Vehicle Performance

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Exercises
Performance criteria

Max speed and gradeability

Exercise 1 : Max speed

- For the vehicle given below, determine an approximation of the maximum power curve of the engine as a function of engine speed.
- Determine the maximum speed of the vehicle on a horizontal road and in the absence of wind if maximum engine power is available.
- Calculate the corresponding optimum reduction ratio (gearbox + differential).
- Determine the maximum speed if the actual reduction ratio of the gearbox and axle is i_b=0.76 and i_d=3.71 respectively.

Exercise 1 : Data

- Vehicle data:
  - $m = 1\ 700$ kg
  - $S = 2.1$ m2
  - $Cx = 0.33$
  - $\rho = 1.2$
  - $f = 0.01 + 4.54 * 10^{-7} V^2$
    - V in [m/s]
  - $m_e = m*(1.04+0.0025\ i^2)$
  - $m_{driver} = 75$ kg
  - $R_e = 0.3$
  - Transmission efficiency= 0.92

- Engine data
  - $P_{max} = 85$ kW @ 5500 min-1
  - $C_{max} = 175$ Nm @ 3750 min-1

Exercise 1 : Acceleration

- Using the optimum reduction ratio calculated in point 2, calculate the time required for the vehicle to accelerate from 90 km/h to 120 km/h.
  - First assume that the vehicle acceleration is constant in this speed range and equal to the vehicle acceleration at 105km/h.
  - Then check the response obtained by the integration method seen in the course.

Approximation of Engine Curves

- Power type approximation

$$\mathcal{P} = \mathcal{P}_1 - A\,|\omega_1 - \omega|^b$$

$$\omega_1 = N_1\,\frac{2\pi}{60} = 5500\,\frac{\pi}{30} = 575,95\,rad/s$$

$$\omega_2 = N_2\,\frac{\pi}{30} = 3750\,\frac{\pi}{30} = 392,69\,rad/s$$

$$\mathcal{P}_1 = 85.000\,W$$

$$\mathcal{P}_2 = \mathcal{C}_2\,.\,\omega_2 = 68.722\,W$$

$$b = \frac{\omega_1/\omega_2 - 1}{\mathcal{P}_1/\mathcal{P}_2 - 1} = 1,970$$

$$A = \frac{\mathcal{P}_1 - \mathcal{P}_2}{|\omega_1 - \omega_2|^b} = 0,5667$$

Approximation of Engine Curves

- Polynomial approximation (order 3, cubic)

$$\mathcal{P} = \mathcal{P}_1 \left[a_0 + a_1 \left(\frac{\omega}{\omega_1}\right) + a_2 \left(\frac{\omega}{\omega_1}\right)^2 + a_3 \left(\frac{\omega}{\omega_1}\right)^3 \right]$$

$$\mathcal{P}(0) = 0 \qquad a_0 = 0$$

$$\mathcal{P}(\omega_1) = \mathcal{P}_1 \qquad a_0 + a_1 + a_2 + a_3 = 1$$

$$\mathcal{P}(\omega_2) = \mathcal{P}_2 \qquad a_0 + a_1(\omega_2/\omega_1) + a_2(\omega_2/\omega_1)^2 + a_3(\omega_2/\omega_1)^3 = \mathcal{P}_2/\mathcal{P}_1$$

$$\left.\frac{d}{d\omega}\frac{\mathcal{P}(\omega)}{\omega}\right|_{\omega_2} = 0 \qquad -a_0\frac{1}{\omega_2^2} + a_1\,0 + a_2\frac{1}{\omega_1^2} + a_3\frac{2\omega_2}{\omega_1^3} = 0$$

- Solve

$$a_0 = 0$$
$$a_1 = 0,3326$$
$$a_2 = 2,5026$$
$$a_3 = -1,8352$$

Max Top Speed

- Max top speed is solution

$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

$$A = m\, g\, f_0 \cos\theta$$

$$B = 1/2\, \rho\, S\, C_x + m\, g\, f_2 \cos\theta$$

- It comes

$$\eta \mathcal{P}_{max} = 0,92 \times 85000 = 78.200\, W$$

$$A = 1700 \times 9,81 \times 0,01 \times 1. = 166,77$$

$$B = 0,5 \times 1,22 \times 2,1 \times 0,33 + 1700 \times 9,81 \times 4,54\, 10^{-7} \times 1. = 0,4303$$

Max Top Speed

- Max top speed is solution

$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

- Solution using Picard iterative scheme

$$v^{(0)} = 0$$

$$v^{(n+1)} = \left(\frac{\eta \mathcal{P}_{max} - Av^{(n)}}{B} \right)^{1/3}$$

- It comes

$$v^{(0)} = 0,0$$

$$v^{(1)} = 56,643 \,\mathrm{m/s}$$

$$v^{(2)} = 54,264 \,\mathrm{m/s}$$

$$v^{(3)} = 54,268 \,\mathrm{m/s}$$

$$v^{(4)} = 54,363 \,\mathrm{m/s}$$

$$v^{(5)} = 54,363 \,\mathrm{m/s}$$

$$v_{max}^{max} = 54,363 \,\mathrm{m/s} = 195,70 \,\mathrm{km/h}$$

Max Top Speed

- Max top speed occurs for nominal power

$$v_{max}^{max} = 54,363 \text{ m/s} = 195,70 \text{ km/h}$$

$$\omega_m = \omega_1 = 575,959 \text{ rad/s}$$

- So transmission length and optimal reduction ratio are given by

$$\left(\frac{R}{i}\right)^* = \frac{v_{max}^{max}}{\omega_{nom}} = 0,0944 \text{ m} \qquad i^\star = R_e \frac{\omega_{nom}}{v_{max}^{max}} = 3,178$$

- If final drive is 3,71

$$i_d = 3,71$$

$$i_b^\star = 0,8567$$

Top speed on actual gear ratio

- Let's now compute the top speed for the actual reduction ratio

$$\begin{aligned} i_d &= 0,76 \\ i_b &= 3,71 \end{aligned} \qquad i = i_b\, i_d = 3,71 \times 0,76 = 2,82$$

- The actual gear ratio is smaller than the optimized one and the transmission length is longer leading to economical ratio

- The top speed is solution of the coupled equations

$$\begin{cases} \eta\, \mathcal{P}(\omega) = \mathcal{P}_{rés} = A v_{max} + B v_{max}^3 \\ \omega = v\, \frac{\bar{i}}{R_e} \end{cases}$$

- or

$$\mathcal{P}_{RES} = A v_{max} + B v_{max}^3 = \eta\, \mathcal{P}(\frac{\bar{i}}{R} v_{max})$$

Top speed on actual gear ratio

- The top speed is solution of the coupled equations

$$\begin{cases} \eta\,\mathcal{P}(\omega) = \mathcal{P}_{rés} = Av_{max} + Bv_{max}^3 \\ \omega = v\,\frac{\bar{i}}{R_e} \end{cases}$$

- Which is solved the iterative scheme

$$v^{(0)} = v_{max}^{max}$$

$$\omega^{(k)} = v^{(k)}\,\frac{\bar{i}}{R_e}$$

$$\mathcal{P}^{(k)} = \eta\mathcal{P}(\omega^{(k)})$$

$$v^{(k+1)} = \left(\frac{\mathcal{P}^{(k)} - Av^{(k)}}{B}\right)^{1/3}$$

Top speed on actual gear ratio

- The iteration history is the following

$$v^{(0)} = v_{max}^{max} = 54,363\,\mathrm{m/s}$$

$$v^{(0)} = 54,363\,\mathrm{m/s} \quad \omega^{(0)} = \omega_{nom} = 575,95\,\mathrm{rad/s} \quad \eta\mathcal{P}^{(0)} = 78200\,W$$
$$v^{(1)} = 53,848\,\mathrm{m/s} \quad \omega^{(1)} = v^{(1)}\frac{i}{R_e} = 506,17\,\mathrm{rad/s} \quad \eta\mathcal{P}^{(1)} = 76256\,W$$
$$v^{(2)} = 53,792\,\mathrm{m/s} \quad \omega^{(2)} = v^{(2)}\frac{i}{R_e} = 505,64\,\mathrm{rad/s} \quad \eta\mathcal{P}^{(2)} = 75961\,W$$
$$v^{(3)} = 53,786\,\mathrm{m/s} \quad \omega^{(3)} = v^{(3)}\frac{i}{R_e} = 505,59\,\mathrm{rad/s} \quad \eta\mathcal{P}^{(3)} = 75928\,W$$
$$v^{(4)} = 53,785\,\mathrm{m/s} \quad \omega^{(4)} = v^{(4)}\frac{i}{R_e} = 505,58\,\mathrm{rad/s} \quad \eta\mathcal{P}^{(4)} = 75924\,W$$
$$v^{(5)} = 53,785\,\mathrm{m/s} \quad \omega^{(5)} = v^{(5)}\frac{i}{R_e} = 505,58\,\mathrm{rad/s} \quad \eta\mathcal{P}^{(5)} = 75923\,W$$

$$v_{max} = 53,785\,\mathrm{m/s} = 193,62\,\mathrm{km/h}$$

Acceleration 90-120 km/h

- Newton's equation

$$m_e \frac{dv}{dt} = F_w - \sum F_{RES} = F_{net}(v)$$

- Equivalent mass

$$m_{\mathrm{e}} = \gamma_m \, m = m \left(1,04 \; + \; 0,0025 \, i^2\right)$$

$$m_{\mathrm{e}} = 1700 \; \left(1,04 \; + \; 0,0025 \;\, 2,8196^2\right) = 1801,78 \, \mathrm{kg}$$

$$m_t = m_e + 75 = 1876,78 \, \mathrm{kg}$$

- The net force is

$$F_{net}(v) \; = \; F_w(v) \; - \; F_{RES}(v)$$

Acceleration 90-120 km/h

- To evaluation the acceleration time and the distance run since the acceleration start, one has to evaluation the following integrals:

  - Acceleration time

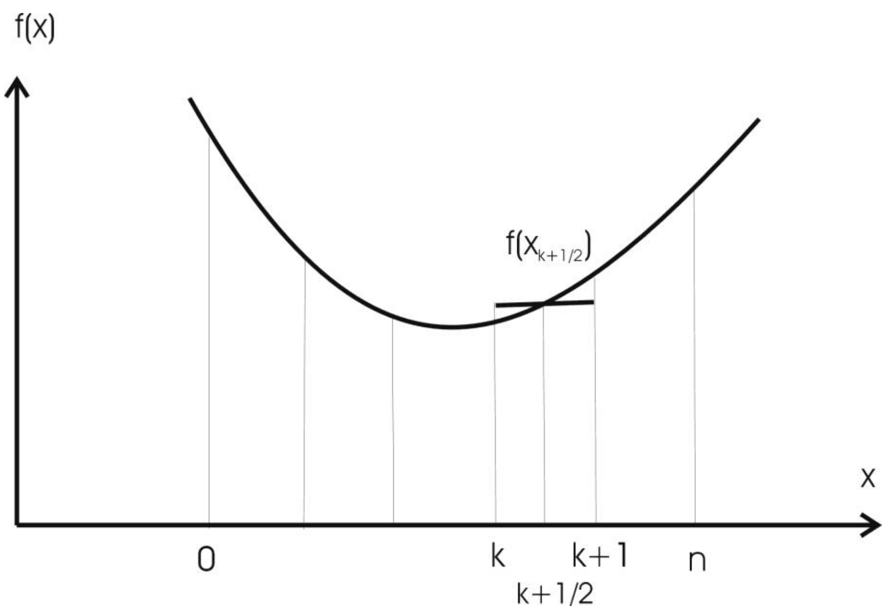
$$\Delta t_{V_1 \to V_2} = \int_{V_1}^{V_2} m_{\mathrm{e}} \frac{dv}{F_{net}(v)}$$

  - Acceleration distance

$$\Delta x_{V_1 \to V_2} = \int_{V_1}^{V_2} m_e \frac{v\, dv}{F_{net}(v)}$$

Acceleration 90-120 km/h

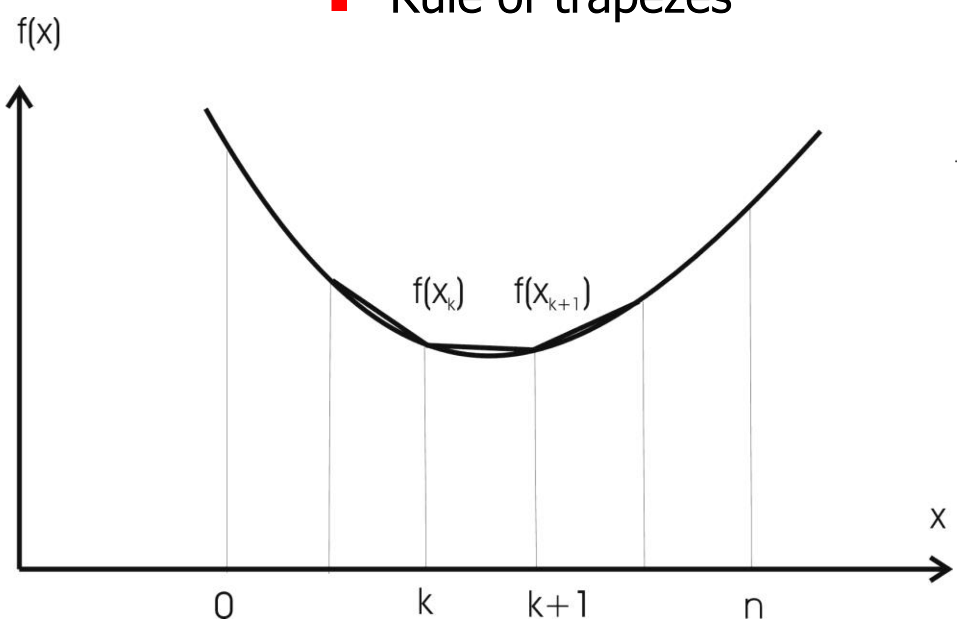
- Numerical evaluation of integrals:
  - Rule of rectangles



$$I = \int_{x_0}^{x_n} f(x)\, dx \simeq \sum_{k=0}^{n} f(x_{k+1/2})\, \Delta x$$

Acceleration 90-120 km/h

- Numerical evaluation of integrals:
  - Rule of trapezes



$$
\begin{aligned}
I &= \int_{x_0}^{x_n} f(x)\,dx \\
&\simeq \sum_{k=0}^{n} \frac{f(x_k)+f(x_{k+1})}{2}\,\Delta x \\
&\simeq \left\{ \frac{f(x_0)}{2} + \left[\sum_{k=1}^{n-1} f(x_k)\right] + \frac{f(x_n)}{2} \right\} \Delta x
\end{aligned}
$$

Acceleration 90-120 km/h

- Tractive forces are determined using the following procedure that is rather computationally effective:
  - Evaluate v in m/s and then the corresponding engine rotation speed

$$V\ [\mathrm{km/h}] \ \rightarrow \ v\ [\mathrm{m/s}] = V/3,6$$

$$\omega_m = v\,\frac{i}{R_e}\ [\mathrm{rad/s}]$$

  - Evaluate the engine power

$$\begin{aligned}\mathcal{P}_m(\omega_m) &= \mathcal{P}_1 - A\,|\omega_1 - \omega_m|^b \\ &= 85000 - 0,567\,|575,959 - \omega_m|^{1,97}\end{aligned}$$

  - Calculate the tractive force as the power at wheels divided by the vehicle speed

$$F_w = \eta\,\mathcal{P}_m/v$$

Acceleration 90-120 km/h

- The road resistance forces are then evaluated

$$F_{RES} = A \, + \, B\, v^2 = 166,77 \, + \, 0,4303\, v^2$$

- And the net force follows from the two evaluations

$$F_{net}(v) \; = \; F_w(v) \; - \; F_{RES}(v)$$

- To evaluate the acceleration time, one has to evaluation the following integration

$$\begin{aligned} \Delta t_{V_1 \rightarrow V_2} \; &= \; m_{\mathrm{e}} \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)} \\ &\simeq \; \left\{ \frac{m_{\mathrm{e}}}{2\, F_{net}(v_0)} + \left[\sum_{k=1}^{n-1} \frac{m_{\mathrm{e}}}{F_{net}(v_k)} \right] + \frac{m_{\mathrm{e}}}{2\, F_{net}(v_1)} \right\} \Delta v \end{aligned}$$

Acceleration 90-120 km/h

- While to evaluate the acceleration distance, one has to evaluation the following integration

$$\begin{aligned}\Delta x_{V_1 \to V_2} &= m_{\mathrm{e}} \int_{V_1}^{V_2} \frac{v\, dv}{F_{net}(v)} \\ &\simeq \left\{ \frac{m_{\mathrm{e}} v_0}{2\, F_{net}(v_0)} + \left[\sum_{k=1}^{n-1} \frac{m_{\mathrm{e}} v_k}{F_{net}(v_k)} \right] + \frac{m_{\mathrm{e}} v_n}{2\, F_{net}(v_n)} \right\} \Delta v\end{aligned}$$

Acceleration 90-120 km/h

- To evaluate these quantities in a systematic way, one can build the following tables

V(km/h)	v(m/s)	omega_m	P_m	F_w	F_res	F_net
90,00	25,00	264,83	38817,04	1428,47	435,71	992,76
100,00	27,78	294,26	47027,58	1557,55	498,79	1058,76
110,00	30,56	323,69	54446,08	1639,32	568,52	1070,80
120,00	33,33	353,11	61069,93	1685,53	644,88	1040,65

Acceleration 90-120 km/h

- It comes the acceleration time and distance

	me/Fnet	me*v/Fnet
	1,81493075	45,3732687
	1,70178967	47,27193522
	1,68264892	51,41427243
	1,73141086	57,71369521
Total	14,33	417,30
	Time	Distance

$$\Delta t_{V_1 \to V_2} = 14,33\ s$$

$$\Delta x_{V_1 \to V_2} = 417,30\ m$$

Exercise 2 : Data

- Vehicle data:
  - m = 1 810 kg
  - C_x = 0.31 – SC_x = 0.69 m²
  - ρ= 1.2 kg/m³
  - f = 0.01
  - m_e= m*(1.04+0.0025 i^2)
  - m_{driver}= 75 kg

- Engine data
  - P_{max}= 225 kW @ 5600 tr.min-1
  - C_{max}= 460 Nm @ [2700-4250] tr.min-1
  - (Suggestion: take C_{max} =460 Nm @ 3475 tr/min)

- Missing data to be estimated by yourself

Exercise 2 : Data

- Transmission:
  - Front engine, longitudinal mounting
  - Rear driven wheels
  - Fixed gear ratio i=3.06:1

1	2	3	4	5	6	7	R
4.38:1	2.86:1	1.92:1	1.37:1	1.00:1	0.82:1	0.73:1	3.42:1

  - Transmission length (km/h per 1000 rpm)

1	2	3	4	5	6	7	R
8.9	13.6	20.3	28.4	38.9	47.5	53.4	11.4

- Missing data to be estimated by yourself
  - R_e= ?
  - Transmission efficiency= ?

Exercise 2 : Max speed

- For the vehicle given below, determine an approximation of the maximum power curve and polynomial fit of order 3 of the engine as a function of engine rotation speed.
- Determine the maximum speed of the vehicle on a horizontal road and in the absence of wind if maximum engine power is available.
- Calculate the corresponding optimum reduction ratio (gearbox + differential).
- Determine the maximum speed when the 5^{th} gear ratio is engaged. What is the corresponding rotation speed of the engine?

Exercise 2 : Max slope - Acceleration

- Compute the maximum slope on the first gear ratio.
- Compute the maximum slope on the 5th gear ratio and the corresponding speed.
- Compute the maximum slope at 100 km/h on the 7th gear ratio.

- Compute the acceleration time from 60 to 90 km/h on the 5th gear ratio.
- Compute the acceleration time to cover 400 m from 40 km/h on the 4th gear ratio.