MECA0525 RIDE & COMFORT

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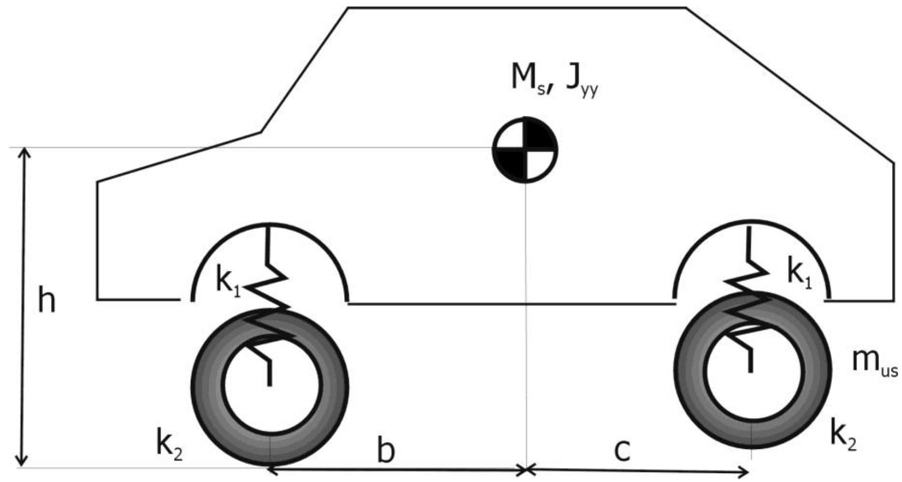
Exercise 1:

Bounce, pitch and wheel hop frequencies

Exercise 1

- A car is characterized by
  - Vehicle body
    - Mass M=1300 kg
    - Inertia J_{yy}=1000 kg.m²
    - L=3,00 m;
    - b=1,5 m; c=1,5 m
  - Suspension stiffness:
    - K_s= 40 kN/m on each axle
    - K_t= 400 kN per tire
  - Unsprung mass
    - M_{us}=30 kg per wheel

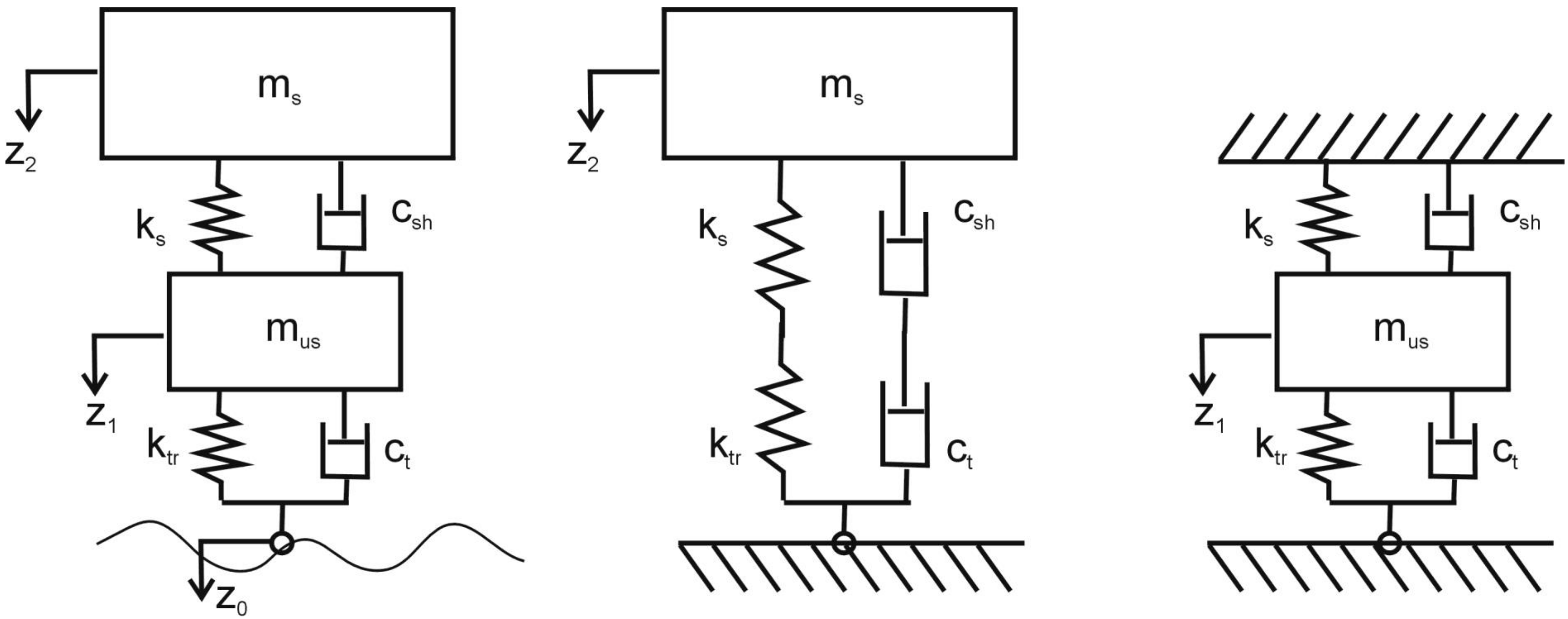


Exercise 1

- It asked to determine:
  - To calculate the eigenfrequencies (exact and approximate) for the wheel hop and the body mode
  - To calculate the eigenfrequency of the pitch and bouncing mode

Exercise 1

- Approximate and “exact” body and wheel hop frequencies



Exercise 1

- Approximate body and wheel hop frequencies
  - Body frequency

$$f_{Body} = \frac{1}{2\pi}\sqrt{\frac{k^*}{m_s}}$$

$$\frac{1}{k^*} = \frac{1}{k_s} + \frac{1}{k_t} = \frac{1}{20\ 10^3} + \frac{1}{400\ 10^3}$$

$$k_s = \frac{40\ 10^3}{2} = 20000\ [N/m]\ !!!$$

$$m_s = 1300/4 = 325\ [kg]$$

$$k^* = 19,048\ 10^3\ [N/m]$$

$$f_{Body} = \frac{1}{2\pi}\sqrt{\frac{19048}{1300/4}} = 1,2184\ [Hz]$$

  - Wheel hop frequency

Exercise 1

- Approximate body and wheel hop frequencies
  - Body frequency
  - Wheel hop frequency

$$f_{Wheel} = \frac{1}{2\pi}\sqrt{\frac{k_s + k_t}{m_{us}}}$$

$$k_s + k_t = 200\ 10^3 + 20\ 10^3 = 220000\ [N/m] \qquad m_{us} = 30\ [kg]$$

$$f_{Wheel} = \frac{1}{2\pi}\sqrt{\frac{220000}{30}} = 13,62\ [Hz]$$

Exercise 1

- "Exact" body and wheel hop frequencies

$$\omega_{n1}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \qquad \omega_{n2}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

$$\begin{aligned} A_1 &= m_s\, m_{us} \\ B_1 &= m_s\, k_s + m_s\, k_{tr} + m_{us}\, k_s \\ C_1 &= k_s\, k_{tr} \end{aligned}$$

$$\begin{aligned} A_1 &= m_s\, m_{us} = 325 \cdot 30 = 9750\ kg^2 \\ B_1 &= m_s\, k_s + m_s\, k_{tr} + m_{us}\, k_s \\ &= 325 \cdot 20\, 10^3 + 325 \cdot 200\, 10^3 + 30 \cdot 20\, 10^3 = 72,10\, 10^6 \\ C_1 &= k_s\, k_{tr} = 200\, 10^6 \cdot 20\, 10^6 = 4,0\, 10^9 \end{aligned}$$

Exercise 1

- "Exact" body and wheel hop frequencies

$$\rho = \sqrt{B_1^2 - 4A_1C_1} = 71.009928\ 10^6$$

- Body frequency

$$\omega_{n1}^2 = \frac{B_1 - \rho}{2A_1} = 55,9010$$

$$\omega_{n1} = 7,4767$$

$$f_{n1} = \frac{1}{2\pi}\omega_{n1} = 1,1899\ [Hz]$$

- Wheel hop frequency

$$\omega_{n2}^2 = \frac{B_1 + \rho}{2A_1} = 7,3389\ 10^3$$

$$\omega_{n2} = 85.6677$$

$$f_{n2} = \frac{1}{2\pi}\omega_{n2} = 13,6344\ [Hz]$$

Exercise 1

- Calculate the approximate frequency for the vehicle body bounce and pitch motion
- Reminder

$$\begin{cases} m_s\,\ddot{z} + k_f\,(z - l_f\,\theta - z_f) + k_r\,(z + l_r\,\theta - z_r) = 0 \\ I_y\,\ddot{\theta} - k_f\,l_f\,(z - l_f\,\theta - z_f) + k_r\,l_r\,(z + l_r\,\theta - z_r) = 0 \end{cases}$$

  - or

$$\begin{cases} \ddot{z} + D_1\,z + D_2\,\theta = 0 \\ \ddot{\theta} + D_3\,\theta + \frac{D_2}{r_y^2}\,z = 0 \end{cases}$$

$$D_1 = \frac{1}{m_s}(k_f + k_r)$$

$$D_2 = \frac{1}{m_s}(k_r\,l_r - k_f\,l_f)$$

$$D_3 = \frac{1}{I_y}(k_f\,l_f^2 + k_r\,l_r^2)$$

Exercise 1

- Calculate the approximate frequency for the vehicle body bounce and pitch motion

$$D_1 = \frac{1}{m_s}(k_f + k_r) = \frac{40\ 10^3 + 40\ 10^3}{1300} = 61,538$$

$$D_2 = \frac{1}{m_s}(k_r\, l_r - k_f\, l_f) = \frac{40\ 10^3 \cdot 1,5 - 40\ 10^3 \cdot 1,5}{1300} = 0$$

$$D_3 = \frac{k_f\, l_f^2 + k_r\, l_r^2}{I_y} = \frac{40\ 10^3\ 1,5^2 + 40\ 10^3\ 1,5^2}{1000} = 180$$

- Since D_2=0, the two modes are uncoupled, and they have the simplified expressions

$$\omega_{n1} = \sqrt{D_1} = 13,4160$$

$$f_{n1} = \frac{1}{2\pi}\omega_{n1} = 1,2485\ [Hz]$$

$$\omega_{n2} = \sqrt{D_3} = 7,8446$$

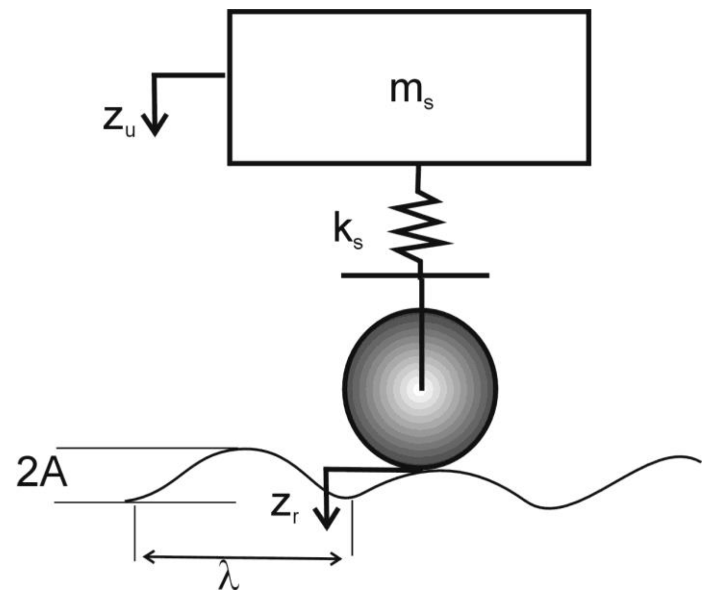
$$f_{n2} = \frac{1}{2\pi}\omega_{n2} = 2,1353\ [Hz]$$

Exercise 2:

Establish a transfer function of the suspension system

Exercise 2

- Derive the expression of the transfer function from road vertical excitation [m] to driver vertical acceleration [m/s²] for the following simplified suspension model.
- Derive the transfer function from road vertical excitation to road contact force [N]
- Calculate the road excitation frequency [Hz] when the vehicle drives at 50 km/h
  - λ=2 m
  - 2A=0,1 m



- Ms = 300 [kg]
- K = 20 [kN/m]
- The tire is very stiff, and its mass is negligible

Exercise 2

- The differential equation of the system writes

$$m_s\, \ddot{z}_u(t) + k_s(z_u(t) - z_r(t)) = M_s\, g$$

- Passing in frequency domain, one gets

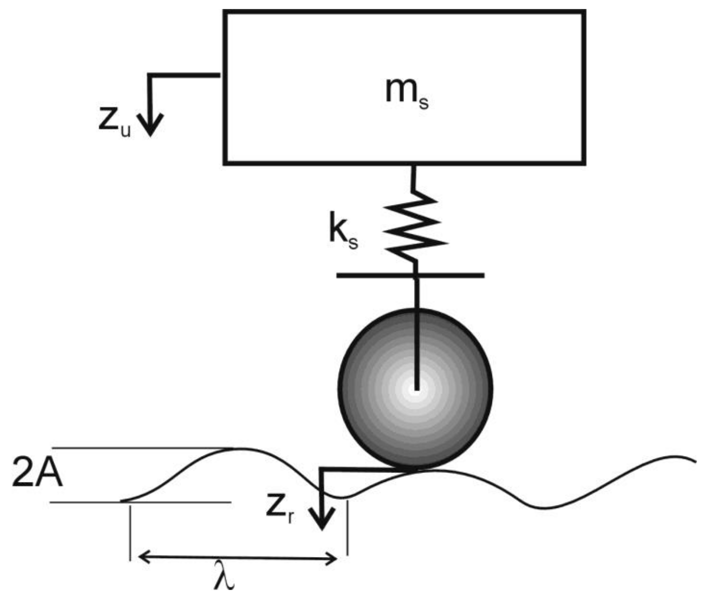
$$z_u(t) = Z_u\, e^{j\omega t}$$

$$z_r(t) = Z_r\, e^{j\omega t}$$

$$\ddot{z}_u(t) = -\omega^2\, Z_u\, e^{j\omega t}$$

- So the homogeneous equation becomes

$$-\omega^2\, m_s\, Z_u + k_s(Z_u - Z_r) = 0$$



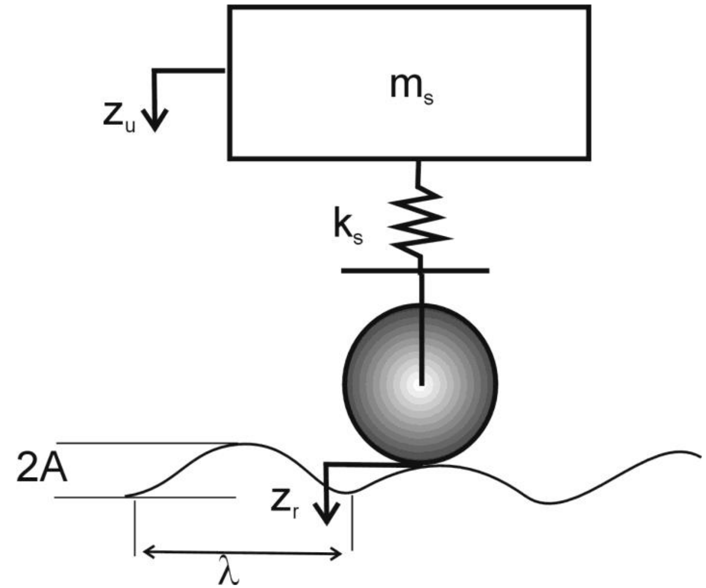
Exercise 2

- The transfer function can be obtained in the following way

$$(-\omega^2 \, m_s + k_s) \, Z_u = k_s \, Z_r$$

- Or

$$H_{Z_r \to Z_u} = \frac{Z_u}{Z_r} = \frac{k_s}{-\omega^2 \, m_s + k_s}$$



- To obtain the transfer function of acceleration

$$H_{Z_r \to \ddot{Z}_u} = \frac{-\omega^2 Z_u}{Z_r} = \frac{-\omega^2 \, k_s}{-\omega^2 \, m_s + k_s} = \frac{\omega^2 \, \omega_n^2}{\omega^2 - \omega_n^2} = \frac{1}{\frac{1}{\omega_n^2} - \frac{1}{\omega^2}}$$

Exercise 2

- Derive the transfer function from road vertical excitation [m] to the road contact force [N]
  - Road contact force

$$F_{rz} = m_{us}\ddot{z}_{us} + mg + k_s(z_r(t) - z_u(t))$$

$$\Delta F_{rz} = k_s(z_r(t) - z_u(t))$$

  - Transfer function between variation of vertical force and road vertical variation

$$H_{Z_r \to \Delta F_{zr}} = k_s \frac{Z_r - Z_u}{Z_r} = k_s \left(1 - \frac{Z_u}{Z_r}\right)$$

  - since

$$H_{Z_r \to Z_u} = \frac{Z_u}{Z_r} = \frac{k_s}{-\omega^2\, m_s + k_s}$$

Exercise 2

- Transfer function between variation of vertical force and road vertical variation

$$H_{Z_r \to \Delta F_{zr}} = k_s \frac{Z_r - Z_u}{Z_r} = k_s \left(1 - \frac{k_s}{-\omega^2\, m_s + k_s}\right)$$

$$H_{Z_r \to \Delta F_{zr}} = k_s \frac{-\omega^2\, m_s + k_s - k_s}{-\omega^2\, m_s + k_s} = \frac{-k_s\, m_s\, \omega^2}{-\omega^2\, m_s + k_s}$$

$$H_{Z_r \to \Delta F_{zr}} = \frac{-k_s\, \omega^2}{\omega_n^2 - \omega^2}$$

Exercise 2

- Calculate the road excitation frequency [Hz] when the vehicle travels at V=50 km/h
  - The spatial wavelength is $$\lambda_s = 2 \ [m]$$
  - The spatial frequency [cycle/m] $$f_s = \Omega = n = 1/\lambda_s = 0,5 \ [cycle/m]$$
  - The Velocity is $$V = \frac{50 \ km/h}{3,6} = 13,8889 \ [m/s]$$
  - The time frequency is $$f_t = \Omega \cdot V = n \cdot V$$ $$f_t = 0,5 \cdot 13,8889 = 6,9444 \ [Hz]$$
  - The angular time frequency $$\omega = 2\pi \ f_t = 43,6333 \ [rad/s]$$

Exercise 2

- When driving under given conditions, calculate the acceleration level at the driver's head

$$H_{Z_r \to \ddot{Z}_u} = \frac{-\omega^2 \, k_s}{-\omega^2 \, m_s + k_s} = \frac{\omega^2 \, \omega_n^2}{\omega^2 - \omega_n^2}$$

$$\omega = 2\pi \, f_t = 2\pi \, 6,9444 \; [Hz] = 43,6333 \; [rad/s]$$

$$\omega_n = \sqrt{\frac{k_s}{m_s}} = \sqrt{\frac{20 \, 10^3}{300}} = 8,1650 \; [rad/s] \qquad f_n = \frac{1}{2\pi}\omega_n = 1,2995 \; [Hz]$$

$$|H_{Z_r \to \ddot{Z}_u}| = \left| \frac{43,6333^2 \, 8,1650^2}{43,6333^2 - 8,1650^2} \right| = 69,0864 \qquad Z_r = \frac{A}{2} = \frac{0,1}{2} = 0,05 \; [m]$$

$$|\ddot{Z}_u| = |H_{Z_r \to \ddot{Z}_u}| \; |Z_r| = 69,0864 \, 0,05 = 3,4543 \; [m/s^2]$$

$$RMS(|\ddot{Z}_u|) = \frac{|\ddot{Z}_u|}{\sqrt{2}} = 2,4425$$

Exercise 2

- When driving under given conditions, calculate road contact force [N] with respect to the given road excitation

$$H_{Z_r \to \Delta F_{zr}} = \frac{-k_s\, \omega^2}{\omega_n^2 - \omega^2}$$

$$\omega = 43,6333 \ [rad/s] \qquad \omega_n = 8,1650 \ [rad/s]$$

$$|H_{Z_r \to \Delta F_{zr}}| = \left| \frac{20\ 10^3\ 43,6333^2}{8,1650^2 - 43,6333^2} \right| = 20725,7490 \ [N]$$

$$Z_r = \frac{A}{2} = \frac{0,1}{2} = 0,05 \ [m]$$

$$|\Delta F_{zr}| = |H_{Z_r \to \Delta F_{zr}}|\ |Z_r| = 20.725,7490\ 0,05 = 1036,2874 \ [N]$$