac{h_r}{t} + 2\,K_\phi\,\frac{\phi}{t}$$

- Comes from two mechanisms

$$2\,F_y\,h_r/t$$

  - Lateral load transfer due to the cornering forces. It depends on the vertical position of the roll center. Independent of the roll angle of the body and roll moment distribution.

$$2\,K_\phi\,\phi/t$$

  - Lateral load transfer due to vehicle roll. It depends on roll dynamics and may lag the changes in cornering conditions. It depends on front / rear roll moment distribution.

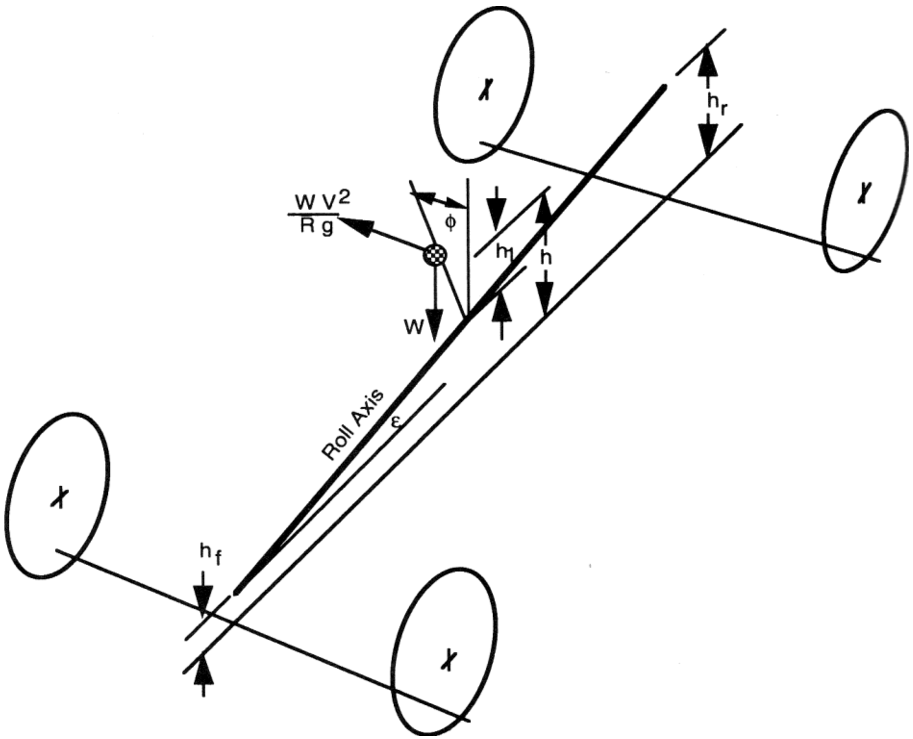
To calculate the load transfer

- To calculate the load transfer

$$F_{zo} - F_{zi} = 2\, F_y\, h_r/t + 2\, K_\phi \phi/t = 2\, \Delta F_z$$

- We need to compute:
  - The roll center height h_r
    - Geometrical properties of the suspension mechanism.
  - The roll angle ϕ under the action of the lateral acceleration
    - This one must be determined for the vehicle equilibrium as a whole. It depends on the front / rear roll moment distribution.

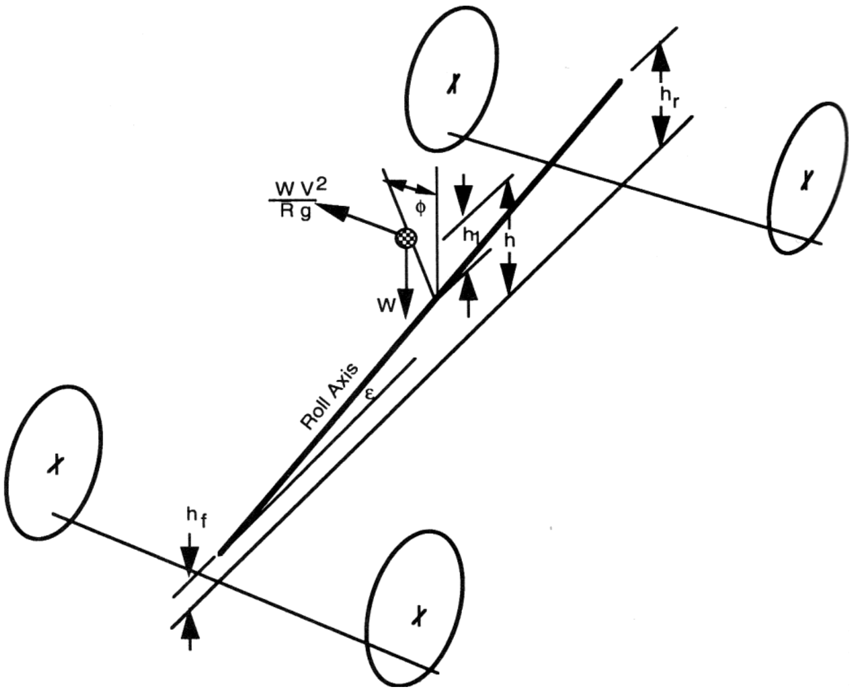
Roll moment distribution



Gillespie Fig. 6.13 Force analysis for roll of a vehicle

- The roll moment distribution requires the total vehicle model
- We define the roll axis as the line connecting the roll centres of the front and rear suspensions.
  - h_f and h_r are the elevations of the front and rear roll centres
  - h_1: height of the centre of mass of the rolling body with respect to the roll axis position

Roll moment distribution



Gillespie Fig. 6.13 Force analysis for roll of a vehicle

- Moment due to the forces acting at CG about the roll axis

$$M_\phi = \left[m \frac{V^2}{R} h_1 \cos\phi + W h_1 \sin\phi \right] \cos\epsilon$$

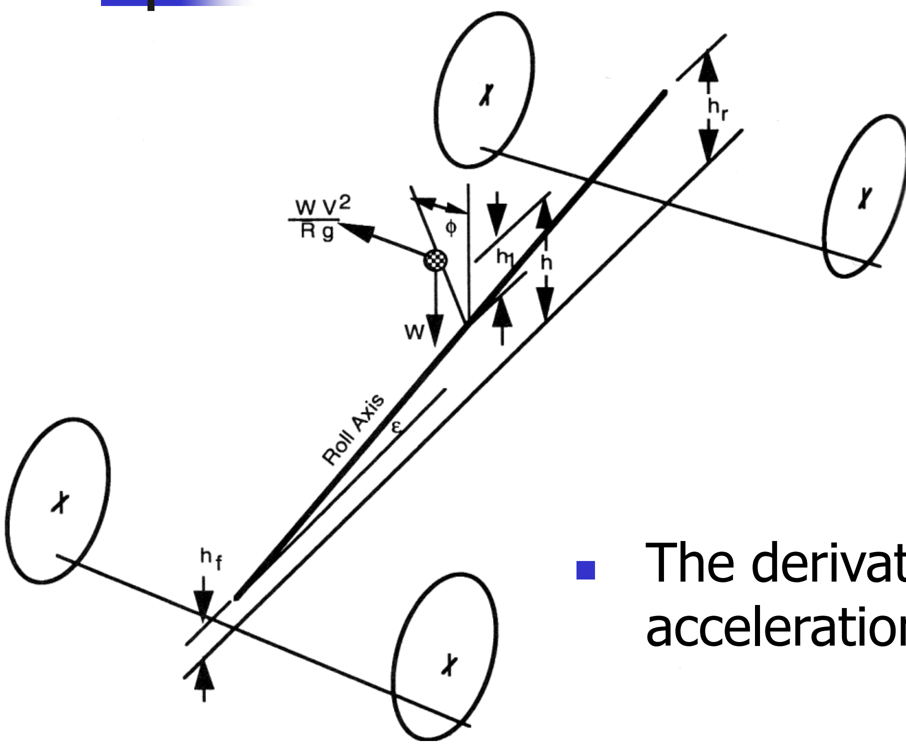
- For small angles,

$$\cos\phi \simeq 1 \qquad \sin\phi \simeq \phi \qquad \cos\epsilon \simeq 1$$

- It comes

$$M_\phi = W h_1 \left[\frac{V^2}{gR} + \phi \right]$$

Roll moment distribution



Gillespie Fig. 6.13 Force analysis for roll of a vehicle

- The roll moment of suspension

$$M_\phi = M_{\phi f} + M_{\phi r} = (K_{\phi f} + K_{\phi r})\phi$$

- Solving for ϕ:

$$\boxed{\phi = \frac{W h_1 \frac{V^2}{gR}}{(K_{\phi f} + K_{\phi r}) - W h_1}}$$

- The derivative of the roll with respect to the lateral acceleration is the roll rate of the vehicle:

$$R_\phi = \frac{\partial \phi}{\partial a_y} = \frac{W h_1}{(K_{\phi f} + K_{\phi r}) - W h_1}$$

- The roll rate is usually in the range of 3 to 7 degres/g on typical passenger cars

Roll moment distribution

- Combining the expression of the roll angle with the expression of the load transfer <u>in each axle</u>,

$$F_y\, h_r \;+\; K_\phi \phi \;=\; \Delta F_z\, t$$

- For instance for the front axle

$$F_{yf} \;=\; m\frac{c}{L}\,\frac{V^2}{R} \;=\; W_f\,\frac{V^2}{gR}$$

$$\phi \;=\; \frac{W h_1 \frac{V^2}{gR}}{(K_{\phi f}+K_{\phi r}) - W h_1}$$

- One gets the expression of the roll moments <u>on the front axle</u>

$$M'_{\phi f} \;=\; K_{\phi f}\,\frac{W h_1 \frac{V^2}{gR}}{(K_{\phi f}+K_{\phi r}) - W h_1} \;+\; W_f h_f \frac{V^2}{gR} \;=\; \Delta F_{zf}\, t_f$$

Roll moment distribution

- One gets the roll moments on the front and rear axles

$$M'_{\phi f} = K_{\phi f} \frac{W h_1 \frac{V^2}{gR}}{(K_{\phi f} + K_{\phi r}) - W h_1} + W_f h_f \frac{V^2}{gR} = \Delta F_{zf}\, t_f$$

$$M'_{\phi r} = K_{\phi r} \frac{W h_1 \frac{V^2}{gR}}{(K_{\phi f} + K_{\phi r}) - W h_1} + W_r h_r \frac{V^2}{gR} = \Delta F_{zr}\, t_r$$

- while

$$\Delta F_{zf} = F_{zfo} - W_f/2 = -(F_{zfi} - W_f/2)$$
$$\Delta F_{zr} = F_{zro} - W_r/2 = -(F_{zri} - W_r/2)$$

Roll moment distribution

- In general, the roll moment distribution tends to be biased towards the front wheels:
  - Relative to the load, the front spring rate K_{sf} is usually slightly lower than the rear K_{sr} (for flat ride / comfort) ➔ bias towards higher roll stiffness at the rear. However, on independent rear suspension, the higher distance t_s between the suspension springs enhances the front roll stiffness.
  - Designers usually strive for a higher front roll stiffness to ensure understeer in the limit cornering
  - Stabilizer bars are often used on the front axles to obtain higher front roll stiffness. <u>If stabilizer bars are needed to reduce the body roll, they must be installed on the front or on the front + the rear</u>. Warning: a stabilizer bar on the rear only can cause unwanted oversteer.

Understeer gradient due to lateral load transfer

- Tire sensitivity to load transfer
- Understeer gradient due to load transfer

Tire load transfer sensitivity

- We have determined the roll moments for front and rear axles and the difference in load between the left and right wheels.
- To translate the lateral load transfer into an effect on understeer gradient, we need data about the sensitivity of tires with respect to the slip angle and load.
- The difference in the change of slip angles between the front and the rear when load transfer is present represents the understeer effect.

Tire load transfer sensitivity

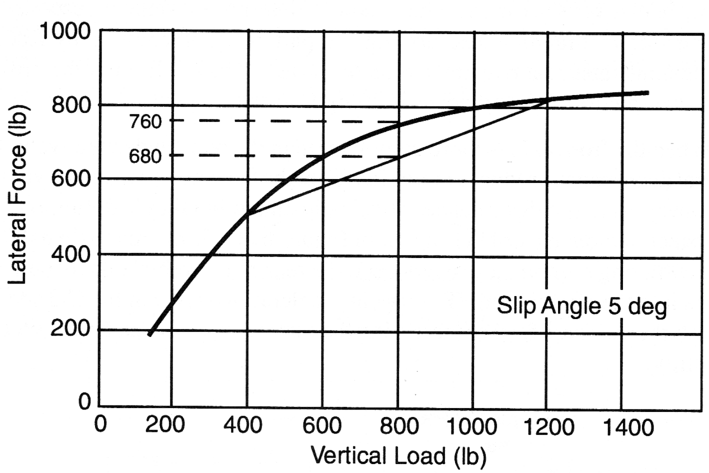
- Express the cornering forces of each tires

$$F_y = C_\alpha \, \alpha$$

- The sensitivity to the load can be modeled using a second order polynomial for the cornering coefficient

$$F_y = C_\alpha \, \alpha = (a \, F_z - b \, F_z^2) \, \alpha$$

  - a : first coefficient in the cornering stiffness polynomial [N/N/deg]
  - b: second coefficient in the cornering stiffness polynomial [N/N²/deg]



Gillespie Fig 6.11 Lateral force-vertical load characteristics of tires

Tire load transfer sensitivity

- For a vehicle cornering, the lateral force of the two tires of the axle is given by:

$$F_y = \left(aF_{zo} - bF_{zo}^2 + aF_{zi} - bF_{zi}^2\right)\alpha$$

- Introduce the lateral load transfer

$$F_{zo} = F_z + \Delta F_z \qquad F_{zi} = F_z - \Delta F_z$$

- It comes after some algebra

$$F_y = \left(2aF_z - 2bF_z^2 - 2b\Delta F_z^2\right)\alpha$$

- and

$$F_y = \left(C_\alpha - 2b\Delta F_z^2\right)\alpha \qquad C_\alpha = 2\left(aF_z - bF_z^2\right)$$

Understeer gradient due to lateral load transfer

- Remind that the equation between the steering angle and the slip angles on front and rear axles is:

$$\delta = \frac{L}{R} + \alpha_f - \alpha_r$$

- For the front and the rear tire, we can write

$$F_{yf} = (C_{\alpha f} - 2b\Delta F_{zf}^2)\,\alpha_f = W_f\,V^2/(gR)$$

$$F_{yr} = (C_{\alpha r} - 2b\Delta F_{zr}^2)\,\alpha_r = W_r\,V^2/(gR)$$

- Substituting, we gets

$$\delta = \frac{L}{R} + \frac{W_f V^2/(gR)}{(C_{\alpha f} - 2b\Delta F_{zf}^2)} - \frac{W_r V^2/(gR)}{(C_{\alpha r} - 2b\Delta F_{zr}^2)}$$

Understeer gradient due to lateral load transfer

- This equations can simplify by using a Taylor expansion

$$\frac{1}{(C_\alpha - 2b\Delta F_z^2)} = \frac{1}{C_\alpha(1 - \frac{2b\Delta F_z^2}{C_\alpha})} \simeq \frac{1}{C_\alpha}\left(1 + \frac{2b\Delta F_z^2}{C_\alpha}\right)$$

- The steering angle equation becomes

$$\delta = \frac{L}{R} + \left[\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right) + \left(\frac{W_f}{C_{\alpha f}}\frac{2b\Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\frac{2b\Delta F_{zr}^2}{C_{\alpha r}}\right)\right]\frac{V^2}{gR}$$

Understeer gradient from nominal cornering stiffness

Understeer gradient contribution From Lateral Load Transfer

Understeer gradient due to lateral load transfer

- The gradient from lateral load transfer

$$K_{LLT} = \frac{W_f}{C_{\alpha f}} \frac{2b\Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \frac{2b\Delta F_{zr}^2}{C_{\alpha r}}$$

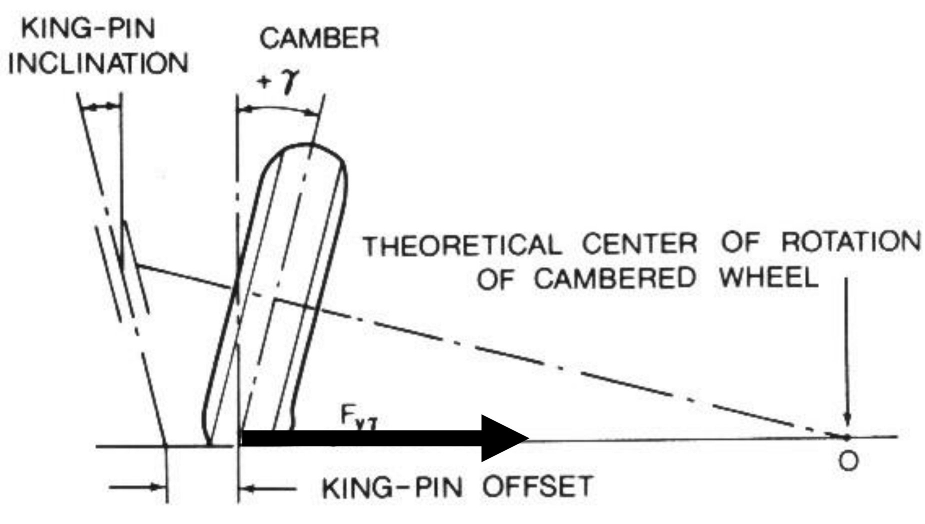
- The lateral load transfer are given by the equilibrium equations
- All the six variables are positive, so that the contribution from front axle is always understeer while the contribution from rear axle is always oversteer

Understeer gradient due to camber change

- Camber thrust
- Camber change
- Understeer gradient due to camber change

Camber thrust

- The inclination of the wheel outward from the body is called the camber angle.
- Camber on a wheel will produce a lateral force known as camber thrust.
- The camber thrust is oriented toward to the intersection point of the wheel rotation axis with the ground

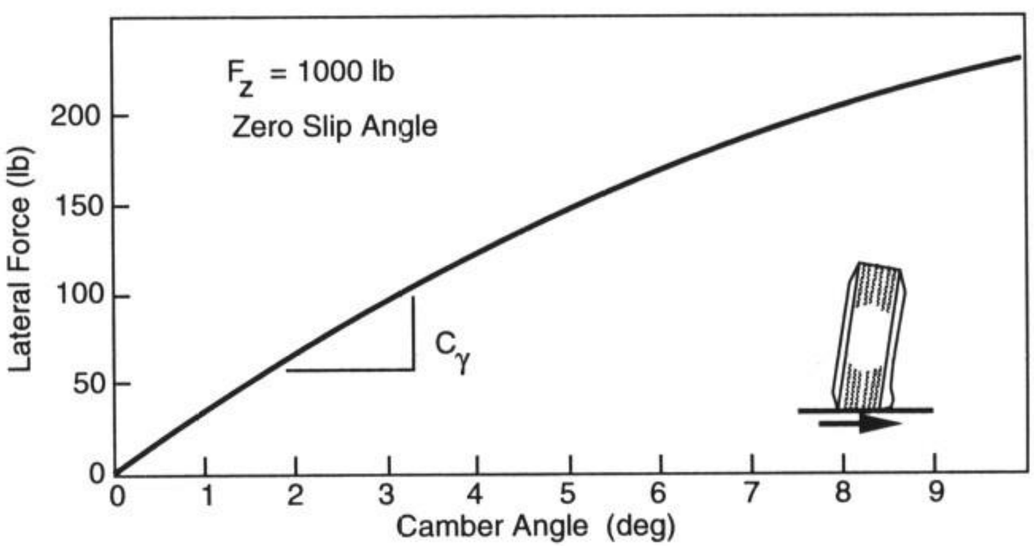


Camber thrust

- In its linear part, (small camber angles), the camber thrust can be approximated by the linear relation with respect to camber angle:

$$F_y(\gamma) = C_\gamma \, \gamma$$

  - C_γ is the camber stiffness



$$C_\gamma = \frac{\partial F_y}{\partial \gamma}\Big|_{\gamma=0}$$

Gillespie Fig. 6.14

Camber thrust

- The camber thrust produces much smaller lateral forces than an equivalent slip angle.
  - About 4 to 6 degrees of camber produce the same lateral force as 1 degree of side slip for a bias tyre.
  - For radial tyres, as much as 10 to 15 degrees are necessary for 1 degree of slip
  - This can be understood because the side slip produces a much bigger deformation of the tyre tread. For radial tyre, the sidewall are even softer and the belts are very rigid.

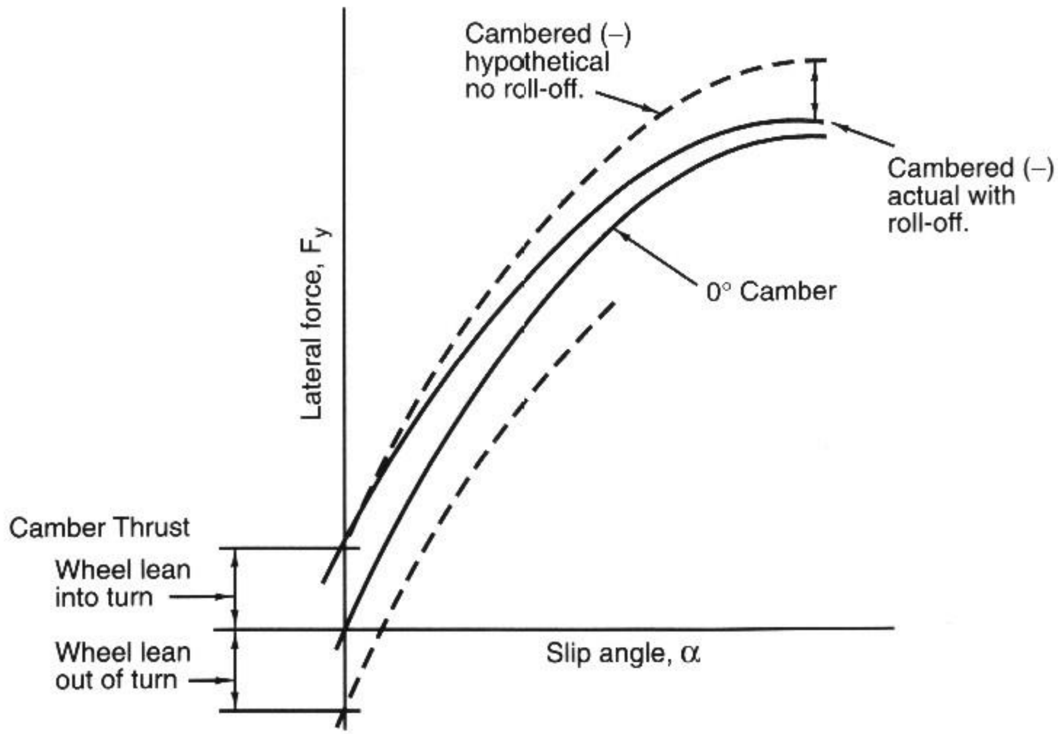
- For passenger car, the camber thrust is fading out for camber thrust over 5 to 10 degrees
- For motorbikes tyres with round profiles and bias structures, the camber thrust can be generated up to 50 degrees

Camber thrust

- Camber thrust is additive to cornering forces from slip angle (at least for small angles).

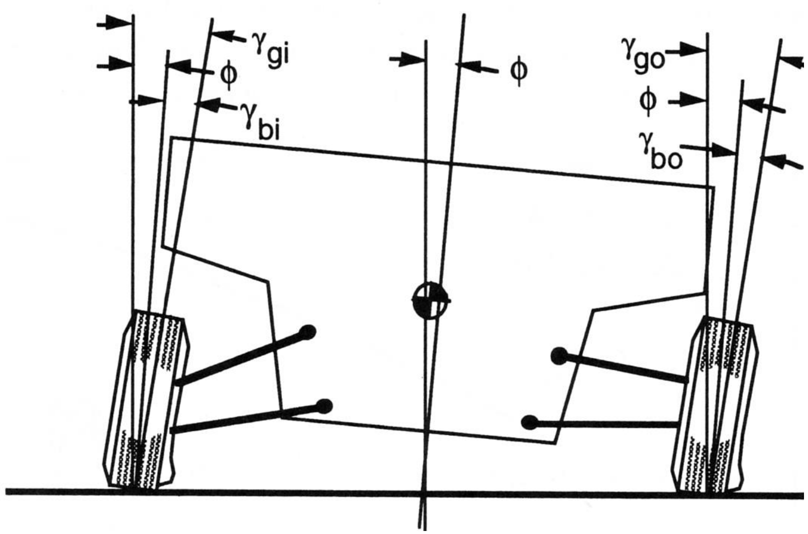
$$F_y = -C_\alpha \alpha + C_\gamma \gamma$$

- For large angles, both phenomena interferes and a saturation is observed



Camber thrust and slip angles
Milliken Fig 2.24

Camber change



Gillespie Fig 6.15: Camber change in the cornering of the vehicle

- On solid axles camber angles are small and, at maximum, they can change the lateral forces by 10%
- On independent wheel suspensions, the camber can play an important role in cornering.
- Camber changes result from both the body roll and from the camber change due to wheel travel in contact with the road (bounce / jounce)

$$\gamma_b = \gamma_{w/b} + \phi$$

Camber change

- The camber angle arising from the suspension kinematics is a function of the roll angle because of the wheel travel in jounce for the inside wheel or in rebound for the outside wheel.
- Given a suspension geometry and type, one can obtain the derivative of the camber change with respect to the roll angle from the kinematics analysis of the suspension mechanism:

$$\delta\gamma = \frac{\partial\gamma}{\partial\phi}\,\delta\phi$$

$$\frac{\partial\gamma}{\partial\phi} = \mathrm{f}_{\gamma}(\text{track width,suspension geometry,roll angle})$$

- In turn the roll angle can be related to the lateral acceleration

$$\phi = \frac{W h_1 \frac{V^2}{gR}}{K_{\phi f} + K_{\phi r} - W h_1}$$

Understeer gradient due to camber change

- The influence on cornering comes from the fact that the lateral force results from both the slip angle and the camber angle

$$F_y = C_\alpha \alpha + C_\gamma \gamma$$

- Thus

$$\alpha = \frac{F_y}{C_\alpha} - \frac{C_\gamma}{C_\alpha}\gamma$$

- Both F_y and γ are related to the lateral acceleration

$$\gamma = \frac{\partial \gamma}{\partial \phi}\phi \qquad \alpha_f = \frac{W_f}{C_{\alpha f}}a_y - \frac{C_{\gamma f}}{C_{\alpha f}}\frac{\partial \gamma_f}{\partial \phi}\frac{\partial \phi}{\partial a_y}a_y$$

$$\phi = \frac{\partial \phi}{\partial a_y}a_y \qquad \alpha_r = \frac{W_r}{C_{\alpha r}}a_y - \frac{C_{\gamma r}}{C_{\alpha r}}\frac{\partial \gamma_r}{\partial \phi}\frac{\partial \phi}{\partial a_y}a_y$$

Understeer gradient due to camber change

- Substituting the slip angles into the equation

$$\delta = \frac{L}{R} + \alpha_f - \alpha_r$$

- One gets

$$\delta = \frac{L}{R} + \left[\left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) - \left(\frac{C_{\gamma f}}{C_{\alpha f}} \frac{\partial \gamma_f}{\partial \phi} \frac{\partial \phi}{\partial a_y} - \frac{C_{\gamma r}}{C_{\alpha r}} \frac{\partial \gamma_r}{\partial \phi} \frac{\partial \phi}{\partial a_y} \right) \right] \frac{V^2}{gR}$$

- The understeer coming from the camber angles

$$\boxed{K_{Camber} = - \left(\frac{C_{\gamma f}}{C_{\alpha f}} \frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}} \frac{\partial \gamma_r}{\partial \phi} \right) \frac{\partial \phi}{\partial a_y}}$$

Understeer gradient due to roll steer

- Roll steer
- Understeer gradient due to roll steer
- Rear axle roll steer

Roll Steer

- When the vehicle rolls in cornering, the suspension kinematics may produce some wheel steer.
- Roll steer is defined as the steering motion of the front or rear wheels with respect to the sprung mass due to the rolling motion of the sprung mass.
- Roll steer effects on handling is lagging the steer input following the roll of the sprung roll.

- Roll steer affects the handling since it alters the angle of the wheels with respect to the direction of travel.

Roll Steer

- Let ε the roll steer coefficient on the axle [degree of steer / degree of roll]

$$\epsilon = \frac{\partial \delta}{\partial \phi}$$

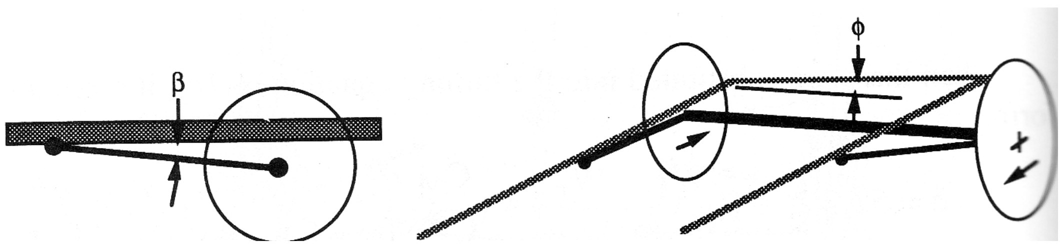
- Following the reasoning that was used before, one can derive the roll steer gradient:

$$\boxed{K_{rollsteer} = (\epsilon_f - \epsilon_r) \frac{\partial \phi}{\partial a_y}}$$

- A positive roll steer coefficient causes the wheels to steer to the right in hand right turn. So as the right-hand roll occurs when the vehicle is turning to the right, positive roll steer on the front axle steers out the turn and is understeer.
- A positive roll steer on the rear wheels lead to oversteer.

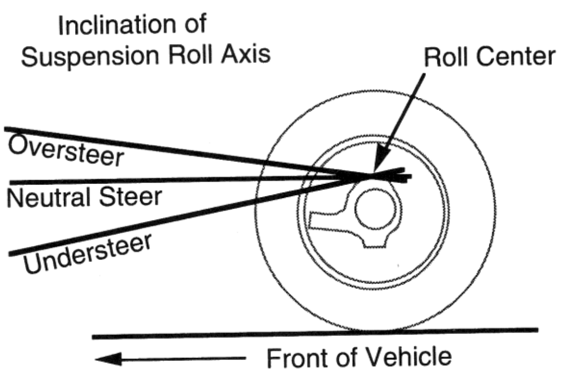
Roll Steer: rear axles

- On a solid axle the suspension allows the axle to roll about an imaginary axis that is inclined with respect to the longitudinal axis of the vehicle
- The kinematics is envisioned as functionally equivalent to leading or trailing arms systems. The roll axis inclination is equivalent to that of the arms β.
- Given an inclination β, as the body rolls positively – turn to the left -, the arm on inside wheel rotates downwards while the arms on the outside wheel rotates upwards



Gillespie Fig 6.16: Roll steer with a solid axle

Roll Steer: rear axles



Gillespie Fig 6.17: Influence of the rear axle trailing arm angle on understeer

- If the orientation of the trailing arm is angled downward, the effect of the trailing arm is
  - To pull the inner wheel forward
  - To push the outer wheel rearward
- This produces roll steer of the solid axle contributing to oversteer
- The roll steer is equal to the inclination angle:

$$\epsilon = \beta$$

- <u>On the rear trailing system, the roll understeer is achieved by keeping the transverse pivots of the trailing arm below the wheel center</u>

Roll Steer

- With independent suspensions, the roll steer coefficient must be evaluated from the kinematics of the suspension.

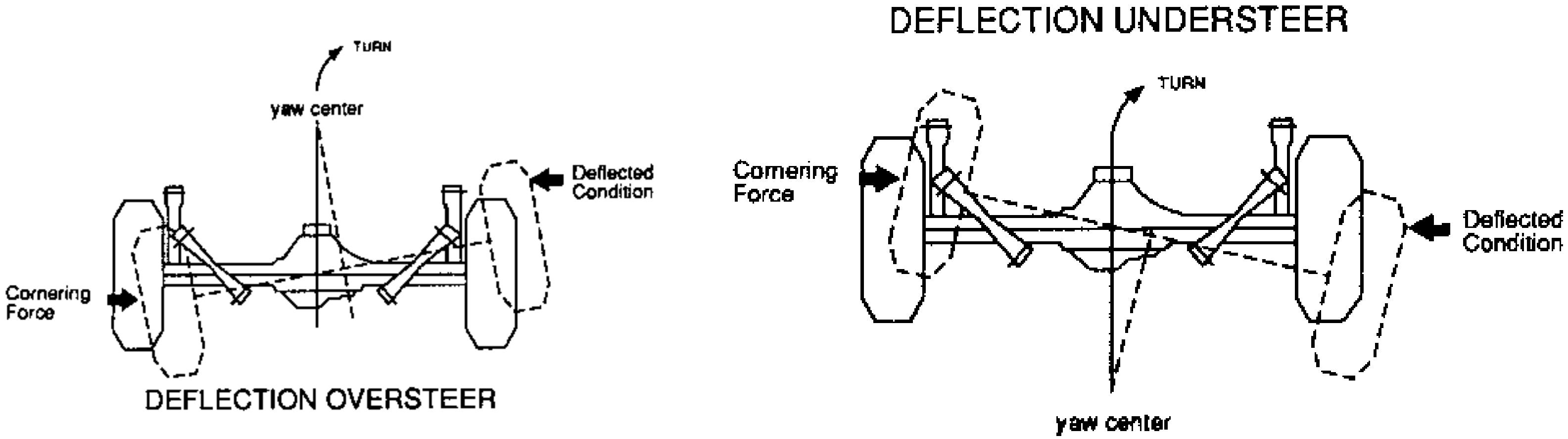
- On steered wheels, the interactions with the steering system must also be taken into account.

Lateral force compliance

- Lateral force compliance
- Understeer gradient due to lateral force compliance

Lateral Force Compliance

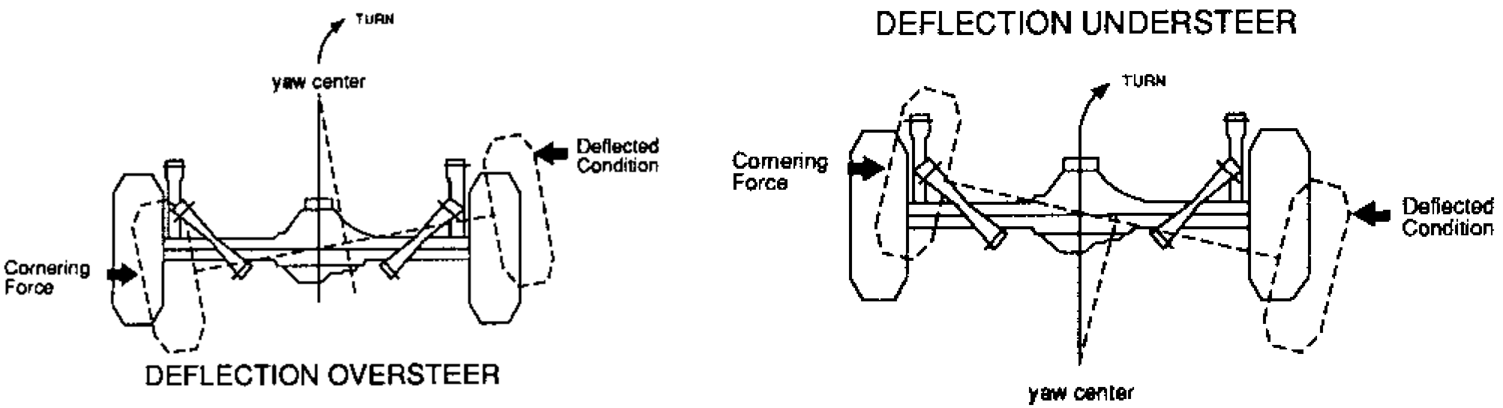
- Because of NVH performance, bushings and elastic joints are used in suspension mechanisms.
- The lateral compliance of the suspension mechanisms can introduce steer displacements.
- With a simple solid axle, the steer motion can be described as a rotation motion about the yaw center.



Gillespie Fig 6.18: Steer due to lateral compliance in the suspension

Lateral Force Compliance Steer

- For a rear axle and a forward yaw center, the lateral forces cause the rear axle to steer outside of the turn and to contribute to oversteer.
- Conversely for a rear yaw center, the lateral compliance under the lateral forces creates understeer
- On a front axle, a rearward yaw center causes oversteer while a forward yaw center is under steer



Gillespie Fig 6.18: Steer due to lateral compliance in the suspension

Lateral Force Compliance Steer

- The influence of lateral compliance steer upon understeer gradient can be characterized using the following coefficient:

$$A = \frac{\delta_c}{F_y}$$

Where δ_c is the steer angle

- The steer due to the lateral force compliance on a solid axle is simply

$$\delta_{cf} = A_f \, W_f \, a_y$$

- As the main understeer effect is related to the steer of the front and rear axles axle, it comes

$$\boxed{K_{LFCS} = -(A_f W_f - A_r W_r)}$$

Lateral Force Compliance Steer

- For independent suspensions mechanisms, the things are more complex, and the linkage mechanisms must be analyzed to determine the coefficients for independent axles

Aligning Torque

Aligning Torque

- Aligning torque tends to reduce the slip angles and to reduce the attempted turn, thus it is a source of understeer effect.

- Aligning torque is related to the position of the resultant lateral force behind the center of the contact patch.
- The pneumatic trail is given by the ratio between the aligning moment and the lateral force.

$$t = \frac{M_z}{F_y}$$

Aligning Torque

- The influence of the aligning torque can be determined by deriving the equations by considering a modified position of application point of the lateral load developed under the contact patch.

$$\tilde{b} = b - t$$

$$\tilde{c} = c + t$$

- It comes

$$\delta = \frac{L}{R} + \left(\frac{m\tilde{c}/L}{C_{\alpha f}} - \frac{m\tilde{b}/L}{C_{\alpha r}} \right) \frac{V^2}{R}$$

$$\delta = \frac{L}{R} + \left(\frac{m(c+t)/L}{C_{\alpha f}} - \frac{m(b-t)}{C_{\alpha r}} \right) \frac{V^2}{R}$$

Aligning Torque

- Finally one can write

$$\delta = \frac{L}{R} + \left(\frac{m\tilde{c}/L}{C_{\alpha f}} - \frac{m\tilde{b}/L}{C_{\alpha r}} \right) \frac{V^2}{R}$$

$$= \frac{L}{R} + \left(\left[\frac{mc/L}{C_{\alpha f}} - \frac{mb/L}{C_{\alpha r}} \right] + \left[\frac{mt/L}{C_{\alpha f}} + \frac{mt/L}{C_{\alpha r}} \right] \right) \frac{V^2}{R}$$

$$= \frac{L}{R} + \left(\left[\frac{mc/L}{C_{\alpha f}} - \frac{mb/L}{C_{\alpha r}} \right] + \left[m \frac{t}{L} \frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}} \right] \right) \frac{V^2}{R}$$

- So that the contribution of the aligning moment to the understeer gradient is

$$\boxed{K_{AT} = \left[m \frac{t}{L} \frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}} \right]}$$

Aligning Torque

- The contribution of the aligning moment to the understeer gradient is always positive and so it clearly reinforces the understeer character of the vehicle

$$K_{AT} = m\frac{t}{L}\frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f}C_{\alpha r}} > 0$$

- This contribution is typically less than 0.5 deg/g.
- However, aligning torque is also indirectly responsible for additional significant understeer effect via its influence upon the steering system compliance. This one is investigated in the section related to the steering system.

Exercises

Exercise

- FORD EXPEDITION MK 2003

$$M = 2590\,kg$$

$$L = 3,018\,m$$

$$b = 1,4890\,m$$

$$c = L - b = 1,5290\,m$$

$$W = mg = 25407,90\,N$$

$$W_f = mg\frac{c}{L} = 12880,7153\,N$$

$$W_r = mg\frac{b}{L} = 12536,4229\,N$$

Exercise based on N.R. Dixit. Evaluation of Vehicle Understeer Gradient Definitions. Master of Science Thesis. Ohio State University. 2009

Exercise

- Tire characteristics

$$C_{\alpha f} = 1802,4191\ N/deg$$
$$C_{\alpha r} = 1826,7956\ N/deg$$
$$bb = 1,7243\ E{-}5\ 1/(N\ deg)$$
$$C_{\gamma f} = 93,3681\ N/deg$$
$$C_{\gamma r} = 91,0106\ N/deg$$
$$R_e = 0,373\ m$$
$$\text{pneumatic trail}\ t = 0,0048\ m$$

- Suspension characteristics

$$\frac{\partial \phi}{\partial a_y} = 4,56\ deg/g$$
$$\frac{\partial \gamma_f}{\partial \phi} = 0,8\ deg/deg$$
$$\frac{\partial \gamma_r}{\partial \phi} = 0,833\ deg/deg$$

- Compliance

$$A_f = 1,2185E-4\ deg/N$$
$$A_r = 0,3743E-4\ deg/N$$

Understeer gradient

- Understeer gradient

$$K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}$$

$$W_f = mg\frac{c}{L} = 12880,7153\ N$$

$$W_r = mg\frac{b}{L} = 12536,4229\ N$$

$$C_{\alpha f} = 1802,7153\ N/deg$$

$$C_{\alpha r} = 1826,7956\ N/deg$$

$$K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} = 0,2839\ deg/g$$

Lateral load transfer

- Lateral Load Transfer

$$K_{LLT} = \frac{W_f}{C_{\alpha f}} \frac{2b\Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \frac{2b\Delta F_{zr}^2}{C_{\alpha r}}$$

$$C_{\alpha f} = 1802,4191\ N/deg$$
$$C_{\alpha r} = 1826,7956\ N/deg$$
$$bb = 1,7243\ E - 5\ 1/(N\ deg)$$

$$\Delta F_{zf} = 837,3332\ N/deg$$
$$\Delta F_{zr} = 646,1931\ N/deg$$

$$K_{LLT} = \frac{W_f}{C_{\alpha f}} \frac{2b\Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \frac{2b\Delta F_{zr}^2}{C_{\alpha r}} = 0,0418\ deg/g$$

Camber Thrust

- Camber thrust

$$K_{\text{Camber}} = -\left(\frac{C_{\gamma f}}{C_{\alpha f}}\frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}}\frac{\partial \gamma_r}{\partial \phi}\right)\frac{\partial \phi}{\partial a_y}$$

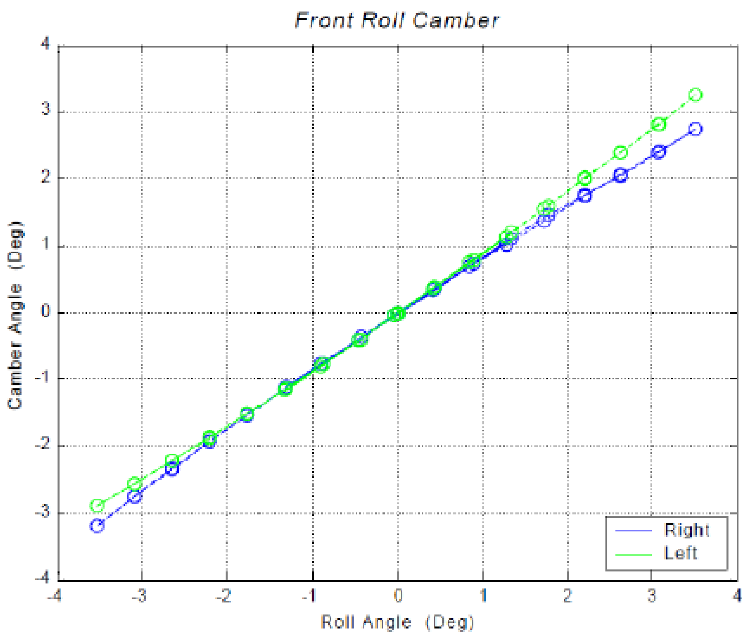
$$C_{\alpha f} = 1802,4191\ N/deg$$
$$C_{\alpha r} = 1826,7956\ N/deg$$
$$C_{\gamma f} = 93,3681\ N/deg$$
$$C_{\gamma r} = 91,0106\ N/deg$$

$$\frac{\partial \phi}{\partial a_y} = 4,56\ deg/g$$
$$\frac{\partial \gamma_f}{\partial \phi} = 0,8\ deg/deg$$
$$\frac{\partial \gamma_r}{\partial \phi} = 0,833\ deg/deg$$

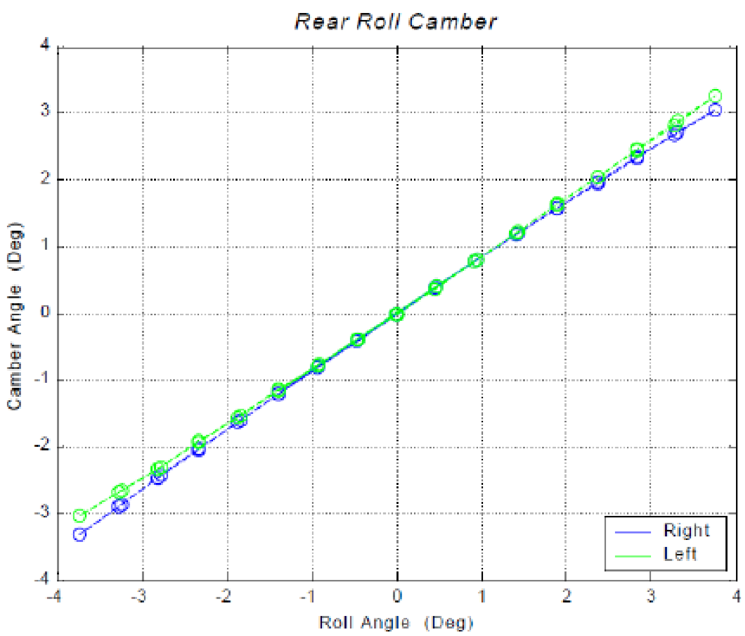
$$K_{\text{Camber}} = -\left(\frac{C_{\gamma f}}{C_{\alpha f}}\frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}}\frac{\partial \gamma_r}{\partial \phi}\right)\frac{\partial \phi}{\partial a_y} = +2,6751E - 4\ deg/g$$

Camber Thrust

- Camber gradient



$$\frac{\partial \gamma_f}{\partial \phi} = 0,8\ deg/deg$$



$$\frac{\partial \gamma_r}{\partial \phi} = 0,833\ deg/deg$$

Roll Steer

- Roll Steer

$$K_{\text{rollsteer}} = (\epsilon_f - \epsilon_r) \frac{\partial \phi}{\partial a_y}$$

$$\varepsilon_f = +0,033 \; deg/deg$$

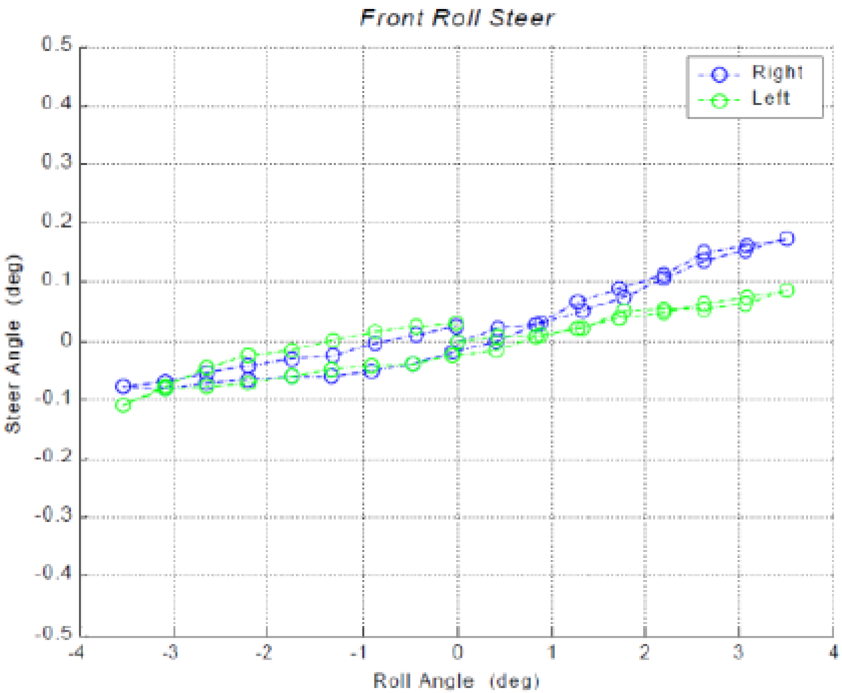
$$\varepsilon_r = -0,025 \; deg/deg$$

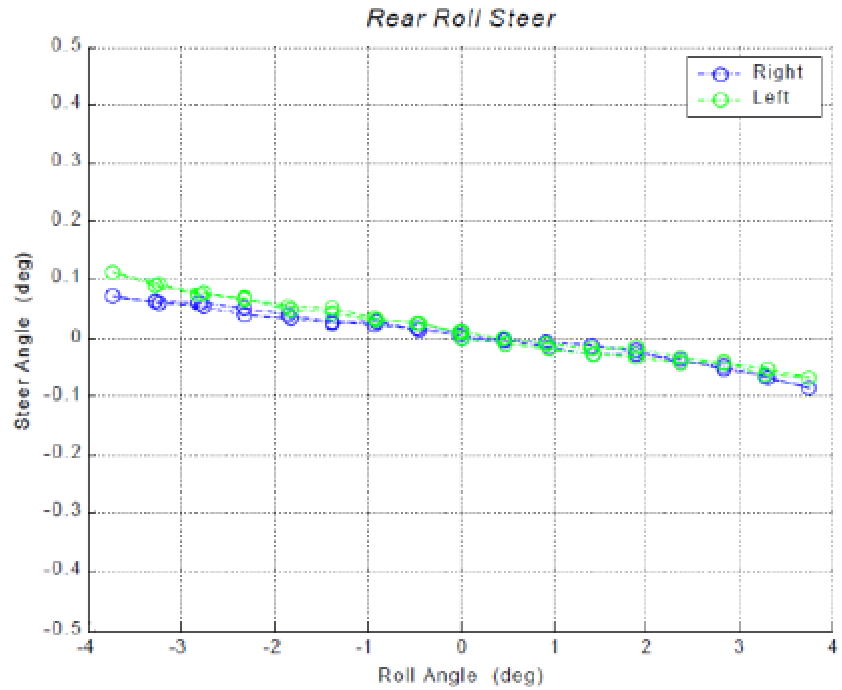
$$\frac{\partial \phi}{\partial a_y} = 4,56 \; deg/g$$

$$K_{\text{rollsteer}} = (\epsilon_f - \epsilon_r) \frac{\partial \phi}{\partial a_y} = 0,26448 \; deg/g$$

Roll Steer

- Roll steer characteristics





$$\varepsilon_r = -0,025\ deg/deg$$

$$\varepsilon_f = +0,033\ deg/deg$$

Lateral Force Compliance Steer

- Lateral force compliance steer

$$K_{LFCS} = -(A_f W_f - A_r W_r)$$

$$A_f = 1,2185E-4\,deg/N$$
$$A_r = 0,3743E-4\,deg/N$$
$$W_f = 12880,7153\,N$$
$$W_r = 12536,4229\,N$$

$$K_{LFCS} = -(A_f W_f - A_r W_r) = -2,0401\,deg/g$$

Steering system compliance steer

- Steering system compliance steer

$$K_{STRG} = W_f \, \frac{R_e \, \nu + t}{K_{ss}}$$

$$R_e = 0,373 \, m$$

$$\nu = 0,129 \, rad \; = \; 7,392 deg$$

$$t = 0,048 \, m$$

$$K_{ss} = 3690 \; N.m/deg$$

$$K_{STRG} = W_f \, \frac{R_e \, \nu + t}{K_{ss}} = 0,3355 \, deg/g$$

Aligning Torque

- Aligning torque

$$K_{AT} = m\,g\,\frac{t}{L}\,\frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}}$$

$$M = 2590\,kg$$

$$L = 3,018\,m$$

$$t = 0,048\,m$$

$$C_{\alpha f} = 1802,4191\,N/deg$$

$$C_{\alpha r} = 1826,7956\,N/deg$$

$$K_{AT} = m\,g\,\frac{t}{L}\,\frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}} = 0,4456\,deg/g$$

Exercise

- Understeer Gradient:

$$K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} = 0,2839\; deg/g$$

$$\begin{aligned} K_{us} &= K_{us0} + K_{LLT} + K_{\text{Camber}} + K_{LFCS} + K_{STRG} + K_{AT} \\ &= -0,7528\; deg/g \end{aligned}$$