MECA0525: COMFORT AND ROAD HOLDING

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References

- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- R. Bosch. « Automotive Handbook ». 5th edition. 2002. Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).

Outlook

- Introduction
- Vibration sources
  - Road profile
  - On-board excitation sources
    - Wheel tire system
    - Transmission line
    - Engine and propulsion system
- Human response to vibrations
- Vehicle modelling for road holding and comfort
- Quarter car model
- Pitch and pumping model

Introduction

Introduction

- When travelling at a given speed, the car is subject to a wide vibration spectrum
- The vibrations impact passengers by visual, haptic or auditive perceptions
- *Ride*
  - Usually refers to tactile or visual sensations
  - Low frequencies 0-25 Hz
- Sound
  - Characteristics of auditive perceptions
  - Higher frequencies: 25 – 20.000 Hz
- Distinction between ride and sound is arbitrary at some points because both frequency ranges are related and may overlap in practice

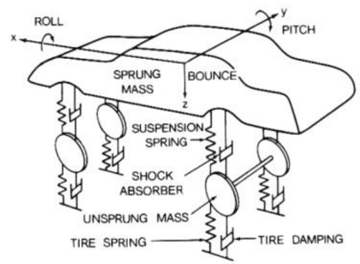
Introduction

- Vibration environment is a major vehicle quality criteria

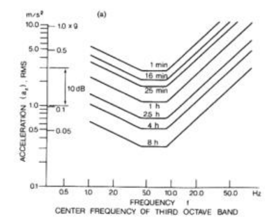
- Difficulties
  - Subjective evaluation criteria
  - Difficulties to propose objective methods to assess the vehicle performance

- Frequency behavior of vehicles
  - Is characterized by a low frequency content, because the vehicle is on its soft rubber wheels
  - Depends on the nature of the excitations
  - Is modified by the frequency response of the vehicle

Introduction



Ride perception



Excitation sources :
- Road roughness
- Tyre / wheel
- Tramisssion ligne
- Engine / motor

Dynamic response of the vehicle

Vibrations

- Investigation of comfort requires
  - Understanding the excitation sources
  - Being able to characterize the perception mechanisms and the compliance of human body subject to vibrations
  - Understanding the frequency response of the vehicle on its wheels

Vibration sources

- One distinguishes
  - The external sources: mostly the road roughness
  - The internal sources: the vibrations of generated by rotating parts of the vehicle:
    - Wheels and tyres,
    - Vibration of the transmission line,
    - Engine and propulsion system.

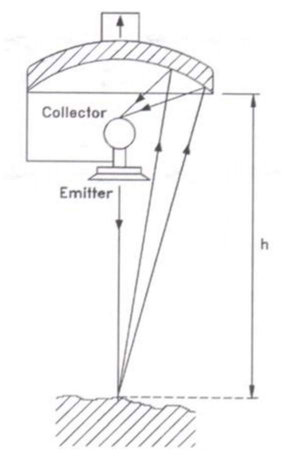
Road roughness

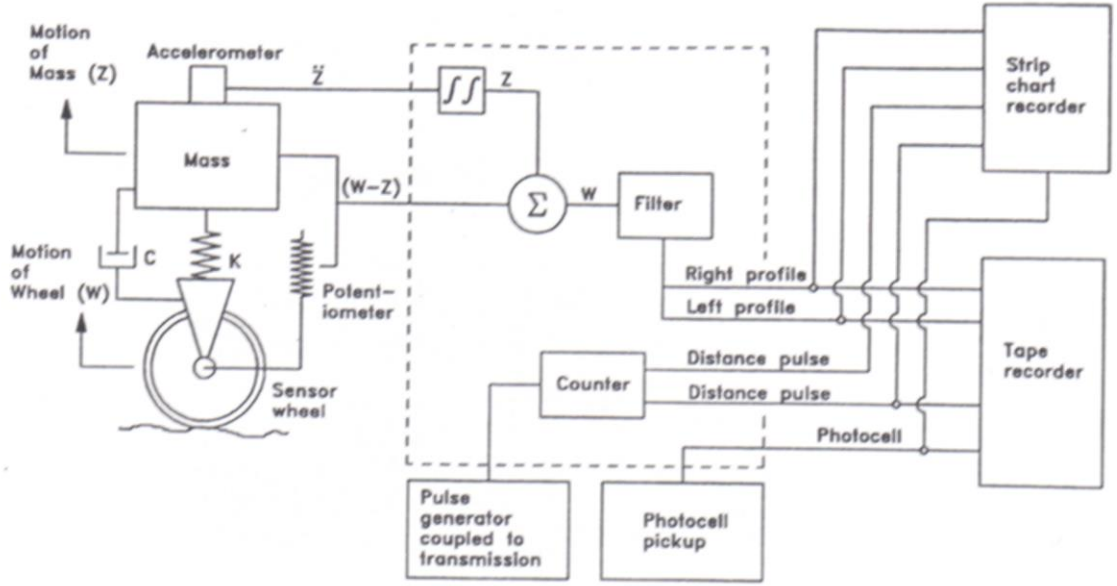
Road roughness

- Encompasses both holes (and potholes, etc.) and random deviations of the road profile resulting from imperfection of the fabrication and reparation process of the road.
- The roughness is described by the *road elevation* along the wheel trajectory.
- It is characterized by a wide spectrum random signal
- The roughness can be either described by the profile itself or by its statistical description. A usual description is the *spectral power density*.
- The space frequency is given in terms of cycles/meter, that is the inverse of the spatial wavelength of roughness imperfections.

Road roughness

- The distribution of the power spectral density can be obtained by a Fast Fourier Transform (FFT) of road elevation profile in terms of the distance.
- The road profile measurement is obtained by a high speed profilometer.





Road roughness

- Signal treatment using FFT

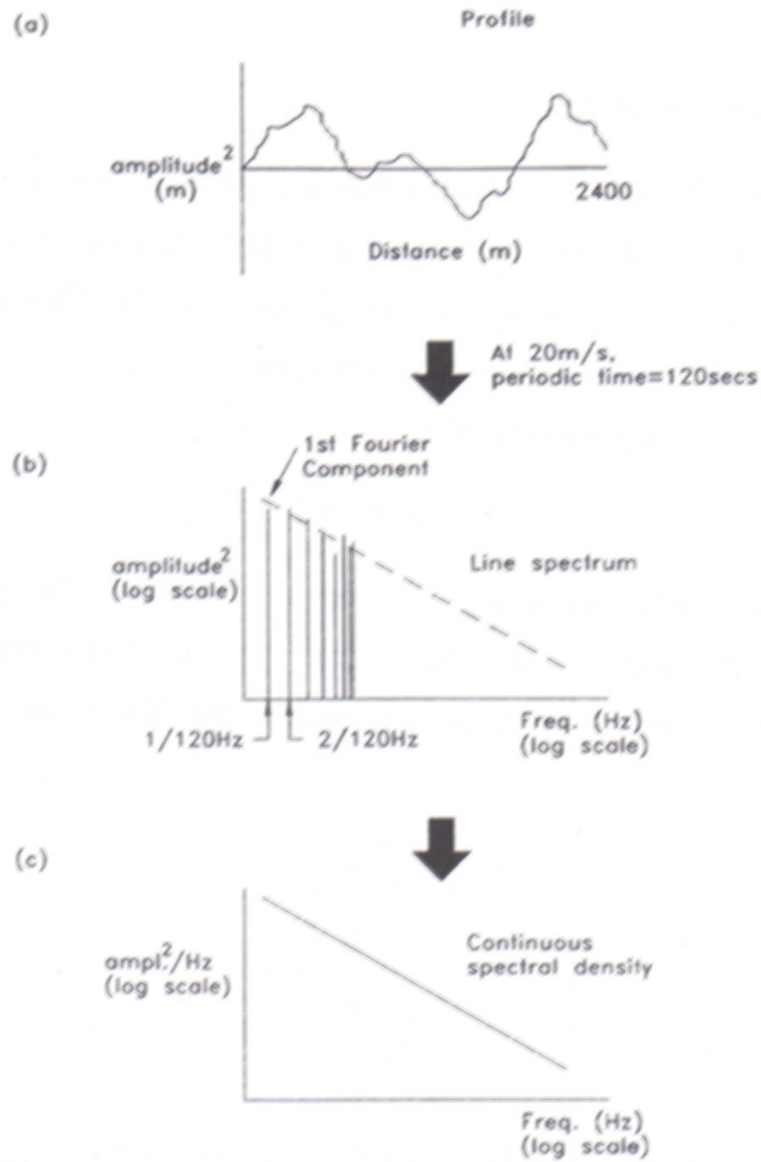
- Discrete spectral density: assign a representative value at the middle of the interval

  Conversion into power spectral density

- Conversion of the time frequency f (Hz) into a spatial density n (cycle/m) based on the vehicle speed V (m/s):

$$n[\text{cycle/m}] = \frac{f[\text{cycle/s}]}{V[\text{m/s}]}$$

- Very compact representation of the road profile and roughness



Road roughness

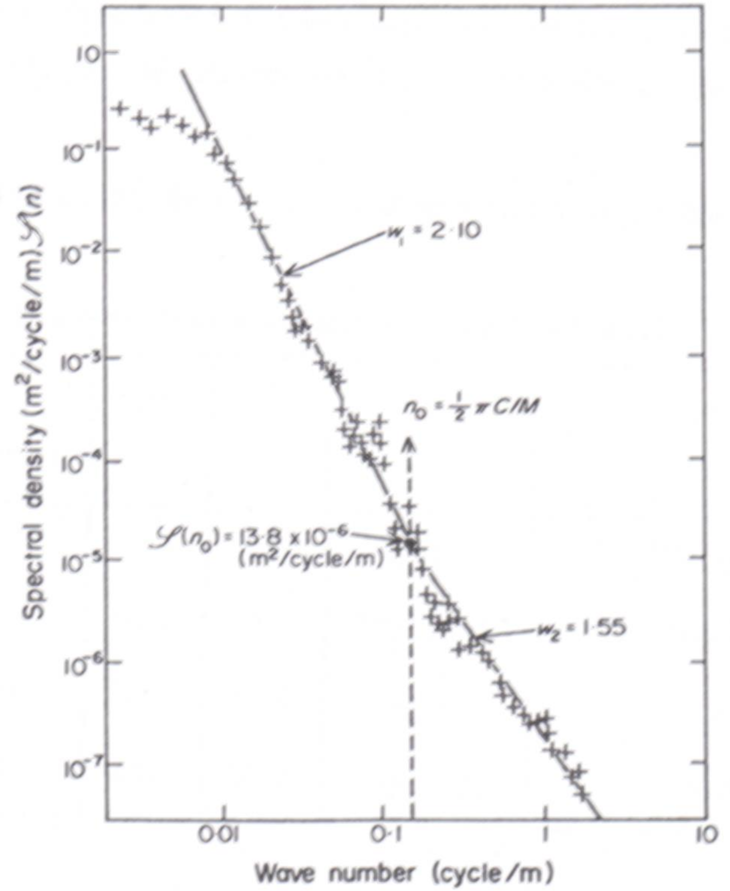
- One can see that for all roads, the PSD curve decreases monotonically with the frequency

- A good approximation of PSD is given by the generic laws

$$S(n) = G\, n^{-p}$$

with p between 2 and 3; The average value p~2,5 is the slope of the line in log-log plot

- Indeed, the deviations with wavelengths of several meters have magnitude of several cm while deviations with wavelength around the meter have magnitudes of a few mm.

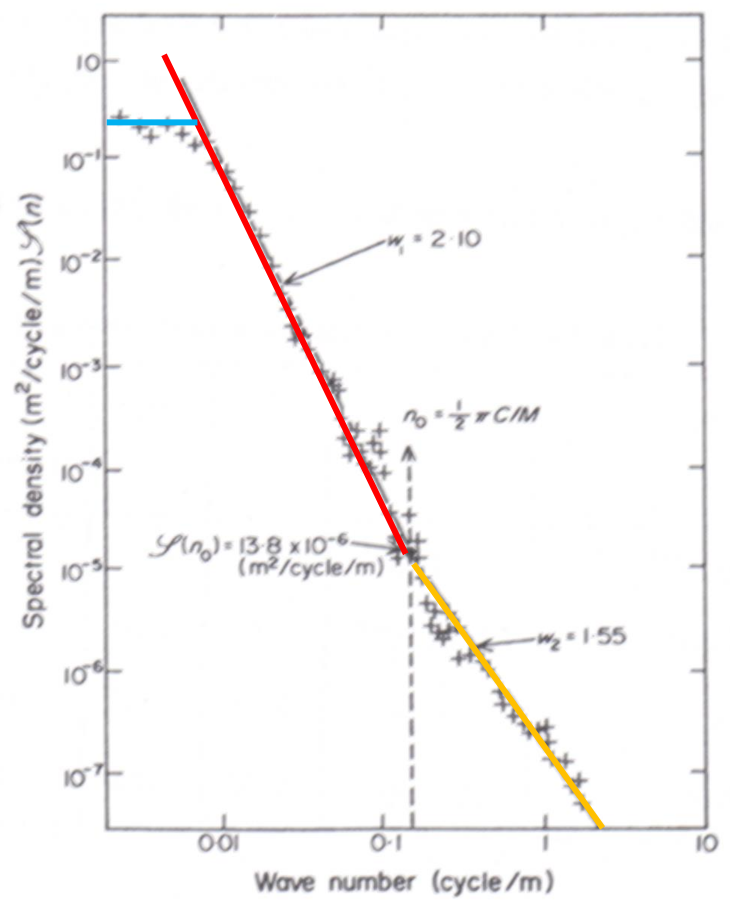


Road roughness

- Very often, we have also a slope discontinuity around a certain frequency n_0. This suggests a power spectral density of the form

$$S(n) = \begin{cases} G\left(\frac{n}{n_0}\right)^{-p_1} & \text{if } n \leq n_0 \\ G\left(\frac{n}{n_0}\right)^{-p_2} & \text{if } n \geq n_0 \end{cases}$$

  with n_0 the discontinuity frequency
- In addition, there might be a cut-off frequency n_{co} below which the power spectral density remains constant



Road roughness

- If the road surface is travelled at constant V (m/s), then there is a relation between the time frequency and the spatial wavelength and the power density can be written in terms of the frequency

$$S(f) = \frac{S(n)}{V} = G\,\frac{V^{p-1}}{f^{p}}$$

- It comes that travelling at a higher speed has the same effect on suspension as driving on road with a higher roughness.
- For linear models, the considered loading can be used as input for the frequency analysis
- For non linear models, it is generally necessary to resort to time-domain and to recreate a road profile on which one drives

Wheels and tires

Tire / wheel assembly

- The tire/wheel assembly is soft and compliant to absorb bumps and is part of the ride isolation system
- Ideally it runs true and does not contribute to the vehicle excitation
- However, in practice there are imperfections in the manufactured tires, wheels, hubs.
- The non uniformities are of three major types:
  - Mass imbalance
  - Dimensional variations
  - Stiffness variations

Tire / wheel assembly

- The non uniformities all combine in the wheel/tire assembly to generate variations of forces and moments at the ground
- These ones are transferred to the axle and act as excitation sources for the ride vibration.
- These variations can be in the vertical (radial) direction, longitudinal (tractive) direction or the lateral direction
- The moment variation (overturning, aligning, rolling resistance) do not significantly contribute as sources of excitations apart from the steering system vibrations).

Tire / wheel assembly: imbalances

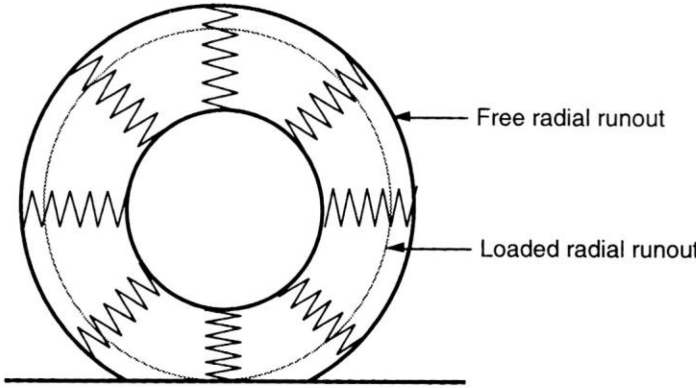
- Imbalances derive from non uniformities in mass distribution of the components along or about the rotation axis
- Asymmetry about the axis of rotation is a static imbalance.
  - Static imbalances result in a rotating force in the wheel plane

$$F_i = (m\,r)\,\omega^2$$

  - An imbalance force creates both radial and longitudinal excitation forces rotating at the rotation frequency.
- A nonuniform and asymmetrical mass distribution along the axis of rotation leads to a dynamic imbalance.
  - Dynamic imbalance creates a rotating torque on the wheel
  - Dynamic imbalance is mostly important on steered wheels because it gives rise to steering vibrations

Tire / wheel assembly: stiffness variations

- The tire is an elastic body which can be modelled as bundle of radial springs
- Variations of the stiffness free length give rise to dimensional non uniformity and so called free radial runout
- Variation of their compressed length at nominal load determines the rolling uniformities and the loaded radial runout



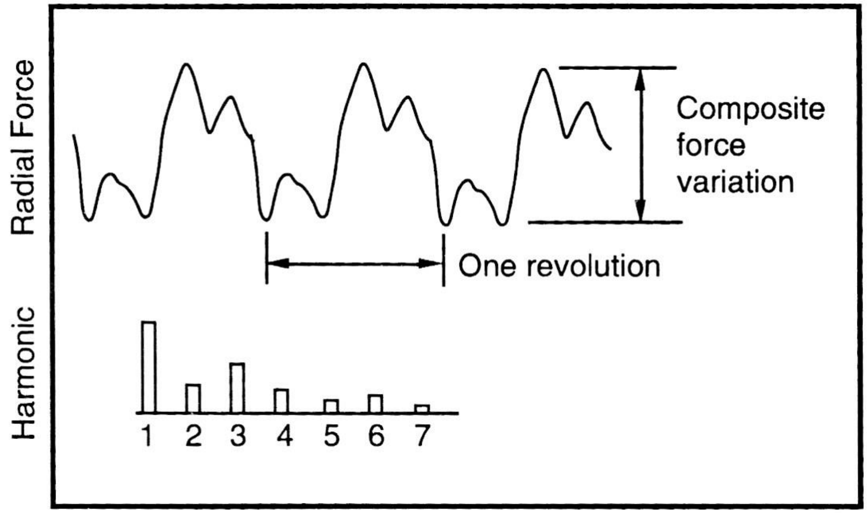
Gillespie. Fig 5.5

- Dimensional runouts of the wheel or of the hub on which is mounted the tire can produce indirectly free and loaded runouts

Tire / wheel assembly: Radial forces

- Nonuniformities in the tire/wheel assembly generate excitation forces and displacements at the axle as the wheel rotates.

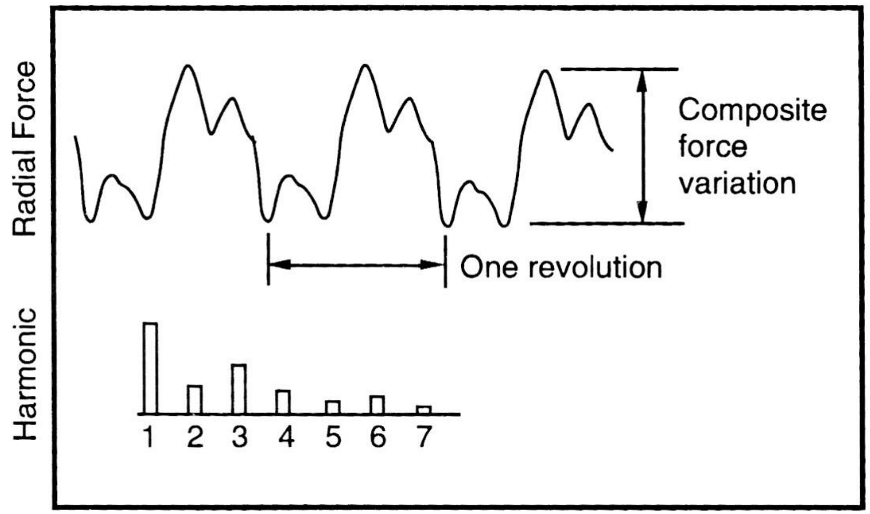
- RADIAL FORCES take typically the form



Gillespie. Fig 5.6: Tire radial force variations

Tire / wheel assembly: Radial forces

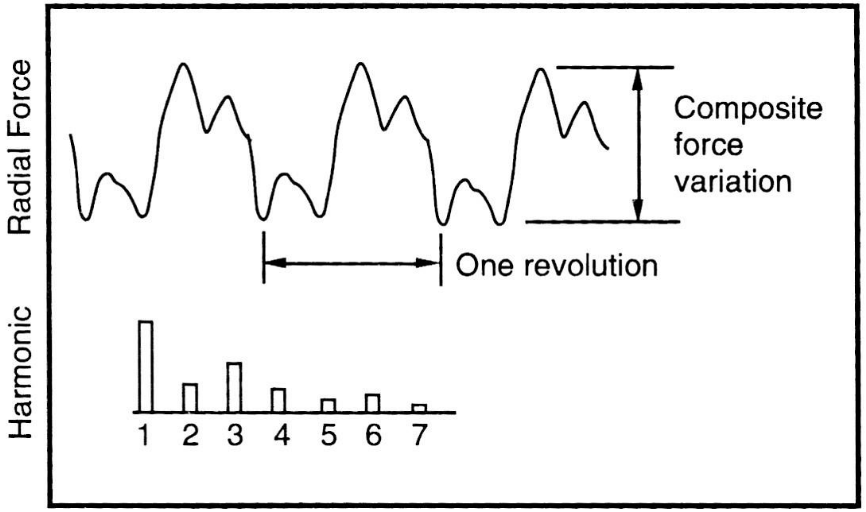
- The peak-to-peak force variation is called composite force variation
- The force variation is decomposed into its harmonics using FFT
- The amplitude of the harmonics is of a great importance while the phase shift is of little interest for the ride phenomena.



Gillespie. Fig 5.6: Tire radial force variations

Tire / wheel assembly: Radial forces

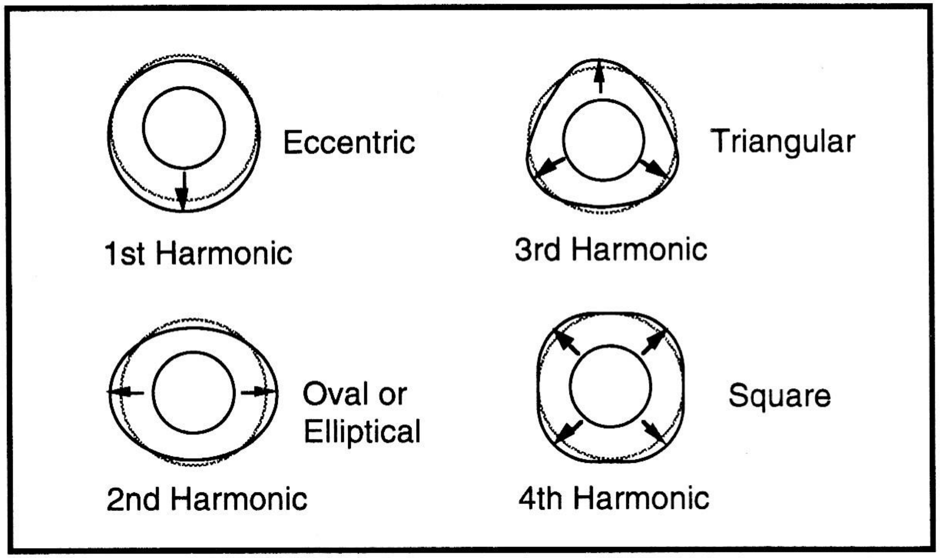
- The first harmonic magnitude contributes most to the composite, while the higher harmonics tend to be of diminishing magnitude (-30% per order typically)
- Intuitively the force variation could be expected to be the runout times the tire stiffness. In practice it is only about 70% of this magnitude



Gillespie. Fig 5.6: Tire radial force variations

Tire / wheel assembly: Radial forces

- The various harmonics of radial non uniformities in a tire/wheel assembly are functionally equivalent to imperfections of the shape



Gillespie. Fig 5.7: Radial non uniformities in a tire

Tire / wheel assembly: Radial forces

- Eccentricity results in first harmonic producing both radial and tractive excitations on the axle
- Excitation has the same frequency as the rotation speed of the wheel (typically 10-15 Hz on highway)
- The magnitude depends on the magnitude of the individual components (tire stiffness, wheel, hub) and their relative positions.
- Eccentricity of one component can compensate at least partially the non uniformity of another if high and low points are matched at the assembly
- The match-mounting techniques tends to minimize the first harmonic non uniformities

Tire / wheel assembly: Radial forces

- Ovality : tires and wheels can have elliptical variations
- They can add or subtract depending on the mounting but match-making is not practical for minimization of the non uniformity
- Tractive and radial excitation forces are produced at twice the rotation frequency of the wheel (typically 20-30 Hz on highway)

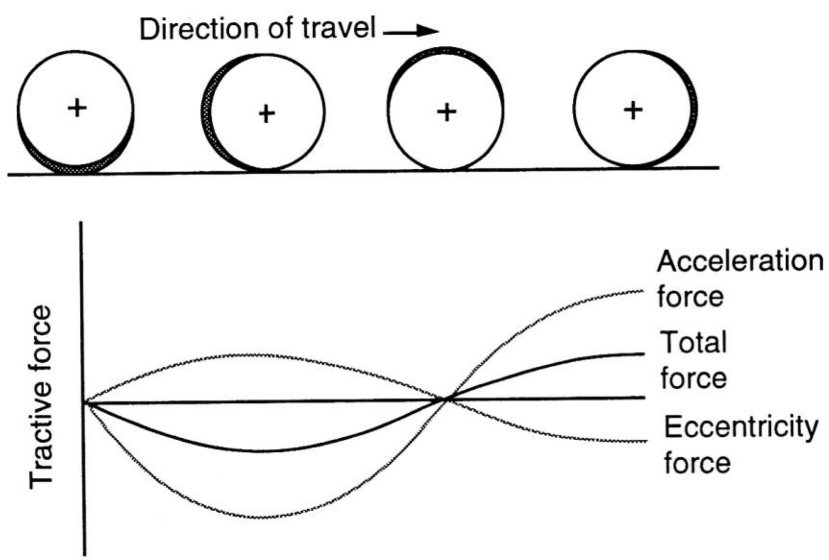
- Higher order radial variations: Third and fourth order variations are predominant in the tire only.
- They stem from construction methods. For instance, for a four-ply belt, ply overlap and drop-off yield increase of stiffness with fourth harmonic distribution.
- They act on the radial and tractive forces with the same multiple of the wheel rotation frequency.

Tire / wheel assembly: Radial forces

- The RADIAL FORCES can be treated as a direct excitation on the axle.
- The magnitude of the radial force variation is independent of the rotation speed
- The frequency of radial forces depends on the rotation speed
- So, the radial force variations can be measured at low speed and constant radius.
- The direct measure of the radial force is the proper way to characterize the radial excitation of the non uniformities.
- Alternatively, one can measure the radial loaded runout and transform it to radial force variations by simply multiplying by the radial spring rate of the tire.

Tire / wheel assembly: Tractive forces

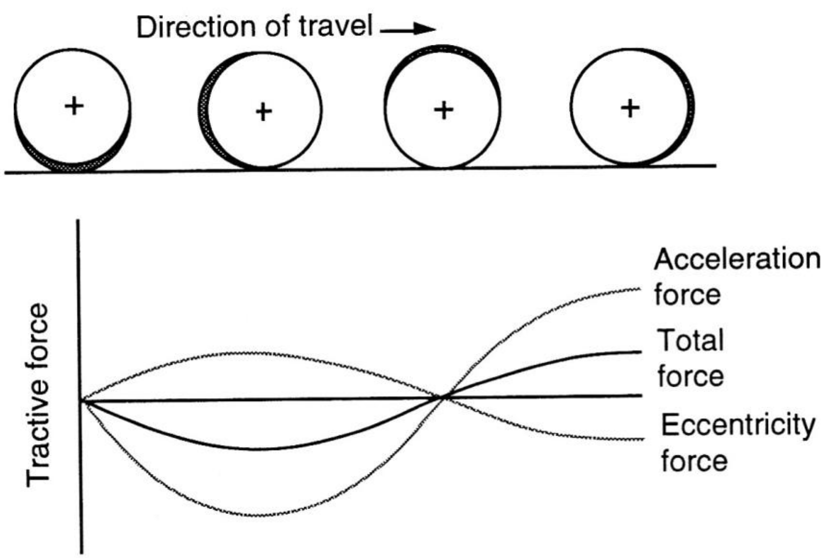
- The TRACTIVE FORCES arise from the dimensional and stiffness non uniformities as a result of the two effects
- As illustrated the axle must roll up and down the hill represented by the variations in radius of the tire wheel assembly
- The tractive forces have to accelerate the rotation speed of the tire



Gillespie. Fig 5.8: Tractive force variations arising from an eccentric wheel

Tire / wheel assembly: Tractive forces

- So, the magnitude of the tractive forces is dependent on the longitudinal stiffness of the tire and on the rotational moment of inertia
- The magnitude varies also with the rotation speed because the acceleration is also dependent on the speed.
- Thus, tractive force variations must be measured at high speed



Gillespie. Fig 5.8: Tractive force variations arising from an eccentric wheel

Tire / wheel assembly: lateral forces

- LATERAL FORCE variations can arise from non uniformities in the tire, but they cannot be related to a lateral runout effect in the wheel or in the hub.
- They are nearly independent of the speed so the measurement can be made at low speed.
- First order lateral variations will cause wobble
- Wobble can cause minor lateral force variations, but also in radial and longitudinal force variations comparable to ovality.
- High-order variations are coming from the tire only and can cause steering vibrations.

- Imperfections in tires and wheels tend to be highly correlated so radial variation will be accompanied by imbalance and tractive force variations.

Human response to vibrations

Human response to vibrations

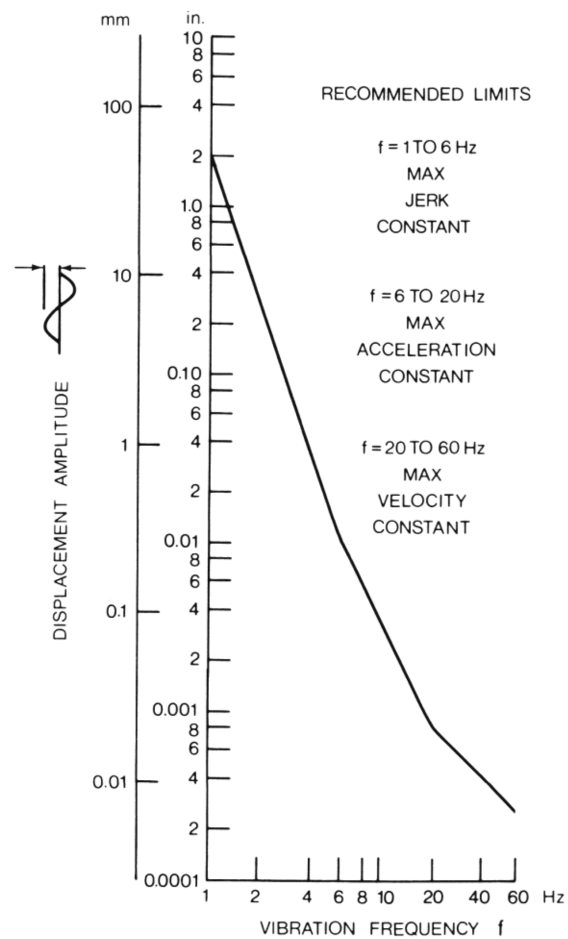
- In general, the boundaries of passenger ride comfort (or discomfort) are difficult to determine because of
  - The variations in individual sensitivity to vibrations
  - Lack of a generally accepted methodology and approach to the assessment of human responses to vibrations
- Considerable research effort has been devoted to define ride comfort limits
- Various methods have been developed:
  - Subjective ride measurements using trained jury
  - Shake table tests
  - Ride simulator tests
  - Ride measurements in vehicles
- Over the years, several comfort criteria have been proposed

Human response to vibrations

Ride simulator tests

Human response to vibrations

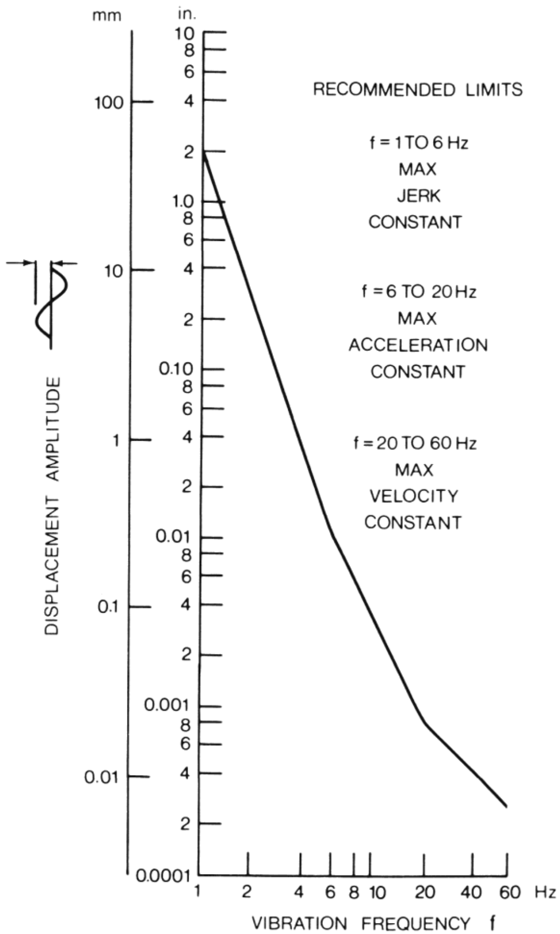
- **Comfort criteria of the vertical vibration described in the *Ride and Vibration Data Manual J6a* of the SAE**, also known as **Janeway comfort criterion**.
- The comfort criterion defines acceptable amplitude of vertical vibration as a function of the frequency.
- As the frequency increases, the allowable amplitude decreases considerably.



Wong Fig 7.1 : Vertical vibration limits for passenger comfort by Janeway

Human response to vibrations

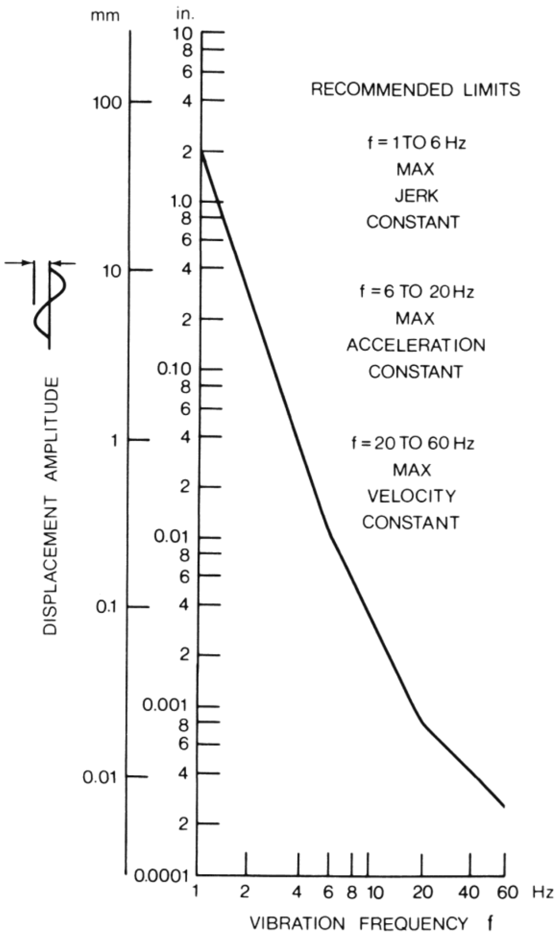
- The Janeway comfort criterion consists of three simple relationships. Each one covers a specific frequency range. In each one, there is a linear simple limitation of the amplitude in terms of the jerk, of the acceleration or of the vertical speed:
  - Range 1-6 Hz: jerk< 12.6 m/s³
  - Range 6-20 Hz : acceleration < 0.33 m/s²
  - Range 20-60 Hz : speed < $2.7\ 10^{-3}$ m/s
- Definitions:
  - jerk = d^3z/dt^3 = amplitude*ω^3,
  - acceleration = d^2z/dt^2 = amplitude * ω^2
  - Speed = dz/dt = amplitude * ω
- Example: amplitude max at 1Hz
  - $12.6 / (2\ \pi * 1\ \text{Hz})^3 = 0.0508$ m



Wong Fig 7.1 : Vertical vibration limits for passenger comfort by Janeway

Human response to vibrations

- Comments:
  - The Janeway comfort criterion is based on a sinusoidal vertical loading with a single frequency
  - When there are several applied frequencies, there is no established basis to assess the resulting effect.
  - Probably, the frequency that represents the highest in comfort will govern the overall sensation
  - The comfort criteria are established on hard seats

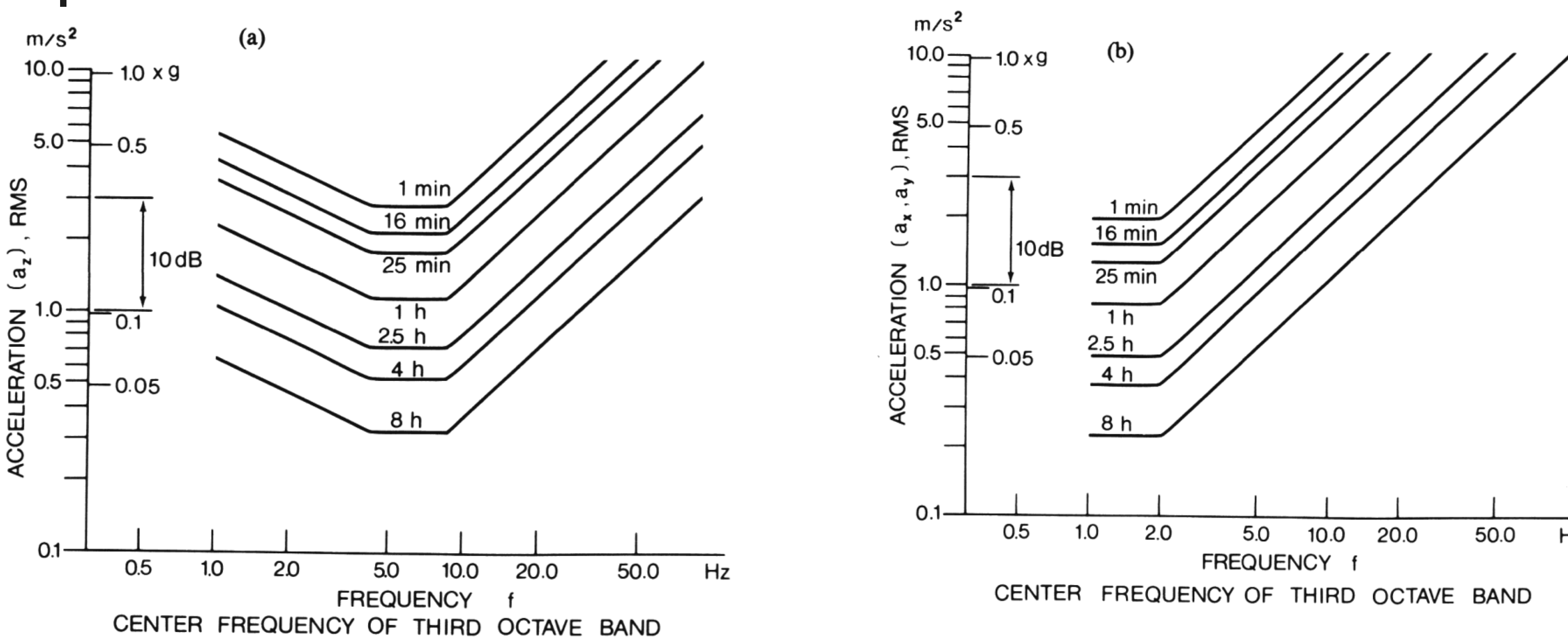


Wong Fig 7.1 : Vertical vibration limits for passenger comfort by Janeway

Human response to vibrations

- **General guide of ISO (ISO 2631)** has been proposed to evaluate human responses to the vibration environment when passengers are subject to motion as a whole
- Applicable to transportation industry but also to general industry
- The ISO guide defines 3 distinct limits for whole-body vibrations in the frequency range 1 to 80 Hz
  - Exposure Limits: related to the preservation of safety (or health) and that should never be exceeded without special justifications
  - Fatigue decreased and proficiency boundaries: related to the preservation of working efficiency and can be applied to such tasks as driving a road vehicle or an off-road vehicle
  - Reduced comfort boundaries, are concerned with the preservation of comfort and, in transport vehicles, it is related to such functions as reading, writing, or eating in vehicles

Human response to vibrations



Wong Fig 7.2 : Vibration bounds for fatigue and decreased proficiency under whole body vibration in vertical (a) and transversal directions (b) following ISO 2631

Human response to vibrations

- ISO bounds are formulated using acceptable RMS acceleration in terms of the loading frequency
- When vibration is applied under several directions, the ISO criterion is applied to each component.
- Exposure limits are obtained from fatigue boundaries by applying a factor two that is +6 dB.
- Reduced comfort limits are deduced from fatigue values by lowering the boundaries by a factor 3.15 or -10 dB.
- Frequencies below 1 Hz deserve a special problem, associated with symptoms such as motion sickness, different from the effects of higher vibrations. A severe discomfort boundary and a reduced comfort boundary for various exposure times in the range 0.1 – 1.0 Hz are recommended by ISO.

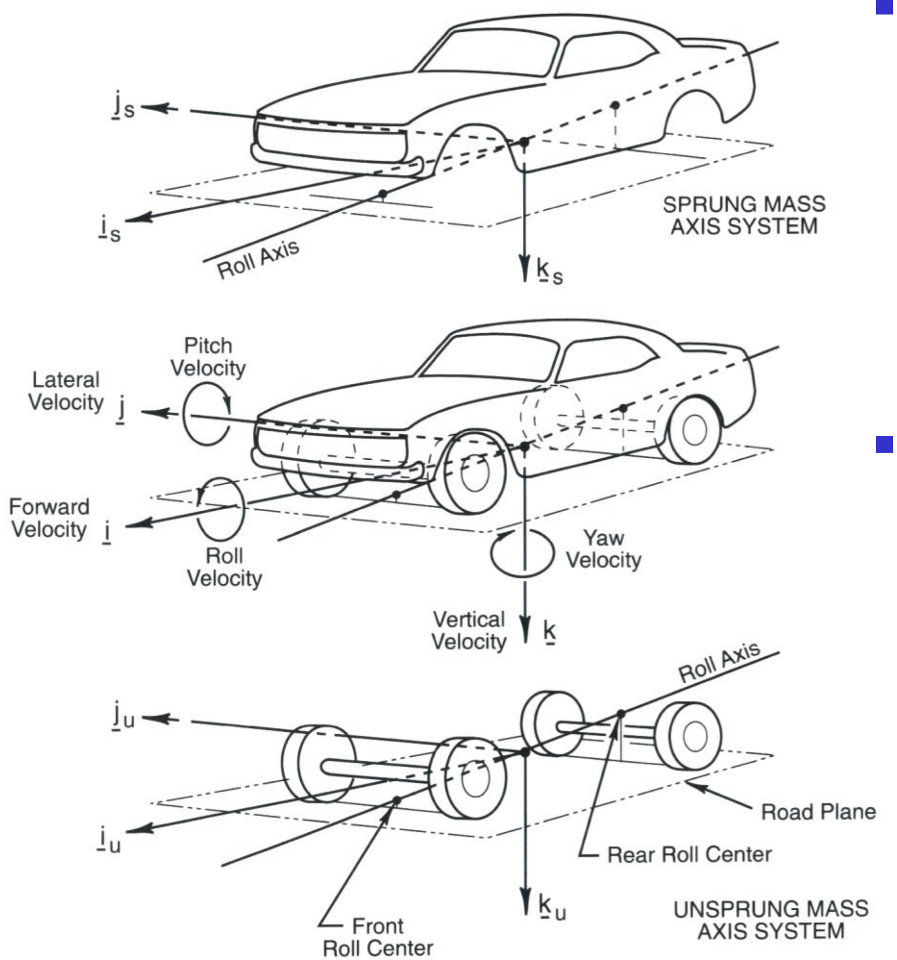
Human response to vibrations

- **The absorbed power** is proposed as the parameter in evaluation of human response for off-road vehicles when negotiating rough terrain.
- Absorbed power is the product of the vibration force by the transmitted velocity. The transmitted power is a measure of the rate at which vibration energy can be absorbed by human body.
- Absorbed power criterion is adopted by US Army AMM-75 Ground Mobility Model and later by NATO Reference Mobility Model for evaluating the ride quality of military vehicles.
- The tolerance limit is taken as <u>6W absorbed at the driver's position</u>.
- The ride-limiting speed is such that the driver's average absorbed power over the total elapsed time reaches the sustained level of 6W.

Comfort vehicle models

Suspended and unsuspended masses

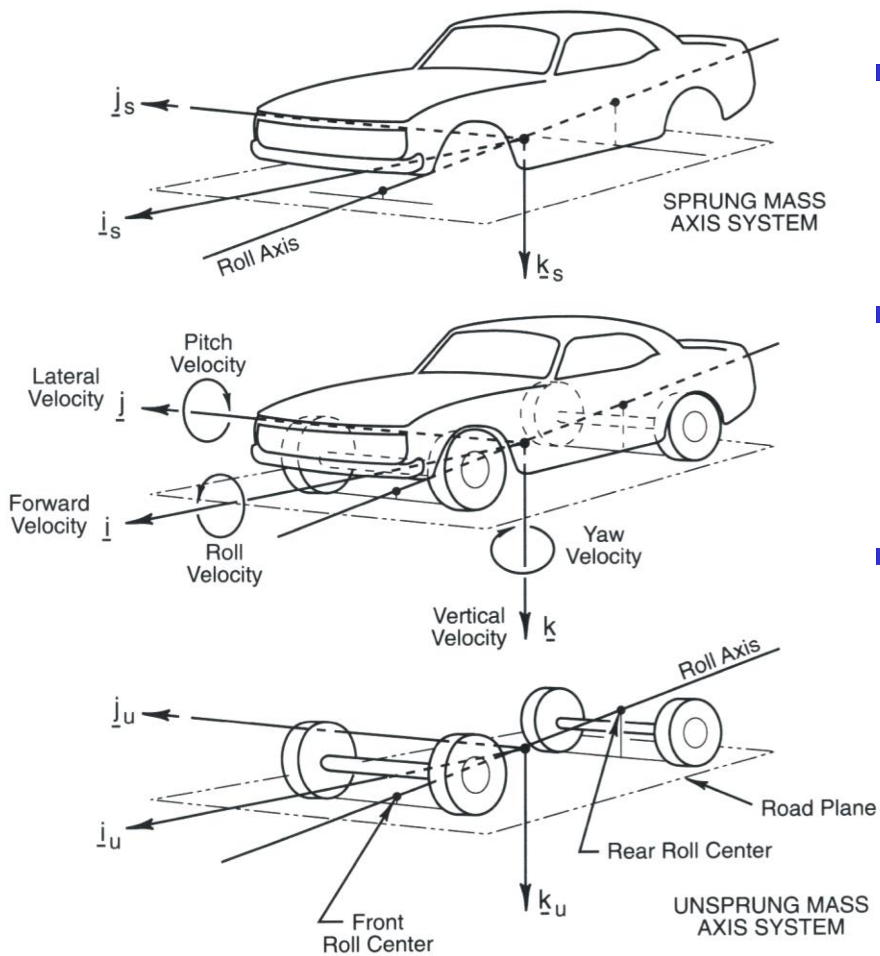
(Milliken, Fig. 4.1)



- **The suspended (sprung) mass** is all components and their mass which are supported by the suspension including a portion of the suspension members and the steering wheels
- **The unsuspended (unsprung) mass** includes all components, which are not carried by above the suspension systems, but directly by the tire and is considered to move with it.

Suspended and unsuspended masses

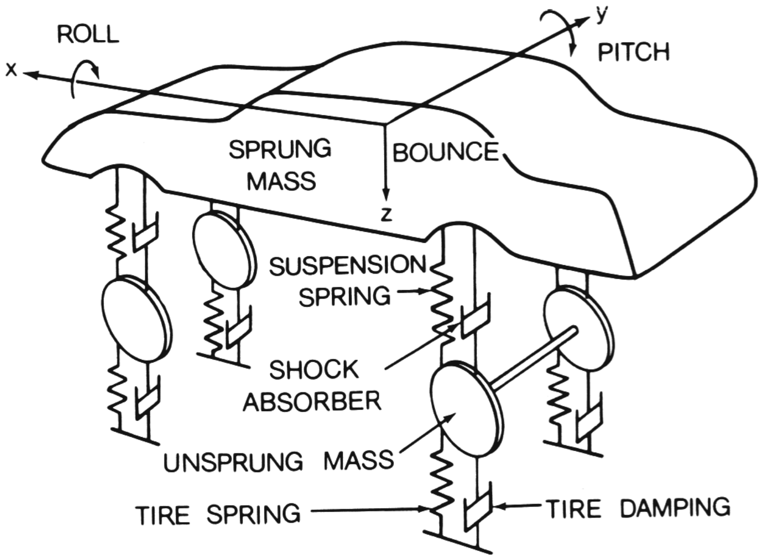
(Milliken, Fig. 4.1)



Vehicle two mass system of Segel (1956)

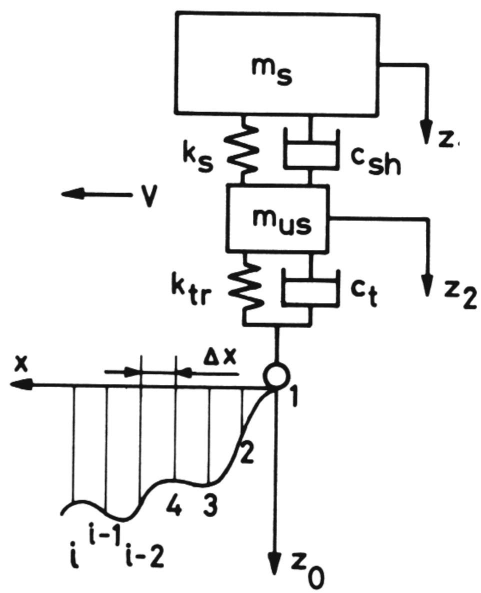
- **The suspended (sprung) mass** is considered a rigid body having equal mass, same center of gravity and moment of inertia as the sprung mass.
- **The suspended (sprung) masses** are the equivalent masses reproducing the inertia forces produced by the the unsprung parts.
- The relative motion between the two masses is a rotation about the roll axis joining the two suspension roll centers. The roll angle describes their relative position.

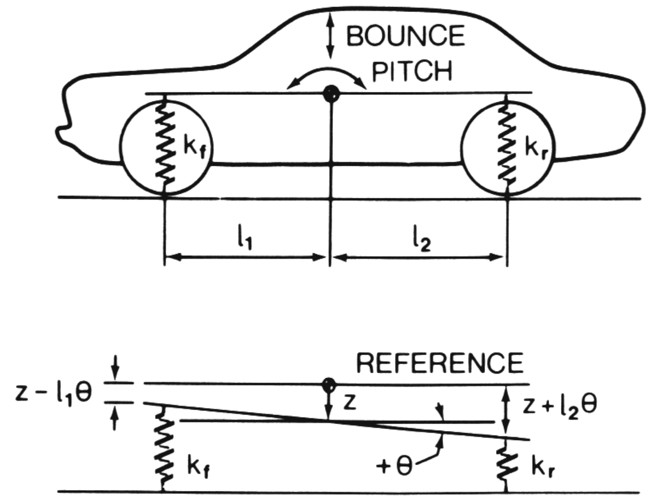
Simple models for vehicle comfort investigation



Wong : Fig 7.3
Model with 7 dof

Wong : Fig 7.6
Model with 2 dof, or quarter car model for vertical comfort





Wong : Fig 7.7
Model with 2 dof for pumping and pitch investigation

Quarter car model

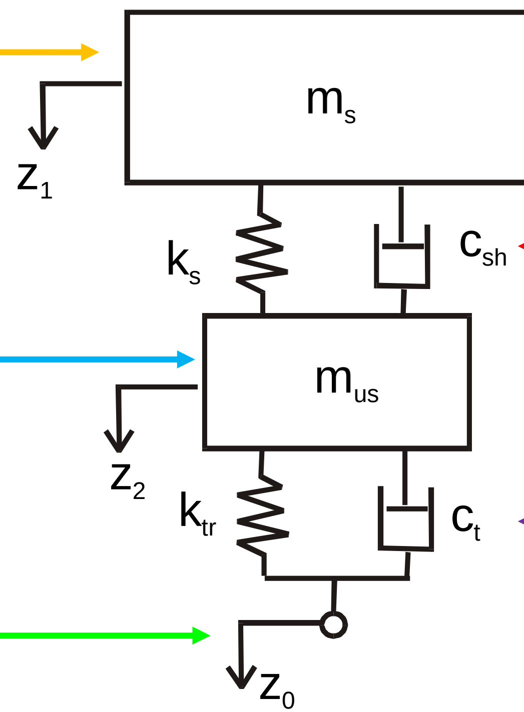
Two-dof model with suspended and non suspended masses

- Quarter car model

M_s: Suspended mass=1/4 of the body mass

M_{us}: unsuspended mass=wheel + tire + part of axle mass

Tire-ground contact point

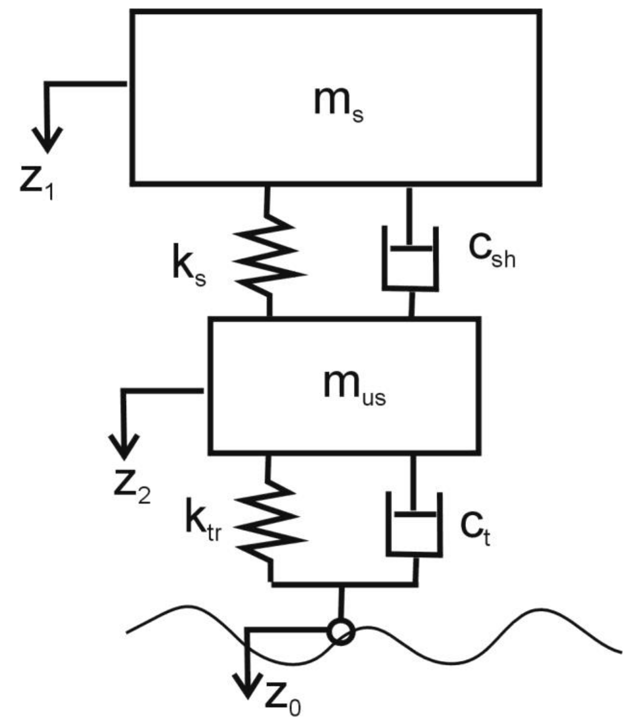


Suspension stiffness k_s and damping c_s.

Tire stiffness k_{tr} and damping c_{tr}.

Two-dof model with suspended and non suspended masses

- Quarter car model



- Dynamic equations of the two dof system

$$m_s \, \ddot{z}_1 + c_{sh} \, (\dot{z}_1 - \dot{z}_2) + k_s \, (z_1 - z_2) \; = \; 0$$

$$\begin{aligned} m_{us}\ddot{z}_2 + c_{sh} \, (\dot{z}_2 - \dot{z}_1) + k_s \, (z_2 - z_1) \\ + c_t \, \dot{z}_2 + k_{tr} \, z_2 \; = \; c_t \, \dot{z}_0 + k_{tr} \, z_0 = \; F(t) \end{aligned}$$

Two-dof model with suspended and non suspended masses

- <u>Investigation of the system natural vibrations without damping</u>

$$\begin{aligned} m_s\,\ddot{z}_1 + k_s\,z_1 - k_s\,z_2 &= 0 \\ m_{us}\,\ddot{z}_2 + (k_s + k_{tr})\,z_2 - k_s\,z_1 &= 0 \end{aligned}$$

- Harmonic solutions

$$\begin{aligned} z_1 &= Z_1 \cos \omega_n t \\ z_2 &= Z_2 \cos \omega_n t \end{aligned}$$

- The system writes

$$\begin{aligned} (-m_s\,\omega_n^2 + k_s)\,Z_1 - k_s\,Z_2 &= 0 \\ -\,k_s\,Z_1 + (-m_{us}\,\omega_n^2 + (k_s + k_{tr}))\,Z_2 &= 0 \end{aligned}$$

- Characteristic equation

$$\omega_n^4\,(m_s\,m_{us}) - \omega_n^2\,(m_s\,k_s + m_s\,k_{tr} + m_{us}\,k_s) + k_s\,k_{tr} = 0$$

Two-dof model with suspended and non suspended masses

- Natural frequencies

$$\omega_n^4\,(m_s\,m_{us}) - \omega_n^2\,(m_s\,k_s + m_s\,k_{tr} + m_{us}\,k_s) + k_s\,k_{tr} = 0$$

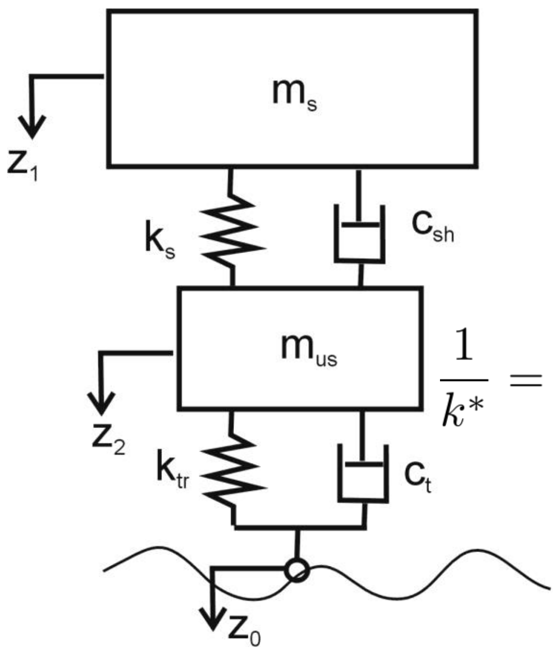
$$\begin{aligned} A_1 &= m_s\,m_{us} \\ B_1 &= m_s\,k_s + m_s\,k_{tr} + m_{us}\,k_s \\ C_1 &= k_s\,k_{tr} \end{aligned}$$

$$\omega_{n1}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \qquad \omega_{n2}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

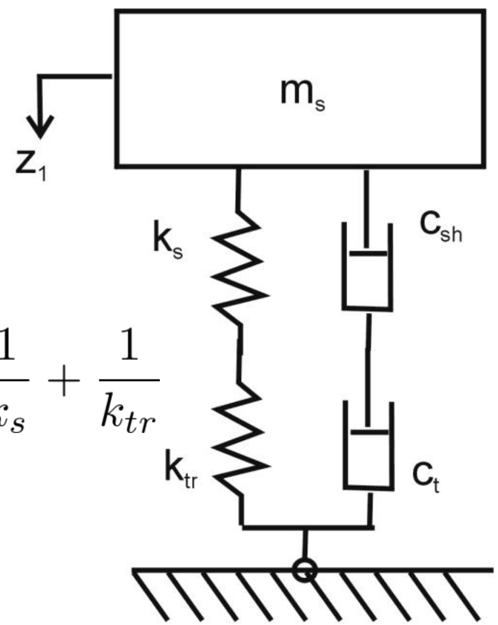
- Remark: given that $m_s > m_{us}$ and that $k_s < k_{tr}$, the eigenfrequencies are closed to the uncoupled frequencies

$$f_{n,s} = \frac{1}{2\pi}\sqrt{\frac{k_s\,k_{tr}/(k_s + k_{tr})}{m_s}} \qquad f_{n,us} = \frac{1}{2\pi}\sqrt{\frac{k_s + k_{tr}}{m_{us}}}$$

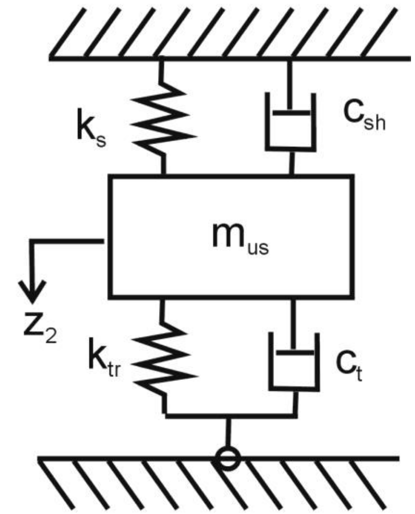
Two-dof model with suspended and non suspended masses



$$\frac{1}{k^*} = \frac{1}{k_s} + \frac{1}{k_{tr}}$$



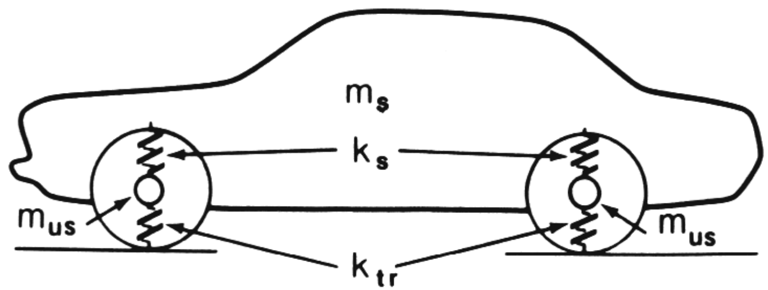
$$k^* = k_s + k_{tr}$$



$$f_n = \frac{1}{2\pi}\sqrt{\frac{k^*}{m_i}} \qquad f_{n,s} = \frac{1}{2\pi}\sqrt{\frac{k_s\, k_{tr}/(k_s + k_{tr})}{m_s}} \qquad f_{n,us} = \frac{1}{2\pi}\sqrt{\frac{k_s + k_{tr}}{m_{us}}}$$

Two-dof model with suspended and non suspended masses

- Exercise:



m_s = 1814 kg, 4000 lb
m_{us} = 181 kg, 400 lb, COMBINED
k_s = 88 kN/m, 500 lb/in., COMBINED
k_{tr} = 704 kN/m, 4000 lb/in., COMBINED

  - f_{n1} = 1.04 Hz
  - f_{n2} = 10.5Hz

Two-dof model with suspended and non suspended masses

- "Exact" body and wheel hop frequencies

$$\omega_{n1}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \qquad \omega_{n2}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

$$\begin{aligned} A_1 &= m_s\, m_{us} \\ B_1 &= m_s\, k_s + m_s\, k_{tr} + m_{us}\, k_s \\ C_1 &= k_s\, k_{tr} \end{aligned}$$

$$\begin{aligned} A_1 &= m_s\, m_{us} = 1814 \cdot 181 = 328.334\ kg^2 \\ B_1 &= m_s\, k_s + m_s\, k_{tr} + m_{us}\, k_s \\ &= 1814 \cdot 88\, 10^3 + 1814 \cdot 704\, 10^3 + 181 \cdot 88\, 10^3 = 1,4526\, 10^9 \\ C_1 &= k_s\, k_{tr} = 88\, 10^6 \cdot 704\, 10^6 = 61,952\, 10^9 \end{aligned}$$

Two-dof model with suspended and non suspended masses

- "Exact" body and wheel hop frequencies

$$\rho = \sqrt{B_1^2 - 4A_1C_1} = 1,4243\;10^9$$

- Body frequency

$$\omega_{n1}^2 \;=\; \frac{B_1 - \rho}{2A_1} = 43,043$$

$$\omega_{n1} \;=\; 6,5608\;rad/s$$

$$f_{n1} = \frac{1}{2\pi}\,\omega_{n1} \;=\; 1,0442\;Hz$$

- Wheel hop frequency

$$\omega_{n2}^2 \;=\; \frac{B_1 + \rho}{2A_1} = 4,3811\;10^3$$

$$\omega_{n2} \;=\; 66,19\;rad/s$$

$$f_{n2} = \frac{1}{2\pi}\,\omega_{n2} \;=\; 10,534\;Hz$$

Two-dof model with suspended and non suspended masses

- Approximate body and wheel hop frequencies
  - Body frequency

$$f_{Body} = \frac{1}{2\pi}\sqrt{\frac{k^*}{m_s}}$$

$$\frac{1}{k^*} = \frac{1}{k_s} + \frac{1}{k_t} = \frac{1}{88\ 10^3} + \frac{1}{704\ 10^3}$$

$$k^* = 78,2222\ 10^3\ \ N$$

$$f_{Body} = \frac{1}{2\pi}\sqrt{\frac{78,222\ 10^3}{1814}} = 1,0451\ \ Hz$$

  - Wheel hop frequency

Two-dof model with suspended and non suspended masses

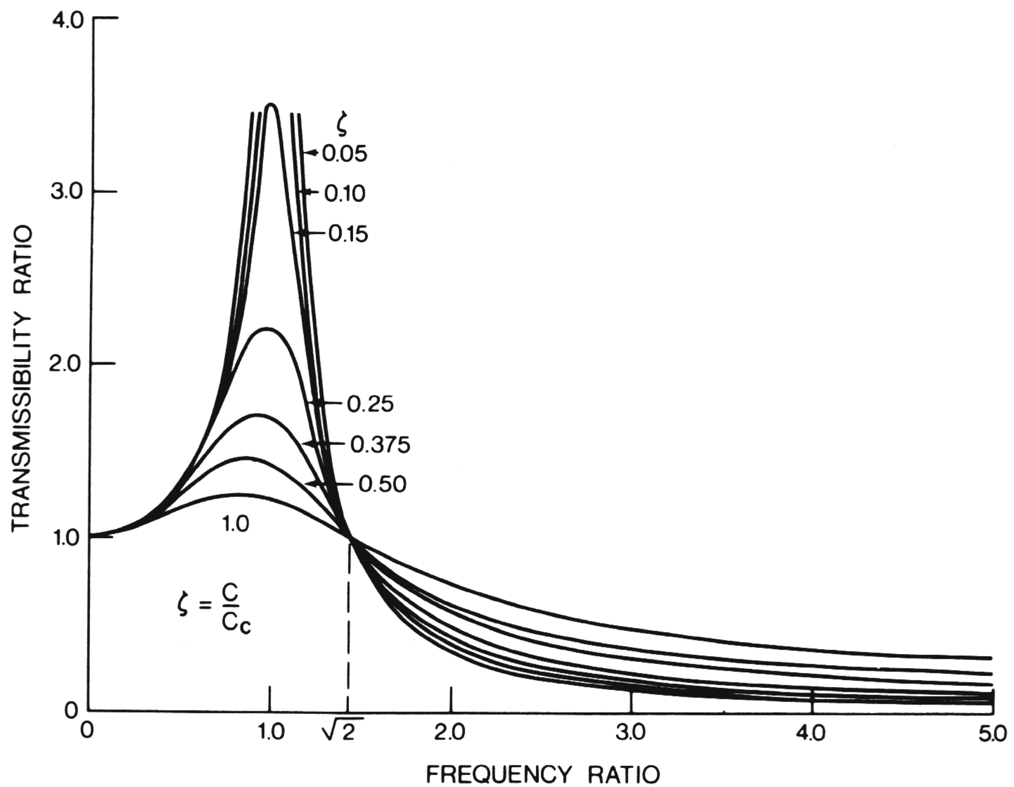
- Approximate body and wheel hop frequencies
  - Body frequency
  - Wheel hop frequency

$$f_{Wheel} = \frac{1}{2\pi}\sqrt{\frac{k_s + k_t}{m_{us}}}$$

$$k_s + k_t = 88\ 10^3 + 704\ 10^3 = 792\ 10^3\ N \qquad m_{us} = 181\ kg$$

$$f_{Wheel} = \frac{1}{2\pi}\sqrt{\frac{792\ 10^3}{181}} = 10,528\ Hz$$

Two-dof model with suspended and non suspended masses



Wong: Fig 7.8: Transmissibility ratio as a function of the frequency ratio for a single dof system

Two-dof model with suspended and non suspended masses

- Sinusoïdal road profile

$$z_0 = Z_0 \cos \omega t \qquad \omega = 2\pi \, V/\lambda_w$$

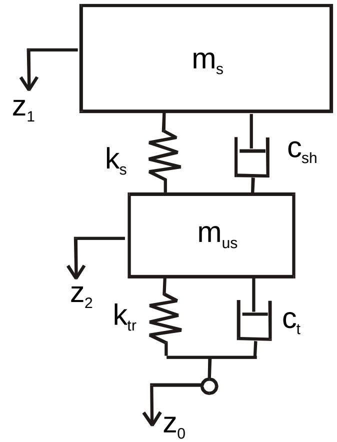
- Transmissibility ratio between the road profile and the magnitude of the motion of the suspended mass

$$\frac{Z_1}{Z_0} = \frac{\sqrt{A_2}}{\sqrt{B_2 + C_2}}$$

$$A_2 = (k_s \, k_{tr})^2 + (c_{sh} \, k_{tr} \, \omega)^2$$

$$B_2 = [(k_s - m_s \omega^2)(k_{tr} - m_{us} \omega^2) - m_s \, k_s \, \omega^2]^2$$

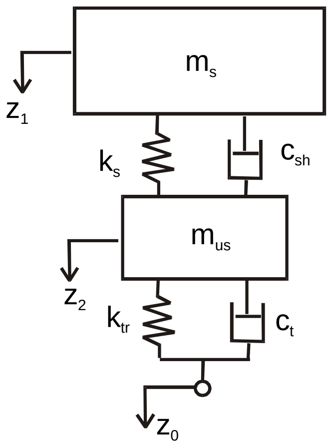
$$C_2 = (c_{sh} \, \omega)^2 \, [m_s \, \omega^2 + m_{us} \, \omega^2 - k_{tr}]^2$$



Two-dof model with suspended and non suspended masses

- Transmissibilty ratio between the road profile and the motion of the non suspended mass

$$\frac{Z_2}{Z_0} = \frac{\sqrt{A_3}}{\sqrt{B_2 + C_2}}$$



$$\begin{aligned}
A_2 &= (k_s\, k_{tr})^2 + (c_{sh}\, k_{tr}\, \omega)^2 \\
B_2 &= [(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2]^2 \\
C_2 &= (c_{sh}\, \omega)^2\, [m_s\, \omega^2 + m_{us}\, \omega^2 - k_{tr}]^2
\end{aligned}$$

$$A_3 = [k_{tr}\, (k_s - m_s\, \omega^2)]^2 + (c_{sh}\, k_{tr}\, \omega)^2$$

Two-dof model with suspended and non suspended masses

- Transmissibility ratios when neglecting the damping i.e. $c_{sh} = 0$

$$
\begin{aligned}
\frac{Z_1}{Z_0} &= \frac{k_s\, k_{tr}}{(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2} \\
&= \frac{k_s\, k_{tr}}{m_s\, m_{us}\, (\omega_{n1}^2 - \omega^2)\, (\omega_{n2}^2 - \omega^2)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{Z_2}{Z_0} &= \frac{k_{tr}\, (k_s - m_s\, \omega^2)}{(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2} \\
&= \frac{k_{tr}\, (k_s - m_s\, \omega^2)}{m_s\, m_{us}\, (\omega_{n1}^2 - \omega^2)\, (\omega_{n2}^2 - \omega^2)}
\end{aligned}
$$

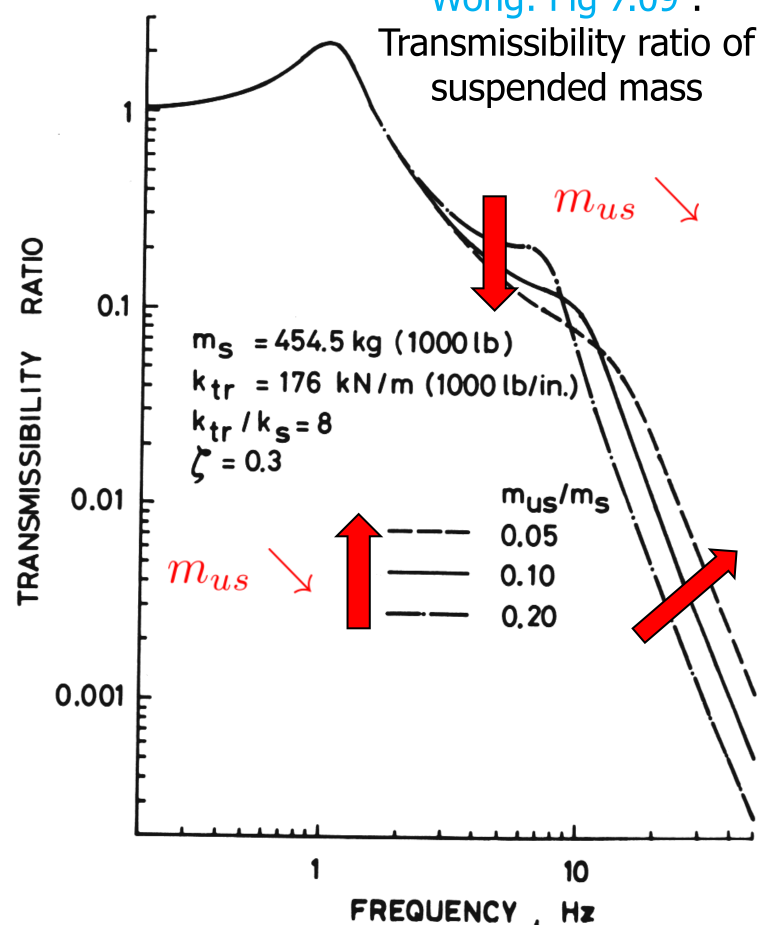
Two-dof model with suspended and non suspended masses

- To assess the performance of the suspension, one must consider three aspects :
  - Vibration isolation: Evaluating the response of the sprung mass to the excitation of the ground. This criterion is usually used to assess the vibration isolation characteristics of a linear suspension system.
  - The suspension travel: measured by the deflection of the suspension spring or by the relative displacement between the sprung and unsprung mass: z_2-z_1. It defines the rattle space required to accommodate the suspension spring movement.
  - The road holding: when the vehicle vibrates, the normal contact force between the tire and the ground fluctuates. The longitudinal and lateral forces developed by the tire are function of the normal load, so its variation impacts directly the handling. The normal force is investigated through the dynamic deflection or by the displacement of the sprung mass relative to the road: z_2-z_0.

Two-dof model: Transmissibility Ratio

- Effect of the non suspended mass m_{us}:
  - Below the eigenfrequency of the unsuspended mass, the transmissibility is decreasing with lower unsprung masses.
  - Over the second wheel hop eigenfrequency, the transmissibility is higher with low unsuspended masses.
- Conclusion: a low mass of the rolling gear and suspension (non suspended mass) is better for the isolation of the suspended mass (passengers) even if there is small penalty at high frequencies

Wong: Fig 7.09 : Transmissibility ratio of suspended mass

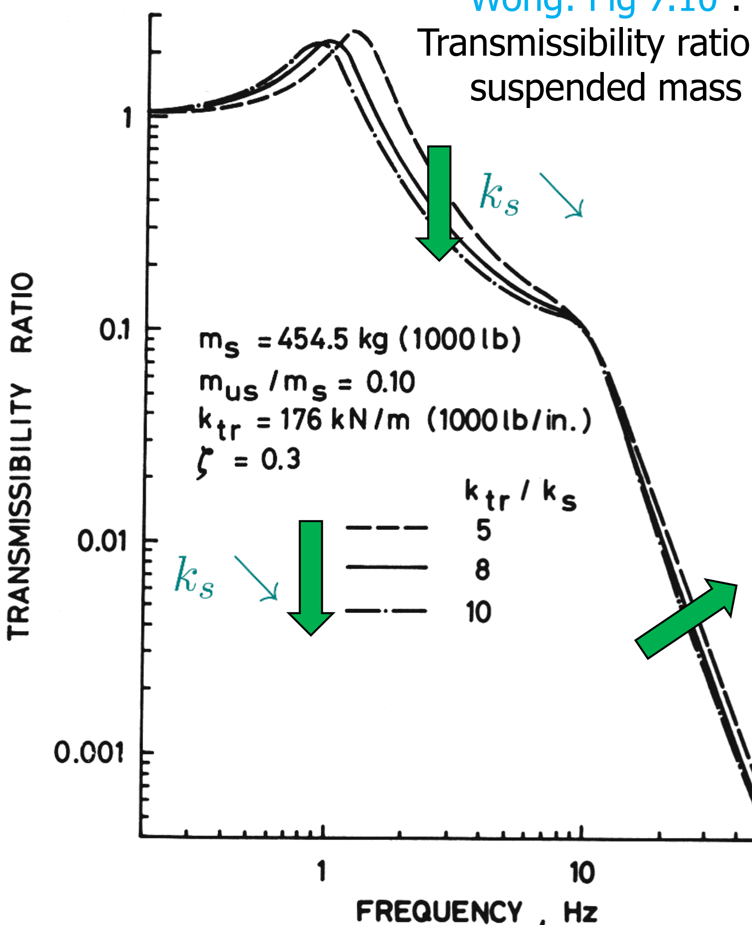


Two-dof model: Transmissibility Ratio

- <u>Effect of the suspension stiffness</u>
  - The tire stiffness k_{tr} is assumed to be given and is our reference

- Conclusion: One has to choose a <u>soft suspension stiffness</u> (k_{tr}/k_s high) to reduce the transmissibility between the two first eigenfrequencies, but there is a small penalty of higher frequencies.

Wong: Fig 7.10 : Transmissibility ratio of suspended mass



Two-dof model: Transmissibility Ratio

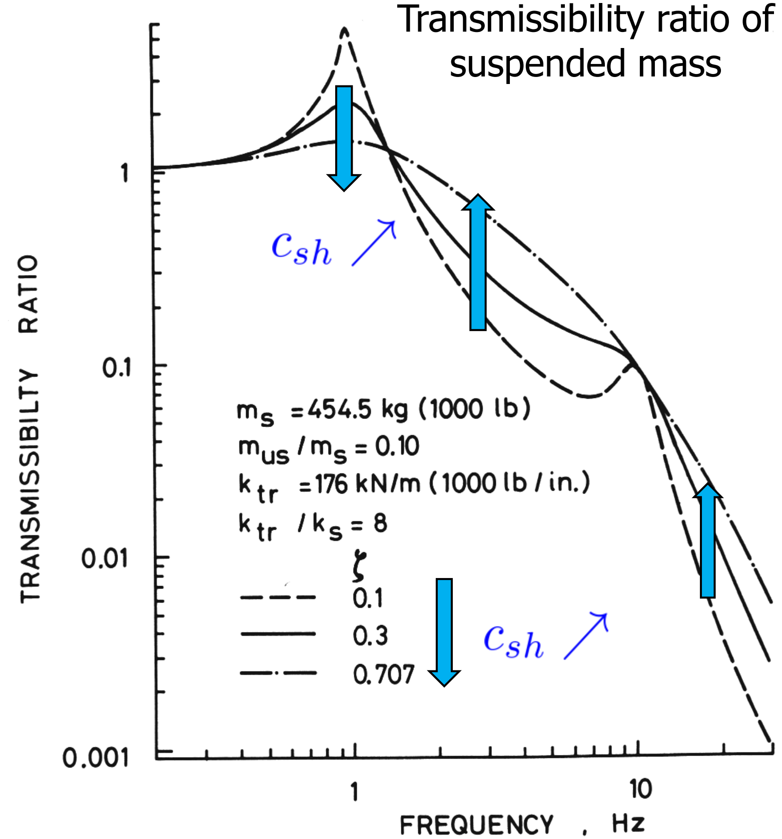
- Effect of the suspension damping ratio

- Conclusion: a medium damping ratio (below critical damping coefficient) is preferred.

  - A good isolation of the vibrations around the natural frequencies of the sprung mass would require increasing the damping

  - In the intermediate range between the natural frequencies, a lower damping ratio is favorable to reduce the transmissibility.

Wong: Fig 7.11 : Transmissibility ratio of suspended mass



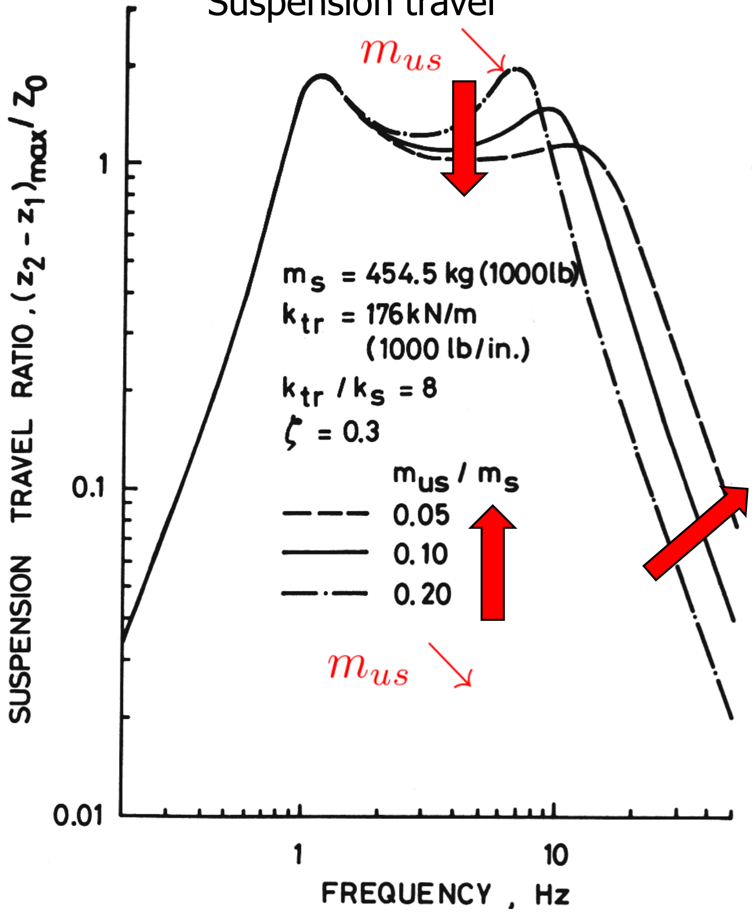
Two-dof model: Suspension travel Z_1-Z_2

- <u>Effect of the non suspended mass m_{us}:</u>

- Conclusion: a <u>small mass of the unsprung mass</u> (rolling gear) is <u>better</u> to reduce the wheel travel even if the conclusion is opposite at higher frequencies.

Wong: Fig 7.12

Suspension travel



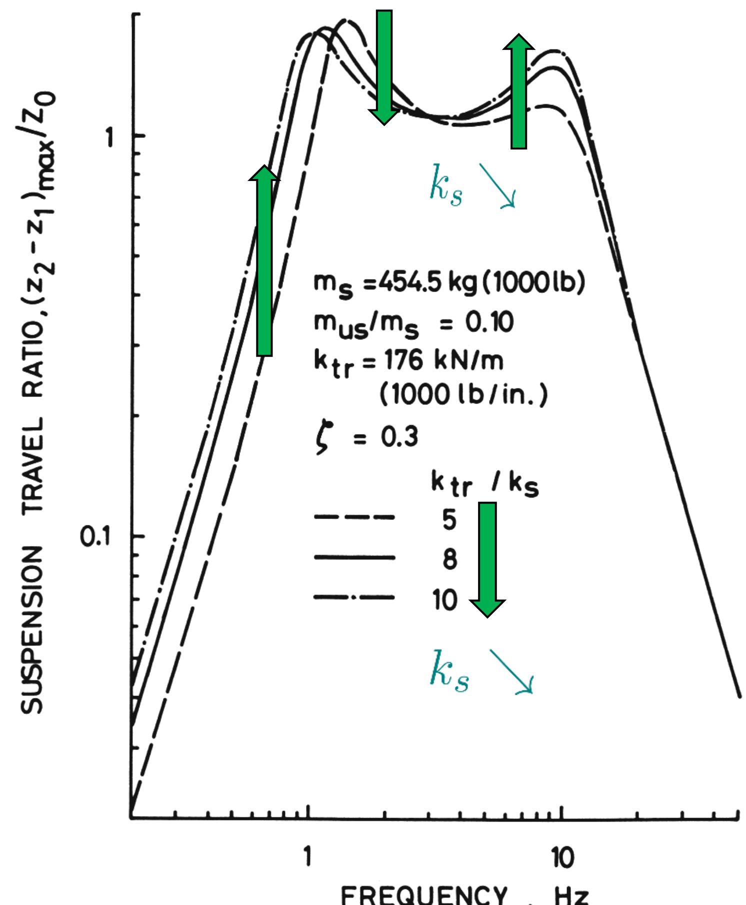
Two-dof model: Suspension travel Z_1-Z_2

- Effect of the suspension stiffness

- Conclusion:
  - Below the natural frequency of the suspended mass, a stiff suspension (k_{tr}/k_s low) can reduce the wheel travel
  - For the intermediate frequency range, there is crossing phenomenon: below the crossing, it is better to adopt a soft suspension stiffness. Over it is preferable to go for stiffer suspension springs
  - A high frequencies, stiffness has little influence

Wong: Fig 7.13

Suspension travel

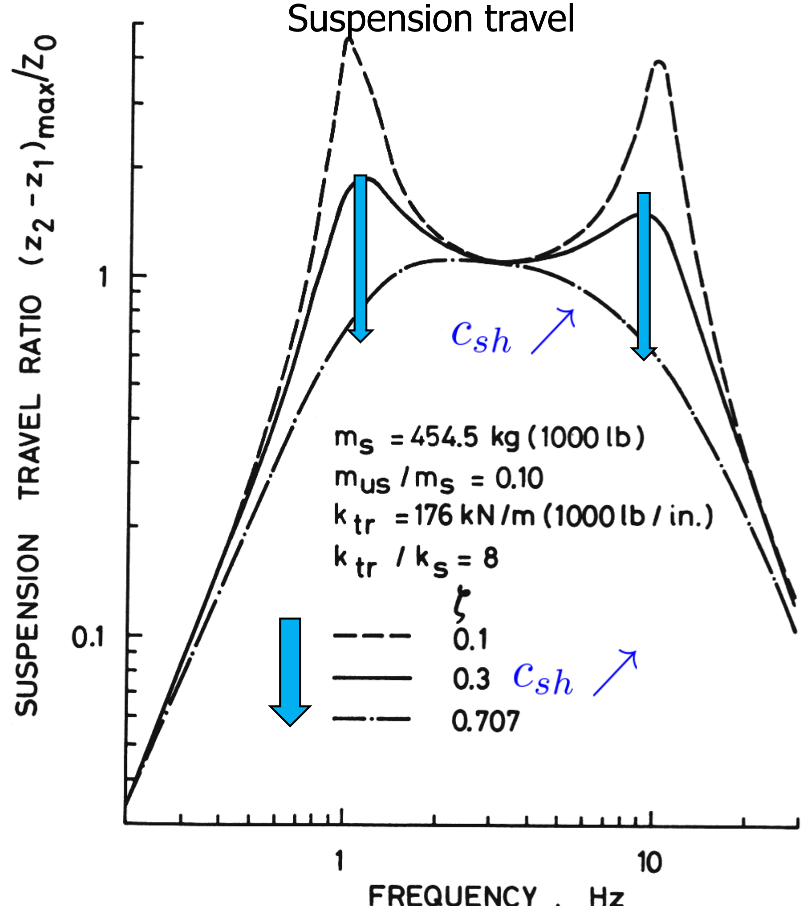


Two-dof model: Suspension travel Z_1-Z_2

- <u>Effect of the suspension damping</u>
- Conclusion: in any cases, to reduce the suspension travel, it is better to adopt a <u>high damping coefficient</u>

Wong: Fig 7.14

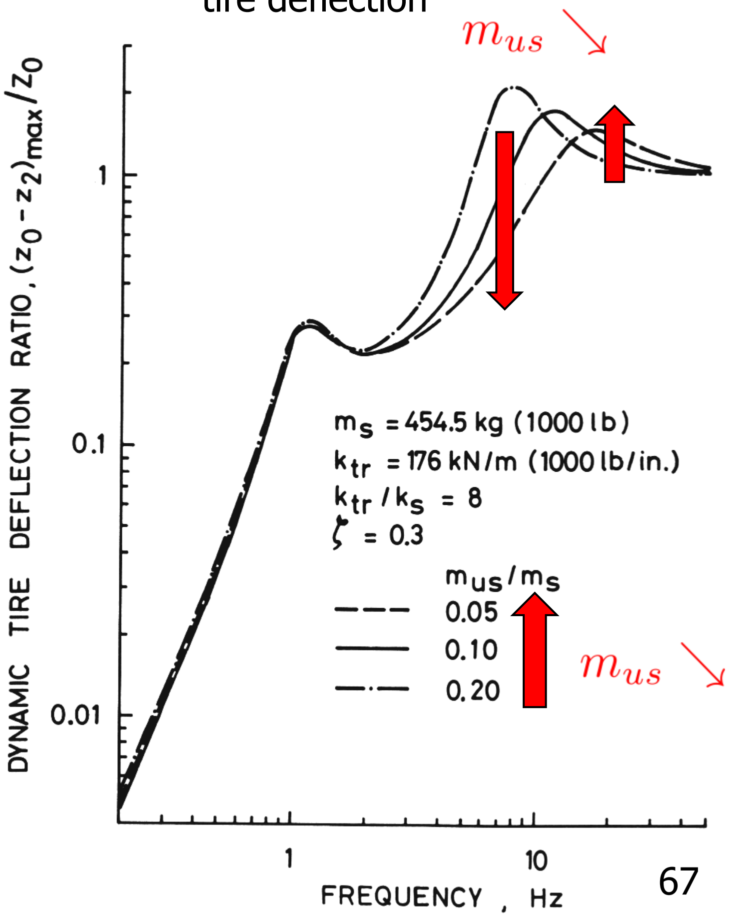
Suspension travel



Two-dof model: Road Holding k_t (Z_2-Z_0)

- Effect od the non suspended mass m_{us}:
- Conclusion: a low non suspended mass reduces the tire dynamic deflection
- Warning: the tire leaves the ground if the dynamic deflection is exceeding the static deflection of the suspension

Wong: Fig 7.15 Dynamic tire deflection



Two-dof model: Road Holding k_t $(Z_2\text{-}Z_0)$

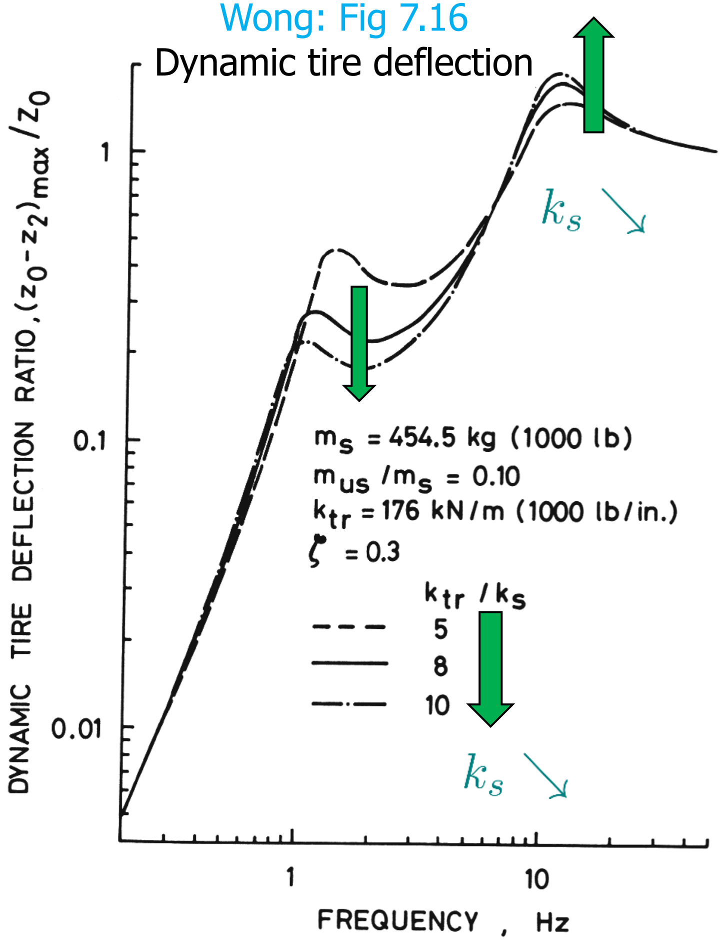
- <u>Effect of the suspension stiffness</u>

- Conclusion: In the frequency range between the first natural frequency and the crossing frequency, a low stiffness (k_{tr}/k_s high) reduces the tire deflection. Over the crossing frequency and around the second frequency of unsprung mass, a stiff suspension (k_{tr}/k_s low) minimizes the tire deflection and so maximizes the road holding

- A high stiffness is better for road holding, while the vibration isolation requires the opposite.

Wong: Fig 7.16

Dynamic tire deflection

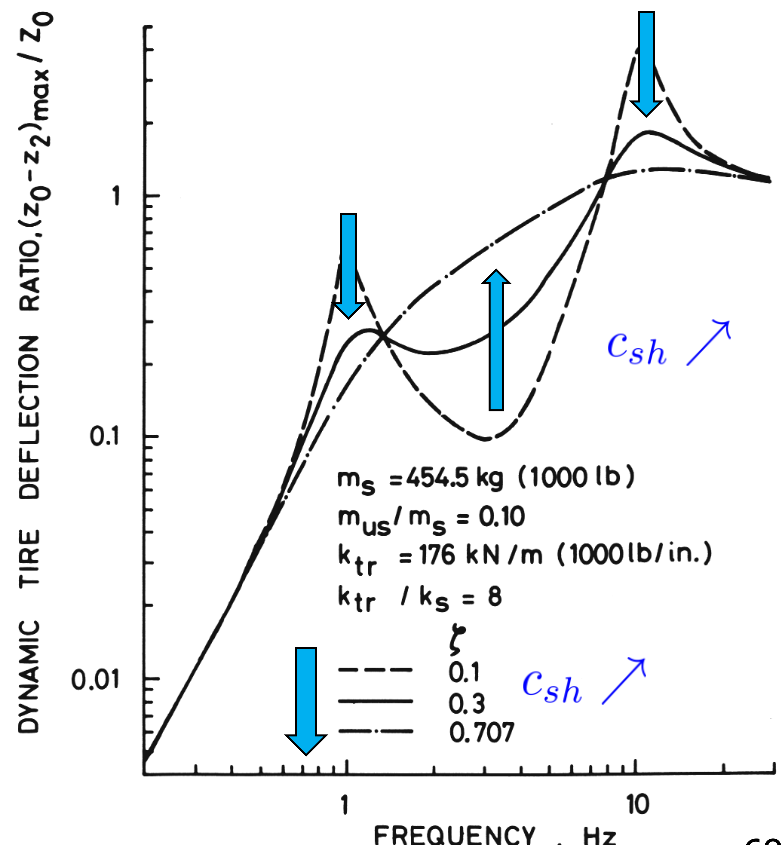


Two-dof model: Road Holding k_t $(Z_2\text{-}Z_0)$

- <u>Effect of the suspension damping</u>

- Conclusion: to reduce the tire dynamic deflection around both eigenfrequencies, one requires a <u>high damping ratio</u>. This penalizes the dynamic tire deflection in the intermediate range.

Wong: Fig 7.17 Dynamic tire deflection



Two-dof model: conclusions

- Suspended / Non-Suspended mass:
  - In all cases, it is preferred to have low non suspended mass ($m_{us} < m_s/10$)
- Stiffness of the suspension springs
  - For vibration isolation: Soft suspension
  - For suspension travel: low stiffness is better at low frequencies but for higher frequencies it is recommended to have higher stiffness
  - For road holding, it is recommended to have high stiffness
  - Conclusion: Soft suspension = comfort favorized, Stiff suspension = focus on road holding
- Damping:
  - Compromise between higher and low values to reach a good comfort and in the same time a good road holding at all frequencies (ζ between 0.2 and 0.4)

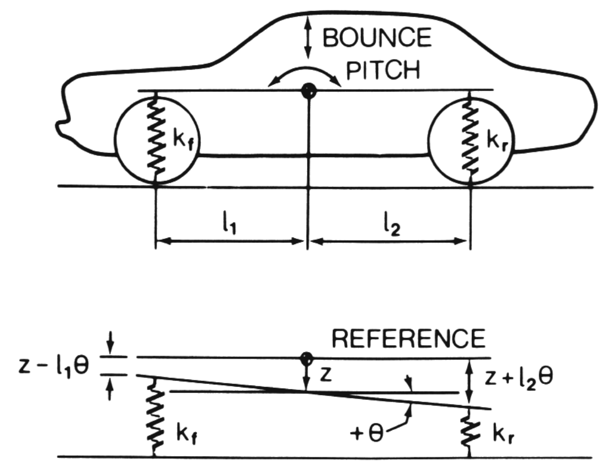
Vehicle model for pitch and bounce motions

Pitch and bounce frequencies

- Adjusting the pitch and bounce frequencies has a direct impact upon ride and road holding performance.
- For most of the vehicles, pitch and bounce motions are coupled. There is generally no pure pitch or bounce modes.
- The behavior in terms of pitch / bounce frequencies and motion center can be investigated using a simple two dof vehicle model.

Two-dof vehicle model for pitch and bounce

- 2 degrees of freedom model
  - The effective stiffness of the suspension and of the tires are considered together
  - The damping of the suspension and of the tires is neglected in a first step
  - The non suspended mass is ignored here



Wong: Fig 7.7 Two dof model for bounce and pitch of the sprung mass

- Equation of system dynamics

$$\begin{cases} m_s\, \ddot{z} + k_f\,(z - l_f\,\theta - z_f) + k_r\,(z + l_r\,\theta - z_r) = 0 \\ I_y\,\ddot{\theta} - k_f\,l_f\,(z - l_f\,\theta - z_f) + k_r\,l_r\,(z + l_r\,\theta - z_r) = 0 \end{cases}$$

Two-dof vehicle model for pitch and bounce

- Let's define

$$D_1 = \frac{1}{m_s}(k_f + k_r) \qquad I_y = m_s\, r_y^2$$

$$D_2 = \frac{1}{m_s}(k_r\, l_r - k_f\, l_f)$$

$$D_3 = \frac{1}{I_y}(k_f\, l_f^2 + k_r\, l_r^2) = \frac{1}{m_s\, r_y^2}(k_f\, l_f^2 + k_r\, l_r^2)$$

The system dynamic equations write

$$\begin{cases} \ddot{z} + D_1\, z + D_2\, \theta = 0 \\ \ddot{\theta} + D_3\, \theta + \frac{D_2}{r_y^2}\, z = 0 \end{cases}$$

Two-dof vehicle model for pitch and bounce

- The equations are <u>uncoupled</u> if

$$D_2 = 0 \quad \Leftrightarrow \quad k_f \, l_f = k_r \, l_r$$

- Then one has two eigenfrequencies of uncoupled motions in pitch and bounce

$$\omega_{n,z} = \sqrt{D_1} = \sqrt{\frac{k_f + k_r}{m_s}}$$

$$\omega_{n,\theta} = \sqrt{D_3} = \sqrt{\frac{k_f \, l_f^2 + k_r \, l_r^2}{I_y}}$$

Two-dof vehicle model for pitch and bounce

- The equations of the coupled motion can be writen using harmonic assumption
  - Harmonic motion

$$\begin{aligned} z &= Z \cos \omega_n t \\ \theta &= \Theta \cos \omega_n t \end{aligned}$$

  - Dynamic equations

$$\begin{cases} \left(D_1 - \omega_n^2\right) Z + D_2 \, \Theta = 0 \\ \left(\frac{D_2}{r_y^2}\right) \; Z + \left(D_3 - \omega_n^2\right) \Theta = 0 \end{cases}$$

  - Characteristic equation

$$\omega_n^4 - (D_1 + D_3)\, \omega_n^2 + \left(D_1 \, D_3 - \frac{D_2}{r_y^2}\right) = 0$$

Two-dof vehicle model for pitch and bounce

- It comes the values of the two eigenfrequencies of the coupled modes mixing pitch and bounce motion

$$\omega_{n1}^2 = \frac{1}{2}(D_1 + D_3) - \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_y^2}}$$

$$\omega_{n2}^2 = \frac{1}{2}(D_1 + D_3) + \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_y^2}}$$

Two-dof vehicle model for pitch and bounce

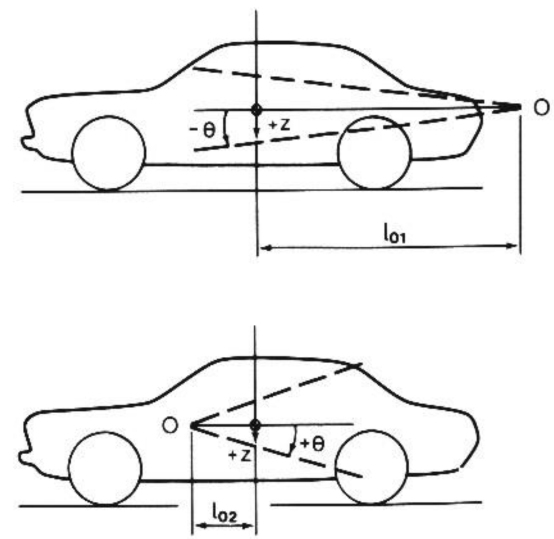
- One defines the ratio of the magnitude of the bounce and pitch parts of the two modes

$$\left.\frac{Z}{\Theta}\right|_{\omega_{n1}} = \frac{D_2}{\omega_{n1}^2 - D_1} \qquad \left.\frac{Z}{\Theta}\right|_{\omega_{n2}} = \frac{D_2}{\omega_{n2}^2 - D_1}$$

- So the concept of <u>oscillation center</u>

$$l_{01} = \frac{D_2}{\omega_{n1}^2 - D_1} < 0$$

$$l_{02} = \frac{D_2}{\omega_{n2}^2 - D_1} > 0$$



Wong: Fig 7.19 Oscillation centers for bounce and pitch of the sprung mass

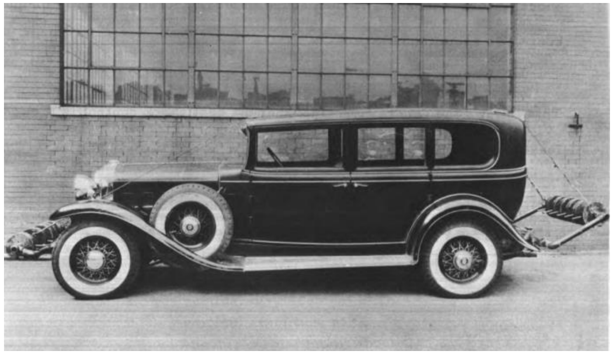
Two-dof vehicle model for pitch and bounce

- Exercise:
  - m_s = 2120 kg r_y = 1.33 m
  - l_f = 1.267 m l_r = 1.548 m
  - k_f = 35 kN/m k_r = 38 kN/m

- Solution:
  - D_1 = 34.43 s^{-2} D_2= 6.83 ms^{-2} D_3= 39.26 s^{-2}
  - $\omega_{n1}{}^2$ = 31.17 s^{-1}, f_{n1} = 0.89 Hz
  - $\omega_{n2}{}^2$ = 42.52 s^{-1} f_{n2} = 1.04 Hz
  - l_{01} = -2.09 m l_{02} = +0.84 m

Pitch and bounce frequencies

- As soon as the 1930ies, Maurice Olley, a pioneer in the vehicle dynamics of modern automobiles has proposed 4 recommendations to design vehicles with a good level of comfort and ride (at least for the lower frequencies, rigid body modes).
- These recommendations stem from experiments conducted on a vehicle with a variable pitch moment of inertia.
- They still remain good rules of thumb to design modern vehicles nowadays.



Pitch and bounce frequencies

- In the 1930ies, Maurice Olley proposed his theory based the idea that the comfort is related to the driving flat evolution.

Pitch and bounce frequencies

- Rule 1: The front suspension should have a 30% lower ride rate *than the rear suspension in other words, the spring stiffness center should be at least 6,5% of the wheelbase behind the CG.*
  - This does not prescribe the natural frequencies of the suspension, because the mass distribution between front and rear is close to 50-50, but it tells us that rear suspension frequency must be greater than the front one.

- Rule 2: *The pitch and bounce frequencies must be close together. The bounce frequency should be less than 1.2 times the pitch frequency. For higher ratio, interference kicks resulting from the superposition of the two motions are likely*
  - It is generally easily encountered by modern cars

Pitch and bounce frequencies

- Rule 3: *No frequency should be greater than 1.3 Hz, which means that the effective static deflection of the car suspension should exceed roughly 6 inches (15 cm).*

- Rule 4: *The roll frequency should be approximately equal to the pitch and bounce frequencies.*
  - In order to minimize roll vibrations, the natural frequency in roll needs to be low just as for the bounce and the pitch modes.
  - It is usually verified by modern cars.

Pitch and bounce frequencies

- The rule that the rear suspension should have a higher suspension spring rate is rationalized by the observation that vehicle bounce is less annoying as a ride motion than pitch.
- If some excitation inputs from the road affect the front wheels first, the higher rear / front frequency ratio will tend to induce more bounce than pitch.

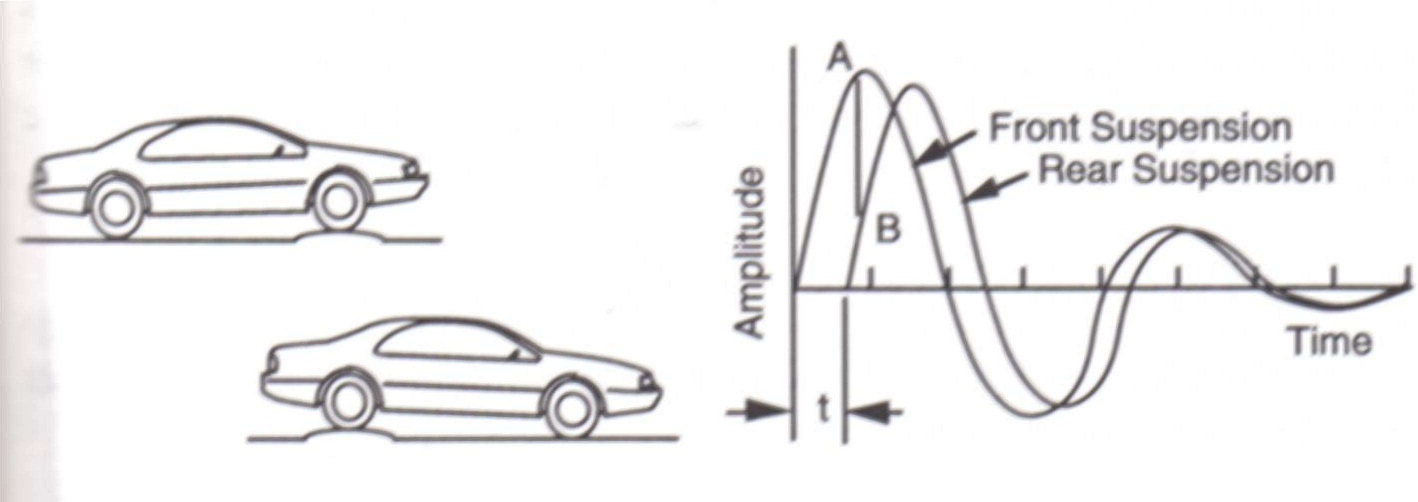
- To illustrate the concept, let's consider a vehicle passing over a road bump. The time lag between the front and rear wheel road inputs at a speed of V is

$$t = L/V$$

  - Where L is the wheelbase of the car

Pitch and bounce frequencies

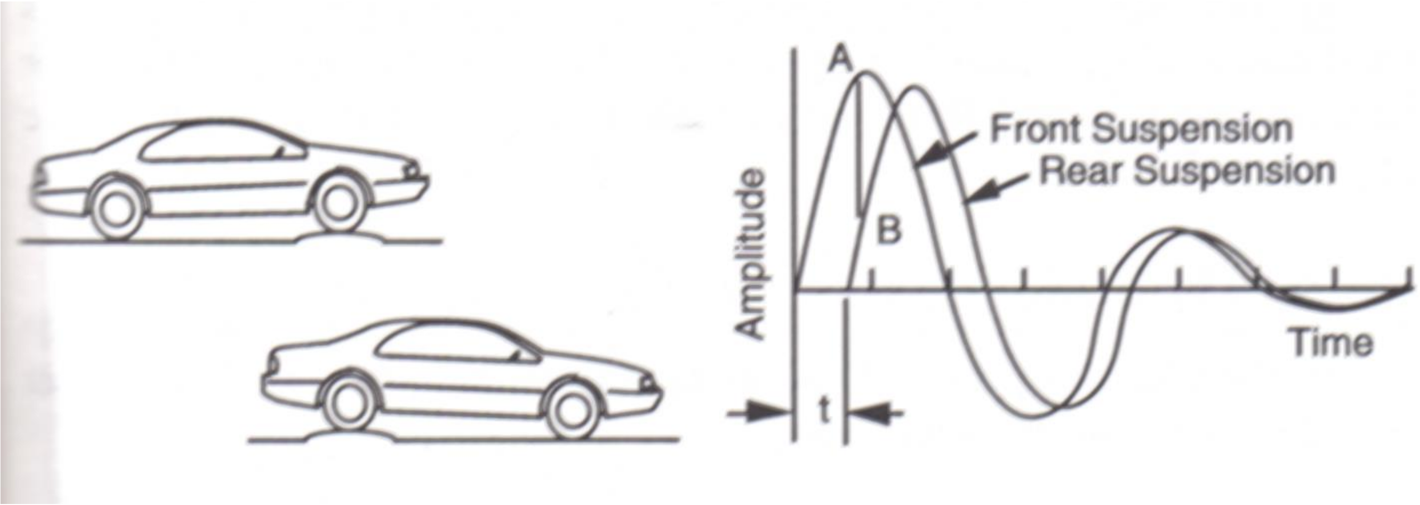
- Soon after the rear wheels have passed over the bump, the vehicle is at worst condition of pitching (points A and B).
- The front is at its maximum upward position while its rear is just beginning to move. Pitch motion is thus maximum.



Gillespie Fig. 5.35 Oscillations of a vehicle passing over a road bump

Pitch and bounce frequencies

- If the front suspension rate is smaller than the rear one, one experiences a flapping phenomenon that tends to reduce the overall pitch motion.
- After 1.5 oscillations of the rear suspension, both ends of the car are moving in phase. The body is merely bouncing up and down until the motion is almost fully damped.



Gillespie Fig. 5.35 Oscillations of a vehicle passing over a road bump

Pitch and bounce frequencies

- The intuition of M. Olley is only partly verified on modern cars, because:
  - The theory does apply only for a particular speed
  - In the US, it is applicable for driving at 55 mph, for cars with a small pitch damping, and road with rather isolated excitations

- Nonetheless, this is not very relevant in practice, especially for European driving conditions
  - The excitation coming from the road contains several frequencies and the vehicle speed is changing
  - The vehicle vibration modes are coupled so that both bounce and pitch modes are simultaneously and mutually excited.
  - At different speeds and for different road geometries, the vehicle response will change.

Pitch and bounce frequencies

- The quarter car model is not able to fully represent the body motions that may occur.
- Because of the distance between the axles, the vehicle is a multi-input system that responds both to pitch and bounce motion.
- Depending on the road and speed conditions, one or the other of the motions may be present.
- Pitch motions are considered objectionable and are primarily source of longitudinal vibrations above the center of gravity.

Pitch and bounce frequencies

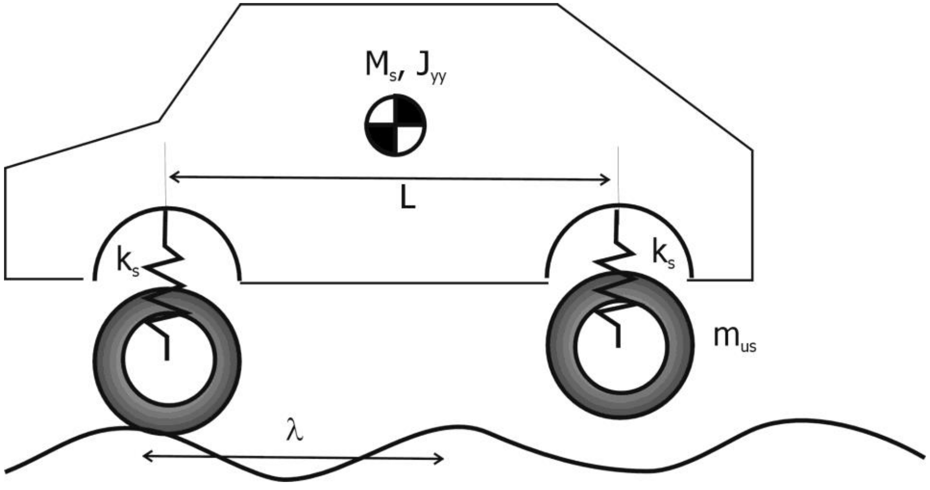
- The road excitation under the different wheels is not independent.
- Rear wheels see the same input profile as the front wheels but delayed in time.
- The time delay is the wheelbase divided by the speed.

$$\Delta t = \frac{L}{V}$$

- The time delay acts as a filter for the bounce and pitch excitation amplitude and has been called a "wheelbase filtering"
- The frequency response curves are shaped by the filtering frequencies of the transmission transfer function

Pitch and bounce frequencies

- To understand the wheelbase filtering, lets consider a vehicle with independent bounce and pitch motions (simplification)
- Let's consider a two-axle vehicle travelling on a sinusoidal shape road



Pitch and bounce frequencies

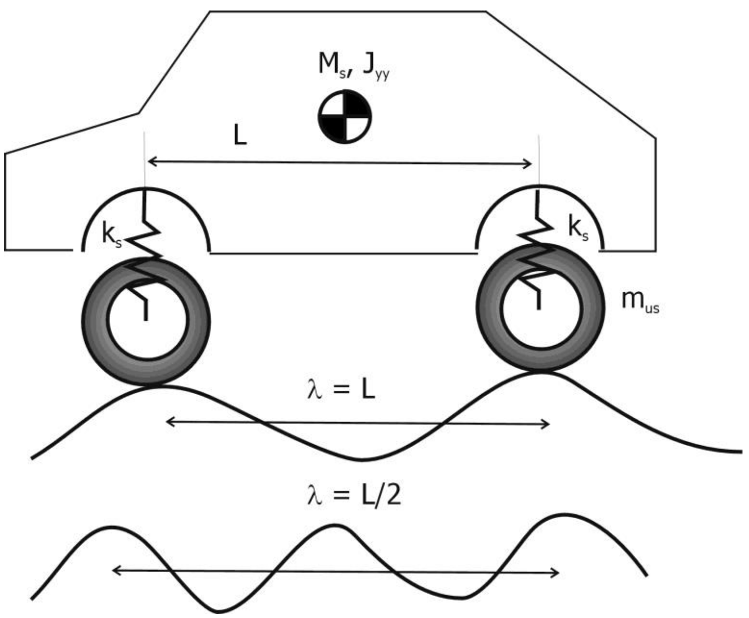
- Only bounce motion input occurs at a wavelength equal to the vehicle wheelbase L.

- The same situation occurs if the wavelength is much longer than the wheelbase or for short road wavelengths which have an integer multiple equal to the base length

$$\lambda_s = L$$

$$\lambda_s \gg L$$

$$\lambda_s = L/k \quad k = 2, 3 \ldots$$

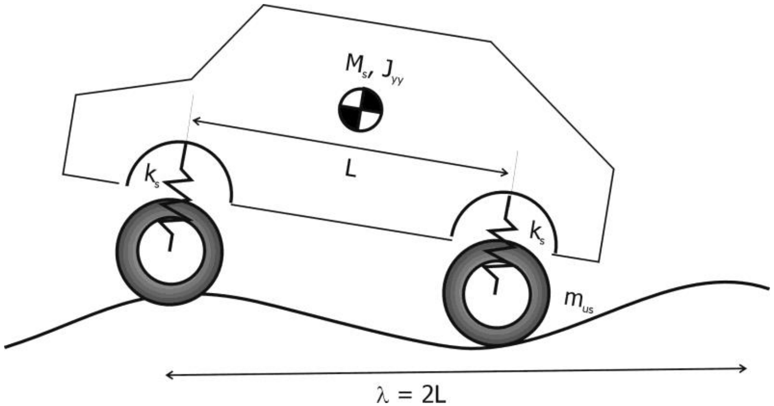


$$z_{rf}(t) = z_{rr}(t)$$

Pitch and bounce frequencies

- Only pitch motion input occurs at a wavelength equal to the twice the vehicle wheelbase 2L.

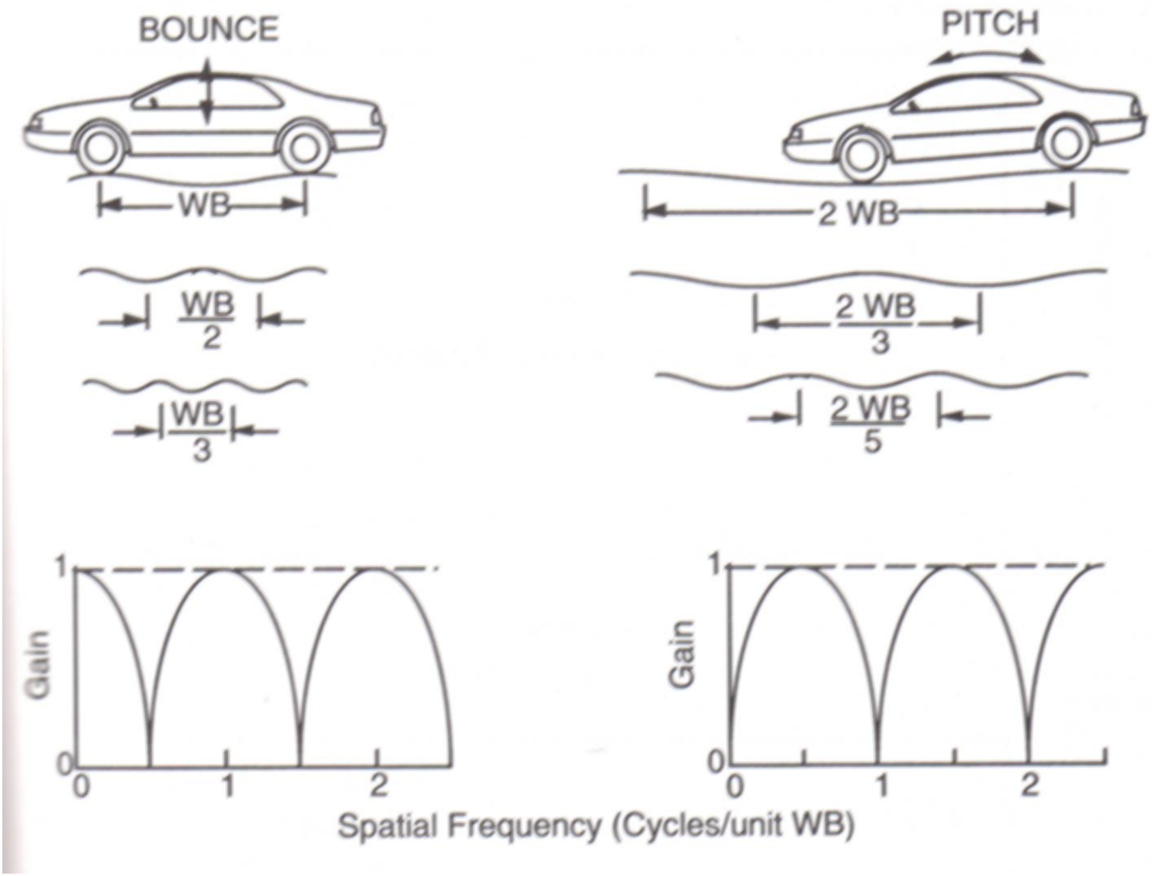
- The same situation occurs if the wavelength for road wavelengths which have an odd integer multiple equal to twice the base length



$$\lambda_s = 2L$$

$$\lambda_s = 2L/k \quad k = 3, 5 \dots$$

Pitch and bounce frequencies



Gillespie Fig. 5.29 The wheelbase filtering mechanism