Vehicle Performance

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Lesson 2
Performance criteria

Outline

- DESCRIPTION OF VEHICLE MOTION
  - Longitudinal motion
- POWER AND TRACTIVE FORCE AT WHEELS
  - Transmission efficiency
  - Gear ratio
  - Expression of power and forces at wheels
  - Power and forces diagram
- VEHICLE RESISTANCE
  - Aerodynamic
  - Rolling resistance
  - Grading resistance
  - General expression of vehicle resistance forces

Outline

- STEADY STATE PERFORMANCES
  - Maximum speed
  - Gradeability and maximum slope
- ACCELARATION AND ELASTICITY
  - Effective mass
  - Acceleration time and distance

Outline

- FUEL CONSUMPTION AND EMISSIONS
  - Specific consumption of power plant
  - Vehicle fuel consumption measures
  - Constant speed consumption
  - Variable speed consumption and driving cycles
  - Chassis dynamometer

References

- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- R. Bosch. « Automotive Handbook ». 5th edition. 2002. Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).
- W.H. Hucho. « Aerodynamics of Road Vehicles ». 4th edition. SAE International. 1998.
- M. Eshani, Y. Gao & A. Emadi. Modern Electric, Hybrid Electric and Fuel Cell Vehicles. Fundamentals, Theory and Design. 2nd Edition. CRC Press.

Max speed and gradeability

Vehicle performances

- Vehicle performance are dominated by two major factors:
  - The maximum power available to overcome the power dissipated by the road resistance forces
  - The capability to transmit the tractive force to the ground (limitation of tire-road friction)

- Performance indices are generally sorted into three categories:
  - Steady state criteria: max speed, gradebility
  - Acceleration and braking
  - Fuel consumption and emissions

Study of performances with tractive force diagrams

- The steady state performances can be studied using the tractive forces / road resistance forces diagrams with respect to the vehicle speed
- Newton equation

$$F_T - F_{AERO} - F_{RR} - F_{SLOPE} = m\frac{dv}{dt}$$

- Stationary condition $$a_x = \frac{dv}{dt} = 0$$

- Then equilibrium writes

$$F_T = F_{RES} = F_{AERO} + F_{RR} + F_{SLOPE}$$
$$\mathcal{P}_T = \mathcal{P}_{RES} = F_{RES}\, v$$

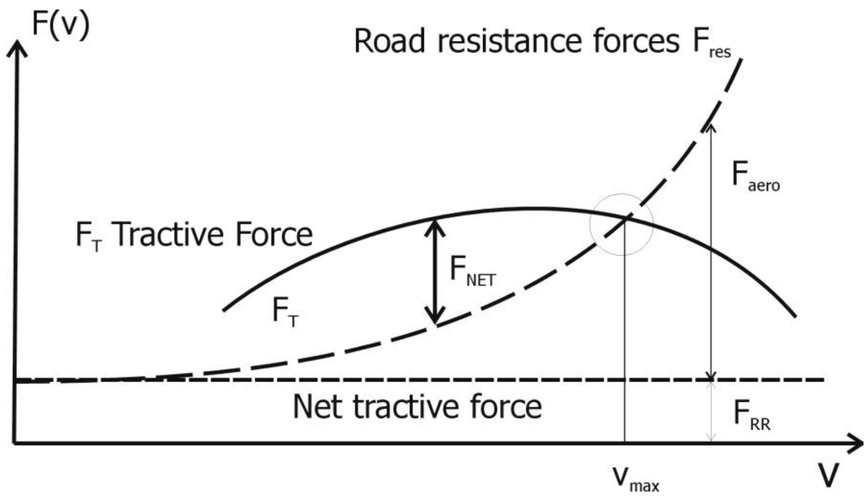
Study of performances with tractive force diagrams

- One generally defines the net force

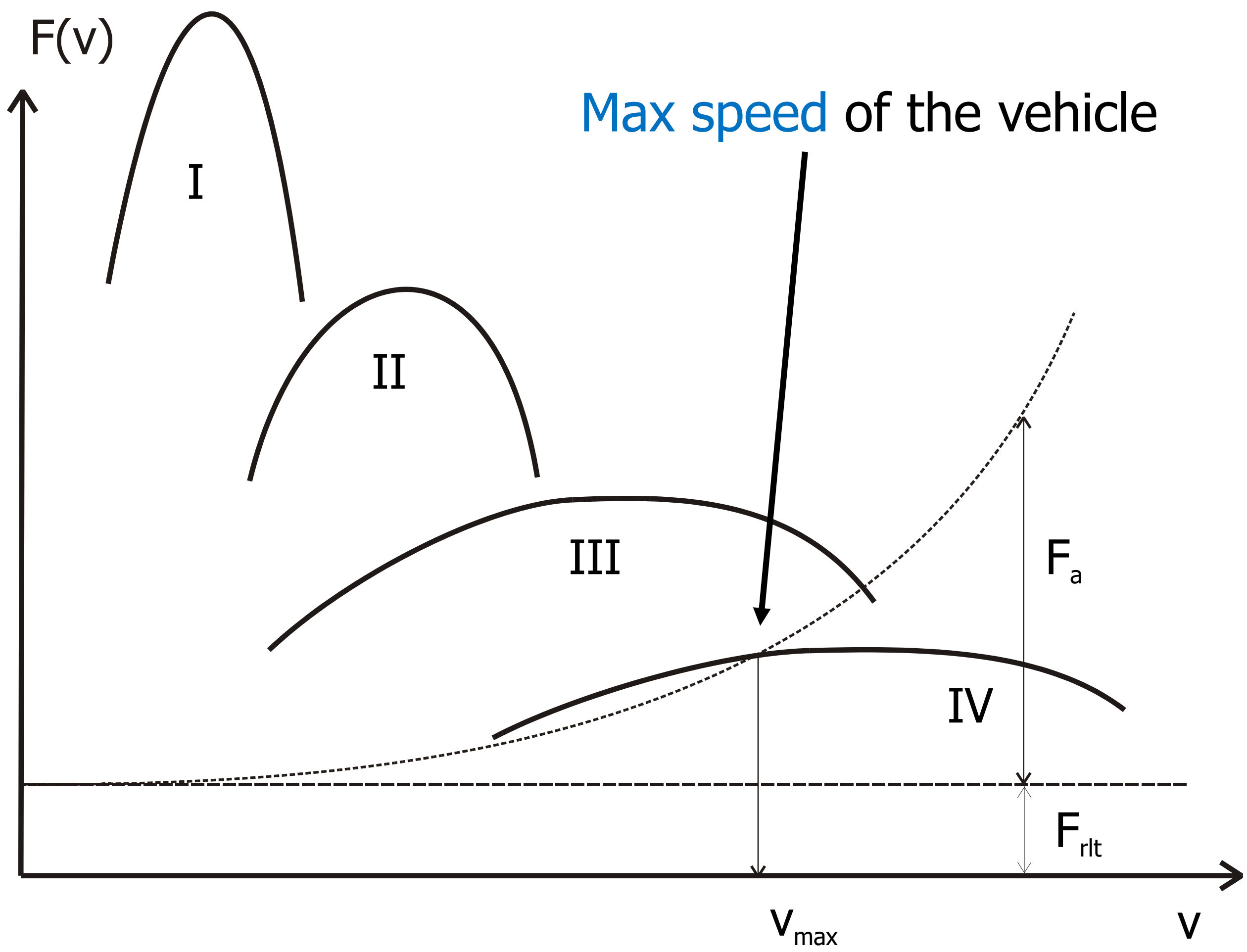
$$F_{NET} = F_T - F_{AERO} - F_{RR} - F_{SLOPE}$$

- One also can use the net force diagram to calculate
  - The maximum speed
  - The maximum slope
  - The reserve acceleration available

$$a_x = \frac{F_{NET}}{m}$$



Study of performances with tractive force diagrams



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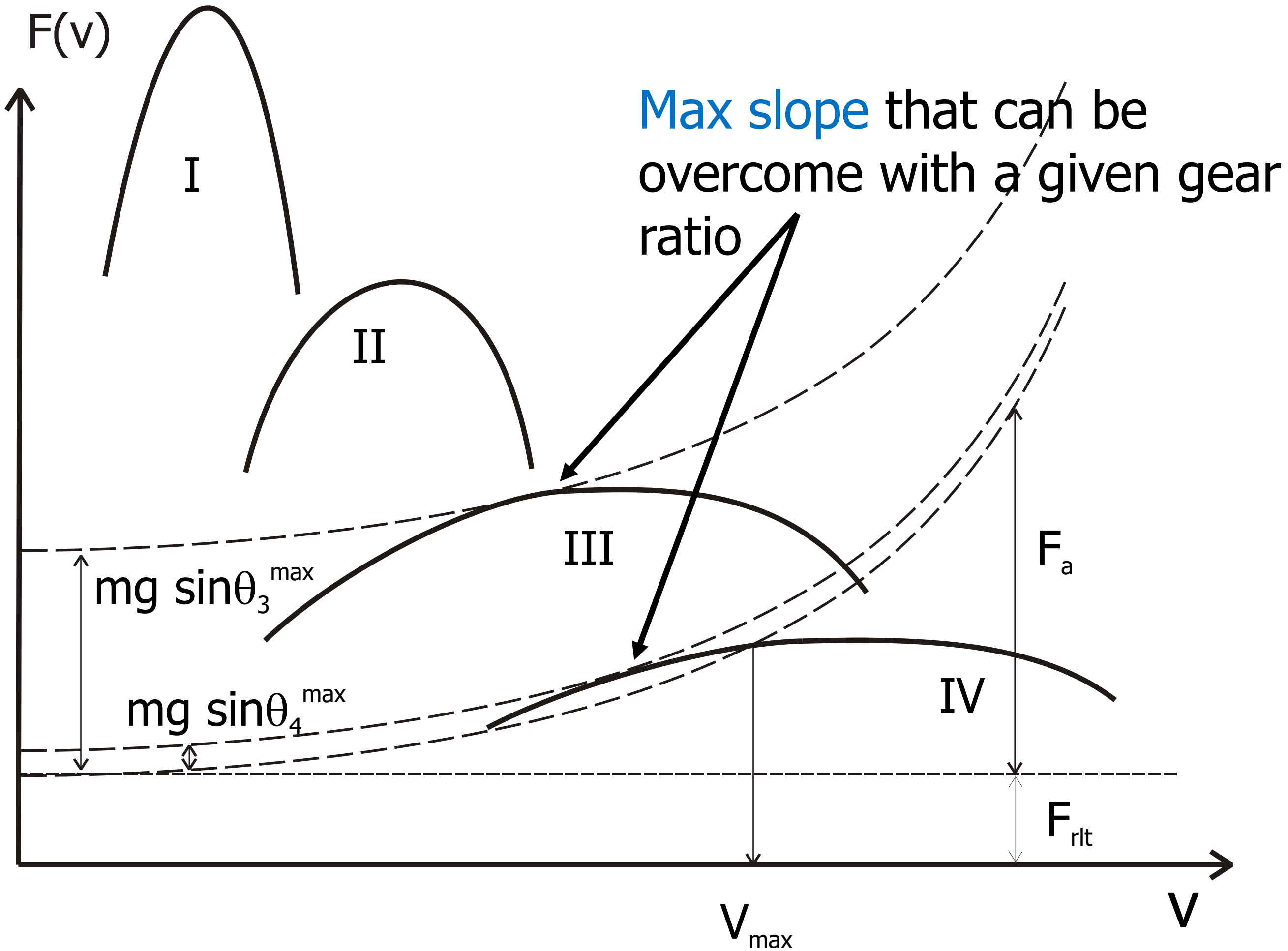
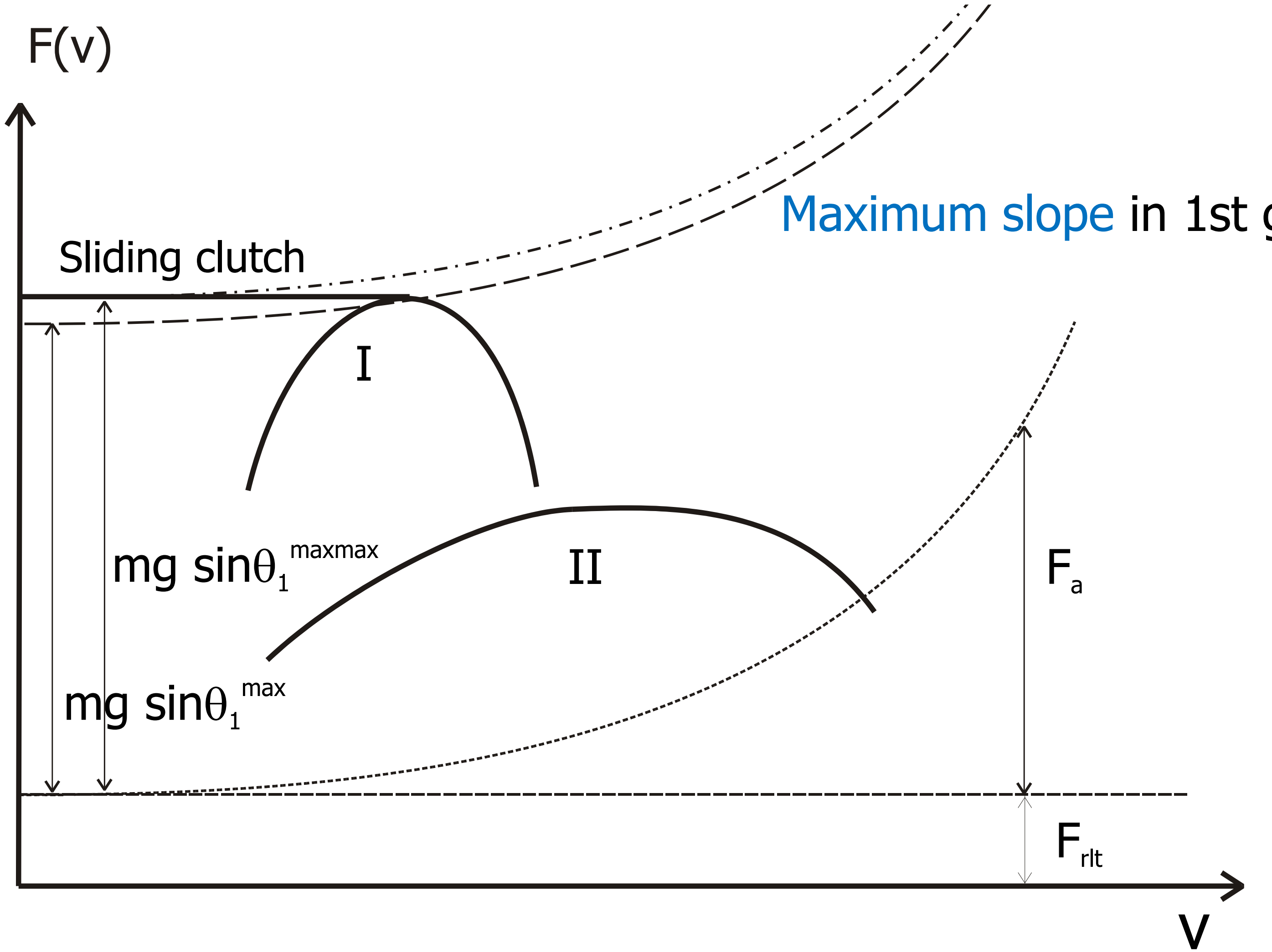


Study of performances with tractive force diagrams



Maximum speed

- For a given vehicle, tires, and engine, calculate the transmission ratio that gives rise to the greatest maximum speed
- Solve equality of tractive power and dissipative power of road resistance

$$\mathcal{P}_t = \mathcal{P}_{RES}$$

  - with

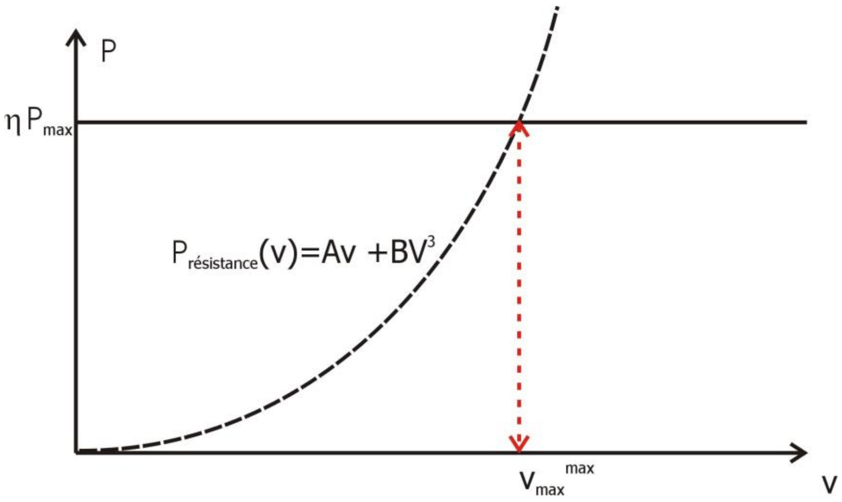
$$\mathcal{P}_{RES} = Av + Bv^3 \quad A,\ B > 0$$

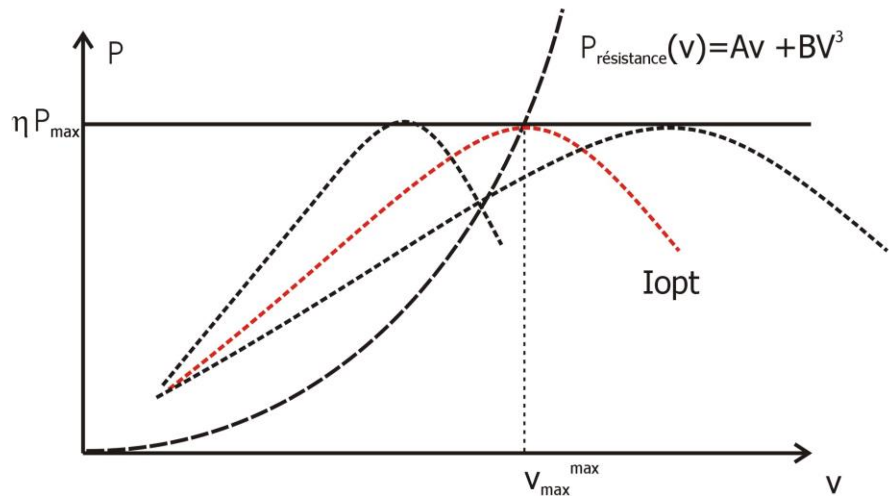
$$\mathcal{P}_t = \eta_t \mathcal{P}_p$$

- As the power of resistance forces is steadily increasing, the maximum speed is obtained when using the maximum power of the power plant

$$Av + Bv^3 = \eta_t \mathcal{P}_{max}$$

Maximum speed





$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

$$\mathcal{P}_t = \mathcal{P}_{RES}$$

$$\left(\frac{R}{i}\right)^* = \frac{v_{max}^{max}}{\omega_{nom}}$$

Maximum speed

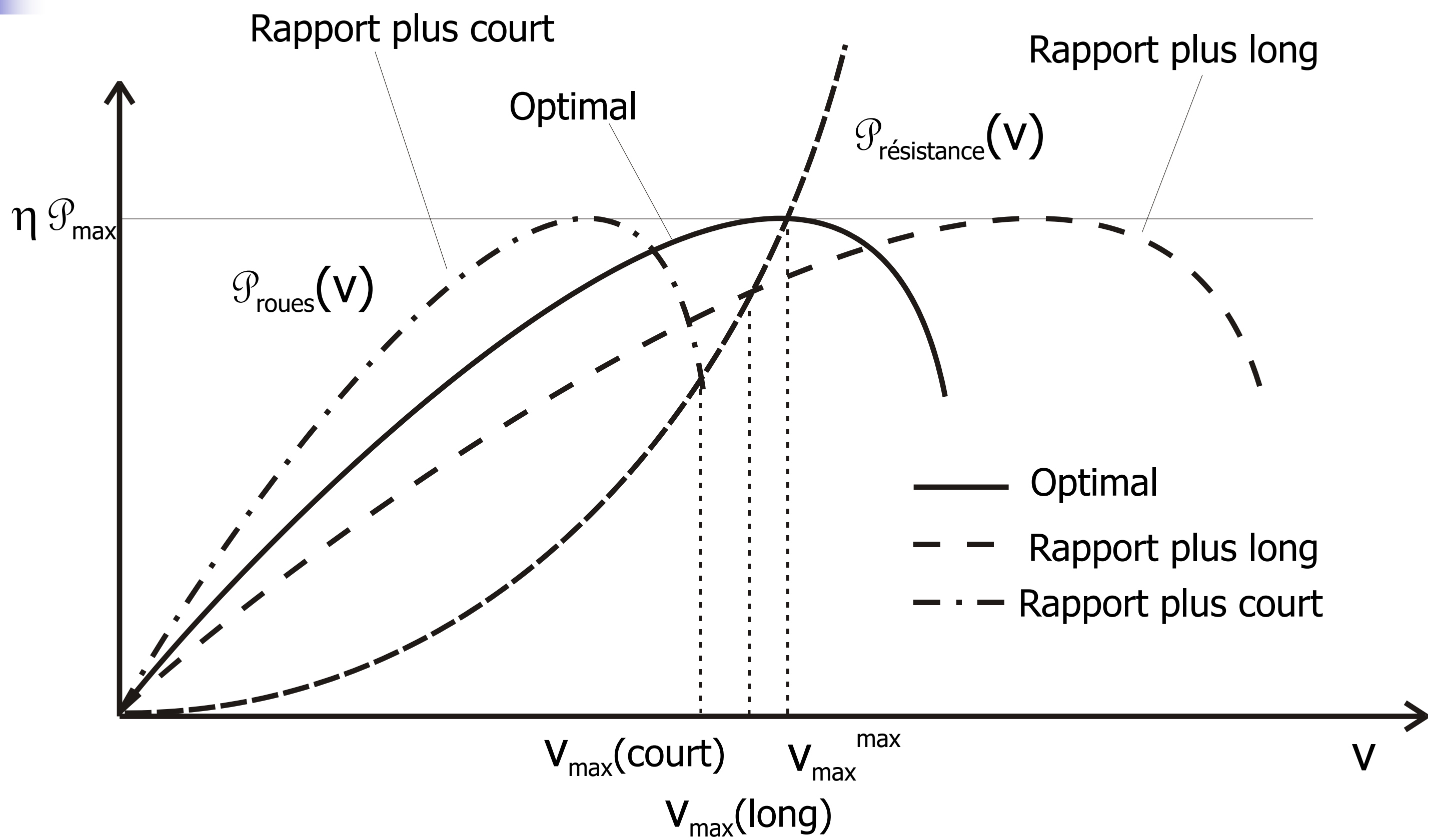
- Iterative scheme to solve the third order equation (fixed point algorithm of Picard) $x^{(k+1)} = f(x^{(k)})$

$$\begin{aligned} v^{(0)} &= 0 \\ v^{(n+1)} &= \left(\frac{\eta \mathcal{P}_{max} - A v^{(n)}}{B} \right)^{1/3} \end{aligned}$$

- Once the maximum speed is determined the optimal transmission ratio can be easily calculated by since it occurs for the nom rotation speed:

$$\left(\frac{R}{i} \right)^{*} = \frac{v_{max}^{max}}{\omega_{nom}} \qquad i^{*} = \frac{\omega_{nom} \cdot R_e}{v_{max}^{max}}$$

Max speed for given reduction ratio



Max speed is always reduced compared to v_{max}^{max}

Max speed for given reduction ratio

- Solve equation of equality of tractive and resistance power, but this time, the plant rotation speed is also unknown.

$$\begin{cases} \eta_t \, \mathcal{P}(\omega) \; = \; \mathcal{P}_{RES} \; = \; A v_{max} \; + \; B v_{max}^3 \\ \omega \; = \; v \, \frac{\bar{i}}{R_e} \end{cases}$$

- Let's eliminate the rotation speed of the engine to find a single nonlinear equation to solve

$$\mathcal{P}_{RES} \; = \; A v_{max} \; + \; B v_{max}^3 \; = \; \eta_t \, \mathcal{P}(\frac{\bar{i}}{R} v_{max})$$

Max speed for given reduction ratio

- Solve equation of equality of tractive and resistance power, but this time, the plant rotation speed is also varying.

$$\mathcal{P}_{RES} \;=\; A v_{max} \;+\; B v_{max}^3 \;=\; \eta_t \, \mathcal{P}(\frac{\bar{i}}{R} v_{max})$$

- Numerical solution using a fixed-point algorithm (Picard iteration scheme)

$$v^{(0)} = 0 \quad \text{ou} \quad v^{(0)} = v_{max}^{max}$$

$$\omega^{(k)} = v^{(k)} \; \frac{\bar{i}}{R_e}$$

$$\mathcal{P}^{(k)} = \eta_t \mathcal{P}(\omega^{(k)})$$

$$v^{(k+1)} = \left(\frac{\mathcal{P}^{(k)} - A v^{(k)}}{B} \right)^{1/3}$$

Selection of the top gear ratio

- Design specifications for the top gear ratio in connection with the topo speed criteria (from Wong)
  - To be able to reach a given top speed with the given engine
  - To be able to maintain a given constant speed (from 88 to 96 km/h) while overcoming a slope of at least 3% with the selected top gear ratio

- These specifications enable to select a proper top gear ratio
  - The first requirement enables to select a first gear ratio
  - The second condition enforces to select a gear ratio that gives rise to an engine rotation speed that is just above the nominal rotation speed (and the max power) in order to save a sufficient power reserve to keep a constant speed while climbing a small slope, overcoming wind gusts or accounting for loss of engine performance with ageing.

Maximum slope

- For the maximum slope the vehicle can climb, two criteria must be checked:

- The maximum tractive force available at wheel to balance the grading force

$$F_t \geq F_{RES} \simeq F_{GRADE} = mg\sin\theta$$

- The maximum force that can be transmitted to the road because of tire friction and weight transfer

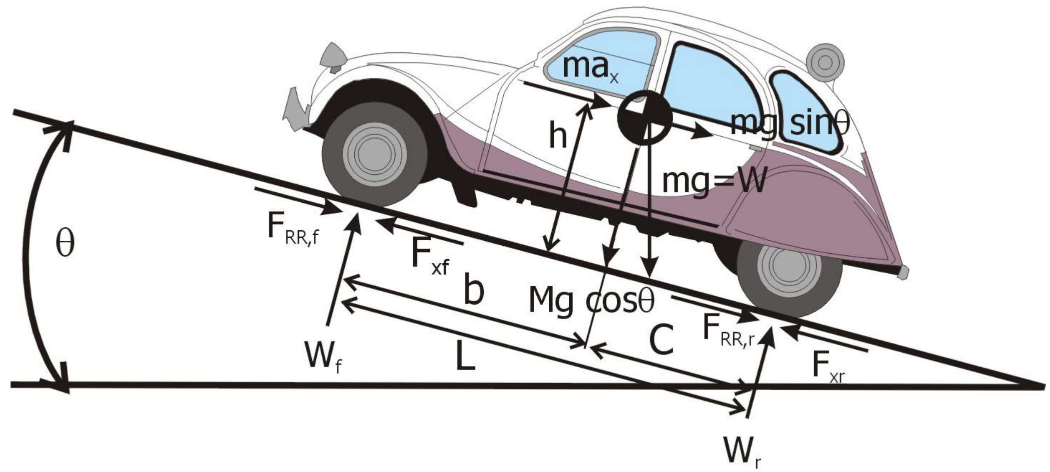
$$F_{w,f} \leq \mu\, W_f \qquad F_{w,r} \leq \mu\, W_r$$

Maximum slope

Surface	Peak Value μ
Asphalt and concrete (dry)	0.8-1.2
Asphalt (wet - 0.2 mm depth)	0.5-0.8
Gravel	0.4
Asphalt (wet - 2.0 mm depth)	0.05-0.5
Earth road (dry)	0.68
Earth road (wet)	0.55
Snow (hard-packed)	0.2
Ice	0.1

Table 1: Average values of friction coefficients for various road conditions

Maximum slope



- Vertical equilibrium

$$m\,g\cos\theta = W_f + W_r$$

- Rotational equilibrium about rear wheels contact point

$$W_f\,L + mg\ \sin\theta\,h + m\,a_x\,h = mg\ \cos\theta\,c$$

- Rotational equilibrium about rear wheels contact point

$$W_r\,L = mg\ \cos\theta\,b + mg\ \sin\theta\,h + m\,a_x\,h$$

Maximum slope

- Limitation due to the friction coefficient

$$F_{w,f} \leq \mu\, W_f \qquad\qquad F_{w,r} \leq \mu\, W_r$$

- Normal forces under the front and rear wheel sets

$$W_f = mg\ \cos\theta\ \frac{c}{L} - mg\ \sin\theta\ \frac{h}{L} - m\, a_x\ \frac{h}{L}$$

$$W_r = mg\ \cos\theta\ \frac{b}{L} + mg\ \sin\theta\ \frac{h}{L} + m\, a_x\ \frac{h}{L}$$

- At low speed and constant speed (a_x=0)

$$W_f = mg\ \cos\theta\ \frac{c}{L} - mg\ \sin\theta\ \frac{h}{L}$$

$$W_r = mg\ \cos\theta\ \frac{b}{L} + mg\ \sin\theta\ \frac{h}{L}$$

Maximum slope

FOUR-WHEEL DRIVE with electronic power split

$$F_p = F_{w,f} + F_{w,r} \leq \mu \left(W_f + W_r \right)$$

$$mg\ \sin\theta + mg\ \cos\theta\ f_{RR} \leq \mu(mg\cos\theta\frac{c}{L} + mg\cos\theta\frac{b}{L})$$

$$\leq \mu\, mg\ \cos\theta$$

Maximum slope $\boxed{\tan\theta \leq (\mu - f_{RR})}$

Maximum slope

FRONT WHEEL DRIVE

$$F_{w,f} \leq \mu W_f$$

$$mg \ \sin\theta + mg \ \cos\theta \ f \leq \mu \ mg(\cos\theta\frac{c}{L} - \sin\theta\frac{h}{L})$$

Maximum slope

$$\boxed{\tan\theta \ \leq \ \frac{\mu c/L - f}{1 + \mu h/L}}$$

Maximum slope

REAR WHEEL DRIVE

$$F_{w,r} \leq \mu\, W_r$$

$$mg\ \sin\theta + mg\ \cos\theta\ f \leq \mu\, mg(\cos\theta \frac{b}{L} + \sin\theta \frac{h}{L})$$

Maximum slope

$$\boxed{\tan\theta\ \leq\ \frac{\mu b/L - f}{1 - \mu h/L}}$$

Selection of first gear ration

- Maximum slope to be overcome, for instance θ_{max} = 25%

$$F_{\mathrm{RES}} = mg\ \sin\theta_{max} + mg\ f_{RR}\ \cos\theta$$

- Tractive force at wheels

$$F_w = \eta_t \frac{i}{R_e} C_p$$

- Sizing of first gear ration

$$i_{max} = \frac{R_e\ F_{RES}}{\eta_t\ C_{max}} \qquad i_{max} = \frac{R_e\ mg \sin\theta_{max}}{\eta_t\ C_{max}}$$

Maximum slope at high speed

- What is the maximum grade that can be overcome at a given speed V is:

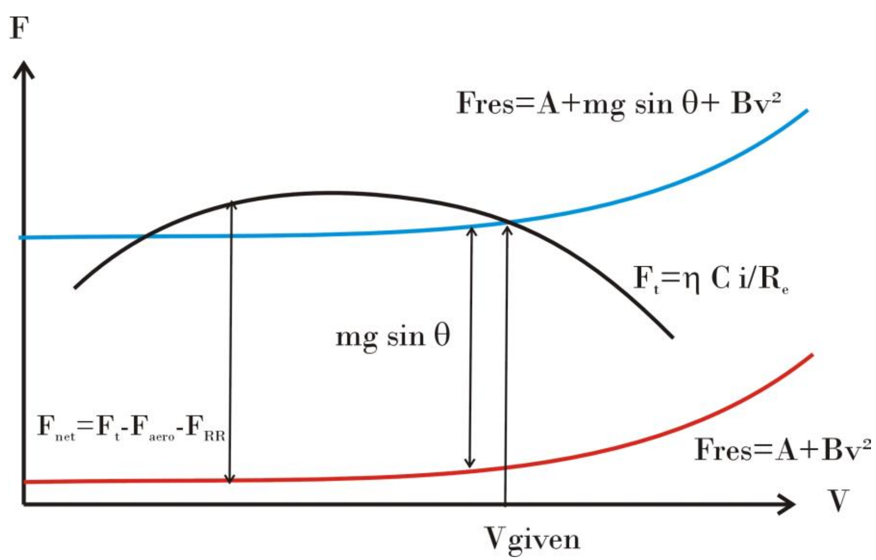
$$\sin\theta = \frac{F_t - F_{RR} - F_{aero}}{mg} = \frac{F_t^{net}}{mg}$$

- Gradeability is ruled by the net tractive force available

$$F_t^{net} = F_t - F_{RR} - F_{aero}$$
$$= F_t - mgf\cos\theta - 0,5\,\rho\, SC_x\, V^2$$

- The tractive force is given by the speed

$$F_t(v) = \eta_t C(\omega = \frac{v}{R_e} i)\frac{i}{R_e}$$



- Check also nonslip condition

$$F_t \le \mu W_f \qquad F_t \le \mu W_r$$

Maximum slope at high speed

- The maximum slope can be evaluated as follows

$$F_t - F_{aero} - F_{RR} - mg\sin\theta = 0$$
$$\Longleftrightarrow F_t - F_{aero} - mg\cos\theta\, f - mg\sin\theta = 0$$

- If we define

$$d = (F_t - F_{aero})/mg$$

- It comes :

$$d - \sin\theta = f \ \cos\theta$$
$$(d - \sin\theta)^2 = f^2 \ \cos^2\theta$$
$$\Longleftrightarrow d^2 - 2d\sin\theta + \sin^2\theta = f^2(1 - \sin^2\theta)$$
$$\Longleftrightarrow (1 + f^2) \ \sin^2\theta - 2d \ \sin\theta + (d^2 - f^2) = 0$$

Maximum slope at high speed

- It a second order equation in sinθ

$$(1+f^2)\ \sin^2\theta - 2d\ \sin\theta + (d^2 - f^2) = 0$$

- Solving for sinθ gives

$$\begin{aligned}\sin\theta &= \frac{2d \pm \sqrt{4d^2 - 4\,(1+f^2)\,(d^2-f^2)}}{2(1+f^2)} \\ &= \frac{d \pm \sqrt{d^2 - d^2 + f^2 - f^2 d^2 + f^4}}{1+f^2} \\ &= \frac{d \pm \sqrt{f^2 - d^2\ f^2 + f^4}}{1+f^2} = \frac{d \pm f\ \sqrt{1 - d^2 + f^2}}{1+f^2}\end{aligned}$$

- It comes :

$$\sin\theta = \frac{d - f\sqrt{1+f^2-d^2}}{1+f^2} \qquad d = (F_t - F_{aero})/mg$$

Selection of gear ratios

- Goal of the selected gear ratio: to adapt the characteristics of engine operation (rotation speed, torque) to the vehicle speed.
- The top and lowest gear ratios are selected to
  - Match a given top speed
  - To be able to drive over given grading conditions, that is to develop sufficiently high tractive forces at wheels
- The distribution of intermediate gear ratios in between the top and lowest gear ratio is made to span the full range of operating speeds more or less smoothl
- In principle, the different gear ratios should render as much as possible the maximum power curve

Accelerations and elasticity

Acceleration performance

- Estimation of acceleration and elasticity is based on the second Newton law

$$F_w - \sum F_{res} = F_{net} = m\frac{dV}{dt}$$

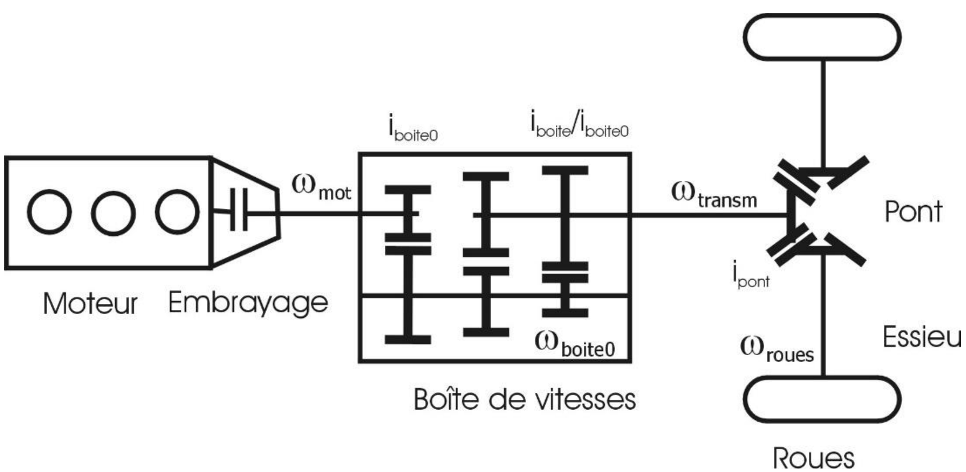
- Warning: when accelerating, the rotation speed of all driveline and transmission components is increasing: wheel sets, transmission shafts, gear boxes and differential, engine…

➔ Effective mass to account for the kinetic energy of all components (translation + rotation)

Effective mass

- Total kinetic energy of the vehicle and its driveline :

$$
\begin{aligned}
T = & 1/2 m\, v^2 + 1/2 (\sum I_{\mathrm{w}} + I_{\mathrm{axle}})\, \omega_{\mathrm{w}}^2 \\
& + 1/2 (I_{\mathrm{transm}} + I_{\mathrm{box2}})\, \omega_{\mathrm{transm}}^2 \\
& + 1/2 (I_{\mathrm{box0}})\, \omega_{\mathrm{box0}}^2 \\
& + 1/2 (I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, \omega_{\mathrm{p}}^2
\end{aligned}
$$



Effective mass

- The rotation speed of the driveline components is linked to the longitudinal speed of the vehicle

$$\omega_{\mathrm{w}} = v/R_e$$

$$\begin{aligned}\omega_{\mathrm{w}} &= \omega_{\mathrm{transm}}/i_{\mathrm{dif}}\\ \omega_{\mathrm{w}} &= \omega_{\mathrm{box0}}/(i_{\mathrm{dif}} * i_{\mathrm{box}}/i_{\mathrm{box0}})\\ \omega_{\mathrm{w}} &= \omega_{\mathrm{p}}/(i_{\mathrm{dif}} * i_{\mathrm{box}})\end{aligned}$$

- The kinetic energy writes

$$\begin{aligned}T =& 1/2 m v^2 + 1/2(\sum I_{\mathrm{w}} + I_{\mathrm{axle}})\, v^2/R_e^2\\ &+ 1/2(I_{\mathrm{transm}} + I_{\mathrm{box2}})\, v^2 i_{\mathrm{dif}}^2/R_e^2\\ &+ 1/2(I_{\mathrm{box0}})\, v^2 (i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2/i_{\mathrm{box0}}^2)/R_e^2\\ &+ 1/2(I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, v^2 i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2/R_e^2\end{aligned}$$

Effective mass

- One defines an effective mass $T = 1/2\, m_{\mathrm{e}}\, v^2$

$$m_{\mathrm{e}} = m + \frac{\sum I_{\mathrm{w}} + I_{\mathrm{axle}}}{R_e^2} + \frac{(I_{\mathrm{transm}} + I_{\mathrm{box2}})\, i_{\mathrm{dif}}^2}{R_e^2} + \frac{(I_{\mathrm{box0}})\, i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2}{i_{\mathrm{box0}}^2\, R_e^2} + \frac{(I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, i_{\mathrm{dif}}^2\, i_{\mathrm{box}}^2}{R_e^2}$$

- The calculation of the effective mass requires the knowledge of the geometry and inertia properties of all the driveline components

- Empirical formula for preliminary design of cars by Wong

$$m_{\mathrm{e}} = m_0 + m_1\, i_{\mathrm{box}}^2 \qquad i = i_{\mathrm{dif}} * i_{\mathrm{box}}$$

Effective mass

- Empirical correction formula to estimate the effective mass of passenger car propelled by piston engines (Wong, 2001)

$$\gamma_m = \frac{m_\mathrm{e}}{m} = 1.04 + 0.0025\, i^2$$

- This estimation formula puts forward the major factors of the corrections :
  - Nearly negligible for low reduction ratios (4th and 5th gear ratios)
  - Rather important for high gear ratios : 1st and 2nd gear ratios
- For railway systems, γ is of an order of magnitude 1,02 to 1,30 for classical train and from 1,30 to 3,50 for rack trains)

Effective mass

- Example: Peugeot 308 1.6 HDi with 5 gear ratios

	i_{boite}	i	γ_m
1	3,95	13,63	1,5043
2	1,87	7,39	1,1764
3	1,16	4,58	1,0925
4	0,82	3,24	1,0662
5	0,66	2,61	1,0570

$i_{dif}=3,95$

Velocity as a function of time

- We now proceed to time integration of Newton equation.

$$m_e \frac{dv}{dt} = F_w - \sum F_{res} = F_{net}(v)$$

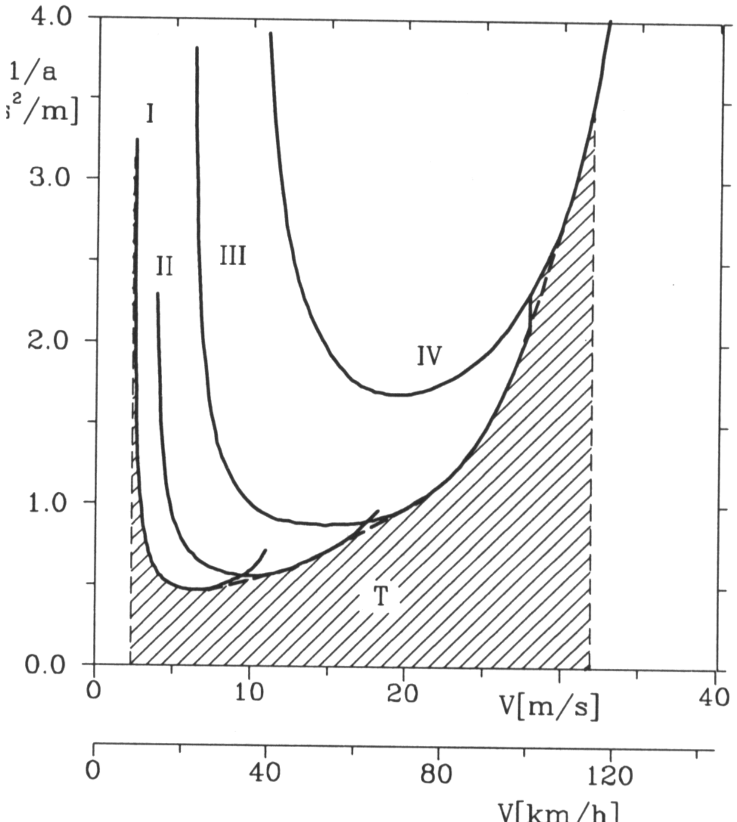
- Time to accelerate form V_1 to V_2.

$$dt = \frac{m_\mathrm{e}\, dv}{F_{net}(v)}$$

$$\Delta t_{V_1 \to V_2} = m_\mathrm{e} \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)}$$

Velocity as a function of time

$$1/a = m_e/F_{net}$$



Genta Fig 4.20 : 1/F as function of time

- Time to accelerate from V_1 to V_2:

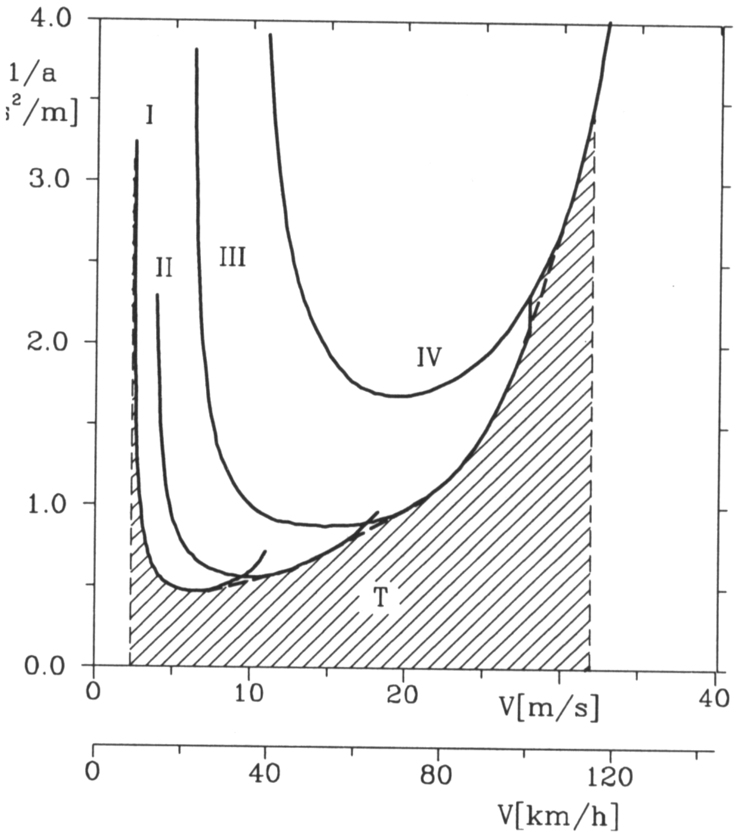
$$\Delta t_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)}$$

- Alternatively

$$F_{net}(v) = \mathcal{P}_{net}(v)/v$$

$$\Delta t_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{v\, dv}{\mathcal{P}_{net}(v)}$$

Velocity as a function of time



Genta Fig 4.20 : 1/F as function of time

- Criteria for gear ratio up shift in order to minimize the acceleration time
- If two curves intersects each other: **change the ratio at curve intersection**
- If there is no intersection, then it is necessary to push the ratio up to maximum rotation speed
- Lower limit is given by an infinite number of gear ratios, that is a Continuous Variables Transmission (CVT)

Velocity as a function of time

- The solution of differential equation yields the time t as a function of the velocity

$$t = f(v)$$

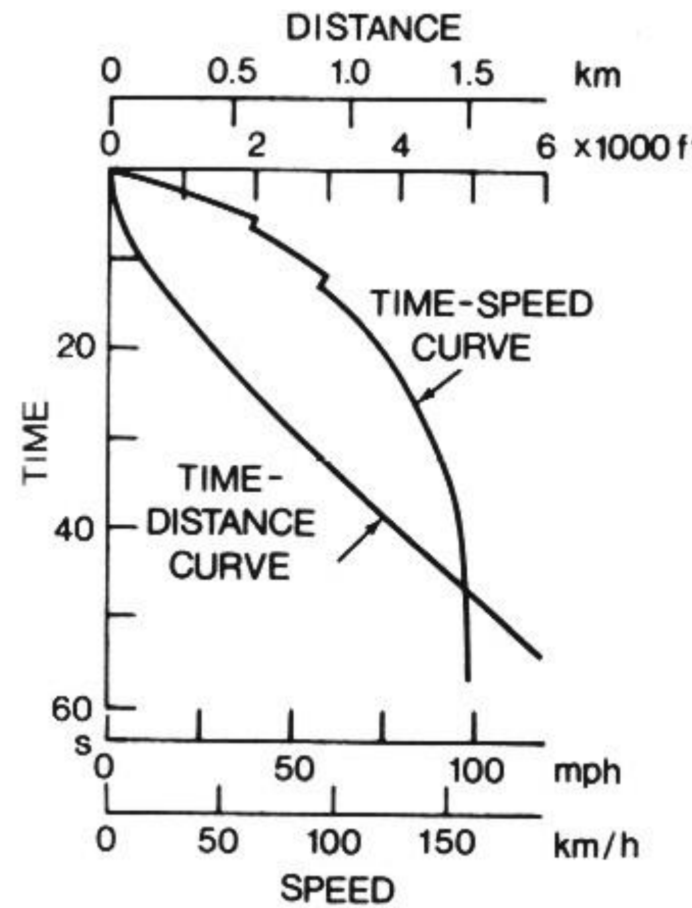
- The reciprocal function

$$v = g(t)$$

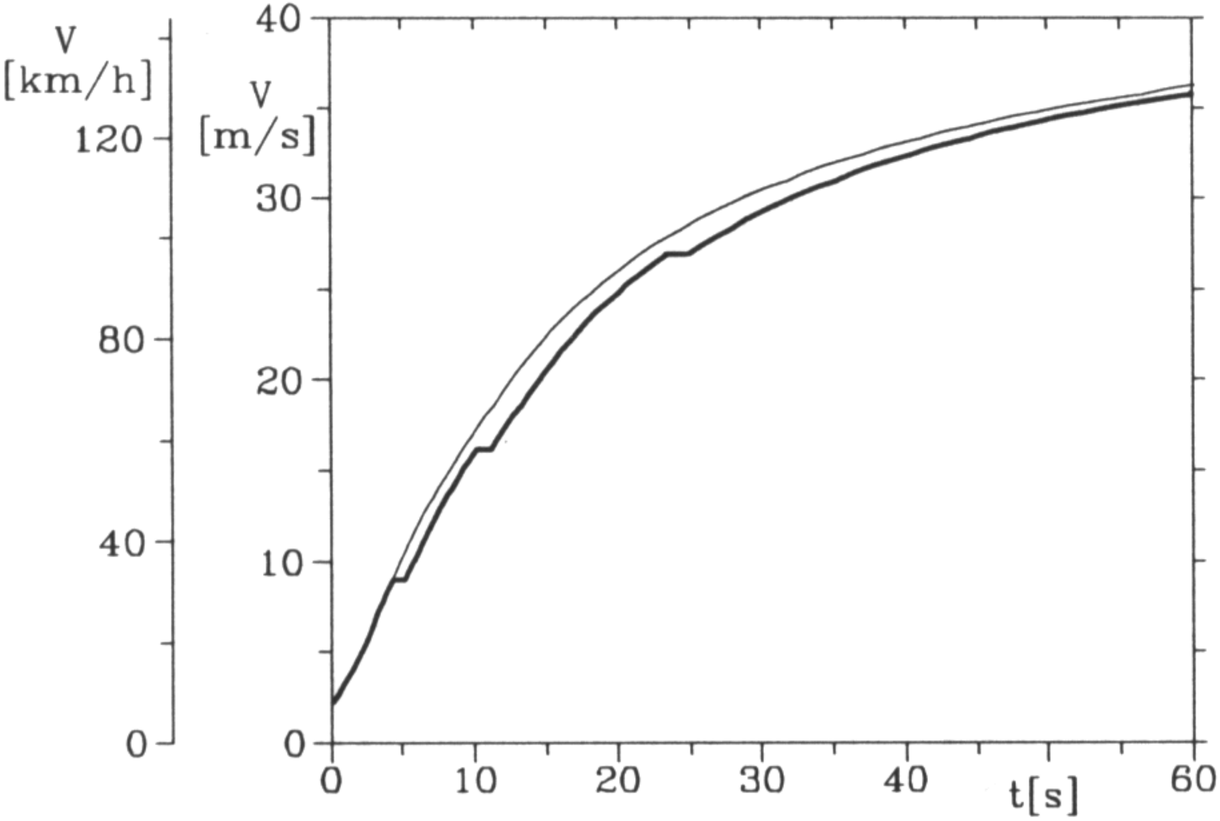
- requires to invert the relation

$$g = f^{-1}$$

- The changes of gear ratio must be taken into account



Velocity as a function of time



G. Genta Fig 4.21

Distance as a function of the speed

- The distance from start can be evaluated by a second integration of the Newton equation

- Velocity and distance are linked by the kinematic relation

$$dx = v\,dt$$

- It comes

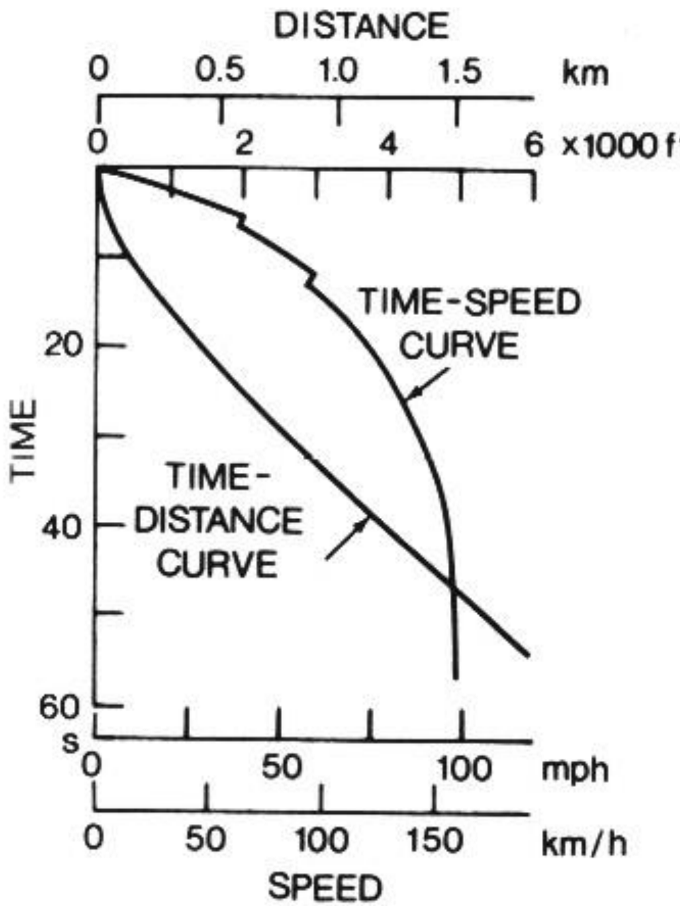
$$\Delta x_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{v\,dv}{F_{net}(v)}$$

Distance as a function of the time

- One can eliminate the velocity V between the two curves t=f(V) and d=h(V)
- On gets the distance as a function of the time:

$$\Delta t = f(\Delta v) \qquad \Delta x = h(\Delta v)$$

$$\Delta x = h(f^{-1}(\Delta v))$$



Change of gear ratio

- Criteria for changing the gear ratio.
- Gear ratio changing is a delicate operation that needs being studied in detail:
  - Changing the gear box ratio takes some time
  - Tractive force is interrupted
  - The vehicle is coasting and slows down
- For an expert driver
  - Small time to change the gear

$$\Delta t \approx 0,8 s$$

  - Reduction of the velocity can be estimated by the first order approximation

$$\Delta v \approx -\frac{F_{rés}(v)}{m_{\mathrm{eff}}} \Delta t$$

Change of gear ratio

- When several gear change are necessary, the integration needs to be carried out by parts
- For instance

$$T_{V_1 \to V_2} = \int_{V_1}^{V_{I \to II}} \frac{m_e(i_1)\, dv}{F_{net}(v)} + \Delta t + \int_{V_{II}}^{V_{II \to III}} \frac{m_e(i_2)\, dv}{F_{net}(v)} + \Delta t + \int_{V_{III}}^{V_2} \frac{m_e(i_3)\, dv}{F_{net}(v)}$$

- with

$$V_{II} = V_{I \to II} - \frac{F_{RES}(V_{I \to II})}{m_{\mathrm{e}}} \Delta t$$

$$V_{III} = V_{II \to III} - \frac{F_{RES}(V_{II \to III})}{m_{\mathrm{e}}} \Delta t$$