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# MECA0525- Performance and Vehicle dynamics

## HOMEWORK 3: VEHICLE DYNAMICS

Modalities

It is requested to solve the following exercise, which is related to the lecture of vehicle dynamics in steady state cornering.

Shape groups of two students. The exercises will be solved with your teammate.

Reports have to be in pdf format and should entitled Name1_Name2_HW3.pdf. Reports can be written using word processor (e.g. Word, Latex…) or they can be manuscript, provided they are clearly written, and they are easy to read, and then scanned in pdf (photos using mobile phones are not accepted). They have to be posted by emails to Pierre Duysinx (p.duysinx@uliege.be) by the 22$^{nd}$ of May 2022 at 12:00 AM CET.

Questions and discussions should be preferably formulated using Discussion Forum on e-Campus to share the information with your classmates.

It is requested to solve the following exercises. Each group will provide a power-point presentation showing their developments by May 10, 2022 at 12:00.

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Question n°1

We consider a test vehicle, Renault Megane Coupé 16V 150 HP. The Ref [1] provides the following data

Vehicle mass M= 1362 kg
The inertia moment about vertical axis is measured as $J_z$= 1623.8 kg/m²
The wheelbase is L = 2.468 m
Distance from front axle to CG: b=0.9552 m
Distance from rear axle to CG: c=1.5128 m
Vertical position of the CG: $h_{CG}$= 0.450

The vehicle is equipped with tires whose cornering stiffness are given by:

- Front: $C_\alpha$ = 84085 N/rad
- Rear: $C_\alpha$ = 87342 N/rad

For a turn with a radius R=60 m and a vehicle speed of 15 m/s, we ask to calculate:

1. The Ackerman angle (deg)
2. The cornering stiffness of the front and rear axles (N/deg)
3. The slip angle under front and rear axles (deg)

4. The drift angle at the centre of gravity (deg)
5. The actual steering angle at front wheels (deg)
6. The vehicle understeer gradient (deg/g)
7. Depending on the case, the characteristic speed or the critical speed (m/s and km/h)
8. The vehicle static margin (in cm and in %)

It is also requested to plot a figure showing the evolution of:

9. The gain in lateral acceleration (g/deg) per degree of steer angle
10. The yaw velocity gain under the same conditions (/sec)

For a speed varying from 0 to 50 m/s per step of 5 m/s.

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Question n°2 : (20 points)

One would like to design a prototype whose front wheels can be controlled either by prescribing a steering angle and by modifying the camber angle. The control mechanism is able to set the same inclination angle on the left and the right wheels. It is assumed that we know all necessary data of the tires and axles characteristics. We consider the case of bias-plies tires.

We ask to study the vehicle stability during steady state cornering. We assume a known circular trajectory of radius R at constant velocity V. We want to expand the single-track model (bicycle model) and at the tire inclination as a second degree of freedom in addition to classic steering angle.

We ask :

1. To draw an extended single-track model accounting for the camber angle degree of freedom. (1 point)
2. To write the behavior equations of the tire (please pay attention to the sign convention) (2 points)
3. To write the vehicle dynamic equilibrium (1 point)
4. To establish the compatibility equations (1 point)
5. To derive the equations relating the steering angle, the inclination angle, the lateral acceleration (as so the velocity V and the radius R and the vehicle geometrical properties) (5 points)
6. To write the transfer function expression as a function of the lateral acceleration (output) with respect to the inclination of the tires when having zero steering angle. Thanks to your knowledge of the tire mechanics, and the order of magnitude between the cornering coefficient and camber thrust coefficient, discuss the gain of the vehicle system with respect to steering or inclination command? (3 points)
7. To derive the expression of the understeer gradient if the inclination angles is now controlled by a feedback loop and remains proportional to the lateral $V^2/R$ that is: $\gamma = k\, V^2/R$. Do we reinforce of weaken the understeer or over steer character? (3 points)

8. To establish the expression of the understeer gradient is we use control system for the inclination angle and that we prescribe a constant relationship to the steering angle : $\gamma = k\ \alpha$. Do we reinforce of weaken the understeer or over steer character? (4 points)

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**References**

[1] R. Di Martino. Modelling and simulation of the dynamic behaviour of the automobile. PhD thesis. Université de haute Alsace – Mulhouse. 2005.