# Vehicle Performance

Pierre Duysinx

Research Center in Sustainable Automotive Technologies of University of Liege

Academic Year 2021-2022

# Approximation of Power and Torque Curves

# Exercise 1

- It is asked to develop approximations of the power and torque curves of a BMW 316i engine

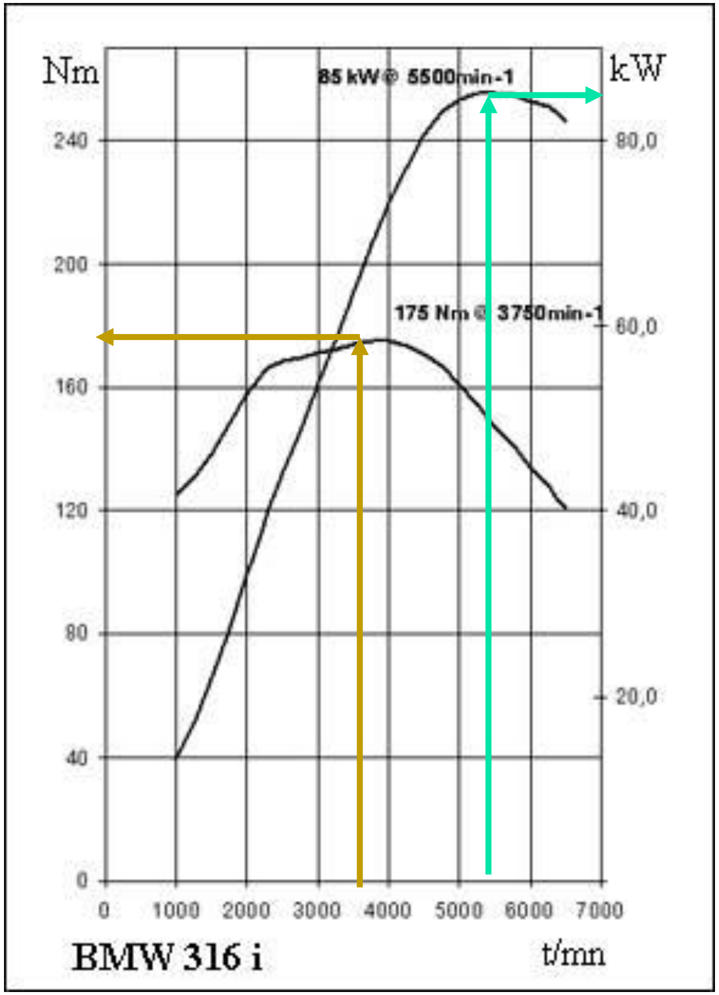
- From the published data, one notices

$$P_1 = P_{max} = 85000\,W$$

$$\omega_1 = \omega_{nom} = 5500\,rpm$$

$$C_2 = C_{max} = 175\,N.m$$

$$\omega_2 = \omega_{Cmax} = 3750\,rpm$$



# Power approximation

- One looks for a power function of the type

$$\boxed{\mathcal{P} \;=\; \mathcal{P}_1 - a\left|\omega - \omega_1\right|^b} \quad \text{with } b > 0$$

- Data $\quad \mathcal{P}(\omega_1) = \mathcal{P}_1 = \mathcal{P}_{max} \qquad \omega = \omega_1$

$$\mathcal{P}(\omega_2) = \mathcal{P}_2 = C_{max}\,\omega_{C_{max}} \qquad \omega = \omega_2$$

$$\left.\frac{d\,C}{d\omega}\right|_{\omega_2} = \left.\frac{d\,(\mathcal{P}/\omega)}{d\omega}\right|_{\omega_2} = 0$$

- We are going to show this yields

$$\boxed{a \;=\; \frac{\mathcal{P}_1 - \mathcal{P}_2}{\left|\omega_1 - \omega_2\right|^b}} \qquad \boxed{b \;=\; \frac{\frac{\omega_1}{\omega_2} - 1}{\frac{\mathcal{P}_1}{\mathcal{P}_2} - 1}}$$

# Polynomial approximation

- Polynomial approximation of order 3

$$\mathcal{P}(\omega)/\mathcal{P}_1 = a_0 + a_1\,(\omega/\omega_1) + a_2\,(\omega/\omega_1)^2 + a_3\,(\omega/\omega_1)^3$$

- Identification of the coefficients

$$\mathcal{P}(0) = 0$$

$$\mathcal{P}(\omega_1) = \mathcal{P}_{max}$$

$$\mathcal{P}(\omega_2) = \mathcal{P}_2 = C_{max}\,\omega_{C_{max}}$$

$$\left.\frac{d\,C}{d\omega}\right|_{\omega_2} = 0$$

$$\begin{aligned} a_0 &= 0 \\ a_1 + a_2 + a_3 &= 1 \\ a_1\,n_2 + a_2\,n_2^2 + a_3\,n_2^3 &= \mathcal{P}_2/\mathcal{P}_1 \\ a_2 + 2\,a_3\,n_2 &= 0 \end{aligned}$$

$$n_2 = \frac{\omega_2}{\omega_1}$$

# Polynomial approximation

- Polynomial approximation of order 4

$$\mathcal{P}(\omega)/\mathcal{P}_1 = a_0 + a_1\,(\omega/\omega_1) + a_2\,(\omega/\omega_1)^2 + a_3\,(\omega/\omega_1)^3 + a_4\,(\omega/\omega_1)^4$$

- Identification of the coefficient

$$\mathcal{P}(\omega_1) = \mathcal{P}_1 = \mathcal{P}_{max} \qquad \omega = \omega_1$$

- Solve the linear system

$$\begin{aligned} &a_1 + a_2 + a_3 + a_4 = 1 \\ &a_1 + 2\,a_2 + 3\,a_3 + 4\,a_4 = 0 \\ &a_1\,n_2 + a_2\,n_2^2 + a_3\,n_2^3 + a_4\,n_2^4 = \mathcal{P}_2/\mathcal{P}_1 \\ &a_2 + 2\,a_3\,n_2 + 3\,a_4\,n_2^2 = 0 \end{aligned}$$

# Exercise 4 : Performance approximations

- Let's calculate the data to proceed to the curve fitting

$$\mathcal{P}(\omega_1) = \mathcal{P}_1 = \mathcal{P}_{max} = 85000\,W \qquad \omega_1 = 5500\,\frac{2\pi}{60} = 575,9587 rad/s$$

$$\begin{aligned}\mathcal{P}(\omega_2) = \mathcal{P}_2 &= C_{max}\,\omega_{C_{max}} \\ &= 175 \cdot 392,6991 \\ &= 68722,33\,W\end{aligned} \qquad \omega_2 = 392,6991\,rad/s$$

# Power approximation

- One looks for a power function of the type

$$\boxed{\mathcal{P} \;=\; \mathcal{P}_1 - a\,|\omega - \omega_1|^b} \quad \text{with } b > 0$$

- It comes

$$\mathcal{P}_2/\mathcal{P}_1 = 0.80850$$

$$\omega_2/\omega_1 = 0.68182$$

$$b \;=\; \frac{\frac{\omega_1}{\omega_2} - 1}{\frac{\mathcal{P}_1}{\mathcal{P}_2} - 1} = 1.9702$$

$$a \;=\; \frac{\mathcal{P}_1 - \mathcal{P}_2}{|\omega_1 - \omega_2|^b} = 0.56607$$

# Polynomial approximation

- Polynomial approximation of order 3

$$\mathcal{P}(\omega)/\mathcal{P}_1 = a_0 + a_1\,(\omega/\omega_1) + a_2\,(\omega/\omega_1)^2 + a_3\,(\omega/\omega_1)^3$$

- Identification of the coefficients

$$\begin{aligned} & a_0 = 0 \\ & a_1 + a_2 + a_3 = 1 \\ & a_1\,n_2 + a_2\,n_2^2 + a_3\,n_2^3 = \mathcal{P}_2/\mathcal{P}_1 \\ & a_2 + 2\,a_3\,n_2 = 0 \end{aligned} \quad \Longrightarrow \quad \left\{ \begin{aligned} a_1 &= 0.33265 \\ a_2 &= 2.50257 \\ a_3 &= -1.83522 \end{aligned} \right.$$

$$n_2 = \frac{\omega_2}{\omega_1} = 0,68182$$

# Polynomial approximation

- Polynomial approximation of order 4

$$\mathcal{P}(\omega)/\mathcal{P}_1 = a_0 + a_1\,(\omega/\omega_1) + a_2\,(\omega/\omega_1)^2 + a_3\,(\omega/\omega_1)^3 + a_4\,(\omega/\omega_1)^4$$

- Solution of the linear system

$$\begin{aligned} &a_1 + a_2 + a_3 + a_4 = 1 \\ &a_1 + 2\,a_2 + 3\,a_3 + 4\,a_4 = 0 \\ &a_1\,n_2 + a_2\,n_2^2 + a_3\,n_2^3 + a_4\,n_2^4 = \mathcal{P}_2/\mathcal{P}_1 \\ &a_2 + 2\,a_3\,n_2 + 3\,a_4\,n_2^2 = 0 \end{aligned} \quad \Rightarrow \quad \left\{ \begin{aligned} a_1 &= -0.43818 \\ a_2 &= 5.53448 \\ a_3 &= -5.75443 \\ a_4 &= 1.65813 \end{aligned} \right.$$

# MATLAB Code

```
P_1 = 85000;
N_1 = 5500;
w_1 = N_1*2*pi/60;
C_max = 175;
N_2 = 3750;
w_2 = N_2*2*pi/60;

n_2 = w_2/w_1;
P_2 = C_max*w_2;

A_3 = [1,1,1;n_2,n_2^2,n_2^3;0,1,2*n_2];
B_3 = [1;P_2/P_1;0];
a_3 = A_3\B_3;

A_4 = [1,1,1,1;n_2,n_2^2,n_2^3,n_2^4;0,1,2*n_2,3*n_2^2;1,2,3,4];
B_4 = [1;P_2/P_1;0;0];
a_4 = A_4\B_4;

b_puis = (w_1/w_2 -1)/(P_1/P_2-1);
a_puis = (P_1-P_2)/(abs(w_1-w_2)^(b_puis));

w=0:1:7000*2*pi/60;
v=0:1:length(w)-1;
P3=P_1*(a_3(1)*(w/w_1)+a_3(2)*(w/w_1).^2+a_3(3)*(w/w_1).^3);
P4=P_1*(a_4(1)*(w/w_1)+a_4(2)*(w/w_1).^2+a_4(3)*(w/w_1).^3+a_4(4)*(w/w_1).^4);
PP=P_1-a_puis*abs(w-w_1).^b_puis;
```

# MATLAB Code

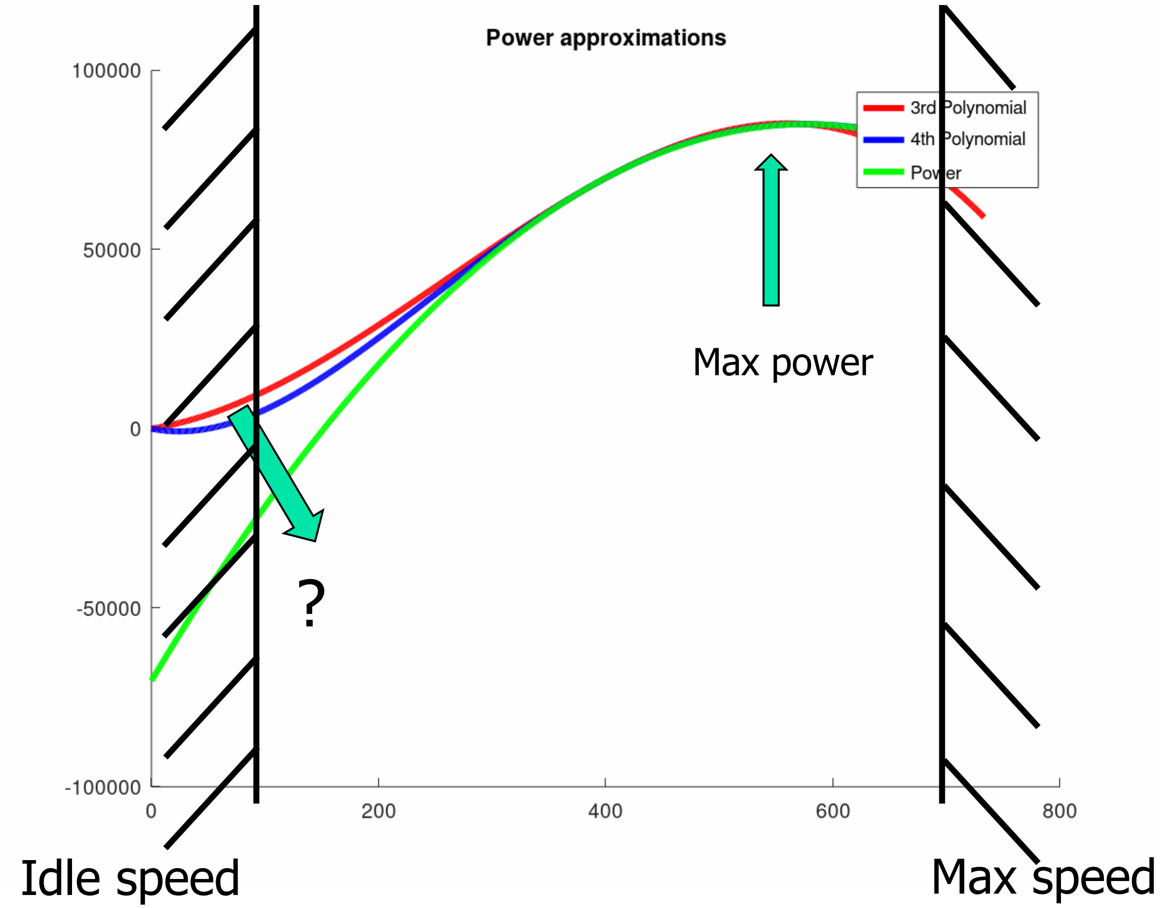
```
w=0:1:7000*2*pi/60;
v=0:1:length(w)-1;
P3=P_1*(a_3(1)*(w/w_1)+a_3(2)*(w/w_1).^2+a_3(3)*(w/w_1).^3);
P4=P_1*(a_4(1)*(w/w_1)+a_4(2)*(w/w_1).^2+a_4(3)*(w/w_1).^3+a_4(4)*(w/w_1).^4);
PP=P_1-a_puis*abs(w-w_1).^b_puis;


figure
hold on
plot(v,P3,'LineWidth',3,'Color','red')
plot(v,P4,'LineWidth',3,'Color','blue')
plot(v,PP,'LineWidth',3,'Color','green')
title('Power approximations')
legend('3rd Polynomial', '4th Polynomial', 'Power')
hold off

figure
hold on
plot(v,P3./w,'LineWidth',3,'Color','red')
plot(v,P4./w,'LineWidth',3,'Color','blue')
plot(v,PP./w,'LineWidth',3,'Color','green')
ylim([0 200])
title('Torque approximations')
legend('3rd Polynomial', '4th Polynomial', 'Power')
hold off
```

# Comparison of Power approximations

- Power approximations



# Comparison of Torque approximations

- Torque approximations