# Vehicle Performance


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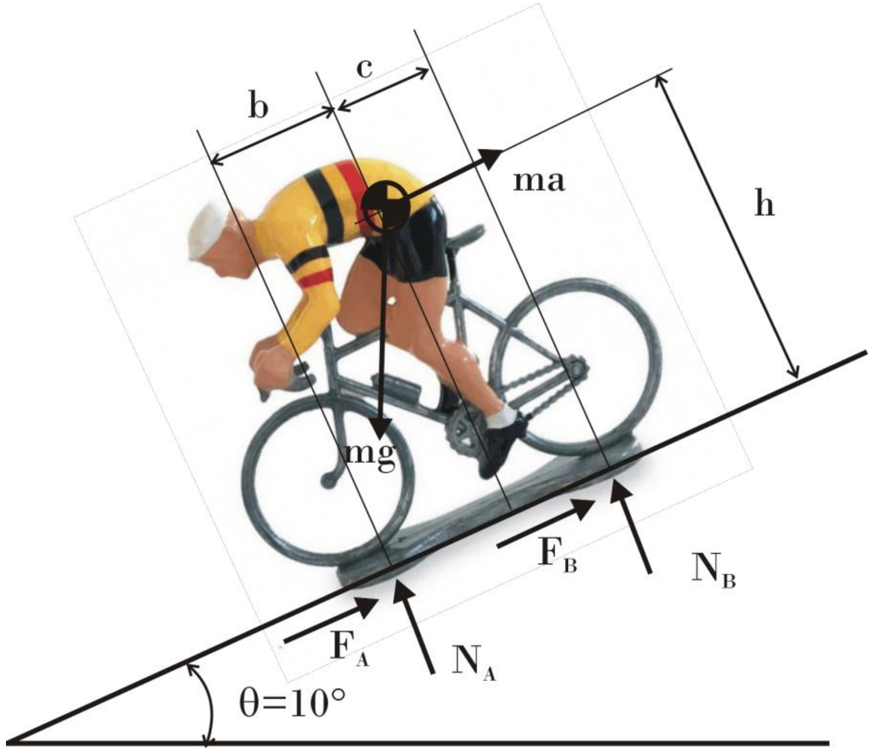
# Longitudinal equilibrium

# Exercise 1 : Weight transfer

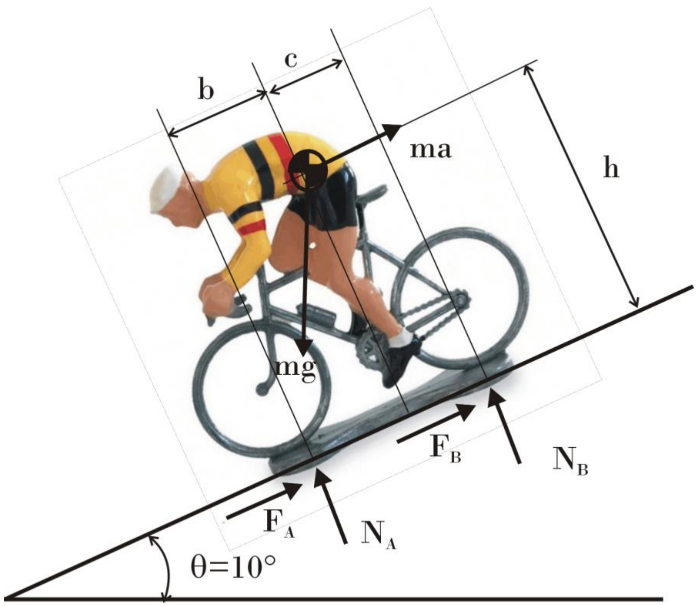
- A cyclist goes down a road at 10° and he is braking.
- What is the maximum deceleration rate that he can apply before passing over his handlebar?




$$m = 80\ kg$$
$$h = 0.90\ m$$
$$b = 0.625\ m$$
$$c = 0.375\ m$$

# Exercise 1 : Weight transfer

- Equilibrium equations




$$\sum F_x = 0 \qquad -(F_A + F_B) + mg\sin\theta = -m|a|$$

$$\sum F_z = 0 \qquad N_A + N_B = mg\cos\theta$$

$$\sum M_y(CG) = 0 \qquad N_A \cdot b - N_B \cdot c - F_A \cdot h - F_B \cdot h = 0$$

# Exercise 1 : Weight transfer

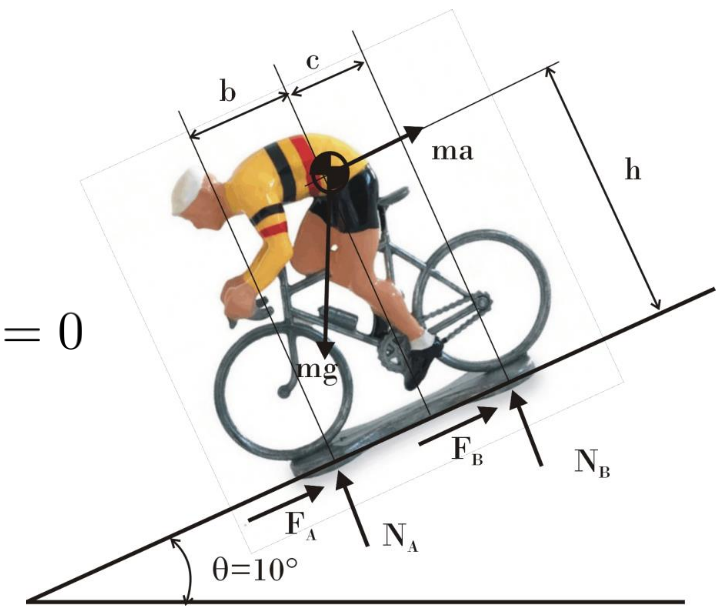
- Solving for $N_A$ using vertical equilibrium

$$N_A = mg\cos\theta - N_B$$

- Inserting into the rotation equilibrium

$$N_A \cdot b - N_B \cdot c - F_A \cdot h - F_B \cdot h = 0$$

$$(mg\cos\theta - N_B)\, b - N_B \cdot c - (F_A + F_B) \cdot h = 0$$




- The value of the braking force is given by the longitudinal equation

$$-(F_A + F_B) = -m|a| - mg\sin\theta$$

# Exercise 1 : Weight transfer

- Substituting into the vertical equation,

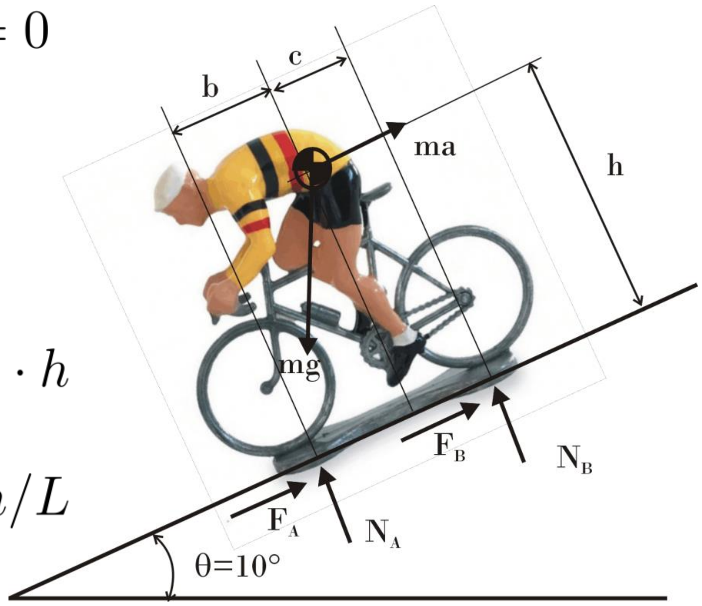
$$(mg\cos\theta - N_B)\, b - N_B \cdot c - (F_A + F_B) \cdot h = 0$$

$$-(F_A + F_B) = -m|a| - mg\sin\theta$$

- It comes

$$b \cdot mg\cos\theta - N_B\,(b + c) = (m|a| + mg\ \sin\theta) \cdot h$$

$$N_B = b/L \cdot mg\cos\theta - (m|a| + mg\ \sin\theta) \cdot h/L$$



# Exercise 1 : Weight transfer

- The biker will pass over his handlebar if

$$N_B = 0$$

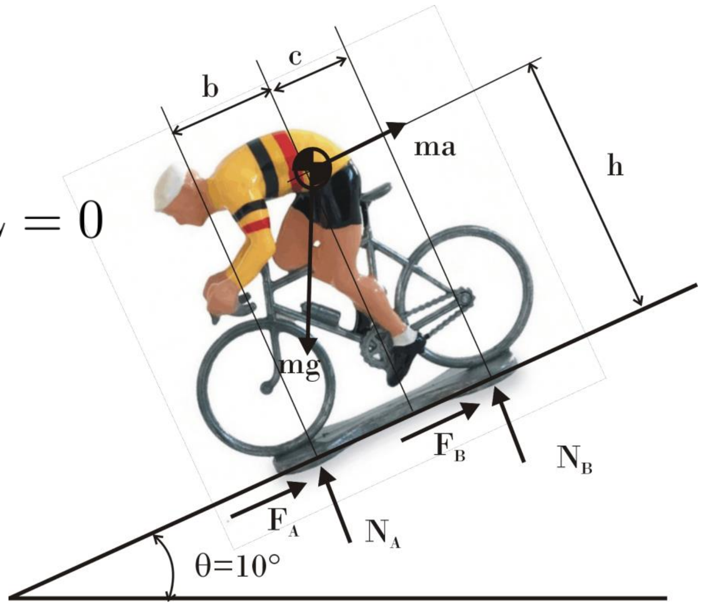
- Which yields

$$N_B = b/L \cdot mg\cos\theta - (m|a| + mg\ \sin\theta) \cdot h/L = 0$$

$$\frac{b \cdot mg\cos\theta - mg\,h\ \sin\theta}{L} = m|a|$$

- The mass of the biker cancels out

$$\frac{b \cdot \cos\theta - \ h \cdot \sin\theta}{L} = |a|/g$$



# Exercise 1 : Weight transfer

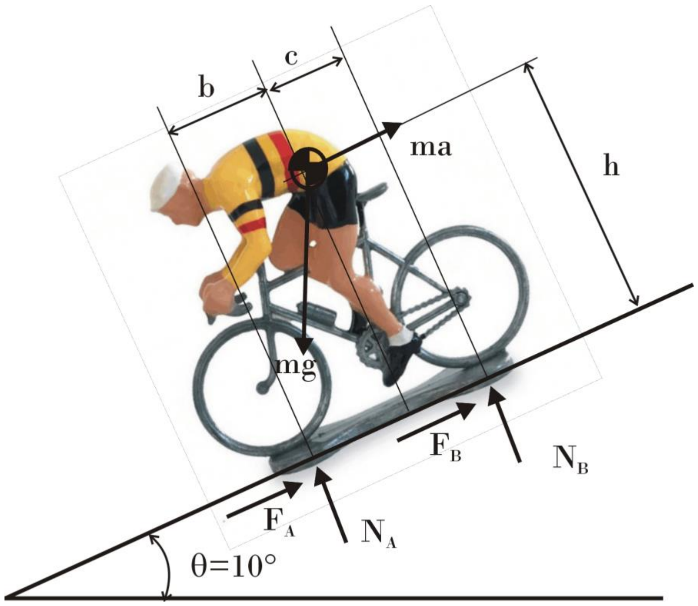
- Finally, one can compute the deceleration rate at which the biker passes of the handlebar

$$\frac{b \cdot \cos\theta - \ h \cdot \sin\theta}{L} = |a|/g$$

$$h = 0.90\ m$$

$$b = 0.625\ m$$

$$c = 0.375\ m$$

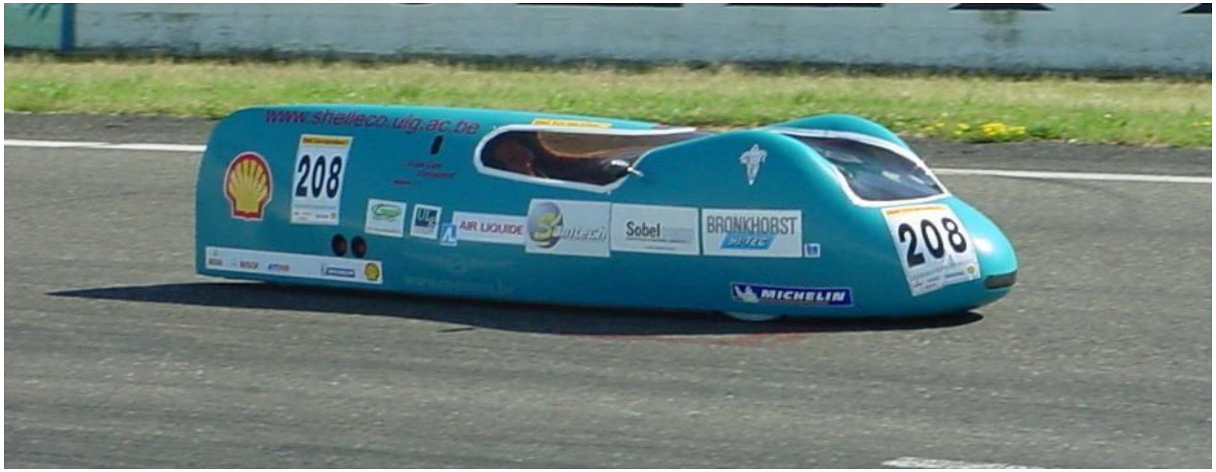



$$\frac{0.625 \cdot \cos 10^\circ - \ 0.9 \cdot \sin 10^\circ}{0.625 + 0.375} = |a|/g = 0.4592$$

$$|a| = 0.4592\, g = 4.5\, m/s^2$$

# Exercise 2 : Shell Eco Marathon

- Let’s consider the Shell Eco Marathon prototype of University of Liege.
- We do some experiment to determine the horizontal and vertical position of the center of gravity of the vehicle.




- Q1/ Calculate the position of the center of gravity of the Ulg Eco Marathon prototype
- Q2/ Calculate the maximum slope assuming infinite engine power.

# Exercise 2 : Shell Eco Marathon

- Vehicle data
  - Wheelbase

$$L = 1.57\ m$$

  - Adherence coefficient with the ground

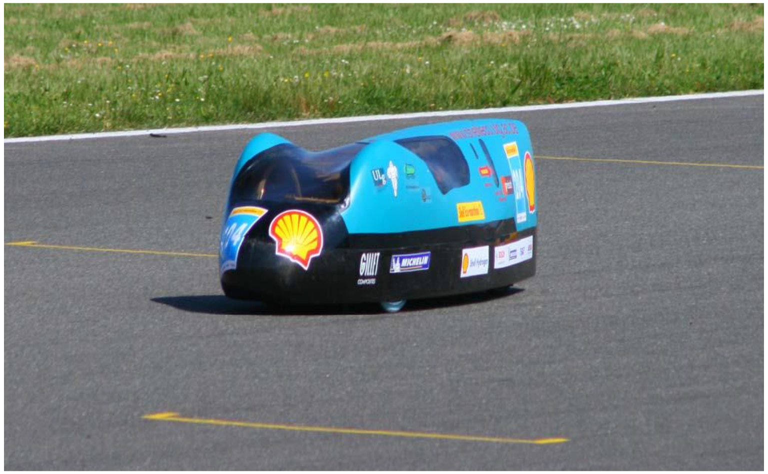
$$\mu = 0.9$$

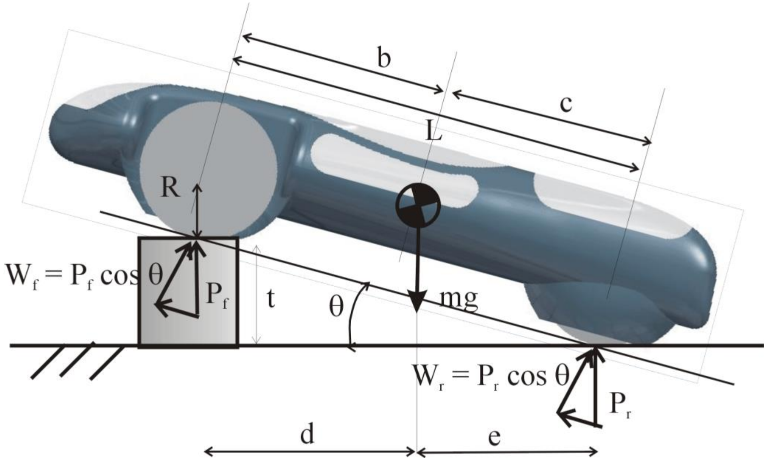
- Experiment: weight measurement
  - On level road (θ=0%)

$$m_f = 64\ kg \qquad m_r = 36.2\ kg$$

  - With a slope (θ=5.25%)

$$m_f = 63\ kg \qquad m_r = 37.2\ kg$$





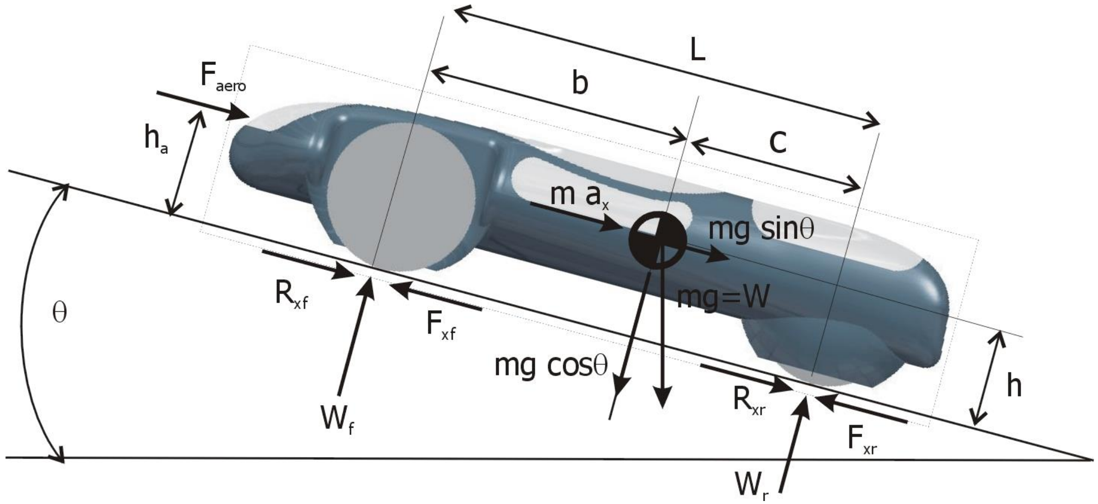
# Exercise 2 : Shell Eco Marathon

- Equilibrium equations

$$\sum F_x = m a_x$$

$$\sum F_z = 0$$

$$\sum M_y(CG) = 0$$




- It comes

$$m\,a_x = \sum_i F_{xi} - \sum_i R_{xi} - mg\sin(\theta) - F_{AERO}$$

$$0 = mg\cos(\theta) - W_f - W_r$$

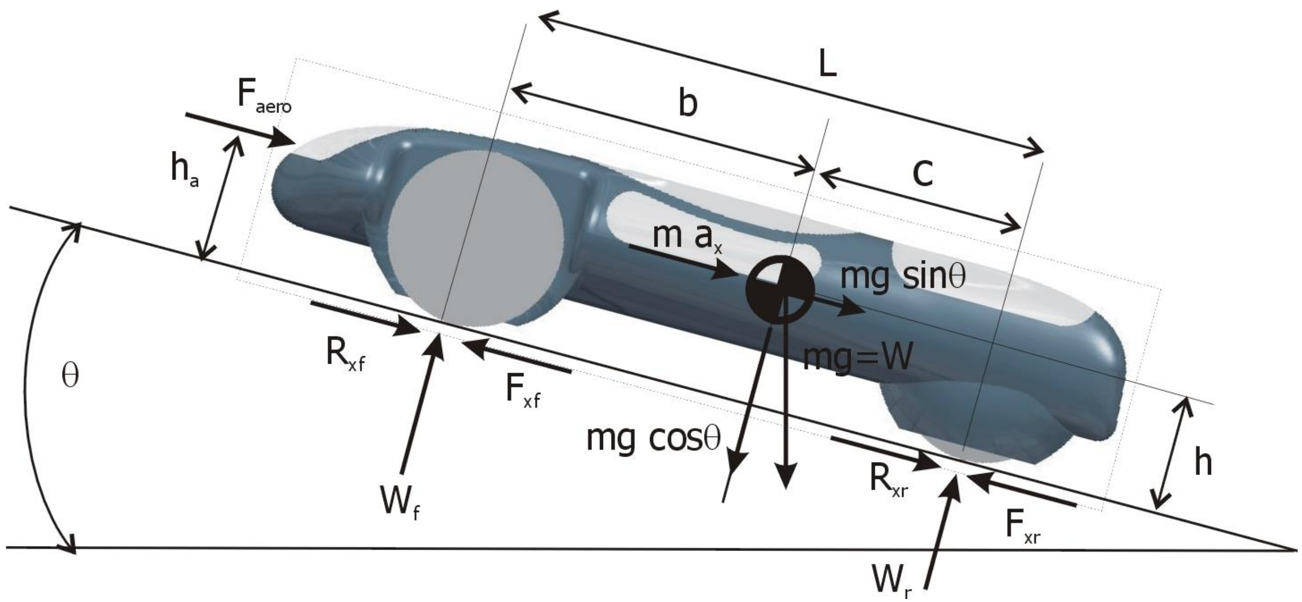
$$0 = W_f\,b - W_r\,c + \sum_i F_{xi}\,h - \sum_i R_{xi}\,h + F_{AERO}\,(h_A - h)$$

# Exercise 2 : Shell Eco Marathon

- Solving for the reaction forces under front and rear axles

$$W_f = mg\cos(\theta)\frac{c}{L} - ma_x\frac{h}{L} - F_{AERO}\frac{h_A}{L} - mg\ \sin(\theta)\ \frac{h}{L}$$

$$W_r = mg\ \cos(\theta)\frac{b}{L} + ma_x\frac{h}{L} + F_{AERO}\frac{h_A}{L} + mg\sin(\theta)\frac{h}{L}$$

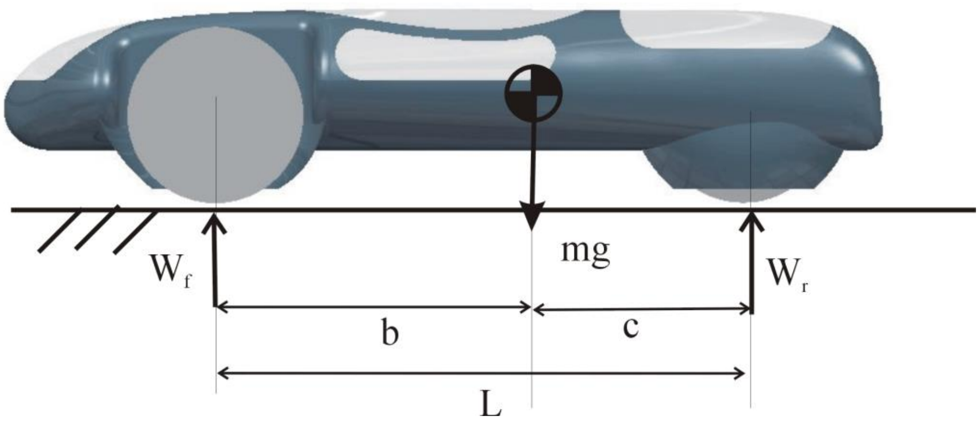


# Exercise 2.1 : Shell Eco Marathon

- In case of static conditions

$$W_f = mg\frac{c}{L}$$

$$W_r = mg\frac{b}{L}$$




- One can determine the CG position

$$W_f = mg\frac{c}{L} \Rightarrow c = \frac{W_f L}{mg} \qquad W_r = mg\frac{b}{L} \Rightarrow b = \frac{W_r L}{mg}$$

- It comes

$$m = m_f + m_r = 64.0 + 36.2 = 100.2\ kg$$

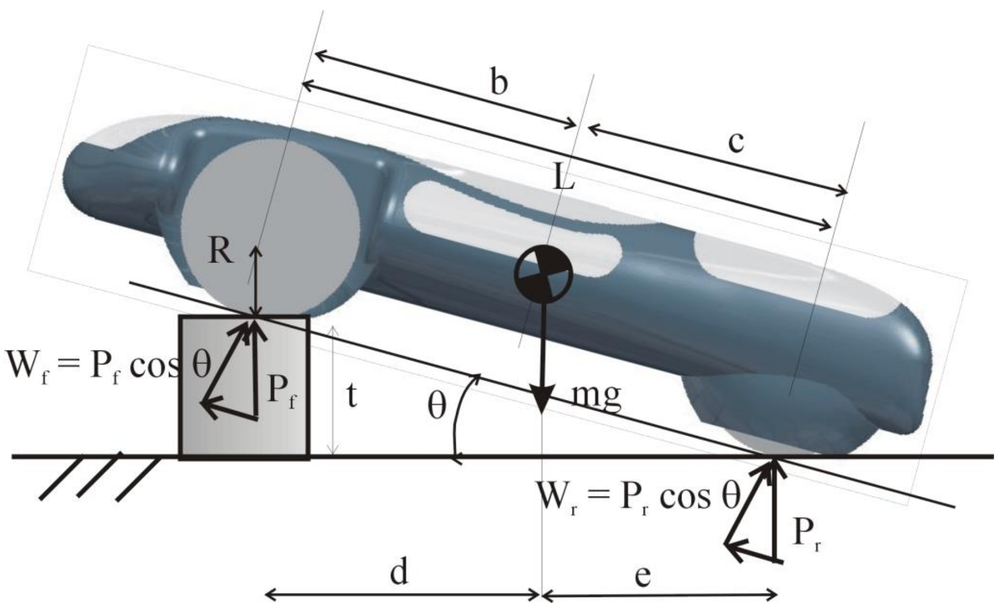
$$c = \frac{64 * 1.57}{100.2} = 1.003\ m \qquad b = \frac{36.2 * 1.57}{100.2} = 0.567\ m$$

# Exercise 2.1 : Shell Eco Marathon

- If we create a slope θ:

$$W_f = mg\cos(\theta)\frac{c}{L} - mg\sin(\theta)\frac{h}{L} = P_f\cos(\theta)$$

$$W_r = mg\cos(\theta)\frac{b}{L} + mg\sin(\theta)\frac{h}{L} = P_r\cos(\theta)$$



# Exercise 2.1 : Shell Eco Marathon

- If we create a slope $\theta$:

$$W_f = mg\cos(\theta)\frac{c}{L} - mg\sin(\theta)\frac{h}{L} = P_f\cos(\theta)$$

$$W_r = mg\cos(\theta)\frac{b}{L} + mg\sin(\theta)\frac{h}{L} = P_r\cos(\theta)$$

- One can identify the elevation of the center of gravity

$$h = \frac{(mg\ \cos(\theta)\ \frac{c}{L} - W_f) * L}{mg\ \sin(\theta)}$$

- In case of a slope of 5.25%

$$\theta = \arctan(\frac{5.25}{100}) = 3.0053^{\circ}$$

$$\frac{h}{L} = \frac{(100.2 * \cos(3.0053) * \frac{1.003}{1.57} - 63 * \cos(3.0053)) * 1.57}{100.2 * \sin(3.0053)} = 0.3024$$

# Exercise 2.1 : Shell Eco Marathon

- If we create a slope $\theta$:

$$W_f = mg\cos(\theta)\frac{c}{L} - mg\sin(\theta)\frac{h}{L} = P_f\cos(\theta)$$

$$W_r = mg\cos(\theta)\frac{b}{L} + mg\sin(\theta)\frac{h}{L} = P_r\cos(\theta)$$

- Alternative formula (see Lecture)

$$\frac{h}{L} = \frac{c\,P_r - b\,P_f}{P_f + P_r}\ \cot\theta \qquad\qquad \theta = \arctan(\frac{5.25}{100}) = 3.0053^\circ$$

- In case of a slope of 5.25%

$$\frac{h}{L} = \frac{0.567 \cdot 37.2 - 63 \cdot 1.003}{100.2}\ \frac{1}{\sin 3.0003^\circ} = 0.3027$$

$$h = 0.4757\ m$$

# Exercise 2.2 : Shell Eco Marathon

- Let's compute the maximum slope, if we assume that the power and the tractive forces are very big.
- The nonslip condition under the live wheels writes

$$F_{w,f} \leq \mu W_f \qquad \boxed{F_{w,r} \leq \mu W_r}$$

- For rear wheel propulsion:

$$F_t = F_{RES} = mg\sin(\theta) + mg\cos(\theta) f \leq \mu\ mg \left( \cos(\theta)\ \frac{b}{L} + \sin(\theta)\ \frac{h}{L} \right)$$

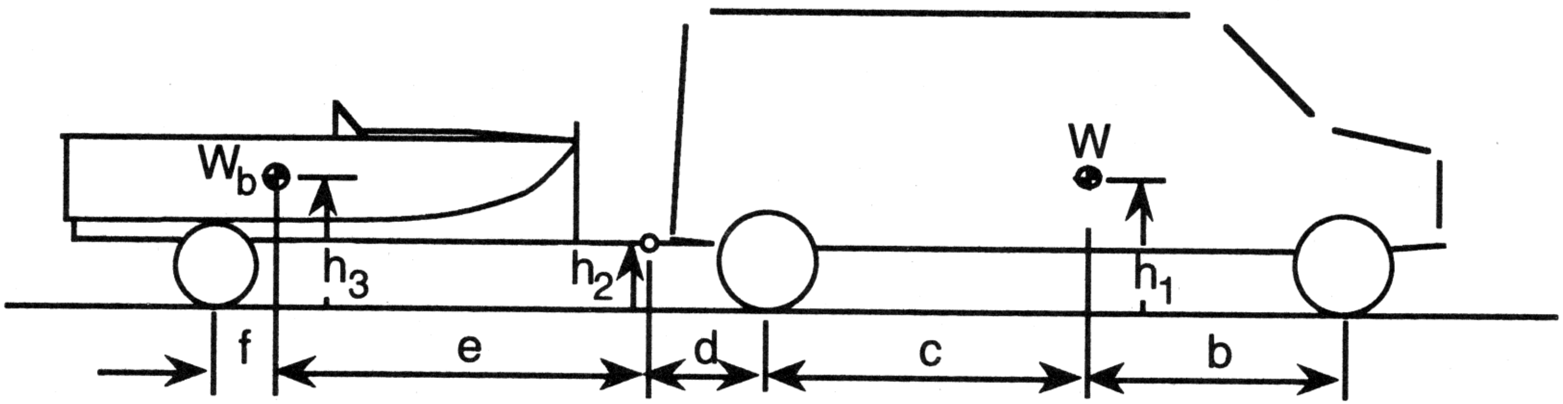
$$\tan(\theta) \leq \frac{\mu\,\frac{b}{L} - f}{1 - \mu\frac{h}{L}}$$

- In the present case, we get

$$\tan(\theta) \leq \frac{0.9\frac{0.567}{1.57} - 0.0136}{1 - 0.9\frac{0.3024}{1.57}} \qquad \theta_{max} = 20.6436\ ^\circ$$

# Exercise 3: Van & Trailer

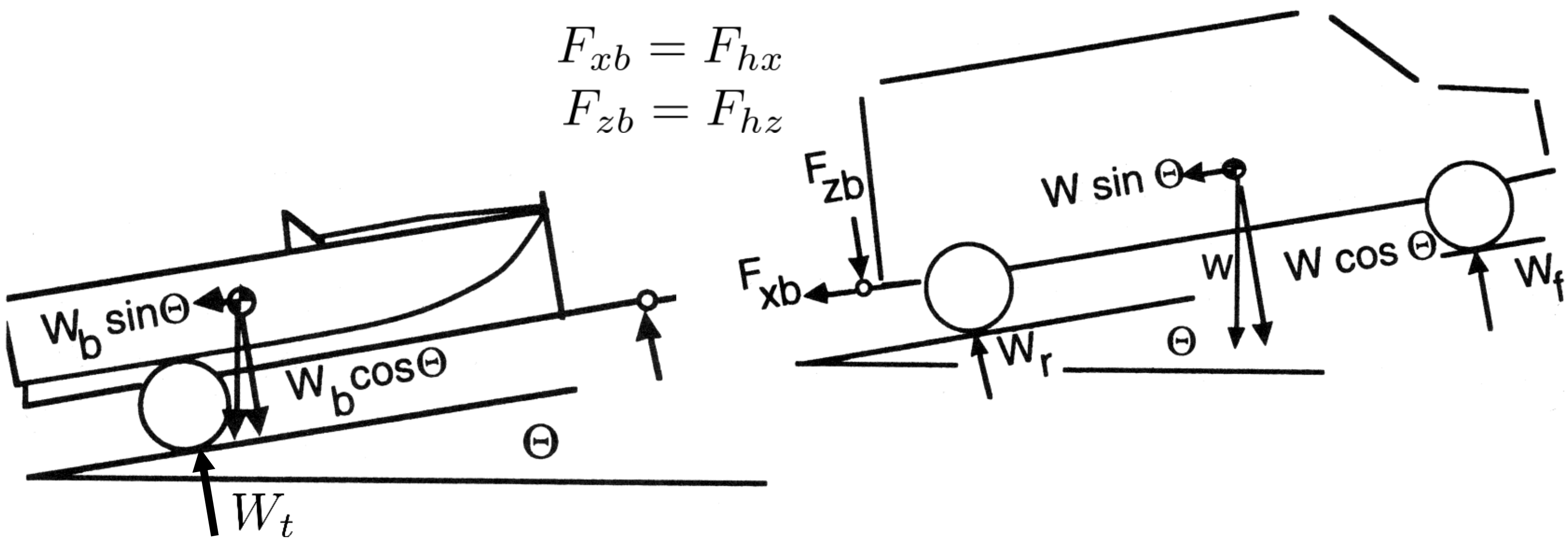
- Compute the maximum slope that the van and its boat trailer can climb without sliding if μ=0.3?



# Exercise 3: Van & Trailer

- Data of the van:
  - $W_f$ = 760 kg
  - $W_r$ = 573 kg
  - Height of CoG h = 61 cm
  - Height of hook $h_h$ = 35 cm
  - Horizontal position of the hook $d_h$=57,5 cm
  - Wheelbase L = 300 cm
- Data of trailer and its payload
  - $W_t$ = 600 kg
  - $F_{h,z}$ = 125 kg
  - Height CoG of the boat $h_t$=27,5cm
  - Wheelbase of the trailer $L_t$=275 cm

# Exercise 3: Van & Trailer




Free body diagram of the van + trailer

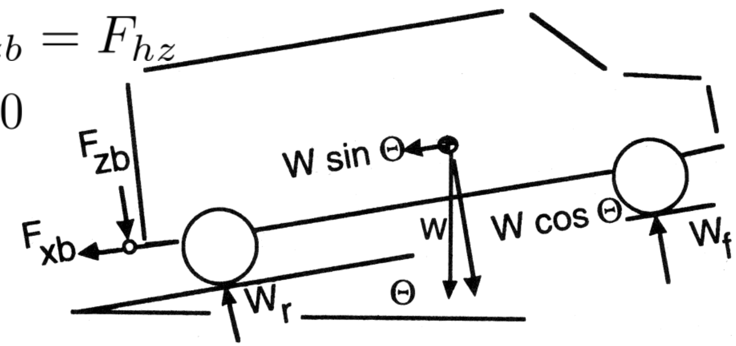
# Exercise 3: Van & Trailer

$$F_{xb} = F_{hx}$$
$$F_{zb} = F_{hz}$$

- Equilibrium of the van

$$m\,a_x = F_{xf} + F_{xr} - F_{RR} - W\sin\theta - F_{hx} = 0$$

$$0 = m_v g\cos\theta + F_{hz} - W_f - W_r$$




- Equilibrium with respect rear wheel

$$W = m_v g$$

$$W\cos\theta\, c - W\sin\theta\, h_1 - W_f\, L - F_{hx}\, h_2 - F_{hz}\, d = 0$$

$$W_f = W\cos\theta\frac{c}{L} - W\sin\theta\frac{h_1}{L} - F_{hx}\frac{h_2}{L} - F_{hz}\frac{d}{L}$$

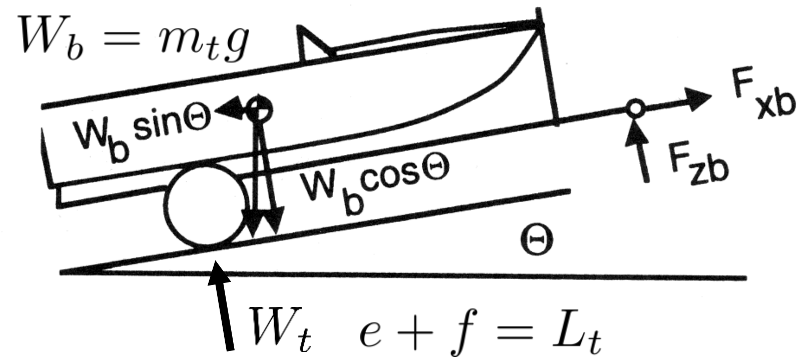
- Equilibrium with respect to front wheel

$$W_r\, L - W\cos\theta\, b - W\sin\theta\, h_1 - F_{hx}\, h_2 - F_{hz}\,(L+d) = 0$$

$$W_r = W\cos\theta\frac{b}{L} + W\sin\theta\frac{h_1}{L} + F_{hx}\frac{h_2}{L} + F_{hz}\frac{L+d}{L}$$

# Exercise 3: Van & Trailer

- Equilibrium of the van




$$m\, a_x = F_{hx} - W_b \sin\theta - F_{RRt} = 0$$

$$0 = W_b \cos\theta - F_{zb} - W_t$$

- Equilibrium with respect to the trailer wheel contact point

$$W_b \cos\theta\, f - W_b \sin\theta\, h_3 + F_{hx}\, h_2 - F_{hz}\, L_t = 0$$

$$F_{hz}\, L_t = W_b \cos\theta\, f - W_b \sin\theta\, h_3 + F_{hx}\, h_2$$

$$F_{hz} = W_b \cos\theta\, \frac{f}{L_t} - W_b \sin\theta\, \frac{h_3}{L_t} + (W_b\ \sin\theta + F_{RRt})\, \frac{h_2}{L_t}$$

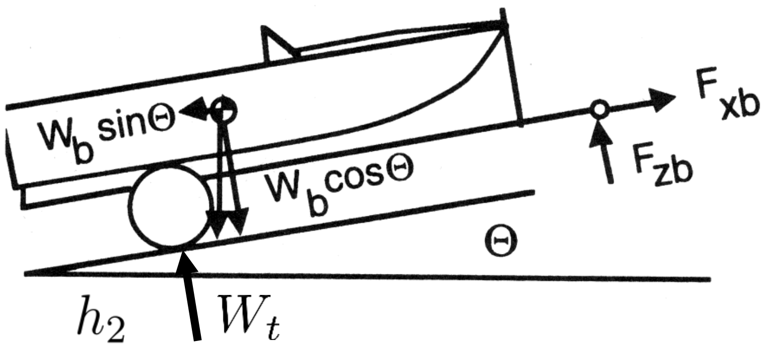
$$F_{hz} = W_b \cos\theta\, \frac{f}{L_t} - W_b \sin\theta\, \frac{h_3 - h_2}{L_t} + f_{RR}\, W_t \frac{h_2}{L_t}$$

# Exercise 3: Van & Trailer

- Equilibrium of the van

$$m\,a_x = F_{hx} - W_b \sin\theta - F_{RRt} = 0$$

$$0 = W_b \cos\theta - F_{hz} - W_t$$




$$F_{hz} = W_b \cos\theta\, \frac{f}{L_t} - W_b \sin\theta\, \frac{h_3 - h_2}{L_t} + F_{RRt} \frac{h_2}{L_t}$$

$$e + f = L_t$$

- Equilibrium with respect to the trailer wheel contact point

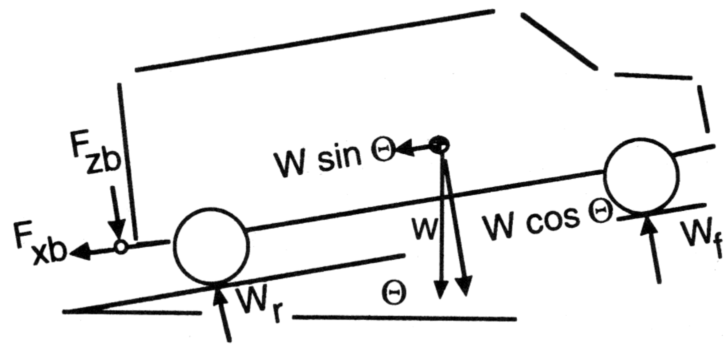
$$\begin{aligned} W_t &= W_b \cos\theta - F_{hz} \\ &= W_b \cos\theta - W_b \cos\theta\, \frac{f}{L_t} + W_b \sin\theta\, \frac{h_3 - h_2}{L_t} - F_{RRt}\, \frac{h_2}{L_t} \\ &= W_b \cos\theta \left(1 - \frac{f}{L_t}\right) + W_b \sin\theta\, \frac{h_3 - h_2}{L_t} - F_{RRt}\, \frac{h_2}{L_t} \end{aligned}$$

$$\Rightarrow \quad W_t = W_b \cos\theta \frac{e}{L_t} + W_b \sin\theta\, \frac{h_3 - h_2}{L_t} - f_{RR} W_t\, \frac{h_2}{L_t}$$

# Exercise 3: Van & Trailer

- FRONT WHEEL DRIVEN VAN:
- Non-slip condition of the van

$$F_{xf} \leq \mu W_f$$




- Tractive force

$$F_{xf} = F_{RR} + W\sin(\theta) + F_{hx}$$

$$F_{hx} - W_b \sin\theta - F_{RRt} = 0$$

- It comes

$$\begin{aligned} F_{xf} &= F_{RR} + W\sin(\theta) + F_{RRt} + W_b \sin\theta \\ &= (W + W_b)\ \sin\theta + (F_{RR} + F_{RRt}) \\ &= (W + W_b)\ \sin\theta + f_{RR}\,(W + W_t)\ \cos\theta \qquad W_t \simeq W_b \end{aligned}$$

# Exercise 3: Van & Trailer

- Reaction under front wheels

$$W_f = W\cos\theta\frac{c}{L} - W\sin\theta\frac{h_1}{L} - F_{hx}\frac{h_2}{L} - F_{hz}\frac{d}{L}$$

$$F_{hx} = W_b\sin\theta + F_{RRt}$$

$$F_{hz} = W_b\cos\theta\,\frac{f}{L_t} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t} + F_{RRt}\frac{h_2}{L_t}$$

- Let's neglect the influence of the rolling resistance $F_{RRt}$ which is a second order term. It comes

$$W_f = W\cos\theta\frac{c}{L} - W\sin\theta\frac{h_1}{L} - W_b\sin\theta\frac{h_2}{L}$$
$$- (W_b\cos\theta\,\frac{f}{L_t} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t})\frac{d}{L}$$

# Exercise 3: Van & Trailer

- Let's insert into the non-slip condition $F_{xf} \leq \mu\, W_f$

$$
\begin{aligned}
(W + W_b)\ \sin\theta + f_{RR}\,(W + W_t)\ \cos\theta \\
\leq\ \mu\ \{W \cos\theta \frac{c}{L} - W \sin\theta \frac{h_1}{L} - W_b \sin\theta \frac{h_2}{L} \\
- W_b \cos\theta\ \frac{f}{L_t} \frac{d}{L} + W_b \sin\theta\ \frac{h_3 - h_2}{L_t} \frac{d}{L}\}
\end{aligned}
$$

- This yields

$$
\begin{aligned}
\sin\theta\ \left((W + W_b) + \mu W \frac{h_1}{L} + \mu W_b \frac{h_2}{L} - \mu\, W_b \frac{h_3 - h_2}{L_t} \frac{d}{L}\right) \\
\leq \cos\theta\ \left(\mu W \frac{c}{L} - \mu W_b \frac{f}{L_t} \frac{d}{L} - f_{RR}(W + W_t)\right)
\end{aligned}
$$

# Exercise 3: Van & Trailer

- We get

$$\tan\theta \;\leq\; \frac{\mu W\,\frac{c}{L} - \mu W_b\,\frac{f}{L_t}\,\frac{d}{L} - f_{RR}(W + W_t)}{(W + W_b) + \mu W\,\frac{h_1}{L} + \mu W_b\,\frac{h_2}{L} - \mu W_b\,\frac{h_3 - h_2}{L_t}\,\frac{d}{L}}$$

- If one denotes by $\zeta$ the ratio of the mass of the trailer and of the vehicle

$$\zeta = \frac{W_b}{W}$$

- To find

$$\tan\theta \;\leq\; \frac{\mu\,\frac{c}{L} - \zeta\,\mu\,\frac{f}{L_t}\,\frac{d}{L} - f_{RR}(1+\zeta)}{1 + \mu\,\frac{h_1}{L} + \zeta\left[1 + \mu\,\frac{h_2}{L} - \mu\,\frac{h_3 - h_2}{L_t}\,\frac{d}{L}\right]}$$

# Exercise 3: Van & Trailer

- Numerical application

$$W_f = 760\ kg \qquad\qquad W_r = 573\ kg$$

$$W = W_f + W_r = 760 + 573 = 1333\ kg$$

$$W_f = \frac{c}{L} W \qquad c = L\,\frac{W_f}{W} = 3000 \cdot \frac{760}{1333} = 3000 \cdot 0.57 = 1710\ mm$$

$$W_r = \frac{b}{L} W \qquad b = L\,\frac{W_r}{W} = 3000 \cdot \frac{573}{1333} = 3000 \cdot 0.43 = 1270\ mm$$

# Exercise 3: Van & Trailer

- Numerical application

$$h = h_1 = 610\ mm$$

$$\frac{h}{L} = \frac{610}{3000} = 0,2033$$

$$d = d_h = 575\ mm$$

$$\frac{d}{L} = \frac{575}{3000} = 0.1916$$

$$\frac{d+L}{L} = \frac{575+3000}{3000} = 1.1916$$

$$h_2 = h_h = 350\ mm$$

$$L_t = 2750\ mm$$

$$h_3 = h_{CG,b} = 875\ mm$$

$$\frac{h_3 - h_2}{L_t} = \frac{875-350}{2750} = 0.1909$$

$$\frac{h_2}{L} = \frac{350}{3000} = 0.1167$$

$$\frac{h_2}{L_t} = \frac{350}{2750} = 0.1273$$

# Exercise 3: Van & Trailer

- Numerical application

$$W_t = 600\ kg \qquad F_{hz} = 125\ kg$$

$$W_b = W_t + F_{hz} = 600 + 125 = 725\ kg \qquad \zeta = \frac{m_b}{m} = \frac{725}{1333} = 0.5439$$

$$F_{hz} = W_b \frac{f}{L_t} + f_{RR}\, W_t \frac{h_2}{L_t} \simeq W_b \frac{f}{L_t} \qquad \Delta F_{hz} = f_{RR}\, W_t \frac{h_2}{L_t}$$

$$W_t = W_b \frac{e}{L_t} \qquad = 0.01 \cdot 600\, \frac{350}{2750}$$

$$= 0,7636\ kg$$

$$f = L_t \frac{F_{hz}}{W_b} = 2750 \cdot \frac{125}{725} = 474.1379\ mm \qquad \frac{f}{L_t} = 0.1724$$

$$e = L_t \frac{W_t}{W_b} = 2750 \cdot \frac{600}{725} = 2275.8620\ mm \qquad \frac{e}{L_t} = 0.8276$$

# Exercise 3: Van & Trailer

- Numerical application: max slope if μ=0.3 $f_{RR}$=0

$$\tan\theta \;\le\; \frac{\mu\frac{c}{L} - \zeta\,\mu\,\frac{f}{L_t}\,\frac{d}{L} - f_{RR}(1+\zeta)}{1+\mu\frac{h_1}{L}+\zeta\left[1+\mu\frac{h_2}{L}-\mu\frac{h_3-h_2}{L_t}\,\frac{d}{L}\right]}$$

$$\tan\theta \;\le\; \frac{0.3\cdot 0.57 - 0.5439\cdot 0.3\cdot 0.1724\cdot 0.1916}{1+0.3\cdot 0.2033+0.5439\,[1+0.3\cdot 0.1167-0.3\cdot 0.1909\cdot 0.1916]}$$

$$\tan\theta \le 0.102387$$

$$\theta \le 5.8442^\circ$$

# Exercise 3: Van & Trailer

- REAR WHEEL DRIVEN VAN
- Non-slip condition of the RWD van

$$F_{xf} \leq \mu\, W_f$$

- Tractive force

$$F_{xr} = (W + W_b)\ \sin\theta + f_{RR}(W + W_t)\cos\theta$$

- Reaction force under the rear axle of the van

$$W_r = W\cos\theta\frac{b}{L} + W\sin\theta\frac{h_1}{L} + F_{hx}\frac{h_2}{L} + F_{hz}\frac{L+d}{L}$$

$$F_{hx} = W_b\sin\theta + F_{RRt}$$

$$F_{hz} = W_b\cos\theta\ \frac{f}{L_t} - W_b\sin\theta\ \frac{h_3 - h_2}{L_t} + F_{RRt}\frac{h_2}{L_t}$$

# Exercise 3: Van & Trailer

- Reaction force under the rear axle of the van if we neglect the rolling resistance contribution

$$W_r = W\cos\theta\frac{b}{L} + W\sin\theta\frac{h_1}{L} + W_b\sin\theta\frac{h_2}{L}$$
$$+ (W_b\cos\theta\,\frac{f}{L_t} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t})\frac{d+L}{L}$$

- Nonslip condition writes

$$(W + W_b)\,\sin\theta + f_{RR}\,(W + W_t)\,\cos\theta$$
$$\leq \;\mu\,\Big[W\cos\theta\frac{b}{L} + W\sin\theta\frac{h_1}{L} + W_b\sin\theta\frac{h_2}{L}$$
$$+ W_b\cos\theta\,\frac{f}{L_t}\frac{d+L}{L} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t}\frac{d+L}{L}\Big]$$

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- Nonslip condition writes

$$\sin\theta \left( (W + W_b) - \mu W \frac{h_1}{L} - \mu W_b \frac{h_2}{L} + \mu W_b \frac{h_3 - h_2}{L_t} \frac{d+L}{L} \right)$$
$$\leq \cos\theta \left( \mu W \frac{b}{L} + \mu W_b \frac{f}{L_t} \frac{d+L}{L} - f_{RR}(W + W_t) \right)$$

- Which gives

$$\tan\theta \leq \frac{\mu W \frac{b}{L} + \mu W_b \frac{f}{L_t} \frac{d+L}{L} - f_{RR}(W + W_t)}{(W + W_b) - \mu W \frac{h_1}{L} - \mu W_b \frac{h_2}{L} + \mu W_b \frac{h_3 - h_2}{L_t} \frac{d+L}{L}}$$

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- Finally, the nonslip condition writes

$$\tan\theta \;\leq\; \frac{\mu\frac{b}{L} + \zeta\,\mu\,\frac{f}{L_t}\frac{d}{L} - f_{RR}(1+\zeta)}{1 - \mu\frac{h_1}{L} + \zeta\left[1 - \mu\frac{h_2}{L} + \mu\,\frac{h_3-h_2}{L_t}\frac{d+L}{L}\right]} \qquad \zeta = \frac{W_b}{W}$$

- Numerical application: max slope if μ=0.3 $f_{RR}$=0

$$\tan\theta \;\leq\; \frac{0.3\cdot 0.43 - 0.5439\cdot 0.3\cdot 0.1724\cdot 1.1916}{1 - 0.3\cdot 0.2033 + 0.5439\,[1 - 0.3\cdot 0.1167 + 0.3\cdot 0.1909\cdot 1.1916]}$$

$$\tan\theta \leq 0.10827$$

$$\theta \leq 6,1796^\circ$$

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- For ALL WHEEL DRIVE (AWD) van, we get

$$F_{xf} + F_{xr} \leq \mu \left( W_f + W_r \right)$$

- Tractive forces (see FWD)

$$F_{xf} + F_{xr} = (W + W_b)\ \sin\theta + f_{RR}(W + W_t)\cos\theta$$

- Reaction under front and rear axles

$$W_f = W\cos\theta\frac{c}{L} - W\sin\theta\frac{h_1}{L} - F_{hx}\frac{h_2}{L} - F_{hz}\frac{d}{L}$$

$$W_r = W\cos\theta\frac{b}{L} + W\sin\theta\frac{h_1}{L} + F_{hx}\frac{h_2}{L} + F_{hz}\frac{L+d}{L}$$

$$W_f + W_r = W\cos\theta\frac{b+c}{L} + F_{hz}\frac{L+d-d}{L}$$

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- The reaction force under the van axles

$$W_f + W_r = W\cos\theta + F_{hz}$$

$$F_{hz} = W_b\cos\theta\,\frac{f}{L_t} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t}$$

- We get

$$W_f + W_r = W\cos\theta + W_b\cos\theta\,\frac{f}{L_t} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t}$$

- The non-slip conditions writes

$$(W + W_b)\ \sin\theta + f_{RR}(W + W_t)\cos\theta \leq \mu\left[W\cos\theta + W_b\cos\theta\,\frac{f}{L_t} - W_b\sin\theta\,\frac{h_3 - h_2}{L_t}\right]$$

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- The non-slip conditions writes

$$\left[(W + W_b) + \mu\, W_b\, \frac{h_3 - h_2}{L_t}\right] \sin\theta$$
$$\leq \left[\mu\, W + \mu\, W_b\, \frac{f}{L_t} - f_{RR}(W + W_t)\right] \cos\theta$$

$$\tan\theta \leq \frac{\mu W + \mu W_b \frac{f}{L_t} - f_{RR}(W + W_t)}{W + W_b + \mu W_b \frac{h_3 - h_2}{L_t}}$$

$$\tan\theta \leq \frac{\mu + \mu\zeta \frac{f}{L_t} - f_{RR}(1 + \zeta)}{1 + \zeta(1 + \mu \frac{h_3 - h_2}{L_t})}$$

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- Numerical application: max slope if μ=0.3 $f_{RR}$=0

$$\tan\theta \leq \frac{\mu + \mu\zeta\frac{f}{L_t} - f_{RR}(1+\zeta)}{1+\zeta(1+\mu\frac{h_3-h_2}{L_t})}$$

$$\tan\theta \leq \frac{0.3 + 0.3\cdot 0.5439 \cdot 0.1724}{1+0.5439(1+0.3\cdot 0.1909)} = 0.2083$$

$$\theta = 11.7681^\circ$$