Vehicle Performance

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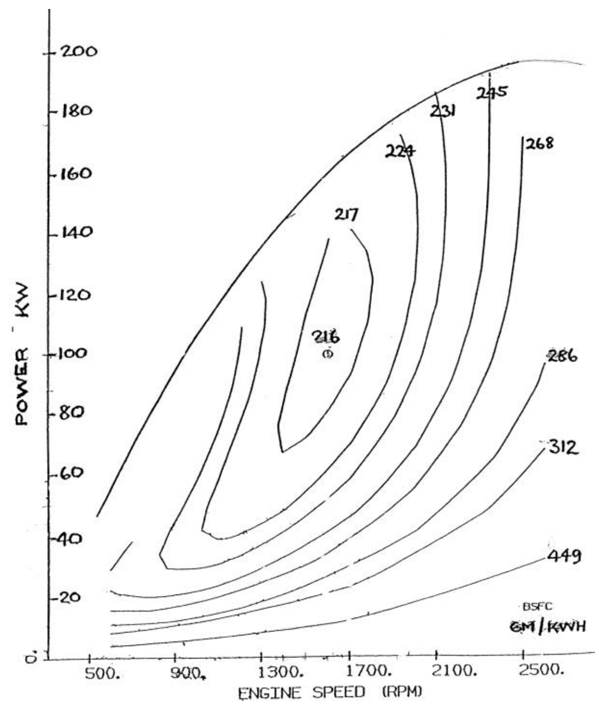
Exercise session:
Fuel consumption

Exercise 1 : Fuel consumption of a truck

- The power curve and the specific fuel consumption map of a truck are given in Fig. 1.
- The truck characteristics are the following
  - Mass: m = 25.000 kg
  - Frontal area: S=8 m²
  - Drag coefficient: C_x = 0,65
  - Rolling Resistance coefficient: f = 0,006
  - Effective rolling radius: R_e= 0,57 m
  - Final drive gear ratio: i_d=4,5
  - Air density: ρ_a= 1,22 kg/m³
  - Transmission efficiency η=0,9
  - Fuel density : ρ_f= 0,8 kg/l
  - Gravity acceleration: g=9,81 m/s²



Exercise 1 : Fuel consumption of a truck

Questions

1. Compute the top speed of the truck on a level road and without wind.
2. The truck speed is 80 km/h on a level road but with a front wind of 20 km/h. Compute the gear reduction ratio required to have the lowest fuel consumption. Express the fuel consumption in L/100km.
3. The same truck drives down a street at 1% with a speed of 100 km/h without wind. Compute the fuel consumption and express it in L/100km if one keeps the same gear ratio as in point 2.

Max Top Speed

- Max top speed is solution

$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

$$A = m\, g\, f_0 \cos\theta + m\, g\, \sin\theta$$
$$B = 1/2\, \rho\, S\, C_x$$

- From power diagram

$$\mathcal{P}_{max} = 190\, kW$$

- It comes

$$\eta \mathcal{P}_{max} = 0,90 \times 190.000 = 171.000\, W$$

$$A = 25000 \times 9,81 \times 0,006 \times 1. = 1471,5\, [N]$$
$$B = 0,5 \times 1,22 \times 8,0 \times 0,65 = 3,1720\, [N/(m/s)^2]$$

Max Top Speed

- Max top speed is solution

$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

- Solution using Picard iterative scheme

$$\begin{aligned} v^{(0)} &= 0 \\ v^{(n+1)} &= \left(\frac{\eta \mathcal{P}_{max} - Av^{(n)}}{B} \right)^{1/3} \end{aligned}$$

- It comes

$$\begin{aligned} v^{(0)} &= 0,0 \\ v^{(1)} &= 37,7764 \,\mathrm{m/s} \\ v^{(2)} &= 33,1364 \,\mathrm{m/s} \\ v^{(3)} &= 33,7774 \,\mathrm{m/s} \\ v^{(4)} &= 33,6903 \,\mathrm{m/s} \\ v^{(5)} &= 33,7022 \,\mathrm{m/s} \end{aligned}$$

$$\begin{aligned} v_{max}^{max} &= 33,7022 \,\mathrm{m/s} \\ &= 121,32 \,\mathrm{km/h} \end{aligned}$$

Optimized gear reduction ratio

- The truck is driving on a level road at 80 km/h with a front wind of 20 km/h.
- Compute first the road resistance at v=80/3,6 = 22,22 m/s:

$$
\begin{aligned}
F_{AERO} &= \frac{1}{2}\rho_a S C_x (V + V_{Wind})^2 \\
&= \frac{1}{2}\ 1,22\ 8,0\ 0,65\ (\frac{80+20}{3,6})^2 = 2447,53\ N
\end{aligned}
$$

$$
\begin{aligned}
F_{RR} &= m g f_0 \cos\theta \\
&= 25000\ 9,81\ 0,006\ \ \cos 0^\circ = 1471,50\ N
\end{aligned}
$$

$$F_{RES} = F_{AERO} + F_{RR} = 2447,53 + 1471,50 = 3981,03\ N$$

$$\mathcal{P}_{RES} = F_{RES}\, v = 3981,03 \cdot 22,22 = 87.089,57\ W$$

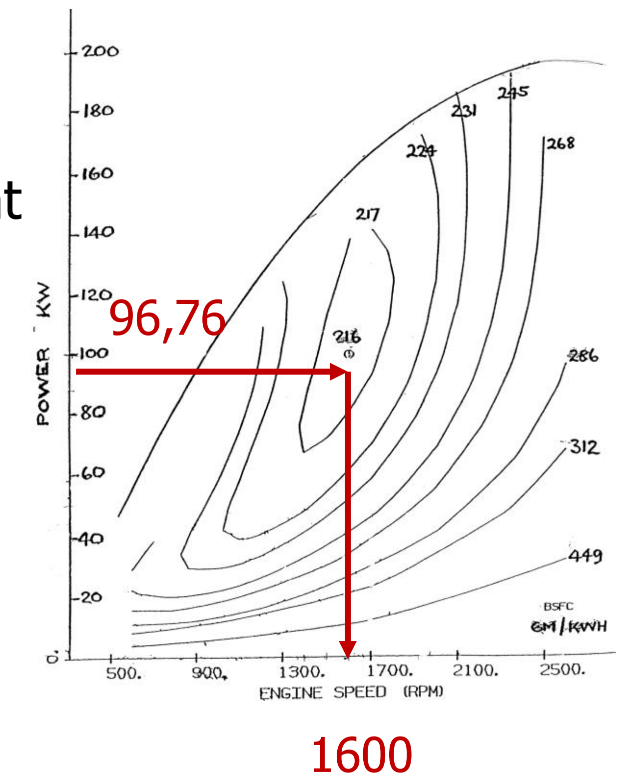
Optimized gear reduction ratio

- Requested engine power to face the road resistance

$$\mathcal{P}_m = \frac{\mathcal{P}_{RES}}{\eta_t} = \frac{87089,57}{0,9} = 96.766,66\ W$$

- Bsfc map shows that best efficient point for 96,76 kW is close to the best efficient of the engine that is 216 g/kWh at ω=1600 rpm

$$\text{bsfc} = 216\ g/kWh \quad @\ \omega_m = 1600\ rpm$$



Optimized gear reduction ratio

- The related fuel consumption is

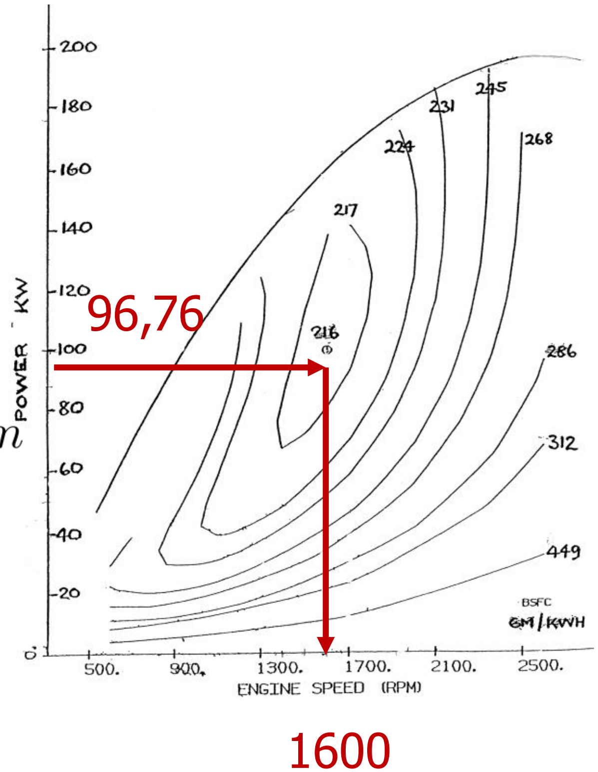
$$B = \text{bsfc}\ \mathcal{P}_m = 216 \cdot 96.7661 = 20.901\ g/h$$

  - In liter/h

$$B = 20,901\ kg/h = \frac{20,901}{\rho_{fuel}}\ l/h = 26,12\ l/h$$

  - In liter / 100 km

$$B = 26,12\ l/h = 26,12\ \frac{100}{v} = 32,66\ l/100\ km$$



Optimized gear reduction ratio

- To calculate the reduction, first compute the wheel rotation speed

$$\omega_w = \frac{v}{R_e} = \frac{22,22}{0,57} = 38,98\ rad/s$$

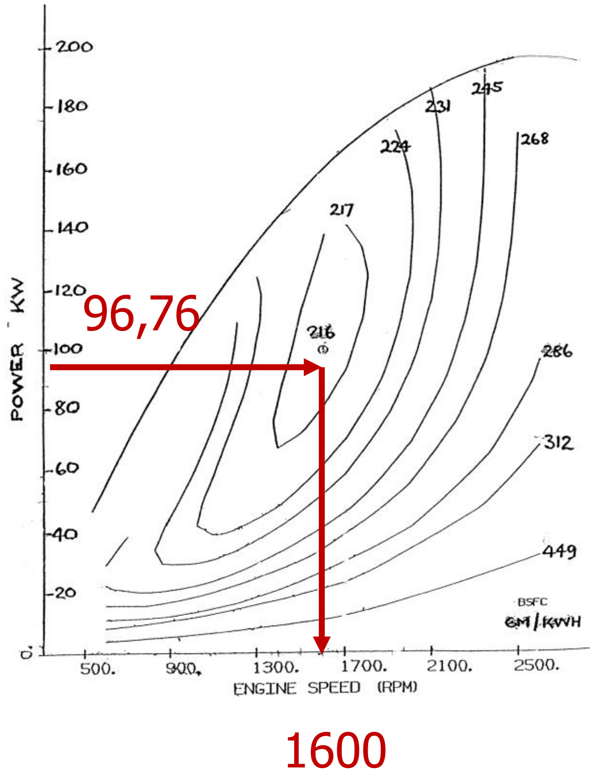
- The engine rotation speed is known

$$\omega_m = 1600\,\frac{2\,\pi}{60} = 167,55\ rad/s$$

- Then the reduction ratio is

$$i_t = i_d\ i_g = \frac{\omega_m}{\omega_w} = \frac{167,55}{38,98} = 4,2981$$

$$i_g = i_t / i_d = 4,2981/4,5 = 0,9551$$



Fuel consumption in given driving conditions

- The truck is driving downhill (slope 1%) at 100 km/h

$$v = 100/3,6 = 27,77 \ m/s$$

$$\theta = \arctan 0,01 = 0,5729^\circ$$

- First compute the power developed by the engine

$$F_{AERO} = \frac{1}{2} \cdot 1,22 \cdot 8,0 \cdot 0,65 \cdot (27,77)^2 = 2447,53 \ N$$

$$F_{RR} = 25000 \cdot 9,81 \cdot 0,006 \cdot \cos(-0,5729^\circ) = 1471,42 \ N$$

$$F_{GRADE} = 25000 \cdot 9,81 \cdot \sin(-0,5729^\circ) = -2452,21 \ N$$

$$\begin{aligned} F_{RES} &= F_{AERO} + F_{RR} + F_{GRADE} \\ &= 2447,53 + 1471,50 - 2452,21 = 1466,74 \ N \end{aligned}$$

$$\mathcal{P}_{RES} = F_{RES} \, v = 1466,74 \cdot 27,77 = 40,742 \ kW$$

Fuel consumption in given driving conditions

- Requested engine power

$$\mathcal{P}_m = \frac{\mathcal{P}_{RES}}{\eta_t} = \frac{40,742}{0,9} = 45,2699 \; kW$$

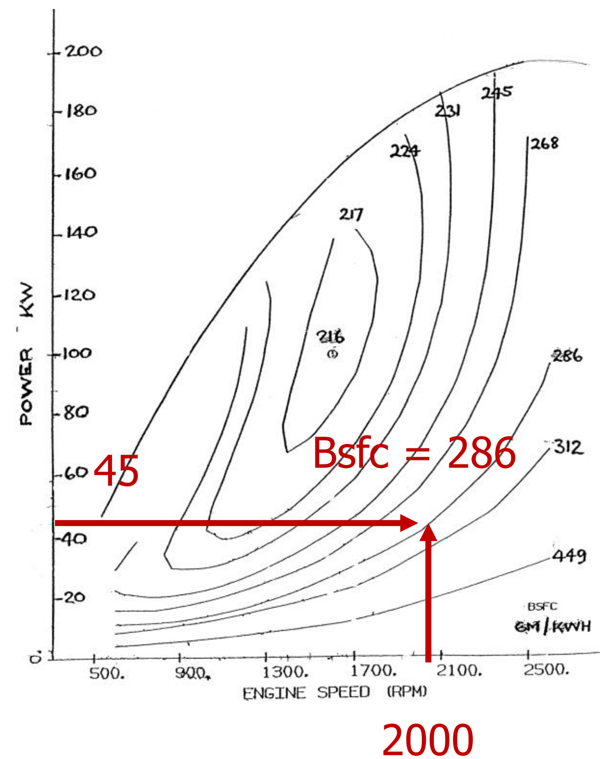
- Engine rotation speed

$$\omega_m = \frac{v}{R_e}\, i = \frac{27,77}{0,57}\, 4,981 = 209,40 \; rad/s$$

$$= 209,40 \cdot \frac{60}{2\,\pi} = 1999,62 \; rpm$$

- On engine maps, one reads

$$\text{bsfc}(\omega_m = 2000, \mathcal{P}_m = 45) = 286 \; g/kWh$$



Fuel consumption in given driving conditions

- The fuel consumption of the truck is

$$B = \text{bsfc}\,\mathcal{P}_m = 286 \cdot 45,28 = 12,950\ kg/h$$

$$B = 12,950\ kg/h\ \frac{1}{\rho_{fuel}}\ \frac{100\,km/h}{V}$$
$$= 12,950\ \frac{1}{0,8}\ \frac{100}{100} = 16,18\ l/100\,km$$

Exercise 2 : Fuel consumption of a SUV

- Let's consider the Volvo XC90 T6. Data about the vehicle can be found on the Volvo Cars website:
  - Maximum motor power: 200 kW @ 5100 rpm
  - Top speed: 210 km/h
  - m = 2131 kg
  - Wheelbase L= 2.857 m
  - C_x = 0.48 and S = 3 m².
  - Effective rolling radius R_e = 0,40 m
  - ρ_{fuel}= 0.734 kg/l

- The following parameters are estimated
  - Rolling resistance coefficient: f = 12 10^{-3}
  - Air density : ρ = 1,206 kg/m³

Exercise 2 : Fuel consumption of a SUV

- 1/ If we assume that the top speed is indeed the one announced by the manufacturer (210 km/h) and that the maximum speed is reached at maximum power, what is the efficiency of the transmission line?

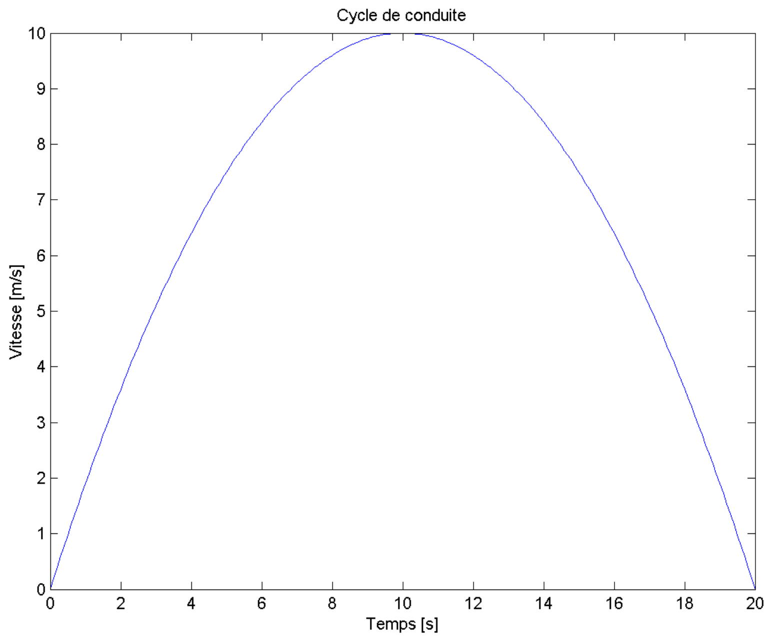
- 2/ What is the reduction ratio of the transmission in the gear for which we have this maximum speed (we will assume that it is the 5th gear of the transmission)?

- 3/ If we now measure a longitudinal slip rate of s_L=0.1 at the moment we transmit the maximum power to the road, what is the effective reduction ratio?

Exercise 2 : Fuel consumption of a SUV

- 4/ To evaluate the fuel consumption of the Volvo XC90 T6, we imagine a new, very simple driving cycle as shown in Fig. 1. The speed over time has the following analytical expression:

$$v(t) = 10. - 0.1 \cdot (t - 10)^2$$



Exercise 2 : Fuel consumption of a SUV

- The vehicle parameters are the same as those given in question 1, with the addition of the following data:
  - Efficiency of the transmission line: η_t= 0.9
  - Specific consumption of the engine: bsfc= 0.350 kg/kWh (assumed to remain constant)
  - Assume that the test is performed with an overall gear ratio i=10 and that the effective mass is given Wong's empirical formula

$$m_e = m\,(1.04 + 0.0025\,i^2)$$

Exercise 2 : Fuel consumption of a SUV

Question 1: estimate the efficiency of the driveline

- The engine power

$$\mathcal{P}_e = 200\ kW$$

- The speed

$$v = 210\ km/h = 210/3.6\ = 58.33\ m/s$$

- The road resistance power

$$\begin{aligned}\mathcal{P}_{RES} &= \frac{1}{2}\rho_a\, S\, C_x\, v^3\ +\ f_{RR}\, m\, g \\ &= \frac{1}{2}\cdot 1.206 \cdot 3 \cdot 0.48 \cdot (58.33)^3 + 0.012 \cdot 2131 \cdot 9.81 \cdot 58.33 \\ &= 172327.95\ +\ 14632.74\ W \\ &= 186.9607\ kW\end{aligned}$$

Exercise 2 : Fuel consumption of a SUV

- The transmission efficiency is the ratio

$$\eta_t = \frac{\mathcal{P}_{RES}}{\mathcal{P}_e} = \frac{186.9607}{200.0000} = 0.935$$

Exercise 2 : Fuel consumption of a SUV

Question 2: reduction ratio of the transmission in the gear for which we have the maximum speed

- The speed

$$v = 58.33 \ m/s$$

- The rolling radius

$$R_e = 0.4 \ m$$

- The rotation speed of the wheels

$$\omega_w = \frac{v}{R_e} = \frac{58.33}{0.4} = 145.825 \ rad/s$$

- The vehicle is supposed to reach the max speed with nominal engine rotation speed

$$\omega_e = \frac{5100 \cdot 2\pi}{60} = 534.71 \ rad/s$$

Exercise 2 : Fuel consumption of a SUV

- The reductio is given by

$$i = \frac{\omega_e}{\omega_w} = \frac{534.071}{145.825} = 3.66$$

Exercise 2 : Fuel consumption of a SUV

Question 3: the actual reduction ratio if we experience a longitudinal slip rate of s_L=0.1

- The longitudinal slip ratio definition

$$s_L = \frac{\omega_w R_e}{V} - 1$$

- The effective (actual) reduction ratio

$$i_{eff} = \frac{\omega_e}{\omega_w}$$

- It comes

$$s_L = \frac{\omega_e R_e}{i_{eff}\, V} - 1$$

Exercise 2 : Fuel consumption of a SUV

- If s_L=0.1, it comes

$$s_L = \frac{\omega_e R_e}{i_{eff}\, V} - 1 = 0.1$$

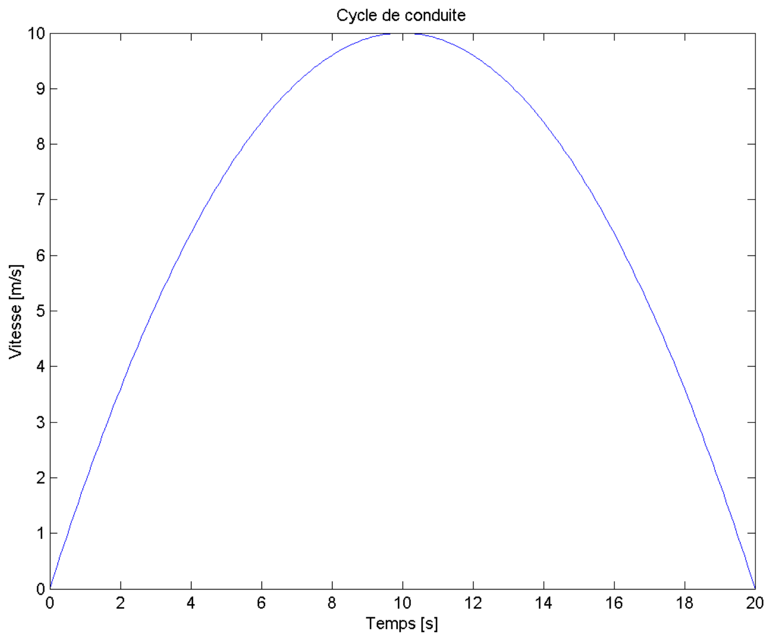
- We can draw

$$i_{eff} = \frac{\omega_e\, R_e}{(1+s_L)\, V} = \frac{i\,\omega_w\, R_e}{(1+s_L)\, V} = \frac{i\, V}{(1+s_L)\, V}$$
$$= \frac{i}{1+s_L} = 3.29$$

Exercise 2 : Fuel consumption of a SUV

<u>Question 4: evaluate the fuel consumption of the Volvo XC90 T6 over the simplified driving cycle</u>

$$v(t) = 10. \; - \; 0.1 \cdot (t - 10)^2$$



Exercise 2 : Fuel consumption of a SUV

- The power to be developed at wheels is given by

$$\mathcal{P}_w = (F_{RES} + m_e \frac{dv}{dt})\, v(t) = \frac{1}{2}\rho_a\, S\, C_x\, v^3 + m\, g\, f\, v + m_e \frac{dv}{dt}\, v(t)$$

- The fuel consumption over a variables speed driving cycle

$$B = \frac{\int_0^T \dot{b}\, dt}{\int_0^T v\, dt}$$

- With

$$\dot{b} = \frac{\mathrm{bsfc}\, \mathcal{P}_e}{\rho_{fuel}}$$

- and

$$\mathcal{P}_e = \frac{\mathcal{P}_w}{\eta_t}$$

Exercise 2 : Fuel consumption of a SUV

- It comes

$$\mathcal{P}_e = \frac{\mathcal{P}_w}{\eta_t}$$

- We can draw

$$B = \frac{\frac{\text{bsfc}}{\eta_t\ \rho_{fuel}} \int_0^T \left[1/2\rho S C_x v^3(t) + mgfv(t) + m_e\ \dot{v}(t)\ v(t)\right]_+\ dt}{\int_0^T\ v\ dt}$$

- We have to consider only tractive power $[]_+$, because in braking phases, power becomes negative, and this would mean that we could flow back energy to the tank!

Exercise 2 : Fuel consumption of a SUV

- We have to adapt the units, it comes

$$B = \frac{\frac{\text{bsfc}}{\eta_t\ \rho_{fuel}} \int_0^T \frac{1}{1000 \cdot 3600} \left[1/2 \rho S C_x v^3(t) + mgf v(t) + m_e \dot{v}(t)\, v(t)\,\right]_+ \, dt}{\left(\int_0^T v\, dt\right) \frac{1}{100000}}$$

- We can now proceed to the numerical evaluation of the integral

$$A = mgf = 0.012 \cdot\ 2131 \cdot 9.81 = 250.8613\ [N]$$

$$B = \frac{1}{2} \rho_a\, S\, C_x = 0.5 \cdot 1.206 \cdot 3.0 \cdot 0.48 = 0.8683\ [N/(m/s)^2]$$

$$\frac{\text{bsfc}}{\eta_t\ \rho_{fuel}} = \frac{0.350}{0.9 \cdot 0.734} = 0.5298\ [l/kWh]$$

$$\begin{aligned} m_e = m\,(1.04 + 0.0025\, i^2) &= 2131 \cdot (1.04 + 0.0025\, 10^2) \\ &= 1.29\, 2131 = 2748.99\, kg \end{aligned}$$

Exercise 2 : Fuel consumption of a SUV

- The driving cycle

$$v(t) = 10. \; - \; 0.1 \cdot (t - 10)^2$$

$$\dot{v}(t) = - \, 0.2 \cdot (t - 10)$$

Exercise 2 : Fuel consumption of a SUV

- Finally, it comes

$$B = \frac{\int_0^T \dot{b}\,dt}{\int_0^T v\,dt} = 17.56\ l/100\ km$$

- If we would have not discarded the negative power during the braking phase, one would get a very different fuel consumption estimation

$$B = \frac{\int_0^T \dot{b}\,dt}{\int_0^T v\,dt} = 4.5684\ l/100\ km\ !!!$$

Exercise 2 : Fuel consumption of a SUV

- How to understand the result?

- Let's make the assumption that we have the average speed t=5 s

$$v(t) = 10. \; - \; 0.1 \cdot (t - 10)^2$$

$$\dot{v}(t) = - \, 0.2 \cdot (t - 10)$$

$$t = 0 \qquad v(t) = 10 - 0.1(0 - 10)^2 = 10 - 100/10 = 0 \; m/s$$

$$\dot{v}(t) = -0.2(0 - 10) = 2 \; m/s^2$$

$$t = 10 \qquad v(t) = 10 - 0.1(10 - 10)^2 = 10 - 0/10 = 10 \; m/s$$

$$\dot{v}(t) = -0.2(10 - 10) = 0 \; m/s^2$$

$$t = 5 \qquad v(t) = 10 - 0.1(5 - 10)^2 = 10 - 25/10 = 7.5 \; m/s$$

$$\dot{v}(t) = -0.2(5 - 10) = 1 \; m/s^2$$

Exercise 2 : Fuel consumption of a SUV

- How to understand the result?

- Let's make the assumption that we have the average speed t=5 s

$$F_{RES} = A + B\,v^2 = 250.8613 + 0.8685\,(7.5)^2 = 299.7031\ N$$

$$\mathcal{P}_{RES} = F_{RES} \cdot v = 299.7031 \cdot 7.5 = 2247.7738\ W$$

$$F_{inertia} = m_e\,\dot{v} = 2748.99\ 1.0 = 2748.99\ N$$

$$\mathcal{P}_{inertia} = F_{inertia} \cdot v = 2748.49 \cdot 7.5 = 20617.43\ W$$

Exercise 2 : Fuel consumption of a SUV

- How to understand the result?
- Let's make the assumption that we have the average speed t=5 s

$$\mathcal{P}_w = \mathcal{P}_{RES} + \mathcal{P}_{inertia} = 2247.77 + 20617.42 = 22865.19\ W$$

Exercise 2 : Fuel consumption of a SUV

- How to understand the result?
- Let's make the assumption that we have the average speed t=5 s

$$\int_0^{20} v(t)\,dt \simeq v(5)\cdot\Delta t = 7.5\ 20 = 150\ m$$

$$\begin{aligned}\int_0^{20} [\mathcal{P}_w(t)]_+\,dt &= \int_0^{20} [\mathcal{P}_w(t)]_+\,dt + \int_0^{20} [\mathcal{P}_w(t)]_+\,dt \\ &\simeq [\mathcal{P}_w(t=5s)]_+\cdot 10 + [\mathcal{P}_w(t=15s)]_+\cdot 10 \\ &\simeq 22865.19\cdot 10 + 0.\cdot 10 = 228651.95\ J\end{aligned}$$

Exercise 2 : Fuel consumption of a SUV

- How to understand the result?
- Let's make the assumption that we have the average speed t=5 s

$$B = \frac{\frac{\text{bsfc}}{\eta_t \, \rho_{fuel}} \int_0^T \frac{1}{1000 \cdot 3600} \left[1/2 \rho S C_x v^3(t) + mgfv(t) + m_e \dot{v}(t) \, v(t)\right]_+ dt}{(\int_0^T v \, dt) \frac{1}{100000}}$$

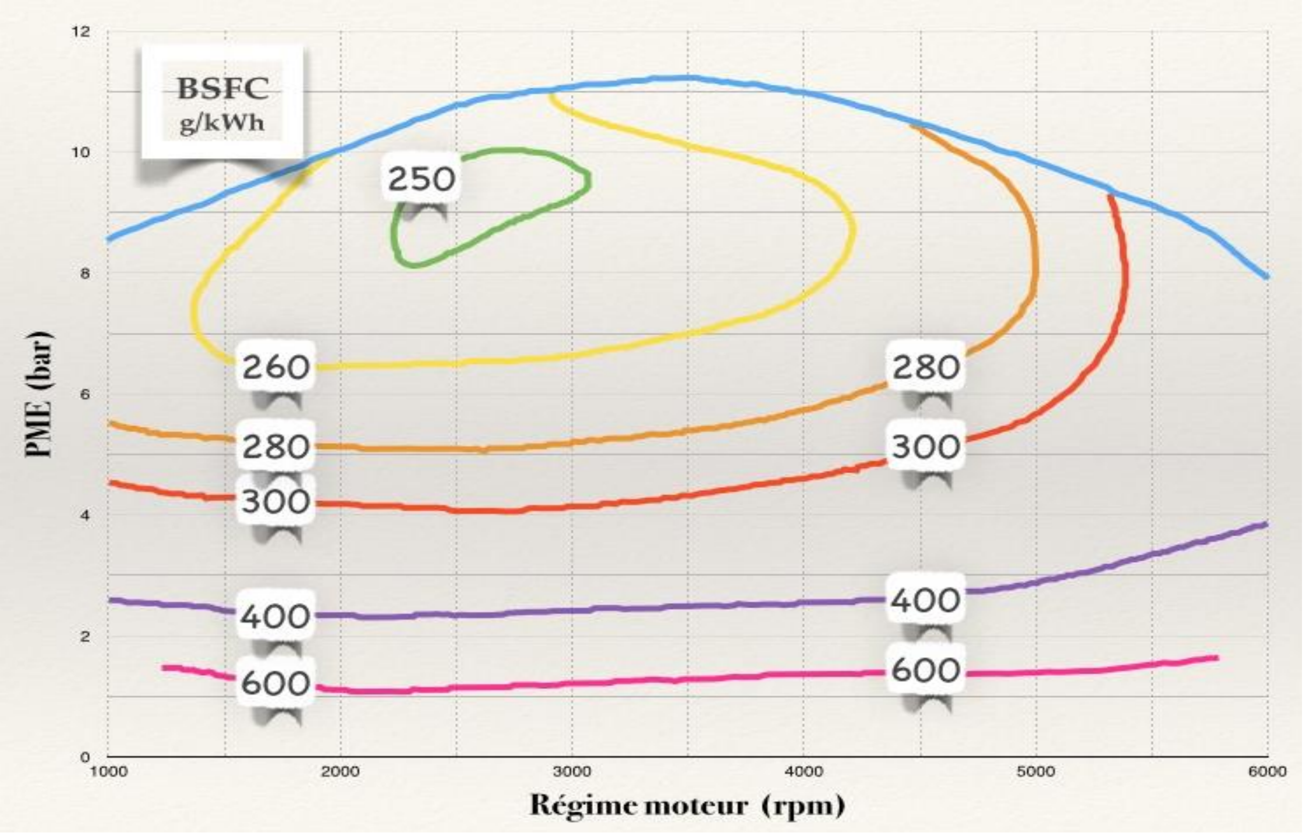
$$\simeq \frac{\frac{\text{bsfc}}{\eta_t \, \rho_{fuel}} \frac{1}{1000 \cdot 3600} \mathcal{P}_w \, \Delta t}{\frac{\bar{v} \, \Delta t}{100000}} = \frac{0.5298 \, \frac{228651.95}{1000 \cdot 3600}}{\frac{150}{100000}} = 22.4332 \frac{l}{100 \, km}$$

Exercise 3: Fuel consumption of a car

- Let’s consider a vehicle that exhibit the following characteristics
  - Four-stroke engine with spark ignition (SI)
  - Displacement V_H= 2.0 l
  - Mass: m = 1050 kg
  - Aerodynamic drag: $S*C_X$= 0.855 m²
  - Coefficient of rolling resistance f = 0.02
  - Effective rolling diameter D_e= 0.6 m
  - Overall gear ratio of the fourth gear i_4= 3.5
  - Air density ρ_{air}= 1.2 kg/m³
  - Transmission efficiency η_t= 0.8
  - Fuel density: ρ_{fuel}= 0.745 kg/l
  - Acceleration of gravity: g = 9.81 m/s².

Exercise 3: Fuel consumption of a car

- We have the following engine map



Exercise 3: Fuel consumption of a car

QUESTIONS

- 1/ Assuming the vehicle is travelling on a level road, at constant speed and in zero wind, calculate the forward drag on the fourth gear ratio and for engine speeds of 1000, 2000, 3000, 4000, 5000 and 6000 rpm. Plot the corresponding points on the engine map.

- 2/ Plot the road resistance curve corresponding to the fourth gear. Determine the maximum speed reached in fourth gear under the given conditions.

- 3/ Determine the overall gear ratio to be given in the fifth gear so that the speed reached at 4000 rpm in the fourth gear is obtained with a specific fuel consumption reduction of 10%. Calculate the consumption obtained in l/100km.

Exercise 3: Fuel consumption of a car

QUESTIONS

- 4/ Driving at maximum speed in the fourth gear, the driver shifts into fifth gear (the speed of the vehicle during the gear change is assumed to remain constant). Immediately after the gear change, the driver continues to hold the accelerator pedal fully depressed. Represent the following operating points on the engine map
  - P1: directly after the gear change
  - P2: stationary state in fifth

- 5/ Determine the maximum slope angle that can be climbed at a speed of 120 km/h in fifths in zero wind.