MECA0527: PERFORMANCE OF ELECTRIC VEHICLES

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Academic Year 2019-2020
References


• Le véhicule électrique. Educauto. www.educauto.org
Performances of Electric Vehicles

- Vehicle driving performance is assessed by
  - Acceleration time
  - Maximum speed
  - Gradeability

- In EV drivetrain design: motor power rating and transmission parameters are selected to meet the performance specifications

- They depend mostly on speed-torque characteristics of the traction motor
Traction motor characteristics

- At low speed: constant torque
  - Voltage supply increases with rotation speed through electronic converter while flux is kept constant

- At high speed: constant power
  - Motor voltage is kept constant while flux is weakened, reduced hyperbolically with the rotation speed

- Base speed: transition speed from constant torque to constant power regime
Traction motor characteristics

- Speed ratio \( X = \) ratio between maximum rotation speed to base speed
  \[
  X = \frac{\omega_{max}}{\omega_B}
  \]
- \( X \sim 2 \) Permanent Magnet motors
- \( X \sim 4 \) Induction motors
- \( X \sim 6 \) Switched Reluctance motors

For a given power, a long constant power region (large \( X \)) gives rise to an important constant torque, and so high vehicle acceleration and gradeability. Thus the transmission can be simplified.
Tractive efforts and transmission requirement

- Remind traction effort and vehicle speed

\[ F_t = \eta_t \frac{C_m i}{R_e} \quad \text{and} \quad \frac{v}{i} = \frac{\omega_m R_e}{i} \]

- The use of multi-gear or single gear transmission depends on the motor speed-torque characteristics.
  - For a given rated power, a long constant power region makes possible to use a single gear transmission, because of high tractive efforts at low speeds.
  - For long constant torque and a given rated power, the available maximum torque is sometimes not sufficient so that a multi gear is generally preferred.
Tractive efforts and transmission requirement

- For a low $x$ ($x=2$) motor, tractive effort is not large enough and 3-gear transmission may be chosen.
- For intermediate $x=4$, a two gear transmission should be preferred.
- For a large $x=6$, a single gear transmission is possible.
- The 3 designs have the same Tractive Force / speed profiles, and so the same acceleration and gradeability performances.
Tractive efforts and transmission requirement

![Graph showing tractive efforts and transmission requirements](image)

- a–b–c: 1st gear operation region
- d–e–f: 2nd gear operation region
- Motor speed ratio: $x = 4$

![Graph showing tractive efforts and transmission requirements](image)

- Single-gear transmission
- Motor speed ratio: $x = 6$

- Rolling resistance + aerodynamic drag
- Tractive effort
- Maximum speed

$V_b$: Vehicle speed

$F_r$: Rolling resistance

$F_d$: Aerodynamic drag

$F_t$: Tractive force

$F_w$: Wind force
Max speed can be evaluated by calculating the intersection between the tractive force curve and the resistance curve or alternatively the tractive power (constant) and the resistance forces power.

\[ \eta_t P_{m}^{max} = A V_{max} + B V_{max}^3 \]

Sometimes the intersection does not exist because it is over the maximum rotation speed of the motor

\[ V_{max} = \frac{\pi N_{m}^{max} R_e}{30 i} \]
Gradeability of EV

- Gradeability is ruled by the net tractive force available
  \[ F_{net}^t = F_t - F_{RR} - F_{aero} \]
  \[ = F_t - mgf \cos \theta - 0.5 \rho SC_x V^2 \]

- The maximum grade that can be overcome at a given speed is:
  \[ \sin \theta = \frac{F_t - F_{RR} - F_{aero}}{mg} = \frac{F_{net}^t}{mg} \]

- At low speed:
  \[ \sin \theta = \frac{d - f \sqrt{1 + f^2 - d^2}}{1 + f^2} \]
  \[ d = (F_t - F_{Aero})/mg \]
EV acceleration

- Acceleration can be evaluated by the time to accelerate from a given low speed (often zero) to a given high speed (e.g. 100 km/h).
- Acceleration performance is often more important for drivers than max speed and gradeability.
- Acceleration performance dictates the power rating of the motor.
EV acceleration

- Newton equation

\[ F_t - \sum F_{res} = F_{net} = m \frac{dV}{dt} \]

- Time to accelerate from 0 to \( V_f \):

\[ \Delta t_{0 \rightarrow V_f} = \int_0^{V_F} \frac{m_e}{F_{net}(v)} \, dv = \int_0^{V_F} \frac{m_e}{F_t(v) - mg - 1/2 \rho SC_x v^2} \, dv \]
EV acceleration

- Tractive force expression of EV
  \[ F_t = \eta_t \frac{C_m i}{R_e} \]

- Under the base speed
  \[ \omega = [0, \omega_B] \quad C = C^{max} \]
  \[ C = C^{max} = \frac{P^{max}}{\omega_B} \]
  \[ F_t = \eta_t \frac{P^{max} i}{\omega_B R_e} = \eta_t \frac{P^{max}}{v_B} = \frac{P_t^{max}}{v_B} \text{Cste} \]
  \[ v_B = \frac{\omega_B R_e}{i} \quad P_t^{max} = \eta_t P^{max} \]
Ev acceleration

- Tractive force expression of EV

\[ F_t = \eta_t \frac{C_m i}{R_e} \]

- Over the base speed

\[ \omega = [\omega_B, \omega_{max}] \quad P = P_{max} \]

\[ C = \frac{P_{max}}{\omega_m} \]

\[ F_t = \eta_t \frac{P_{max} i}{\omega_m R_e} = \eta_t \frac{P_{max}}{v} = \frac{P_{t \max}}{v} \text{Cste} \]

\[ v = \frac{\omega_B R_e}{i} \quad P_{t \max} = \eta_t P_{max} \]
EV acceleration

- Acceleration time can be calculated by the integral

\[
\begin{align*}
t_a &= \int_0^{V_b} \frac{\gamma m}{P_t/V_b - mgf - 0,5\rho C_x V^2} \, dV \\
&+ \int_{V_b}^{V_f} \frac{\gamma m}{P_t/V - mgf - 0,5\rho C_x V^2} \, dV
\end{align*}
\]

- Approximation solution: neglect the rolling and the drag resistances

\[
t_a = \frac{\gamma m}{2P_t} (V_f^2 + V_b^2)
\]
EV acceleration

- Sizing of rated power of electric motor

\[ P_{max} = \frac{\gamma m}{2 t_a \eta_t} (V_f^2 + V_b^2) \]

- However to determine more accurately the rated power, one needs to determine the power consumption of the resistance forces

\[ \bar{P}_{aver}^{Res} = \frac{1}{t_a} \int_0^{t_a} mg f V + 0,5 \rho SC_x V^3 \, dt \]
EV acceleration

- As the power is supposed to be constant in some part of the acceleration, one gets the kinematic relation for constant power force

\[ V(t) = V_f \sqrt{\frac{t}{t_a}} \]

- Inserting into the integral, it yields

\[ \bar{P}_{aver} = \frac{2}{3} m g f V_f + \frac{1}{5} \rho S C_x V_f^3 \]

- It comes the estimated power of the motor

\[ P_t = \frac{\gamma m}{2t_a} (V_f^2 + V_b^2) + \frac{2}{3} m g f V_f + \frac{1}{5} \rho S C_x V_f^3 \]
The result shows that for a given acceleration performance, lower vehicle base speeds will result in smaller motor power rating.

However, the power rating decline rate to the vehicle base speed reduction is not identical.

\[
\frac{dP_t}{dV_b} = \frac{\gamma m}{t_a} V_b
\]
EV acceleration

- Fine estimation of acceleration time requires solving exactly the Newton equation:

\[ t_a = \int_0^{V_f} \frac{\gamma m}{F_t - mg\beta - 0.5 \rho SC_x V^2} \, dV \]

- Parameters:
  - \( P_t = 63 \text{ kW, } x = 4 \)
  - \( M_v = 1200 \text{ kg, } f_r = 0.01 \)
  - \( C_D = 0.3, A_f = 2.0 \text{ m}^2 \)
Acceleration capacity and impedance adaptation

- 1D system with one electric motor connected to the mechanical load via a gear box or reduction ratio \( r \)
  - \( M \) mass of load (here the vehicle)
  - \( J \) inertia of electric motor
  - \( i = \frac{Z_1}{Z_2} \), the gear ratio
  - \( R_e \): tire rolling radius
  - \( r = \frac{R_e}{I} \): transmission length
  - \( a = \frac{dv}{dt} \): acceleration of the load (vehicle)
Acceleration capacity and impedance adaptation

- Newton equation of the vehicle
  \[ F_t = M \frac{dV}{dt} \]

- Tractive force
  \[ F_t = \eta \frac{i}{R_e} \left( C_m - J_m \frac{d\omega_m}{dt} \right) \]

- Relation between engine rotation speed and velocity
  \[ V = \frac{\omega_m R_e}{i} \]

- Equation of motion
  \[ \eta \frac{i}{R_e} \left( C_m - J_m \frac{i}{R_e} \frac{dV}{dt} \right) = M \frac{dV}{dt} \]
Acceleration capacity and impedance adaptation

- Newton equation of the vehicle

\[
C_m = \left( \frac{J_m}{(R_e/i)} + M \frac{R_e}{i} \right) \frac{dV}{dt} \\
= (M + J (i/R_e)^2) \left( \frac{R_e}{i} \right) \frac{dV}{dt}
\]

- Acceleration

\[
a = \frac{dV}{dt}
\]

- Acceleration

\[
a = \frac{C_m (i/R_e)}{M + J (i/R_e)^2}
\]
Acceleration capacity and impedance adaptation

- Derivative of acceleration with respect to gear ratio

$$\frac{\partial a}{\partial (i/R_e)} = \frac{C_m}{M + J(i/R_2)^2} - \frac{C_m(i/R_e)}{(M + J(i/R_e)^2)^2} \cdot 2(i/R_e)J = 0$$

- Optimal gear ratio

$$(i/R_e)^{opt} = \sqrt{\frac{M}{J}}$$

- Optimal acceleration power

$$a_{max} = \frac{1}{2} \frac{C_m}{\sqrt{JM}} = \frac{1}{2} \frac{C_m}{M} (i/R_e)^{opt}$$

- Conclusion: this is the maximum acceleration that can be given to the load by a motor with maximum torque $C_m$. 
EV accelerations in normal operation

- Driving cycle $V(t)$ is given
- Evaluate the acceleration required: differencing the time velocity profile:
  \[
  \frac{dV}{dt} \approx \frac{V(t_{k+1}) - V(t_k)}{t_{k+1} - t_k}
  \]
- Tractive force is given by the net force necessary to follow the driving cycle
  \[
  F_t = mgf \cos \theta + \frac{1}{2} \rho SC_x SV^2 + \gamma m \frac{dV}{dt}
  \]
EV accelerations in normal operation
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EV energy consumption

- In transportation the unit of energy is usually the kWh (kiloWatt hour) (preferred to J ou kJ)
  - ICE with liquid fuels: L/100 km or mpg
  - Gaseous fuel (CH$_4$, H$_2$): kg/100 km

- Advantage: size of batteries given in kWh at battery ports so that the driving range can be calculated immediately.

- Energy consumption results from the time integration of the power output and input at battery terminal.
EV energy consumption

- Energy power output
  - Equal to the resistance power and the power losses in the transmission and motor drive including the power electronic loses

\[
P_{\text{bat}}^{\text{out}} = \frac{V}{\eta_t \eta_m(C_m, \omega_m)} \left( m g f \cos \theta + mg \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma_m \frac{dV}{dt} \right) > 0
\]

- The non traction loads are not included (auxiliary loads) while they can be significantly important, and they should be added to the traction load.
The efficiency of the traction motor varies with the operating points on the speed-torque (speed-power) plane.

Good design: large overlap between maximum efficiency region and the region visited by the most frequent operation points.
The regenerative braking power at battery can be evaluated as

\[
P_{bat}^{in} = \alpha \eta_t \eta_m V \left( mg f \cos \theta + mg \sin \theta + \frac{1}{2} \rho C_x V^2 + \gamma m \frac{dV}{dt} \right) < 0
\]

In which
- road slope \( \sin \theta < 0 \) and/or \( \frac{dV}{dt} < 0 \)
- \( 0 < \alpha < 1 \) is the fraction of energy recovered during braking

The braking factor \( \alpha \) is a function of the applied braking strength and the design and control of braking system.

Typical \( \alpha \) is around 0.3.
EV energy consumption

- The net energy consumption from batteries is:

\[ E_{\text{out}} = \int_{P_t>0} P_{\text{out}} \, dt - \int_{P_t<0} P_{\text{bat}} \, dt \]

- When the net battery energy consumption reaches the total energy in the batteries, measured at terminal, the batteries are empty and need to be charged.

- The traveling distance between two charges is called the effective travel range.

- It is dependent on the battery capacity, the road resistance power, the driving cycle, the effectiveness of regenerative braking, the efficiency of the car and its powertrain.
Preliminary design procedure of EV
EV Design Procedure

- First estimate the rating power of the e-motor. Acceleration time is generally the most critical criteria

\[ P_{\text{max}} = \frac{\gamma m}{2 \eta_t \ t_a} (V_f^2 + V_b^2) \quad V_b = \frac{V_{\text{max}}}{X} \]

- \( V_b \) can be estimated using \( N_{\text{max}} \) and the aspect ratio of the selected e-motor technology (X factor)

- Then the reduction ratio of the transmission can be determined using the target top speed

\[ i = \frac{2\pi \ N_{m}^{\text{max}} \ R_e}{60 \ V_{\text{max}}} \]

- Select one motor from the catalog
**EV Design Procedure**

- Check the top speed

\[ \eta_t P_{m}^{\text{max}} = A V_{\text{max}} + B V_{\text{max}}^{3} \]

- And the grade ability

\[ \sin \theta = \frac{d - f \sqrt{1 - d^2 + f^2}}{1 + f^2} \quad d = \frac{(F_t - F_{\text{Aero}})}{mg} \]

- If not satisfied, adapt the gear ratio \( i \) and if not possible select a second gear ratio to satisfy both specifications
EV Design Procedure

- Adapt the base speed

\[ V_b = \frac{2\pi N_m^{\text{max}} R_e}{60 i X} \]

- Compute a finer estimation of the acceleration power

\[ P_t = \frac{\gamma m}{2t_a} (V_f^2 + V_b^2) + \frac{2}{3} mg f V_f + \frac{1}{5} \rho S C_x V_f^3 \]

- And solve numerically the time integration of the Newton’s equation to quote the acceleration time

\[ t_a = \int_0^{V_f} \frac{\gamma m}{F_t - m g f - 0.5 \rho S C_x V^2} \, dV \]
EV Design Procedure

- Repeat the design cycle till convergence
- Use simulation to estimate energy consumption and size the battery pack to reach the desired range
- Select the relevant driving cycles

\[
P_{bat}^{out} = \frac{V}{\eta_t \eta_m(C_m, \omega_m)} \left( m g f \cos \theta + m g \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma m \frac{dV}{dt} \right)
\]

\[
P_{bat}^{in} = \alpha \frac{V}{\eta_t \eta_m} \left( m g f \cos \theta + m g \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma m \frac{dV}{dt} \right)
\]

\[
E_{n}^{out} = \int_{P_{t}>0} P_{bat}^{out} dt - \int_{P_{t}<0} P_{bat}^{in} dt
\]