OPTIMIZATION IN ENGINEERING

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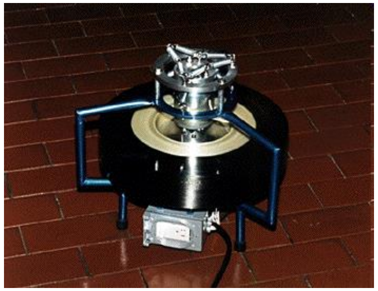
CONTEXT

- CONTEXT
  - Enhancing products performance and their fitness to customers' expectations
  - Shortening time-to-market release of new products
  - Saving mass and energy…

CONTEXT

- Engineering design becomes a more and more complex task.
- OPTIMISATION
  - Relieving designers from the tedious work of re-analysis tasks management
  - Accelerating design process
  - Taking into account a large number of design parameters and design constraints
  - Finding solutions to non intuitive problems with conflicting constraints
  - Generating new and innovative solutions

CONTEXT

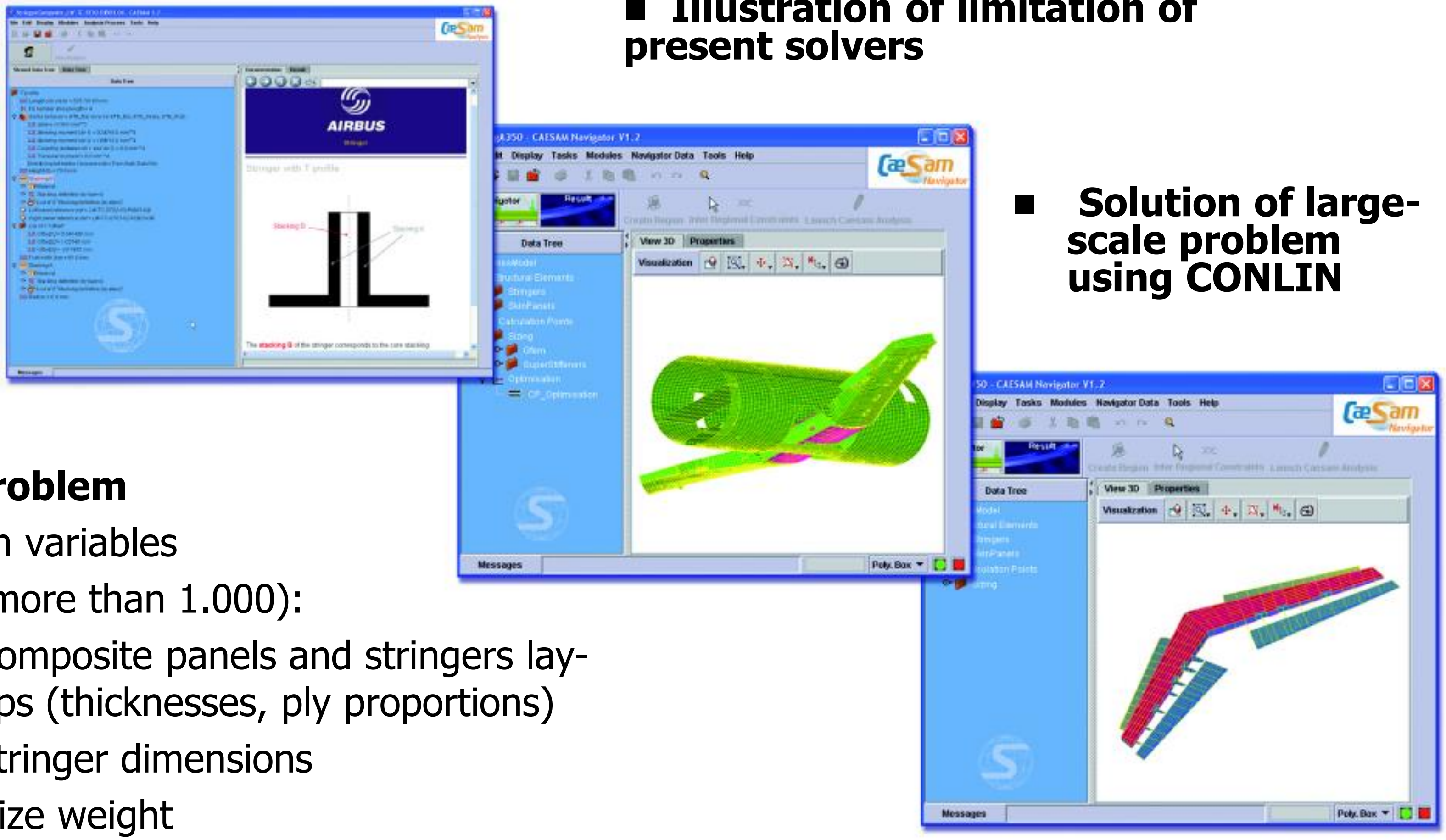
- SS Great Britain (1843)
  - First ship to be built with an iron hull
  - Parts designed and fixed together according to the available technology at the time (wood technology)
  - If you don't know it is metal you would think it is wood!

CONTEXT

- Boeing 787
  - Composite structure BUT…
  - True potential is not fully explored
  - Composite structures are almost direct replicas of metallic structures
  - Some of the issues
    - Repair
    - Failure modes
    - Systems interaction
  - Weight saving =< 5% in current use

CONTEXT

- **Illustration of limitation of present solvers**
- **Solution of large-scale problem using CONLIN**



- **Design problem**
  - Design variables
    - (more than 1.000):
    - Composite panels and stringers lay-ups (thicknesses, ply proportions)
    - Stringer dimensions
  - Minimize weight
  - Constraints (more than 100.000)
    - Reserve factors
    - Buckling, reparability, …

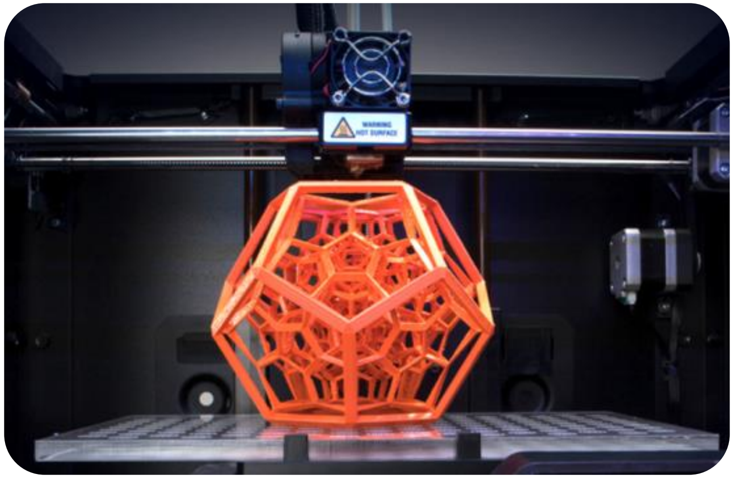
With courtesy by Samtech and Airbus Industries

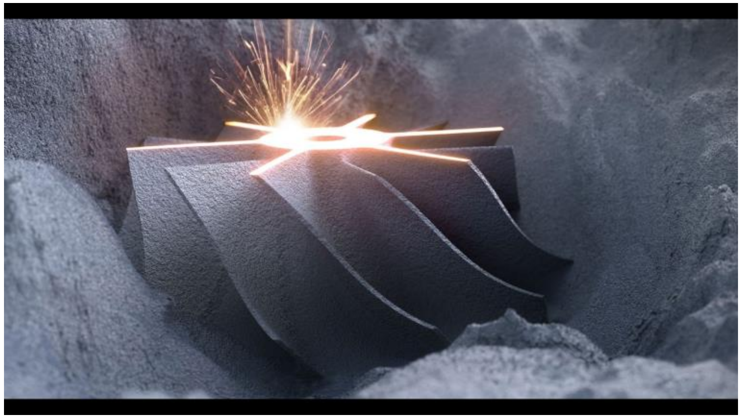
CONTEXT

- Additive manufacturing enables the fabrication of parts with complex geometries

- Fully taking advantage of additive manufacturing (AM) requires to redesign the components

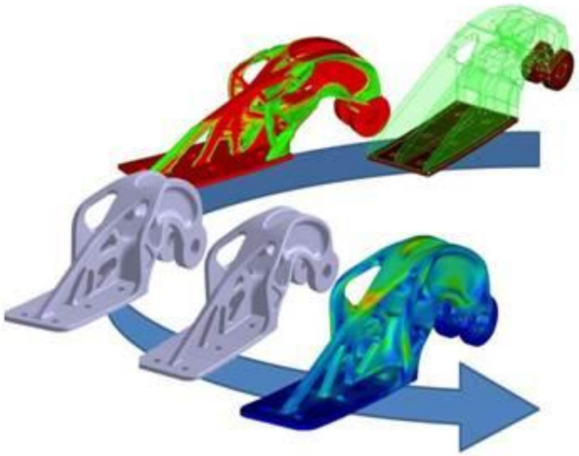
- There is less manufacturing constraints but there are still some other ones, different ones!

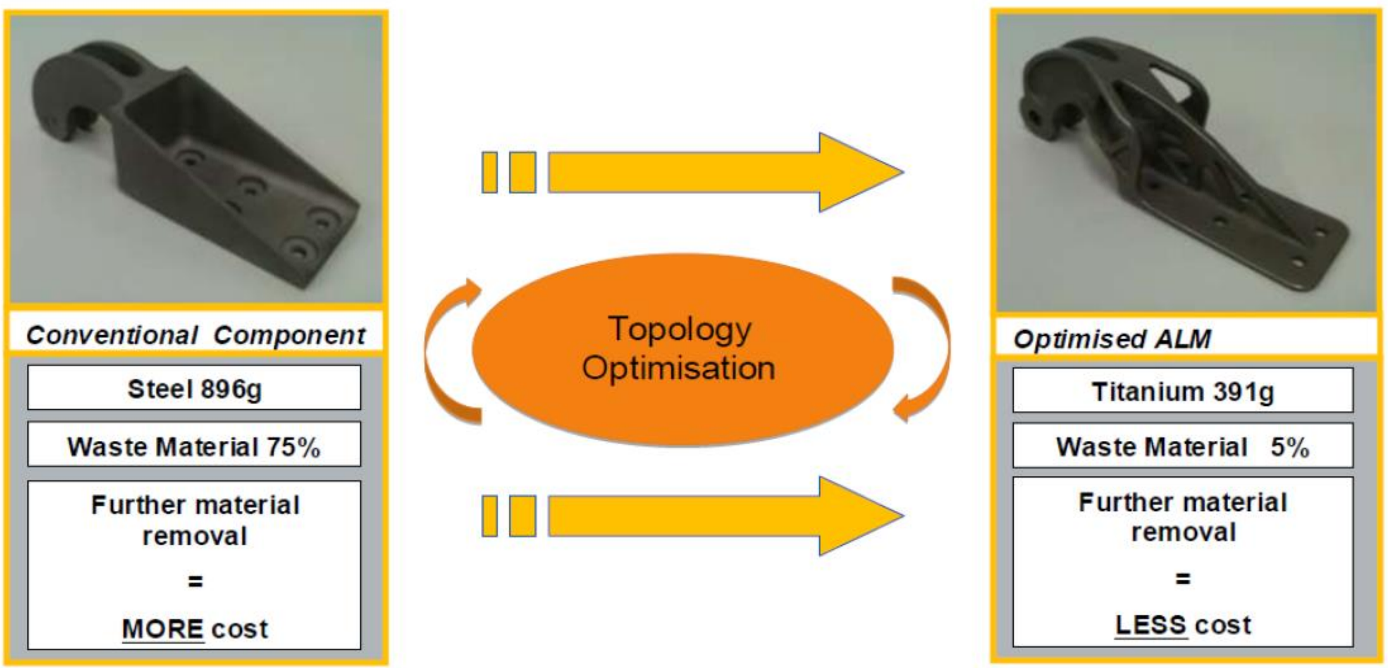




CONTEXT

- Topology optimization allows to discover innovative concepts fully exploiting the liberty offered by additive manufacturing.





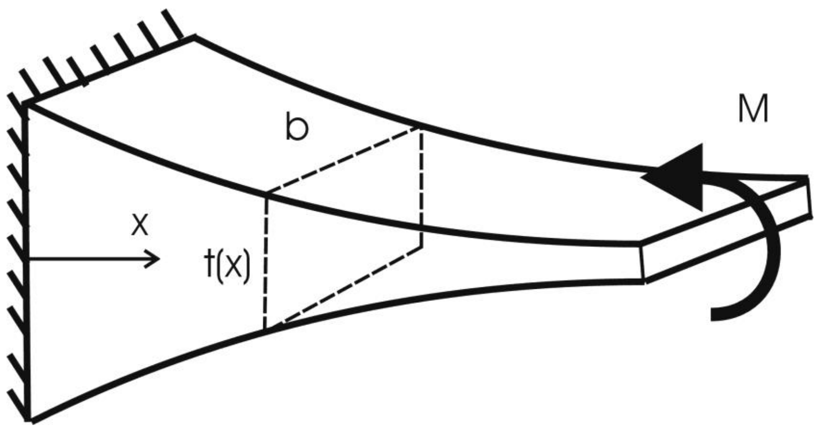
A BRIEF HISTORICAL PERSPECTIVE OF STRUCTURAL OPTIMIZATION

Calculus of variations: 18ies century → …

- From the 18ies century famous mathematicians have developed the Calculus of Variations.
- The work is essentially based on developing analytical solutions for various problems → analytical solution to structural optimization problems.
- The design variables are functions that define the structural properties. The objective functions and the design restrictions are function of the design variable functions.
- The calculus of variations also provides the necessary optimality conditions as (partial) differential equations, i.e. the Euler Lagrange Equations.
- Example: Optimization of the cross section of a bar subject a continuous traction

Calculus of variations: 18ies century → …

- Example: Optimization of the cross section of a beam subject a bending moment at its free end



  - Analytical solution $t(x) = c\sqrt[4]{x}$

Optimization with discrete systems: 1960ies →…

- Later on during the 20ies century, the mathematical programming or numerical optimization was developed in parallel to the soar of numerical solutions in mechanics using finite element method (FEM) or boundary element methods (BEM).

- Application of numerical optimization to structural design can be traced back to the pioneer work by Schmidt (1960).
  - L. Schmit. Structural design by systematic synthesis. In 2nd ASCE Conf Electronic Computing (Pittsburgh, PA), pages 139–149, 1960

- With computational mechanics and the FEM or BEM methods, the unknowns of the physical problem (mechanical, electrical, thermal...) are discrete values of the displacement and stress fields (e.g. nodal values).

- In a similar way, the optimization problem is stated in terms of a finite number of design parameters resulting from a natural or discretization process of the design functions.

Optimization with discrete systems: 1960ies →…

- Numerical optimization or mathematical programming is the branch of mathematics dealing with optimization problems with a finite number of variables.

- The variables are generally collected in a finite dimension vector in R^n.

$$x_i \ (i = 1 \dots n) \longrightarrow \mathbf{x} = \{x_1 \, x_2 \dots x_n\}^T$$

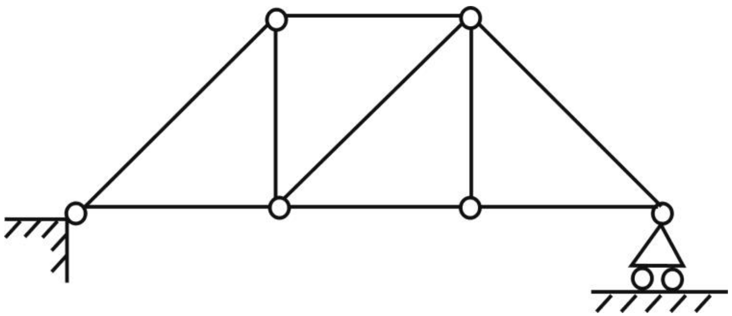
- The objective functions and the design restrictions are implicit or explicit functions which depend on the discrete state variables **q** and of the discrete design variables **x**.

$$\mathbf{x} \in \mathbb{R}^n, \ \mathbf{q} \in \mathbf{R}^p \longrightarrow f(\mathbf{x}, \mathbf{q})$$

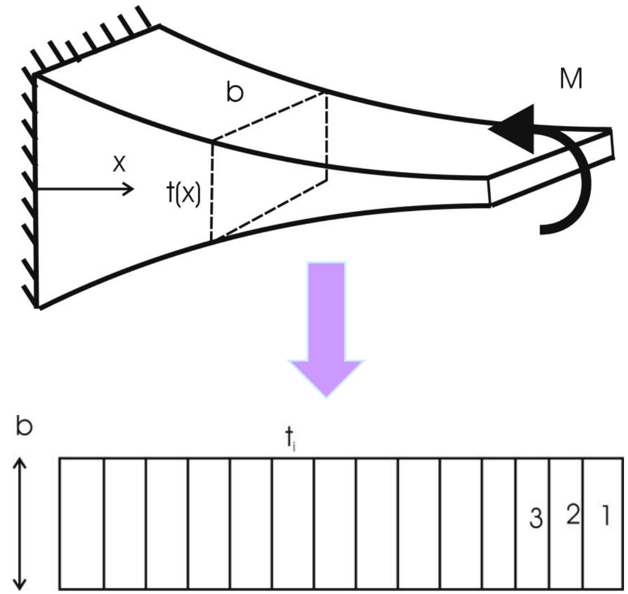
- Mathematical programming provides the necessary optimality conditions of the minimization / maximization problem as algebraic equations.

Optimization with discrete systems: 1960ies →…

- Typical simple examples of structural optimization are mass minimization of truss structures. The state and design variables are naturally discrete, respectively the nodal displacements and the bar cross sections. One can also discretize continuous system into a finite dimension vector of variables.



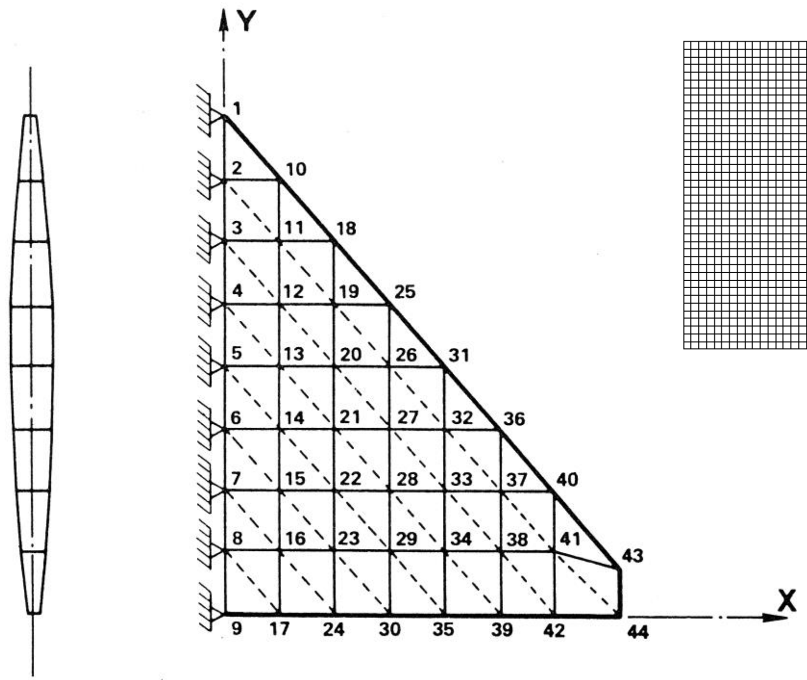
Truss: naturally discrete value problem



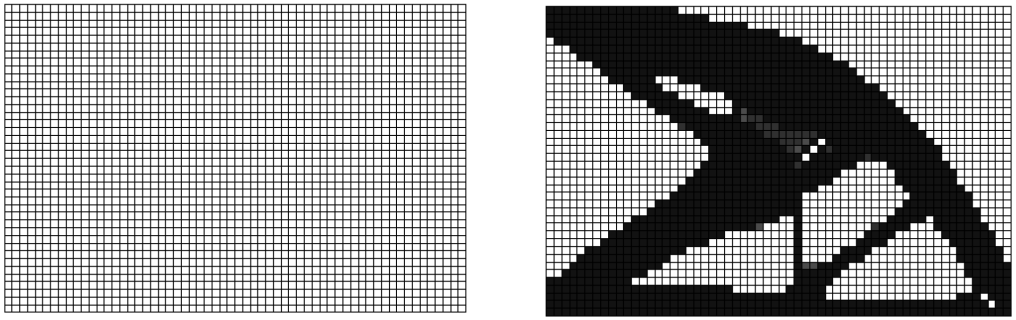
Discretization of the beam long its span

Optimization with discrete systems: 1960ies →…

- Later, many problems which are discretized because of numerical solution have also been investigated. The state and design variables are element properties or groups of element properties.



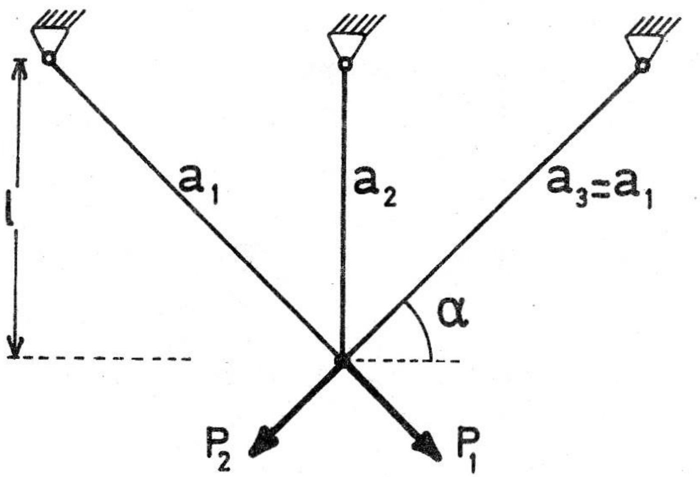
Discretization of the mass minimization problem of a wing



Discretization of the density distribution in topology optimization

Example: three-bar-truss problem

- Typical example of finite dimension optimization problem



module d'Young	$E = 7000\ kg/mm^2$
masse volumique	$\rho = 2.8\ 10^{-6}\ kg/mm^3$
tension maximale admissible:	
traction	$\bar{\sigma} = 50\ kg/mm^2$
compression	$\underline{\sigma} = -25\ kg/mm^2$
hauteur du treillis	$l = 500\ mm$
angle des barres	$\alpha = 45°$

cas de charge 1 : $P_1 = 1000\ kg$

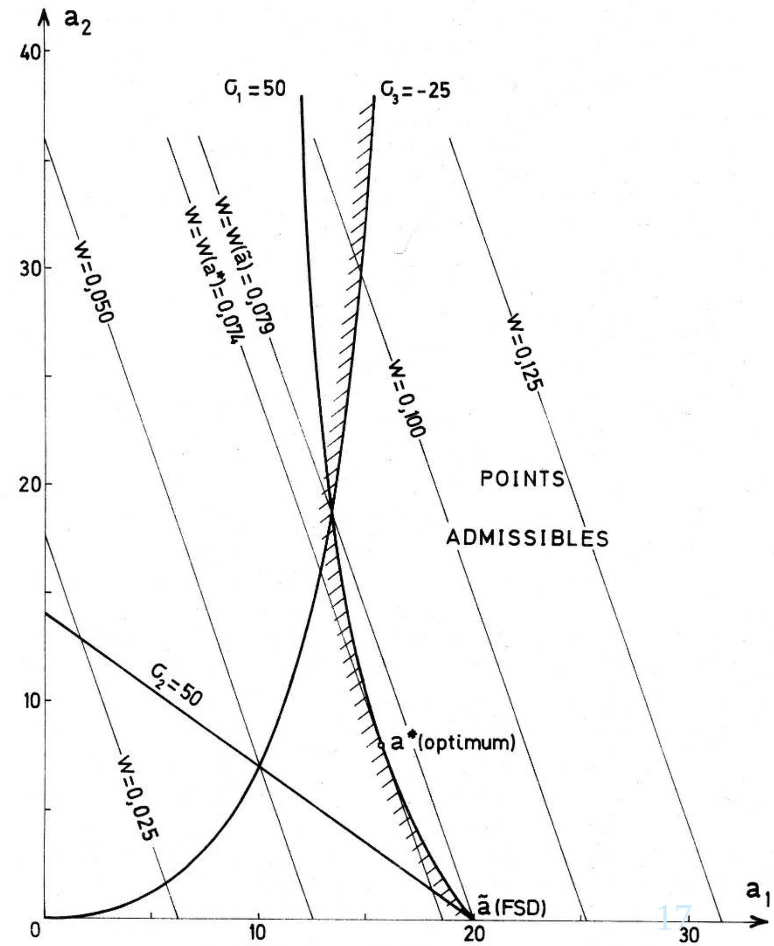
// // 2 : $P_2 = 1000\ kg$

$\Big\}$ (symetriques)

Example: three-bar-truss problem

- It is possible to express explicitly the functions and the problem writes

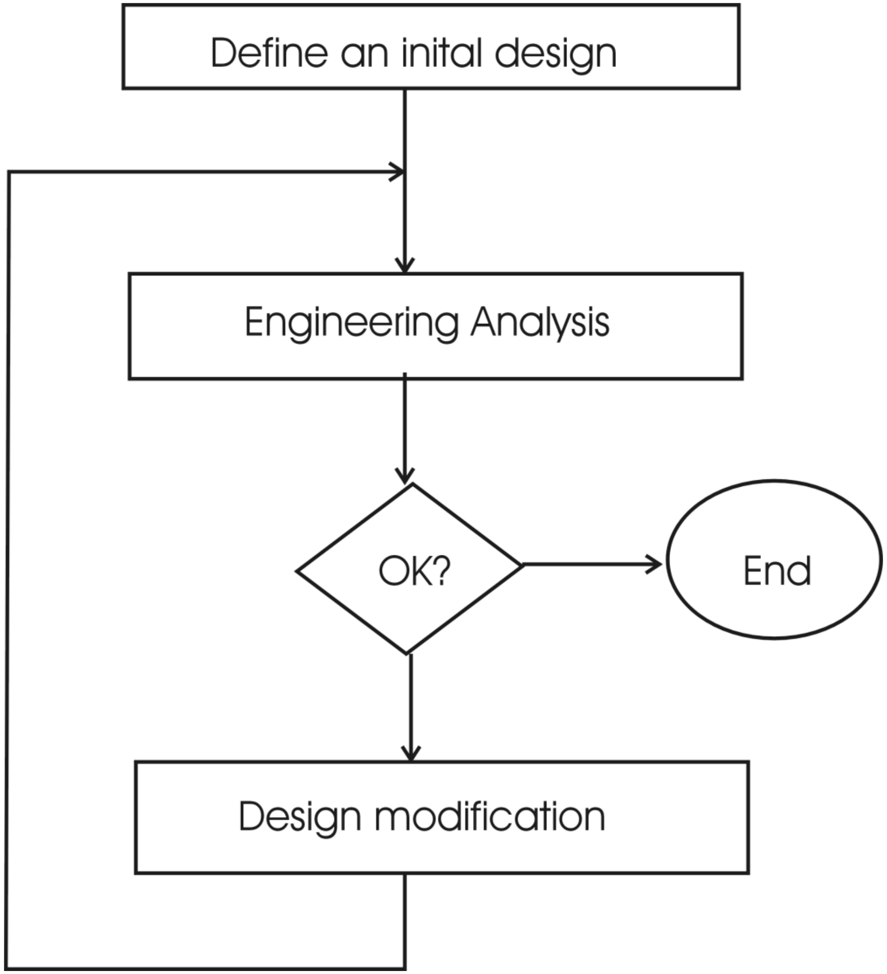
$$
\begin{aligned}
\min \quad & W(x) = \rho l (2\sqrt{2} a_1 + a_2) \\
\text{s.t.} \quad & \sigma_1 = \frac{P}{\sqrt{2} a_1} \frac{\sqrt{2} a_1 + a_2}{a_1 + \sqrt{2} a_2} \leq 50 \\
& \sigma_2 = \frac{P}{\sqrt{2} a_1 + a_2} \leq 50 \\
& -\sigma_3 = \frac{P}{\sqrt{2} a_1} \frac{a_2}{\sqrt{2} a_1 + a_2} \leq 25 \\
& a_1 \geq 0 \quad a_2 \geq 0
\end{aligned}
$$



ENGINEERING DESIGN: AN OPTIMIZATION PROBLEM

OPTIMIZATION LOOP IN ENGINEERING

- Design process is intrinsically an iterative process



Define an initial design
Determine failure modes
Select design variables
Devise an appropriate analysis scheme

Closed form solution
Experimental
Numerical e.g. finite elements

Design criteria are satisfied?

Change the design variables

OPTIMIZATION LOOP IN ENGINEERING

- Design is a complex process mixing various skills and resulting from a compromise between various factors. Some of them are perfectly rationale and can be expressed in numerical criteria. Some other considerations are fuzzier typically inspired by experience, sometimes intuition, subject to some aesthetic or ethic considerations.

- With the soar of computers in engineering, a growing part of the design task is carried out by mathematical modeling and computer simulation.

- A lot of efforts have been devoted to identify in the design process the tasks which can be precisely described by mathematical modelling in order to take advantage of the growing capability of computers to predict more and more accurately the complex behavior of large problems even before having built physical prototypes.
  - ➔ all digital design concept

OPTIMIZATION LOOP IN ENGINEERING

- TOWARDS A "COMPUTER AIDED DESIGN" = ALL DIGITAL ENGINEERING

  - Numerical **CAD model**: CATIA, SolidWorks, NX, Pro Engineer…
    - Parametric models + model updating

  - **Analysis**
    - Generally Finite Element model (80% = Linear Static Analysis)
    - Responses: Stresses, displacements, weight, eigenfrequency, buckling loads, etc.

  - **Redesign**: automatic and rational redesign tools
    - Optimality criteria
    - Mathematical programming algorithms
    - Heuristic algorithms (e.g. Genetic Algorithms)

OPTIMIZATION LOOP IN ENGINEERING

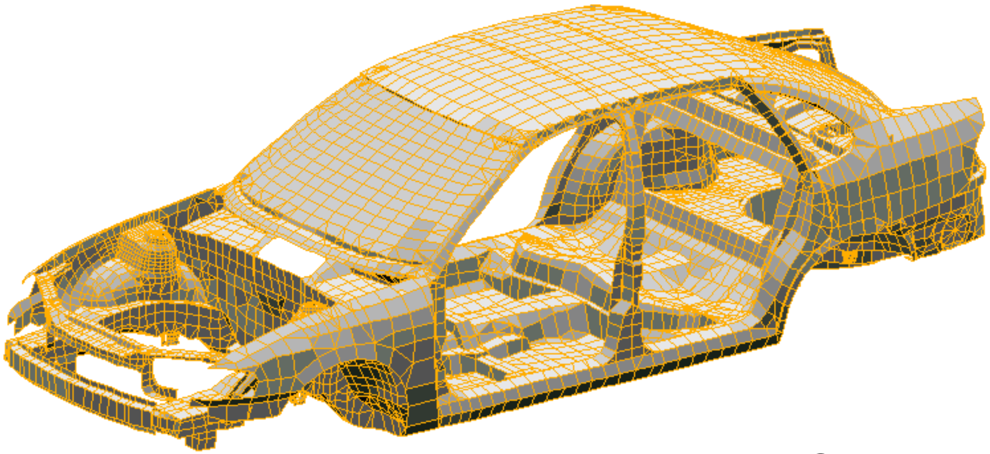
- For a long time, the focus was given to improve the analysis step using computer simulation. However design process can not be reduced to the sole analysis step.

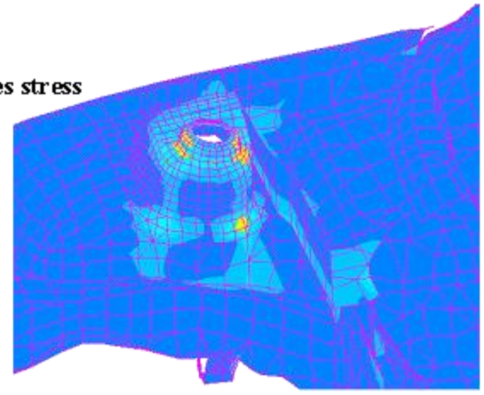
- The design process is even more concerned with modifying the concepts and details of the solutions.

- At least for its rational part, the redesign process can be cast as a mathematical problem belonging to the field of optimization.

- Typically engineers can translate their design problem into the structured formalism of an optimization problem which can later be solved efficiently using the rationale tools of numerical optimization.

ENGINEERING PROBLEM FORMULATION

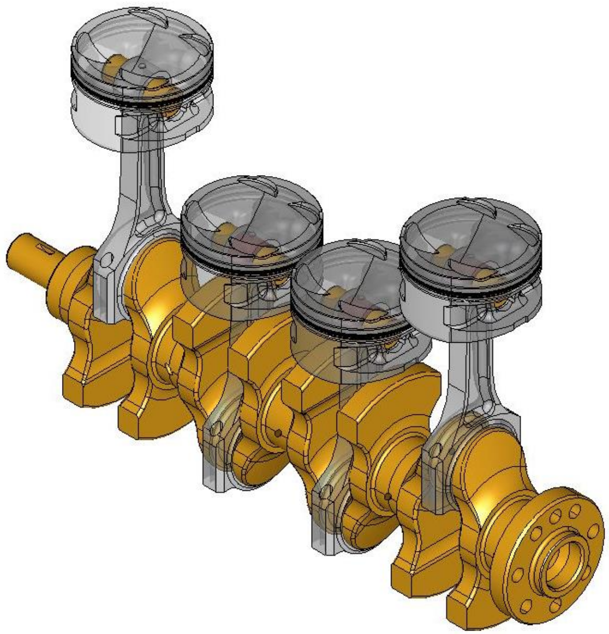


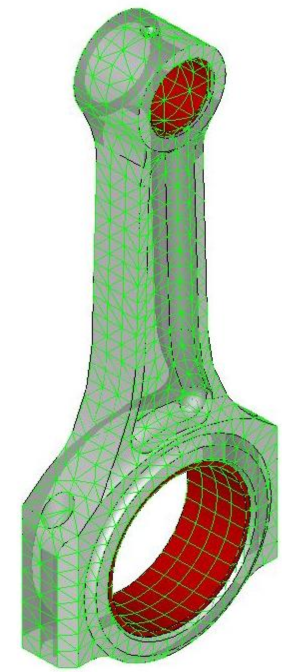


Courtesy of Samtech and PSA

- Design of a car body
  - Maximize stiffness against several load cases
  - Maximize natural frequencies of car body
  - Constrain stress criteria of various kinds including failure
  - Satisfy manufacturing and assembly constraints
  - Minimum weight and cost…

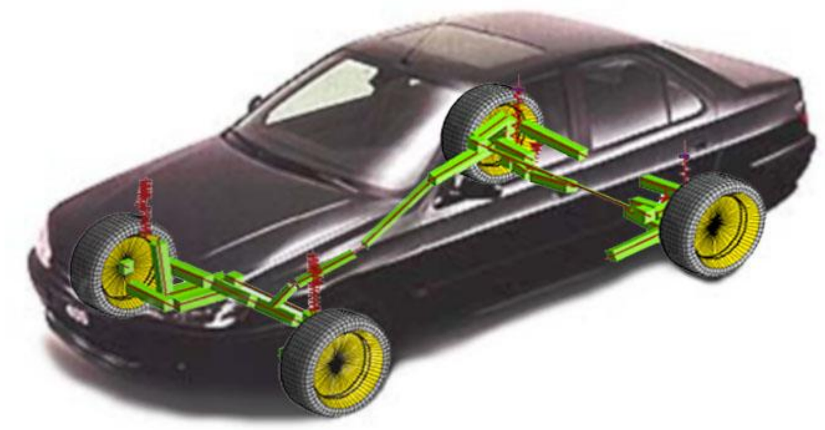
ENGINEERING PROBLEM FORMULATION

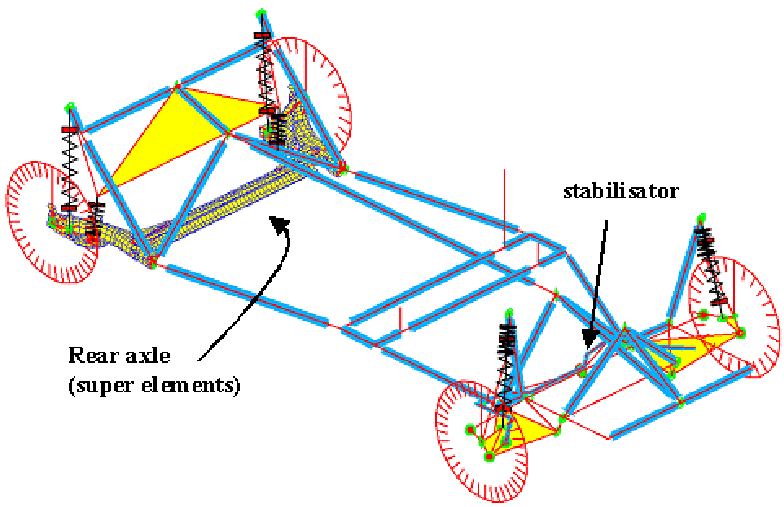




- Design of an engine connecting rod at 6000 rpm
  - Maximize dynamic response
  - Restriction on stress constraints and fatigue life
  - Constraint on maximum deformation, buckling load…
  - Calculate bold prestressing, lubrication, balancing…

ENGINEERING PROBLEM FORMULATION





- Vehicle dynamics
  - Verify suspension kinematics
  - Evaluate various vehicle stability tests
  - Verify the rolling behavior
  - Maximize both ride and comfort
  - Include active suspension and active safety systems…

ENGINEERING PROBLEM FORMULATION

- When being educated in the structural optimization, engineers can easily 'think' their design problem as minimizing / maximizing a selected objective function subject to some design constraints.

- The design variables are the parameters of the system that the design is ready to modify to improve his/her design.

- Engineers have to be able to identify adequately the quantity of interest that must be minimized/maximized while translating the design specifications into numerical criteria which are the design restrictions of the problem.

ENGINEERING PROBLEM FORMULATION

- The engineering design problem can be cast into a mathematical programming problem to be solved in an efficient and rational way

$$
\begin{array}{lccc}
\min\limits_{x_i} & f(x) & & \\
s.t. & g_j(x) \leq 0 & & j = 1 \ldots m \\
 & h_k(x) = 0 & & k = 1 \ldots l \\
 & \underline{x}_i \leq x_i \leq \bar{x}_i & & i = 1 \ldots n
\end{array}
$$

  - Enable solutions based on mathematical optimization methods
  - Mathematical background ➔ convergence guaranties
  - Standard and general approach
  - Open systems approach
  - Structural and multidisciplinary problems

ASSUMPTIONS AND DEFINITIONS

DEFINITIONS: Design variables

- **Design variables**: any idealized engineering system can be described by a finite set of quantities:
  - Material parameters (Young modulus…)
  - Dimensions (thickness, area, etc.)
  - Shape parameters (control points, dimensions, angles)
  - Layout of components (presence/absence of members, local density variables…).

- Some quantities are fixed: the prescribed parameters

- Some quantities can be modified to improve the design: the design variables

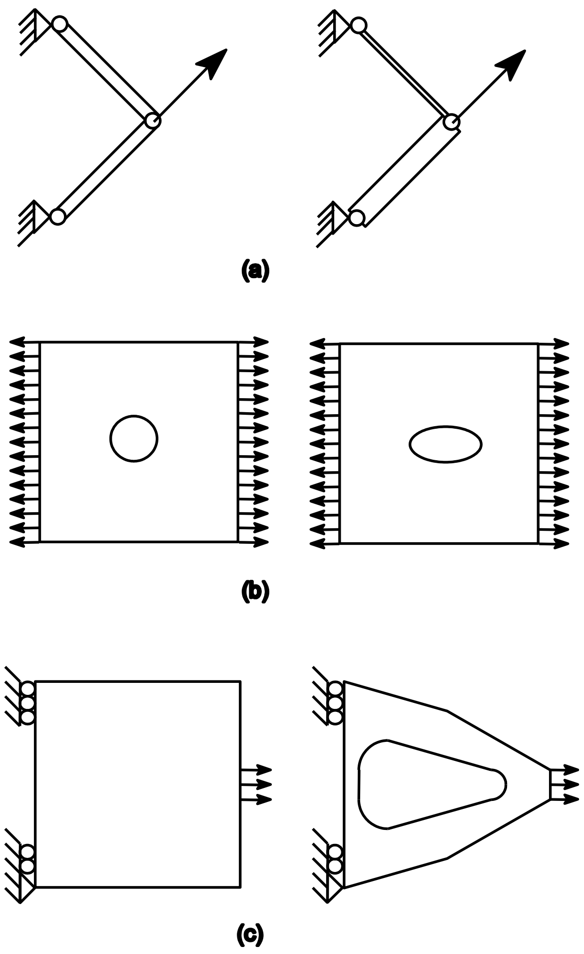
Hierarchy of design variables

- In the 1990ies with the soar of topology optimization, [Jog, Haber and Bendsoe, 1996] sort the design variables hierarchy into 3 families:

  a/ Sizing
  - Cross section, thickness, Young modulus...

  b/ Shape
  - Lengths, angles, control point positions…

  c/ Topology
  - Presence or absence of holes,
  - Connectivity of members and joints...



ASSUMPTIONS AND DEFINITIONS

- Design variables are denoted by x
- We are going to work with **finite dimension problems**
  - Naturally discrete structures (e.g. truss)
  - Discretized structures (FEM, Finite volume, BEM…)
- The numerical solution of this kind of optimization problems resorts to mathematical programming methods
- The design variables are collected in design variable vector x of dimension n:

$$x_i \quad (i = 1 \ldots n) \quad \longrightarrow \quad \mathbf{x} \in R^n$$

DEFINITIONS: Design variables

- Most often the design variables are assumed to vary continuously between a lower and an upper limit: these are the **side constraints**.

$$\underline{x}_i \leq x_i \leq \bar{x}_i$$

  - Side constraints are generally related to technological, manufacturing and physical bounds

- In some cases the design variables can only take on **discrete values** from a given set: these are discrete variables

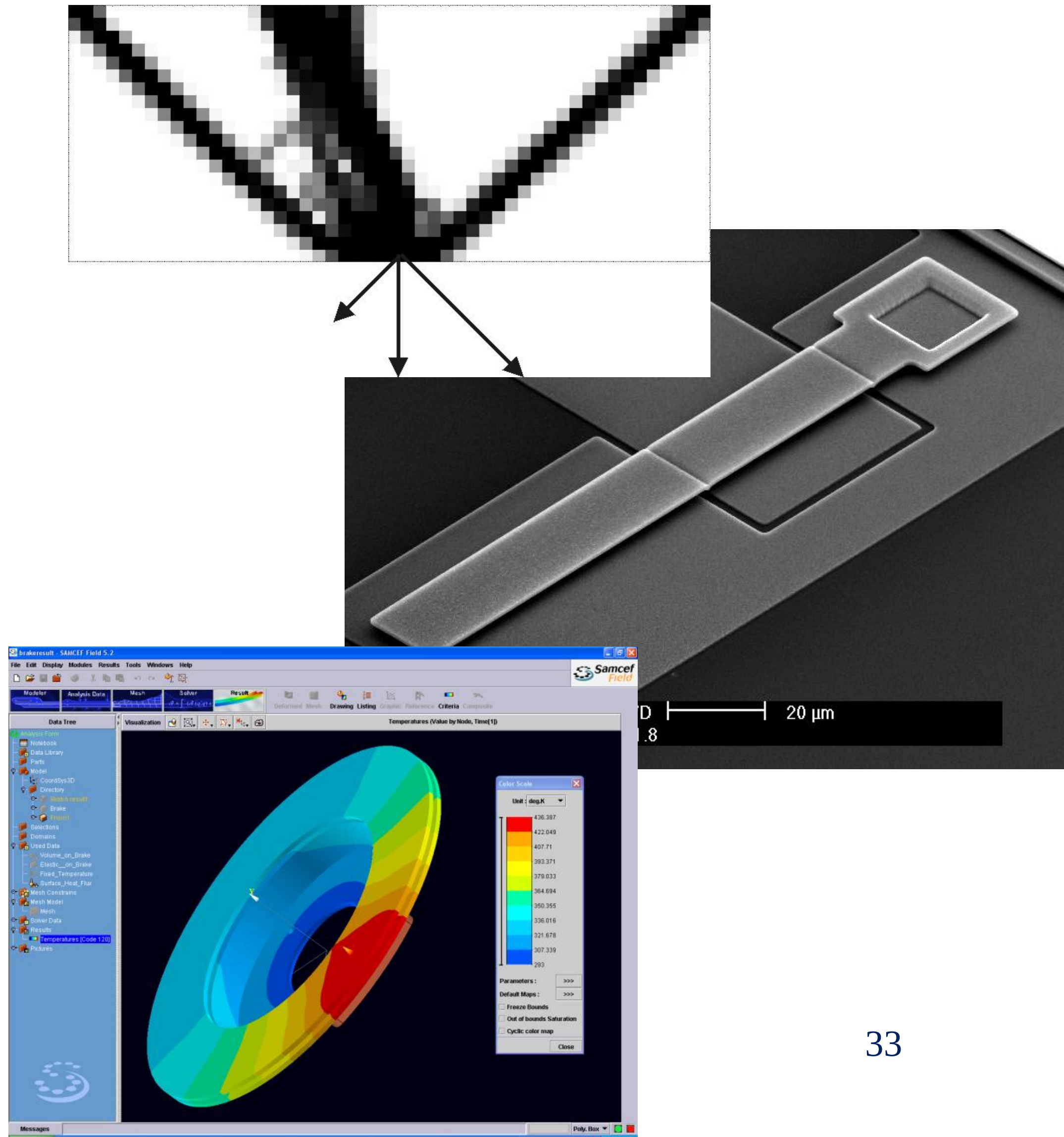
$$x_i \in X_i = \left\{ x_i^k | k = 1 \ldots n^k \right\}$$

  - The discrete design variables can be selected from a catalog or are naturally discrete (e.g. number of stiffeners)

- Real problems usually involve mixed continuous-discrete variables

Types of analysis and optimization

- TYPES OF OPTIMISATION
  - Structural
  - Multidisciplinary
    - Structural,
    - Aerodynamics,
    - Thermal,
    - Electromagnetic,
    - Manufacturing…



DESIGN PROBLEM FORMULATION

OBJECTIVE FUNCTION

- Define a function that is a quantity of interest, a criterion able to quantify and to compare the performance of the design. It is a function of the design variables **x** and also of the state variables **q** (displacements…)

$$\mathbf{x} \in \mathbb{R}^n,\ \mathbf{q} \in \mathbf{R}^p \longrightarrow f(\mathbf{x}, \mathbf{q})$$

  - Examples of objective functions
    - Weight of a wing
    - Compliance of a machine tool
    - Operating cost of a heat exchanger
    - Time to move to a position within a given tolerance for a robot

- Represented by its contours of constant values in the design space

DESIGN PROBLEM FORMULATION

- Standard formulation of optimization problem is minimization problem of single objective function

$$\min_{\boldsymbol{x} \in S} \quad f(\boldsymbol{x})$$

- But one may also have to solve other kinds of problems that can be cast into an equivalent minimization problem with a single objective function.

- Maximize f(x) is equivalent to minimize –f(x) or 1/f(x)

$$\max_{\boldsymbol{x} \in S} \quad f(\boldsymbol{x}) \quad \Longleftrightarrow \quad \min_{\boldsymbol{x} \in S} \quad -f(\boldsymbol{x})$$

$$\max_{\boldsymbol{x} \in S} \quad f(\boldsymbol{x}) \quad \Longleftrightarrow \quad \min_{\boldsymbol{x} \in S} \quad \frac{1}{f(\boldsymbol{x})}$$

DESIGN PROBLEM FORMULATION

- Several design problems involve the minimization of the max value of a set of function.
  - For instance one may want to minimize the maximum of the equivalent stress in several points of the domain.

$$\min_{\boldsymbol{x} \in S} \quad \max_{k} f_k(\boldsymbol{x})$$

- Max function is non smooth, and it can be replaced by continuous approximations. Several famous techniques exist

- Introduce an addition variable

$$\min_{\boldsymbol{x} \in S} \quad \max_{k} f_k(\boldsymbol{x}) \quad \Longleftrightarrow \quad \begin{array}{ll} \min & \beta \\ \boldsymbol{x} \in S, \beta & \\ \text{subject to} & f_k(\boldsymbol{x}) \leq \beta \end{array}$$

DESIGN PROBLEM FORMULATION

- One can also use smooth approximation of max operator. The most famous ones are

Average

Disjunctive

P mean

$$P_m = \left(\frac{1}{N} \sum_k (f_k)^p \right)^{1/p}$$

P norm

$$P_m = \left(\sum_k (f_k)^p \right)^{1/p}$$

KS-L

$$KS\text{-}l = \frac{1}{p} \ln \left(\frac{1}{N} \sum_k^N \mathrm{e}^{(p\, f_k)} \right)$$

KS-U

$$KS\text{-}u = \frac{1}{p} \ln \left(\sum_{k=1}^N \mathrm{e}^{(p\, f_k)} \right)$$

  - Are tailored to aggregate sets of data whose values are in [0,1]!
  - Parameter p has to be chosen as high as possible but not too high for sake of computational degeneracy. In practice p: 4 → 30?
  - Have special properties and behaviors see Ref. Mesiar et al (2015) for a nice review

DESIGN PROBLEM FORMULATION

- If using p-norm, maximum is replaced by the following problem

$$\min_{\boldsymbol{x}\in S} \quad \max_{k} f_k(\boldsymbol{x}) \quad \Longleftrightarrow \quad \min_{\boldsymbol{x}\in S} \left(\frac{1}{N} \sum_{k} (f_k(\boldsymbol{x}))^p \right)^{1/p}$$

DESIGN PROBLEM FORMULATION

CONSTRAINTS or DESIGN RESTRICTIONS

- Establish and quantify the requirements and the specifications to be satisfied by the final design
  - For example:
    - Structural responses
      - Upper / lower limits on stresses
      - Maximum displacements
      - Lower limit on eigen frequencies
      - Lower limit on buckling loads
    - Geometric quantities
      - Weight, volume, CG position…
    - Manufacturing
      - Minimum thickness or variables linking
      - Molding / unmolding…

DESIGN PROBLEM FORMULATION

CONSTRAINTS or DESIGN RESTRICTIONS

- In engineering design, the design restrictions are generally inequality constraints
  - Maximum displacement, stresses
  - Volume resources…

$$g_j(x) \leq \bar{g}_j \quad (j = 1 \ldots m)$$

- Sometimes equality constraints
  - E.g. tangency or symmetry conditions

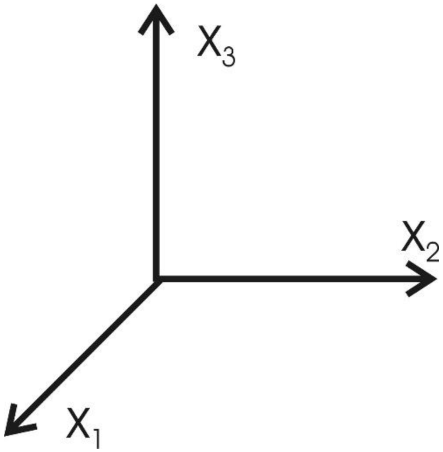
$$h_k(x) = 0 \quad (k = 1 \ldots l)$$

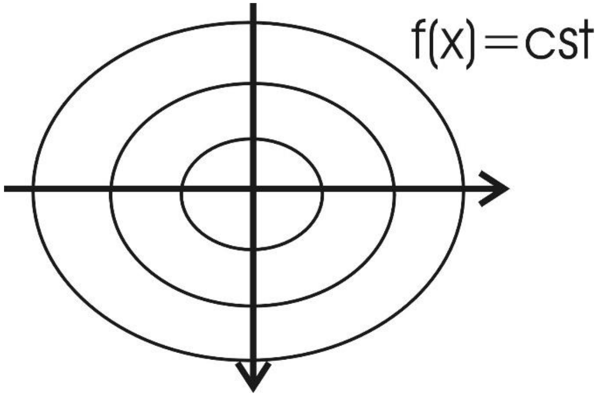
- Represented in the design space, by the surface of their trace

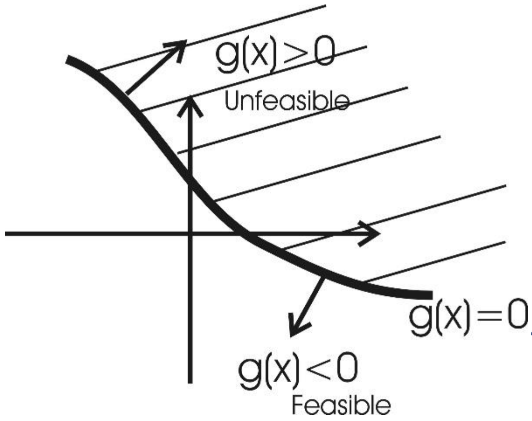
$$g_j(x) - \bar{g}_j = 0$$

THE DESIGN SPACE

- We have a set of n design variables x_i (i=1…n). One can imagine a n dimensional space, each axis corresponding to one design variables. This is the **design space**
- The design space is very useful to grasp what is the nature of the optimization problem
- The objective function is represented by its contours of constant value (level sets)
- The constraints are represented by the restraint surface on which the constraint function is equal to its bound

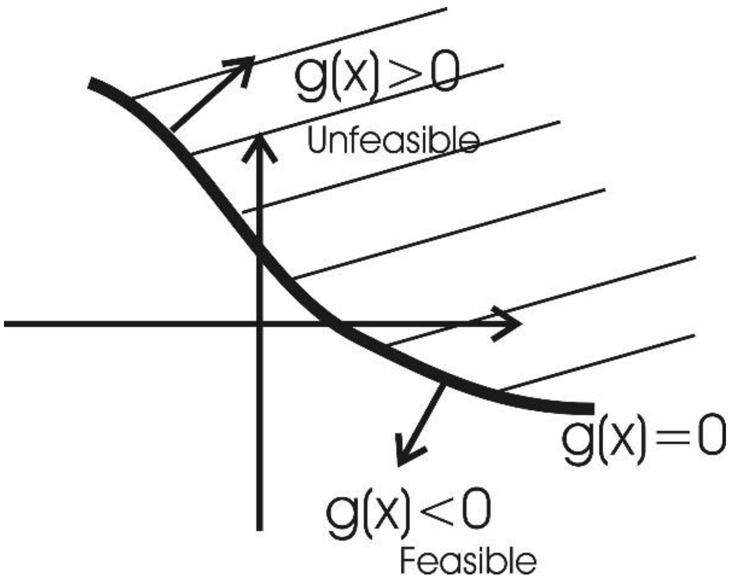






THE DESIGN SPACE

- Feasible point:
  - A design point is **feasible** if all constraints are satisfied meaning that the design specifications and restrictions are fully satisfied
  - An unfeasible point is characterized by one or several constraints that are not satisfied.
- A constraint that is not satisfied is said to be violated
- When a constraint is satisfied as an equality, the constraint is said to be **active or tight**.



The design space: example

- Non-linear constraint

$$g_1(x) = x_1^2 - x_2 \quad \bar{g}_1 = 0$$

- Linear constraint

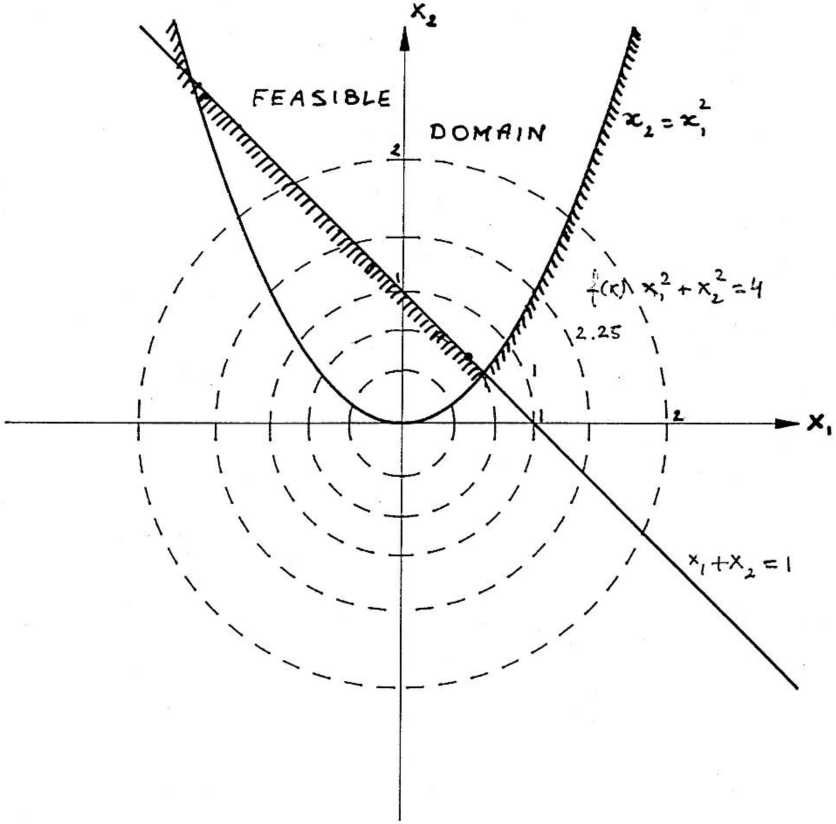
$$g_2(x) = -x_1 - x_2 \quad \bar{g}_2 = -1$$

- Objective function (quadratic)

$$f(x_1, x_2) = x_1^2 + x_2^2$$

- Optimization problem

$$\begin{aligned} \min \quad & x_1^2 + x_2^2 \\ \text{s.t.} \quad & x_1^2 - x_2 \leq 0 \\ & x_1 + x_2 \geq 1 \end{aligned}$$



minimize $f(x) \equiv x_1^2 + x_2^2$

subject to

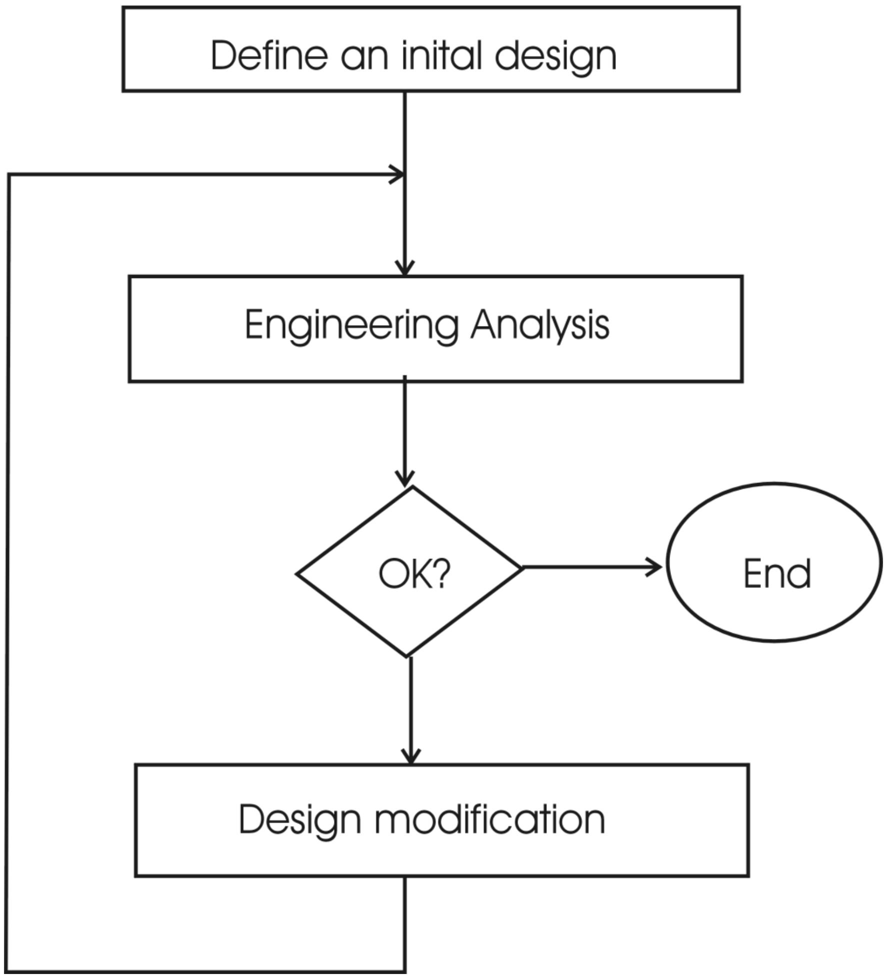
$h_1(x) \equiv x_1^2 - x_2 \leq 0$

$h_2(x) \equiv x_1 + x_2 \geq 1$

THE OPTIMIZATION DESIGN LOOP

OPTIMIZATION LOOP IN ENGINEERING

- The target: the fully computerized design cycle



Define an initial design
Determine failure modes
Select design variables
Devise an appropriate analysis scheme

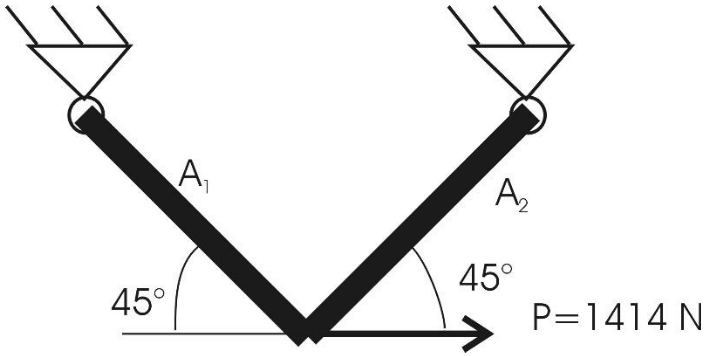
Closed form solution
Experimental
Numerical e.g. finite elements

Design criteria are satisfied?

Change the design variables

Simple example: 2 bar truss

- Max allowable stresses
  - Compressive σ = 25 daN/mm²
  - Tensile σ = 50 daN/mm²
- Initial design
  - Cross sectional areas 10 mm²
- Analysis $A_1 = A_2 = 10\, mm^2$



$$\begin{aligned} (-Q_1 + Q_2)\tfrac{\sqrt{2}}{2} + P &= 0 \\ (Q_1 + Q_2)\tfrac{\sqrt{2}}{2} &= 0 \end{aligned}$$

$$\Rightarrow \quad Q_1 = -Q_2 = P\frac{\sqrt{2}}{2} = 1000\, daN$$

- Stresses

$$\sigma_1 = \frac{Q_1}{A_1} = \frac{1000}{10} = 100\, daN/mm^2 \qquad \sigma_1 = 100\, daN/mm^2 > 50\, daN/mm^2$$

$$\sigma_2 = \frac{Q_2}{A_2} = \frac{-1000}{10} = -100\, daN/mm^2 \qquad |\sigma_2| = 100\, daN/mm^2 > 25 daN/mm^2$$

Simple example: 2 bar truss

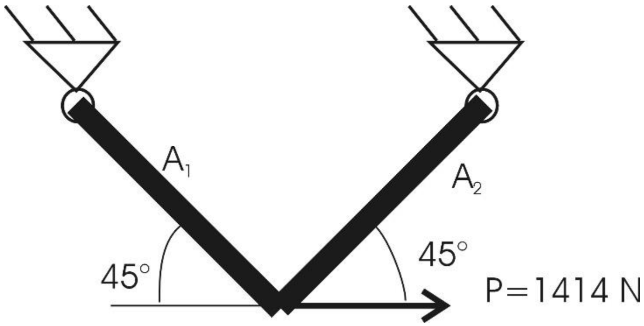
- Redesign

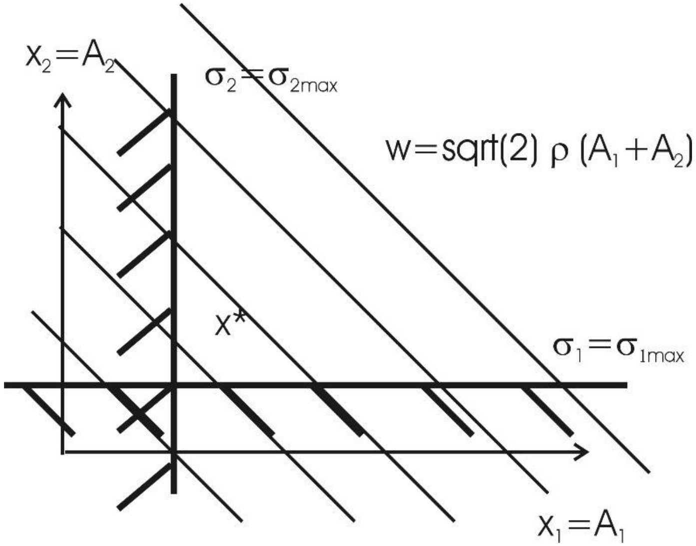
$$A_1 = \frac{Q_1}{\bar{\sigma}_1} = \frac{1000}{50} = 20\, mm^2$$
$$A_2 = \frac{Q_2}{\bar{\sigma}_2} = \frac{1000}{25} = 40\, mm^2$$

- Reanalysis

$$\sigma_1 = \frac{Q_1}{A_1} = \frac{1000}{20} = 50 = \bar{\sigma}^t\, daN/mm^2$$
$$\sigma_2 = \frac{Q_2}{A_2} = \frac{-1000}{40} = -25 = \bar{\sigma}^c\, daN/mm^2$$

Exactly the tensile and
Compressive stresses
Limits





Optimum design in the sense of minimum
Weight design

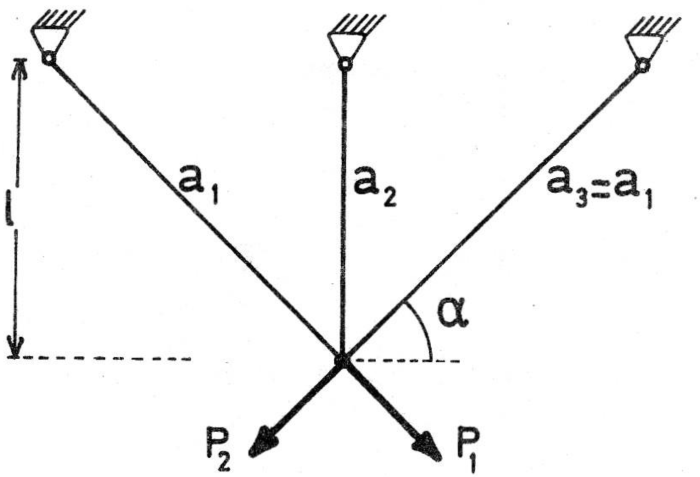
OPTIMIZATION LOOP IN ENGINEERING

- For the two-bar truss: very simple situation: after one redesign, one finds the true optimum solution

- Generally: much more complicated situation!
  - Several iteration steps are necessary to come to a stationary solution, which is an approximate solution within a certain tolerance
  - No analytical solution: the analysis generally requires an expensive computer analysis (FE analysis)
  - The constraints are implicit and very non-linear functions of the design variables
  - The function evaluation is expensive and requires a run of a full FE (or numerical) model

Example: three-bar-truss problem

- Let’s investigate a more complex problem to understand the problem!



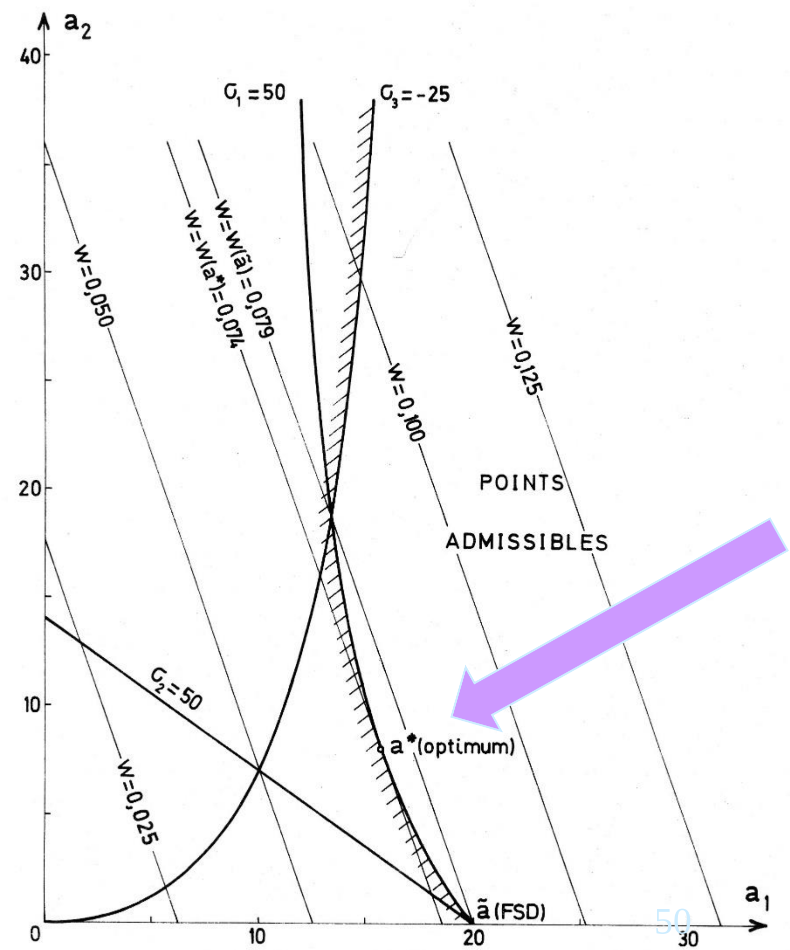
module d'Young	$E = 7000\ kg/mm^2$
masse volumique	$\rho = 2.8\ 10^{-6}\ kg/mm^3$
tension maximale admissible :	
traction	$\bar{\sigma} = 50\ kg/mm^2$
compression	$\underline{\sigma} = -25\ kg/mm^2$
hauteur du treillis	$l = 500\ mm$
angle des barres	$\alpha = 45°$

cas de charge 1 : $P_1 = 1000\ kg$
// // 2 : $P_2 = 1000\ kg$
(symetriques)

Example: three-bar-truss problem

- It is possible to express explicitly the functions and the problem writes

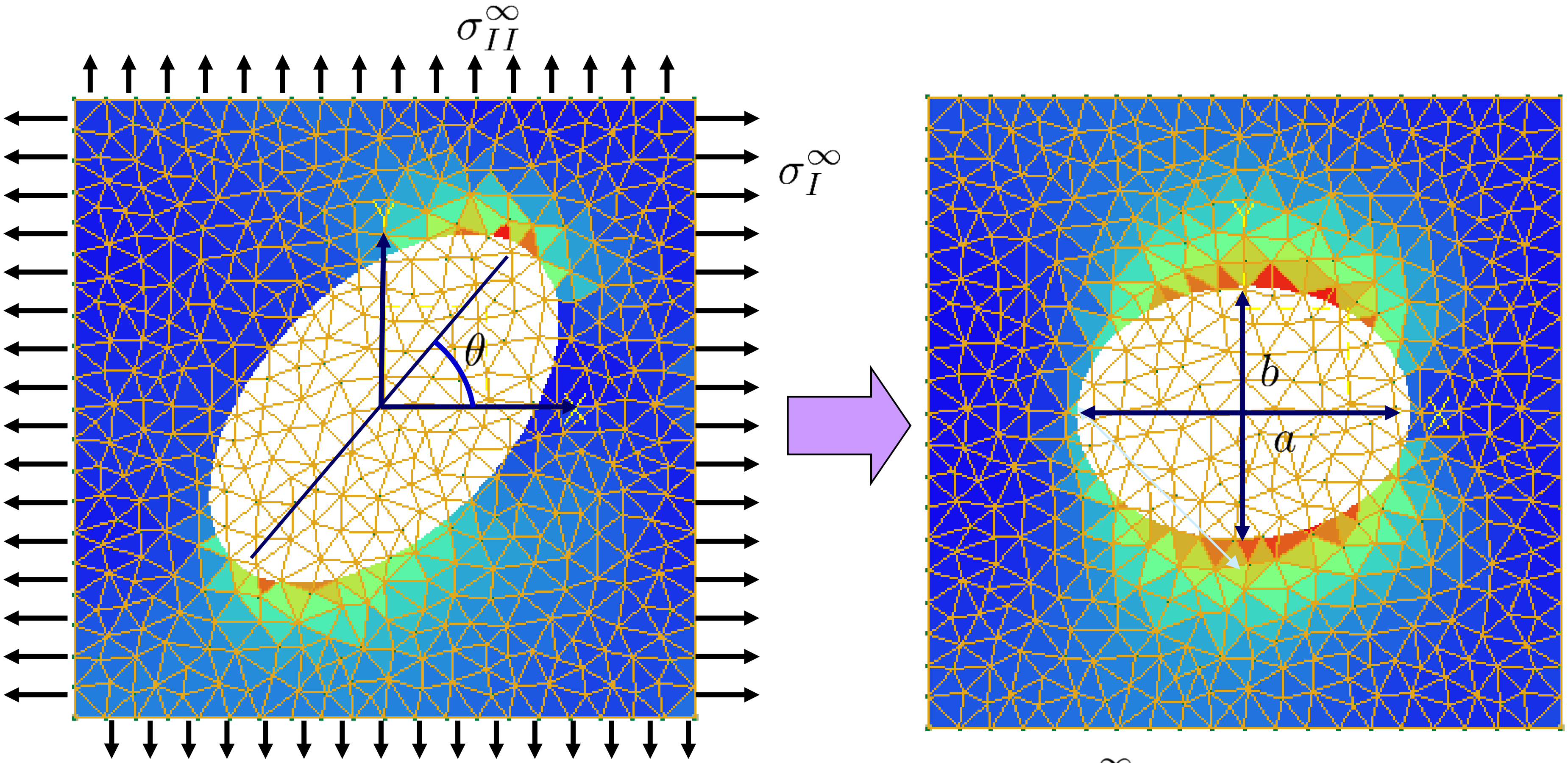
$$
\begin{aligned}
\min \quad & W(x) = \rho l (2\sqrt{2}a_1 + a_2) \\
\text{s.t.} \quad & \sigma_1 = \frac{P}{\sqrt{2}a_1}\frac{\sqrt{2}a_1 + a_2}{a_1 + \sqrt{2}a_2} \leq 50 \\
& \sigma_2 = \frac{P}{\sqrt{2}a_1 + a_2} \leq 50 \\
& -\sigma_3 = \frac{P}{\sqrt{2}a_1}\frac{a_2}{\sqrt{2}a_1 + a_2} \leq 25 \\
& a_1 \geq 0 \quad a_2 \geq 0
\end{aligned}
$$



- Only one single stress constraint is active at the optimum (σ_1)
- Question: how to find a trajectory leading to the optimum?
- Requires a periodic evaluation of the constraints

OPTIMIZATION LOOP IN ENGINEERING

- Shape optimization problem of a hole in plate under bi-axial stress field

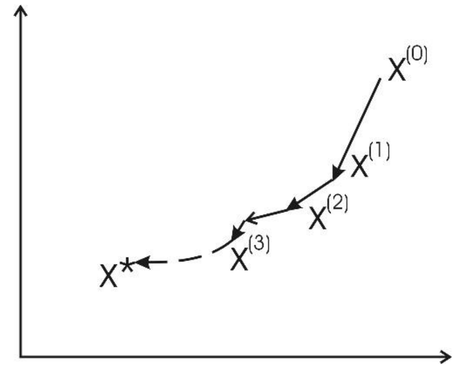


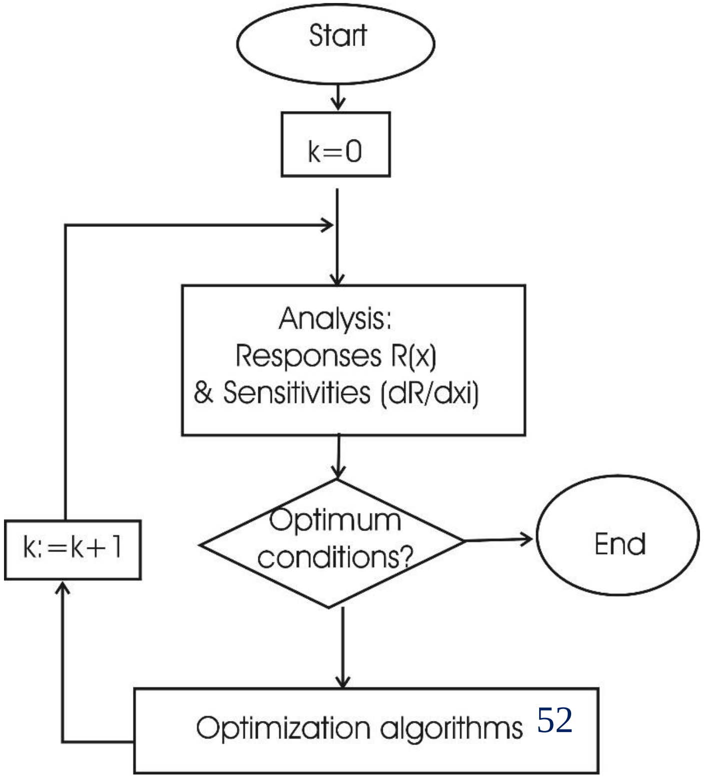
$a = ?\quad b = ?\quad \theta = ?$

$$\frac{a}{b} = \frac{\sigma_{II}^{\infty}}{\sigma_{I}^{\infty}} \qquad \theta = 0$$

ITERATIVE OPTIMIZATION PROCESS

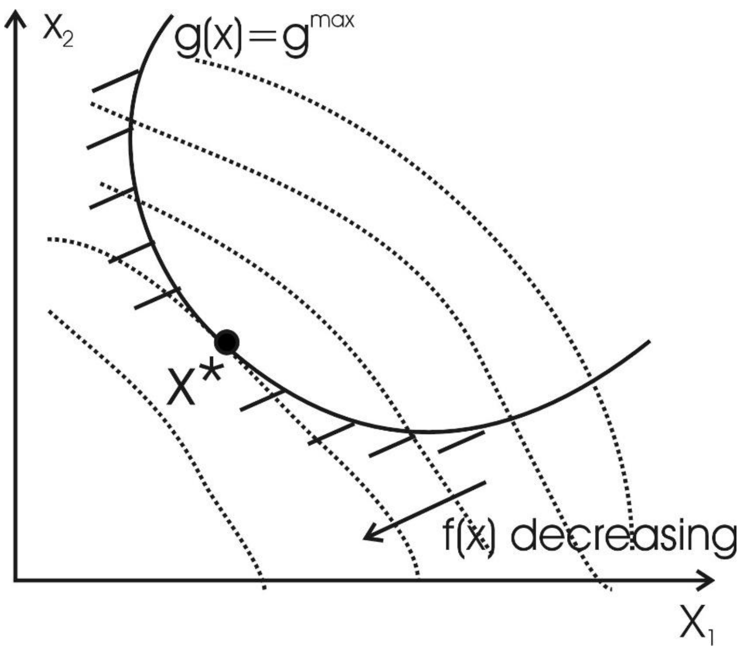
- The functions are generally implicit, non-linear and non-convex.
- To solve a non-linear optimization problem, one has to resort to an iterative process.
- At each stage k with a design $x^{(k)}$, the optimization procedure aims at determining a better design $x^{(k+1)}$.
- From an ideal point of view, one would like to have a sequence of steadily feasible and continuously improved designs.
- At each step the objective function and the constraints must be evaluated.
- Moreover, many optimization algorithms require the computation of the derivatives: it is the **sensitivity analysis**



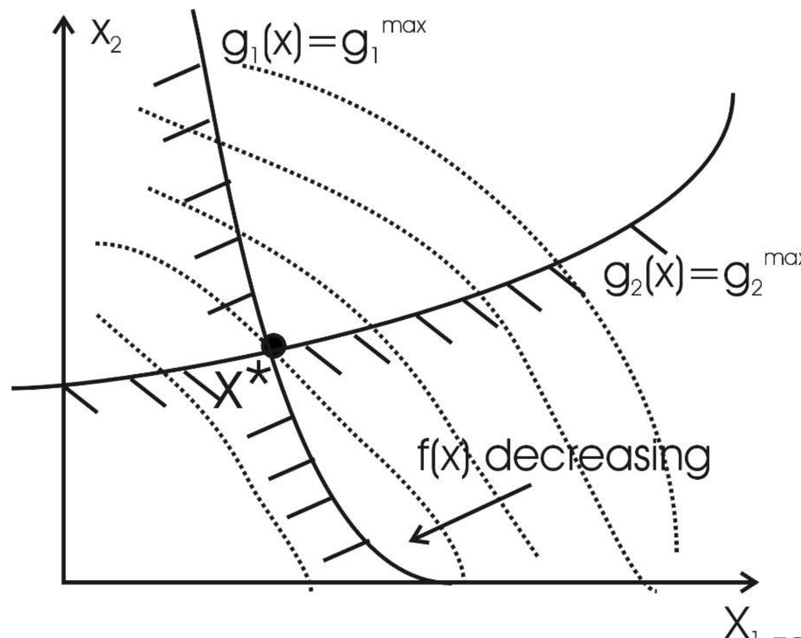


OPTIMUM DESIGN CHARACTERIZATION

- At the **optimum design point x***, it is impossible to make further progress without
  - Either violating at least one constraints
  - Or increasing the value of the objective function
- The following two situations are possible (constrained minimum)



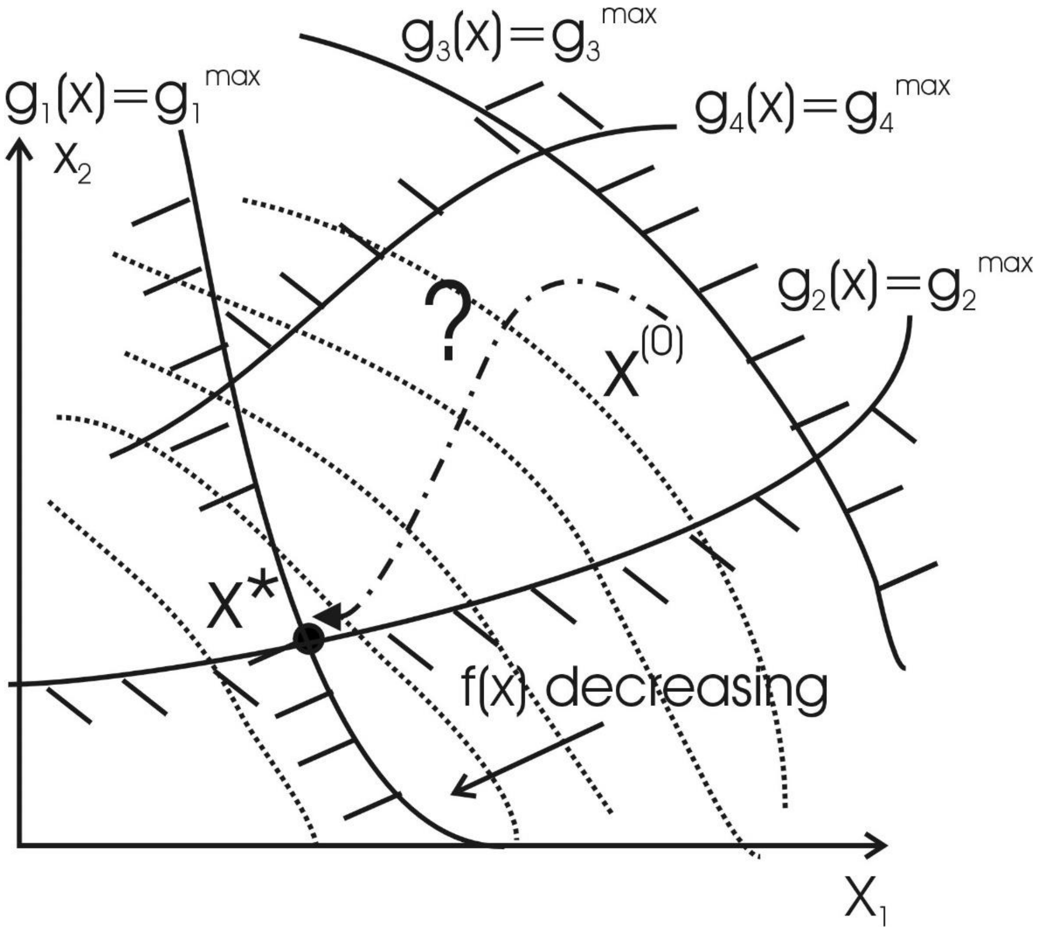
Tangency



Vertex

OPTIMUM DESIGN CHARACTERIZATION

- Numerical optimization problem: how to go from $x^{(0)}$ to x^*?



ITERATIVE OPTIMIZATION PROCEDURE

- Optimization procedure requires an alternating evaluation
  - Structural analysis (costly) + Sensitivity analysis
  - Weight minimization ➔ new design point prediction

- Mathematical programming
  - Rigorous
  - General
  - Stable, monotonic convergence
  - Local optimum?
  - Large number of re-analyses growing with the number of d.v.

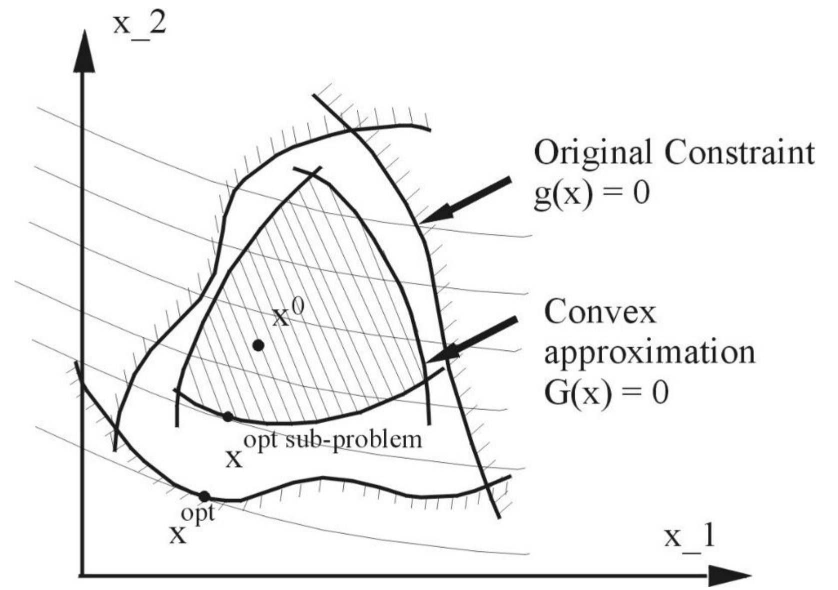
- Optimality criteria
  - Intuitive
  - Specific
  - Uncertain convergence
  - Lead not necessarily to an optimum point
  - Small number of reanalyses independent of the number of d.v.

UNIFIED APPROACH : SEQUENTIAL CONVEX PROGRAMMING + STRUCTURAL APPROXIMATIONS

SEQUENTIAL CONVEX PROGRAMMING APPROACH

Direct solution of the original optimisation problem which is generally non-linear, implicit in the design variables

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_j(x) \leq \bar{g}_j \quad j = 1 \ldots m \end{array}$$



is replaced by a sequence of optimisation (convex explicit) sub-problems

$$\begin{array}{ll} \min & F(x) \\ \text{s.t.} & G_j(x) \leq \bar{g}_j \quad j = 1 \ldots m \end{array}$$

by using approximations of the responses F, G_j and using powerful mathematical programming algorithms

SEQUENTIAL CONVEX PROGRAMMING APPROACH

- Two basic concepts:
  - Structural approximations: replace the implicit problem by an explicit optimisation sub-problem using convex, separable, conservative approximations; e.g. CONLIN, MMA
  - Solution of the convex sub-problems: efficient solution using dual methods algorithms or SQP method.
- Advantages of SCP:
  - Optimised design reached in a reduced number of iterations: 10 to 20 F.E. analyses
  - Efficiency, robustness, generality, and flexibility, small computation time
  - Large scale problems in terms of number of design constraints and variables

SENSITIVITY ANALYSIS

- FINITE DIFFERENCES:
  - Redo a full FE analysis for each (perturbated) variable

$$\frac{\partial f(x)}{\partial x_i} \simeq \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i}$$

  - General approach: available for any analysis programme, linear or non linear problems
  - Short development time
  - Can be expensive from a computation time point of view

- ANALYTIC AND SEMI-ANALYTIC APPROACHES:
  - Based on the derivation of the state equations
  - Derivatives are evaluated as additional results of the analysis programmes
  - More efficient and less expensive (computation time)
  - Only available for a restricted set of analysis programmes

SENSITIVITY ANALYSIS OF THE GENERALISED DISPLACEMENTS

- Discretised equilibrium (Finite Elements)

$$\mathbf{K}\mathbf{q} = \mathbf{g}$$

with **K** the stiffness matrix, **q** the generalised displacements, and **g** the load vector

- The sensitivity of the displacements with respect to variable x:

$$\frac{\partial \mathbf{q}}{\partial x_i} = \mathbf{K}^{-1} \left(\frac{\partial \mathbf{g}}{\partial x_i} - \frac{\partial \mathbf{K}}{\partial x_i} \mathbf{q} \right)$$

- The semi-analytic method: calculate the derivatives of the stiffness matrix and of the load vector by finite differences:

$$\frac{\partial \mathbf{K}}{\partial x_i} \simeq \frac{\mathbf{K}(x_i + \delta x_i) - \mathbf{K}(x_i)}{\delta x_i} \qquad \frac{\partial \mathbf{g}}{\partial x_i} \simeq \frac{\mathbf{g}(x_i + \delta x_i) - \mathbf{g}(x_i)}{\delta x_i}$$

NUMERICAL APPLICATIONS: SIZING, SHAPE, AND TOPOLOGY OPTIMIZATION

SIZING OPTIMIZATION

A typical sizing optimization problem

- Design variables are some cross sections or plate thickness parameters
  - The design variables do not change the FE model

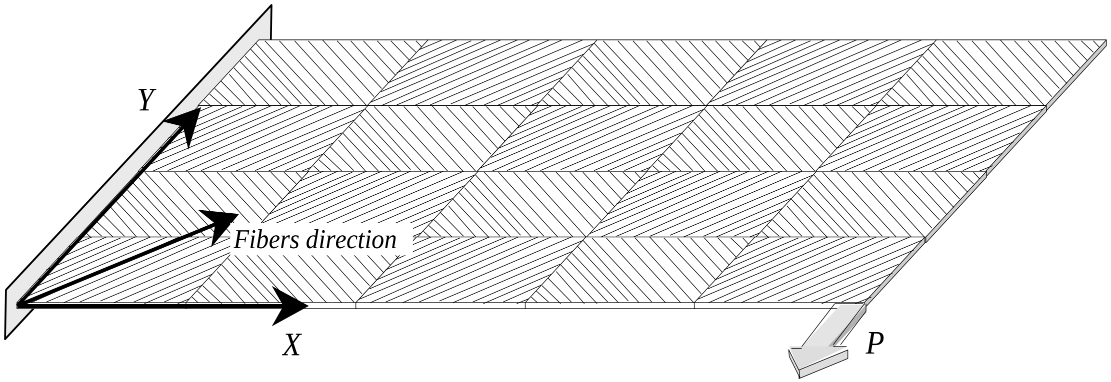
- Minimum weight design (or equivalently the volume)

$$W(x) = \sum_{i=1}^{n} w_i \, x_i \rho$$

- The behavior constraints: limitations on the static / dynamic / stability responses
  - Displacement $u_j(x) \leq \bar{u}_j$
  - Stress $\sigma_e(x) \leq \bar{\sigma}_e$
  - Frequency $\omega_k(x) \geq \underline{\omega}_k$
  - Buckling load $\lambda_l(x) \geq \underline{\lambda}_l$

- Side Constraints $\underline{x}_i \leq x_i \leq \bar{x}_i$

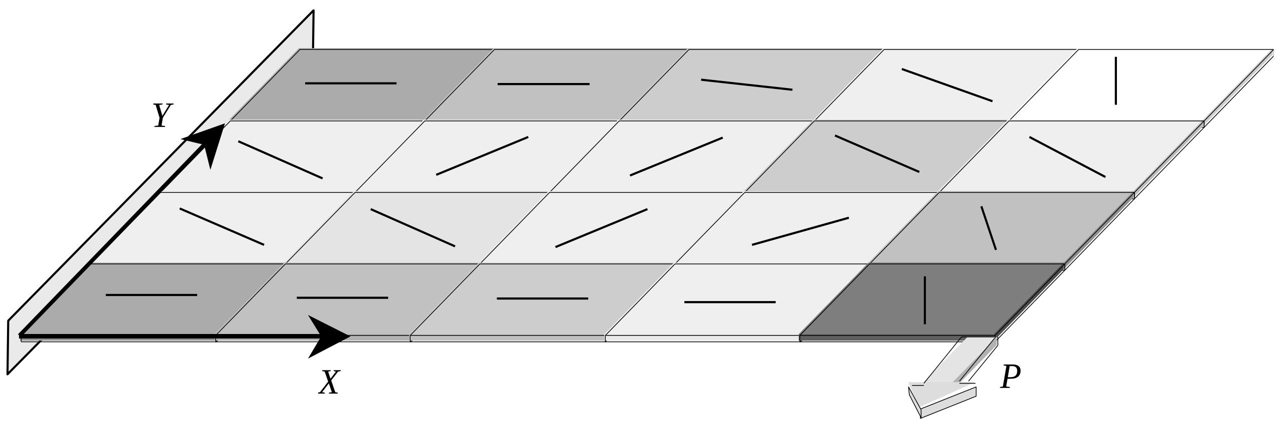
Strength optimization of a composite plate



$$\min_{\theta,t} Compliance$$

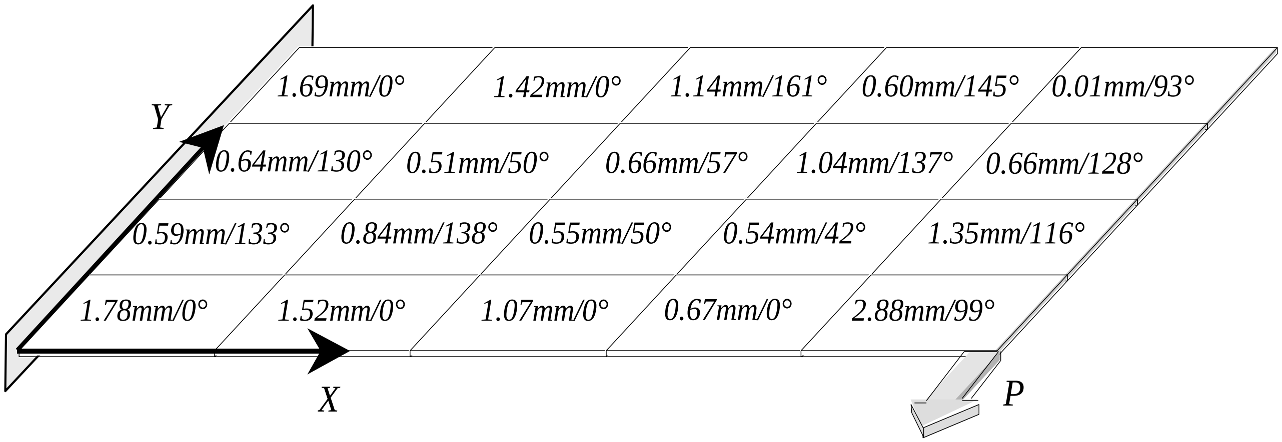
$$weight \le 1.5kg$$

$$\min\left(\frac{1}{\sqrt{T.H.}} - 1\right) \ge 0.15$$



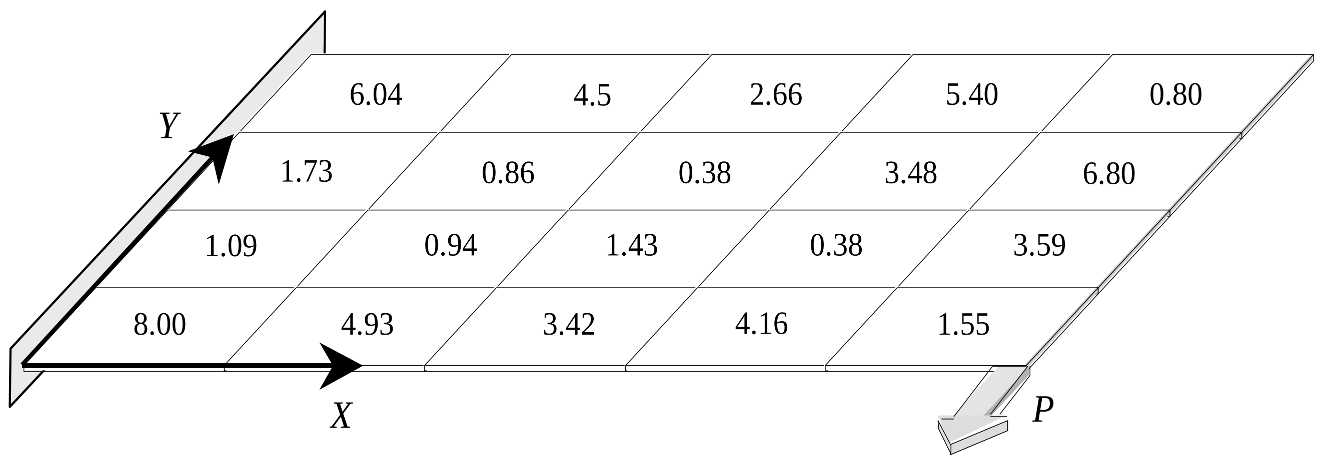
Optimized solution

Strength optimization of a composite plate

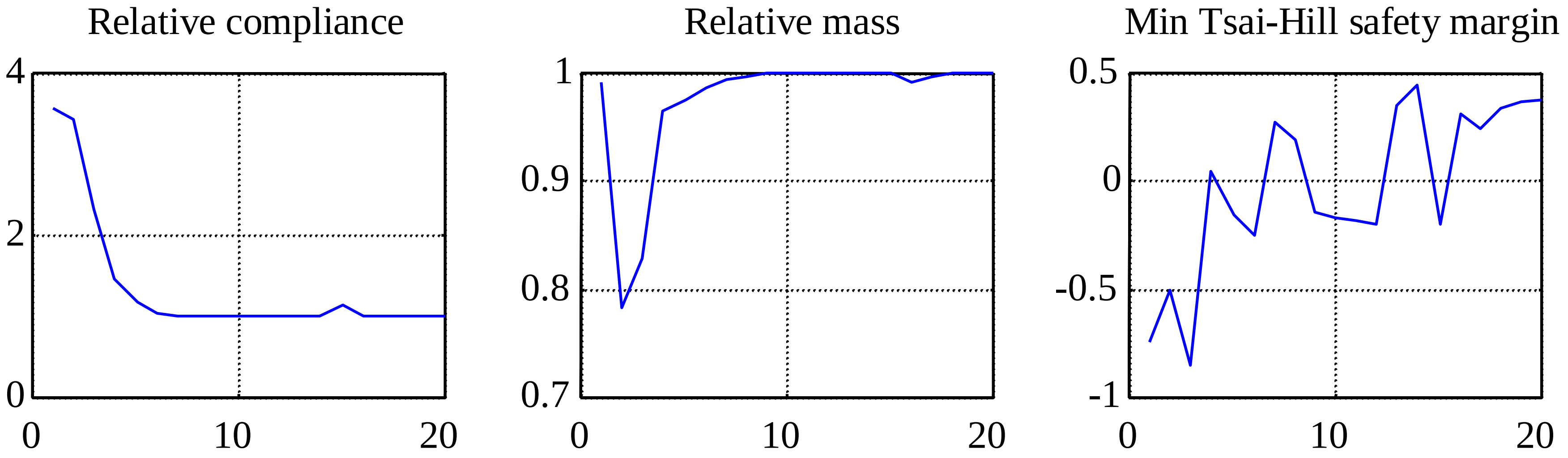


Optimized solution

Tsai-Hill safety margins

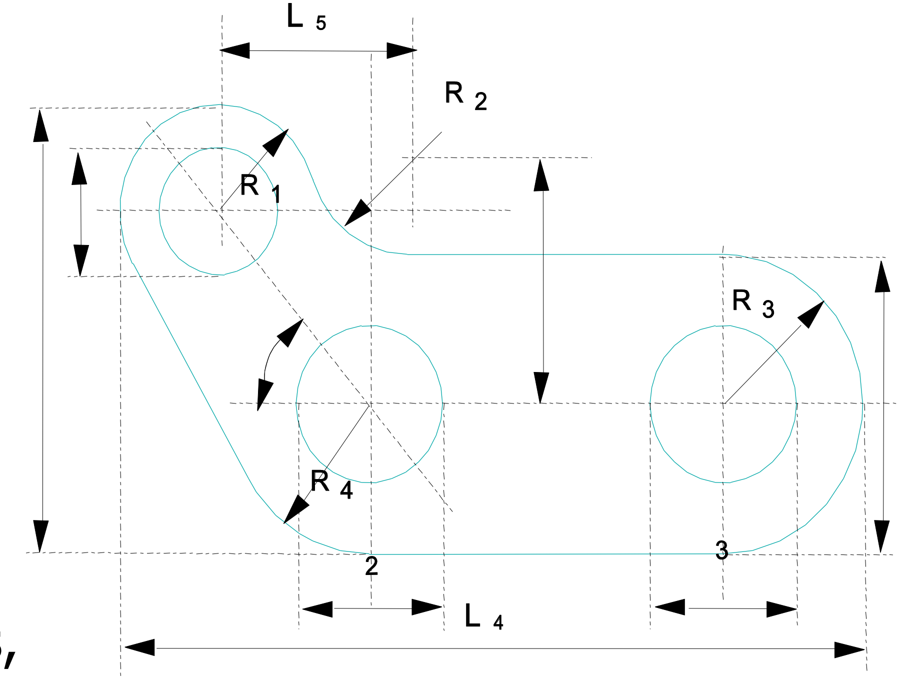


Strength optimization of a composite plate



SHAPE OPTIMIZATION

- Modification of external or inner boundaries
- Key issue: definition of a consistent parametric CAD model
  - Geometrical constraints (tangency, linking of points)
  - Geometrical features: straight lines, circles, NURBS, surfaces, etc.

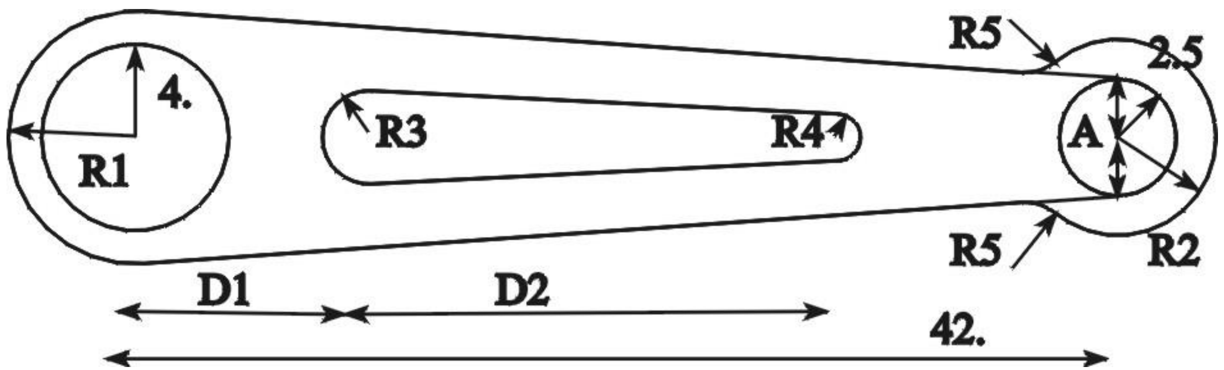


Zhang, Duysinx, Fleury (1993)

- Implementation issue: API to and from CAD systems (CATIA, Pro E, etc.)

- Design variables = a set of independent CAD model parameters

Example : Shape Optimisation of a Torque Arm



- Statement of the design problem:

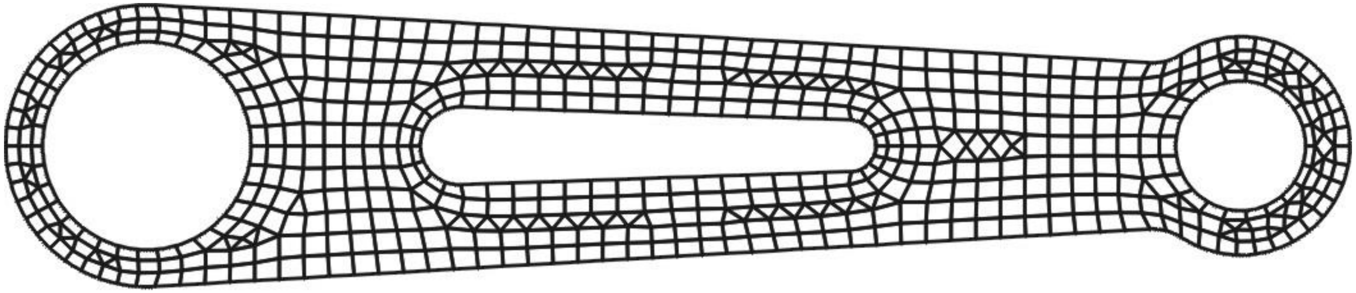
  Minimise Weight

  8 design parameters

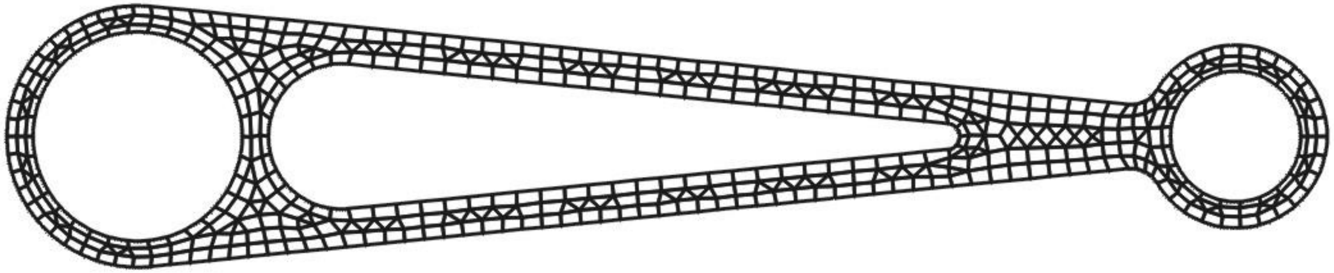
  s.t. Von Mises equivalent stress under 80000 N/mm²

  Geometry constraints (thickness of members > 1 cm)

Example : Shape Optimisation of a Torque Arm



Iteration 0

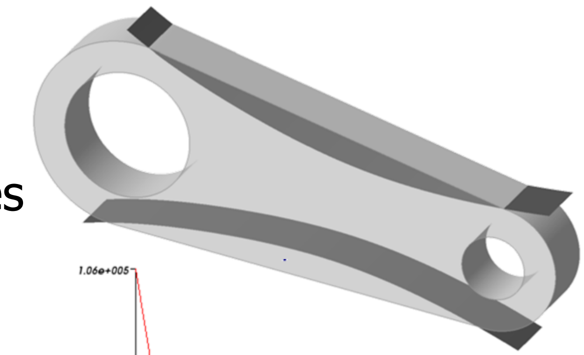


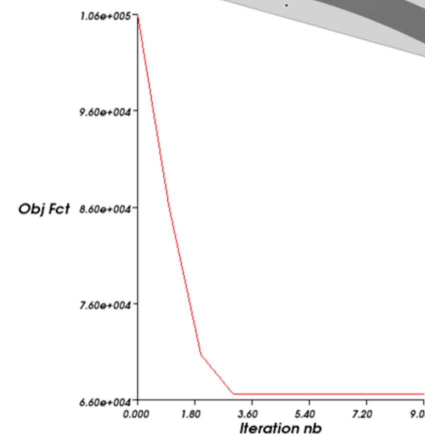
Iteration 12

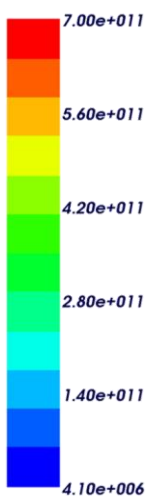
Generalized Shape Optimization With XFEM

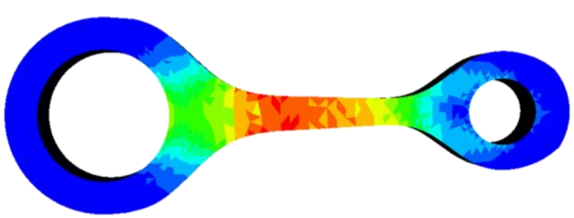
[Van Miegroet et al., 2007]

- Connecting rod problem :
  - 2 Level Set 3D surface defined by NURBS curves
  - Parameters : Control points of the NURBS
    - Variables (12) : Mvt. of K_i along y axis
  - Objective function : min Volume
  - S.t. Constraints : Von Mises<70 Mpa
    - 65000 Elements ~ 30000 constraints
  - Volume reduction ~ 50%

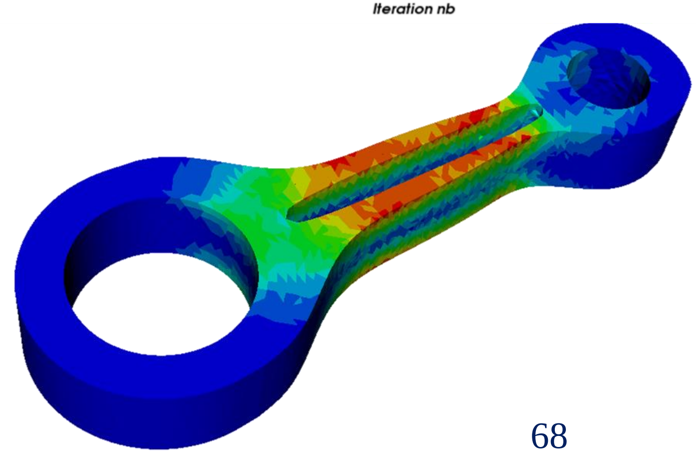






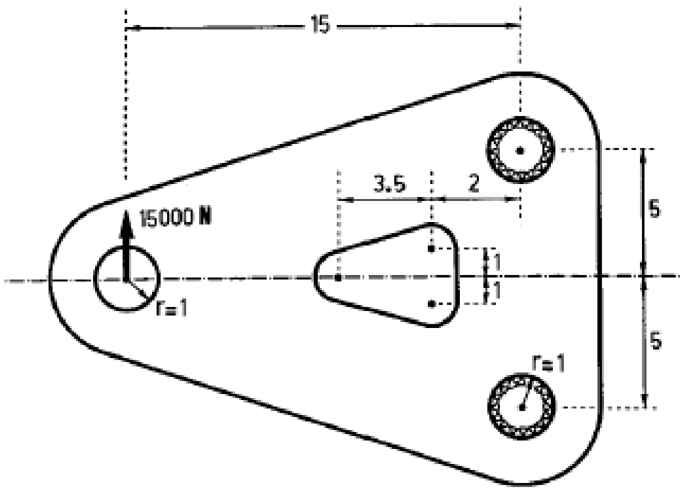


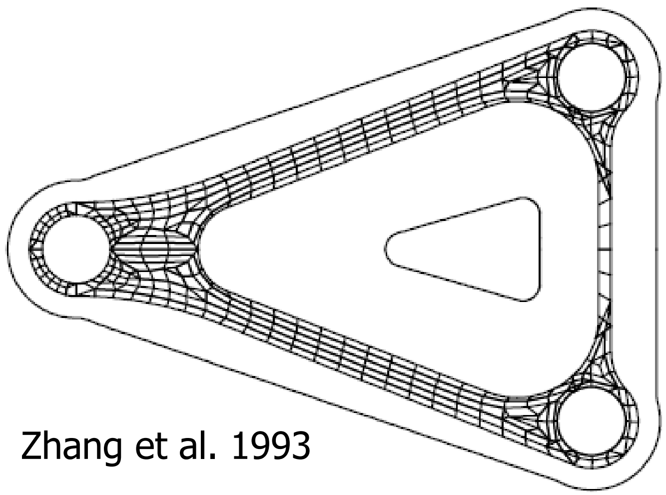
Addition of a superellipse at center (same mesh) : gain of 3% volume ➔



Why topology optimization?

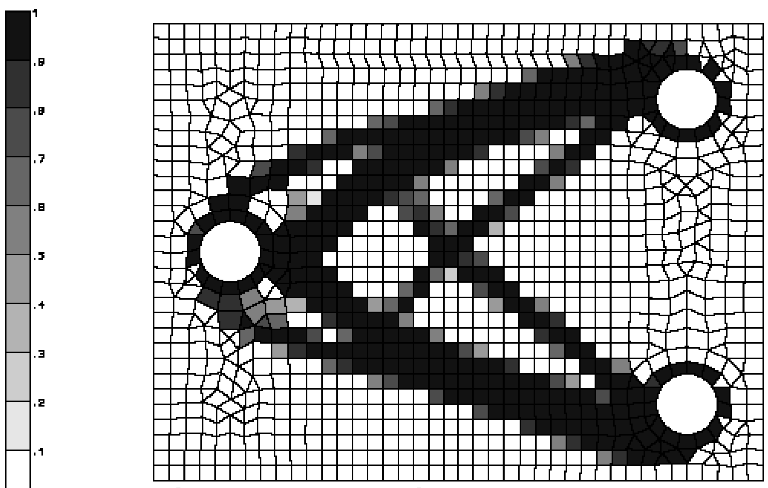
- CAD approach does not allow topology modifications





Zhang et al. 1993

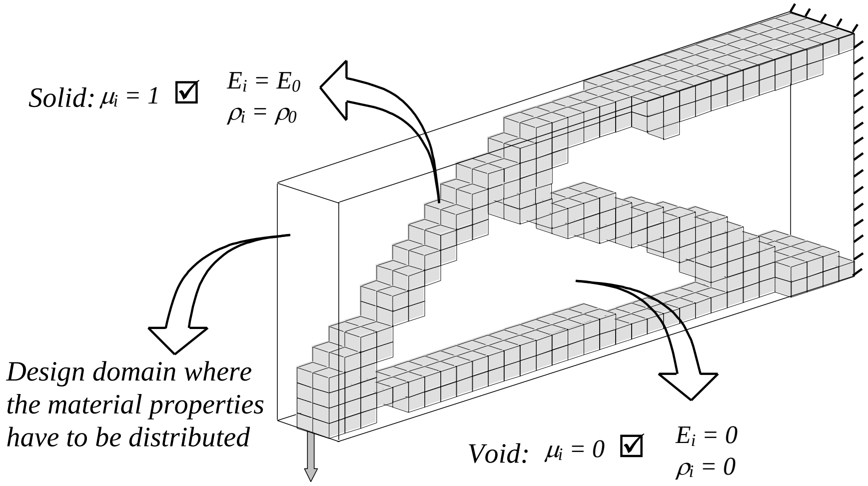
A better morphology by topology optimization



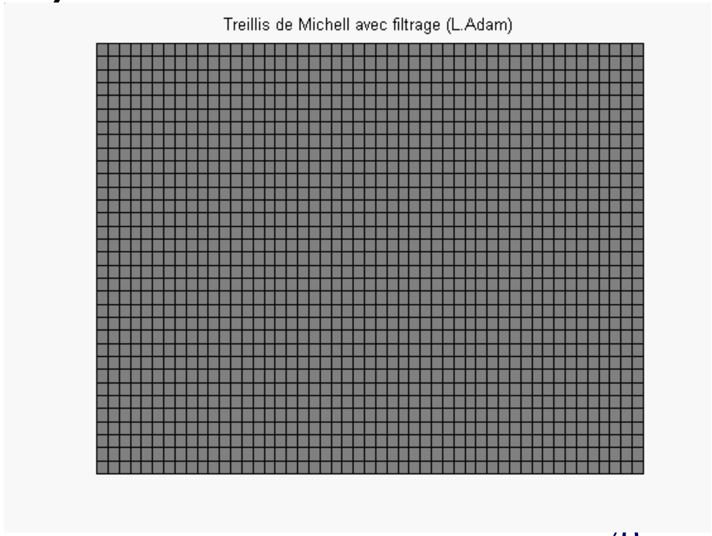
(Duysinx, 1996)

Topology optimization

- Optimal material distribution formulation (Bendsøe & Kikuchi, 1988)
  - Optimal topology without any a priori $\quad E = x^p\, E_0$
  - Fixed mesh $\quad \rho = x\, \rho_0$
  - Design variables = Local density parameters $\quad p > 1$
  - Homogenization law for continuous interpolation of effective properties (e.g. SIMP / power law)

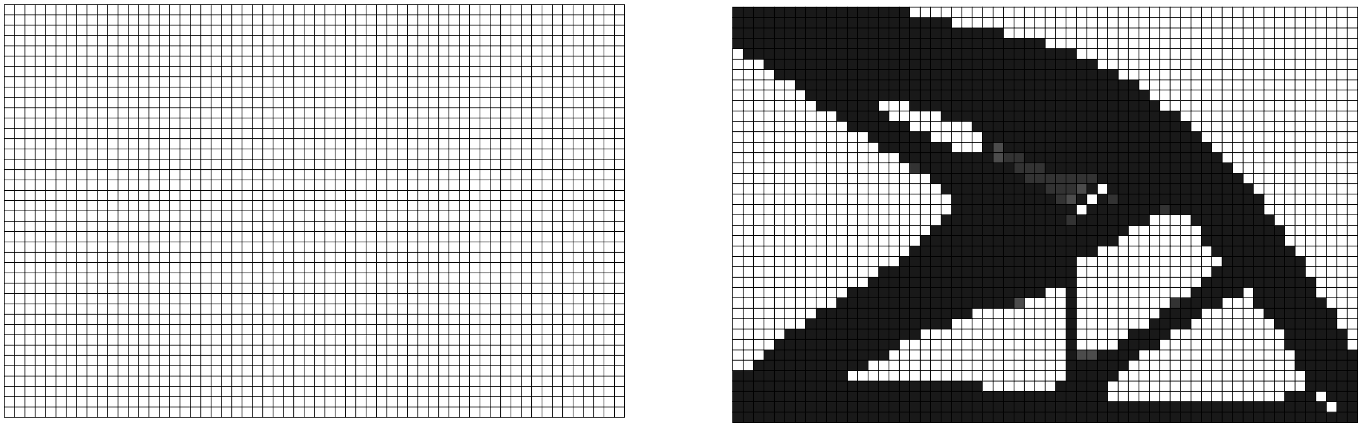


Bruyneel & Duysinx (2004)



Duysinx (1996)

NUMERICAL FORMULATION



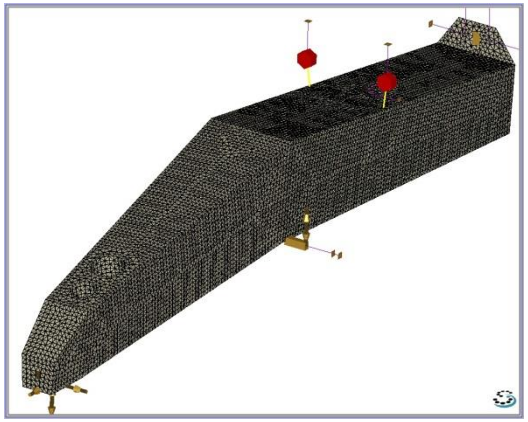
- Finite elements (F.E.) discretisation of the design domain
- Density in each element = design variable
- Discrete variable 0-1 problem ➔ Continuous variable problem with penalisation
- SIMP interpolation of material properties

$$E = x^3 \, E^0 \qquad \rho = x \, \rho_0$$

$$0 \leq x \leq 1$$

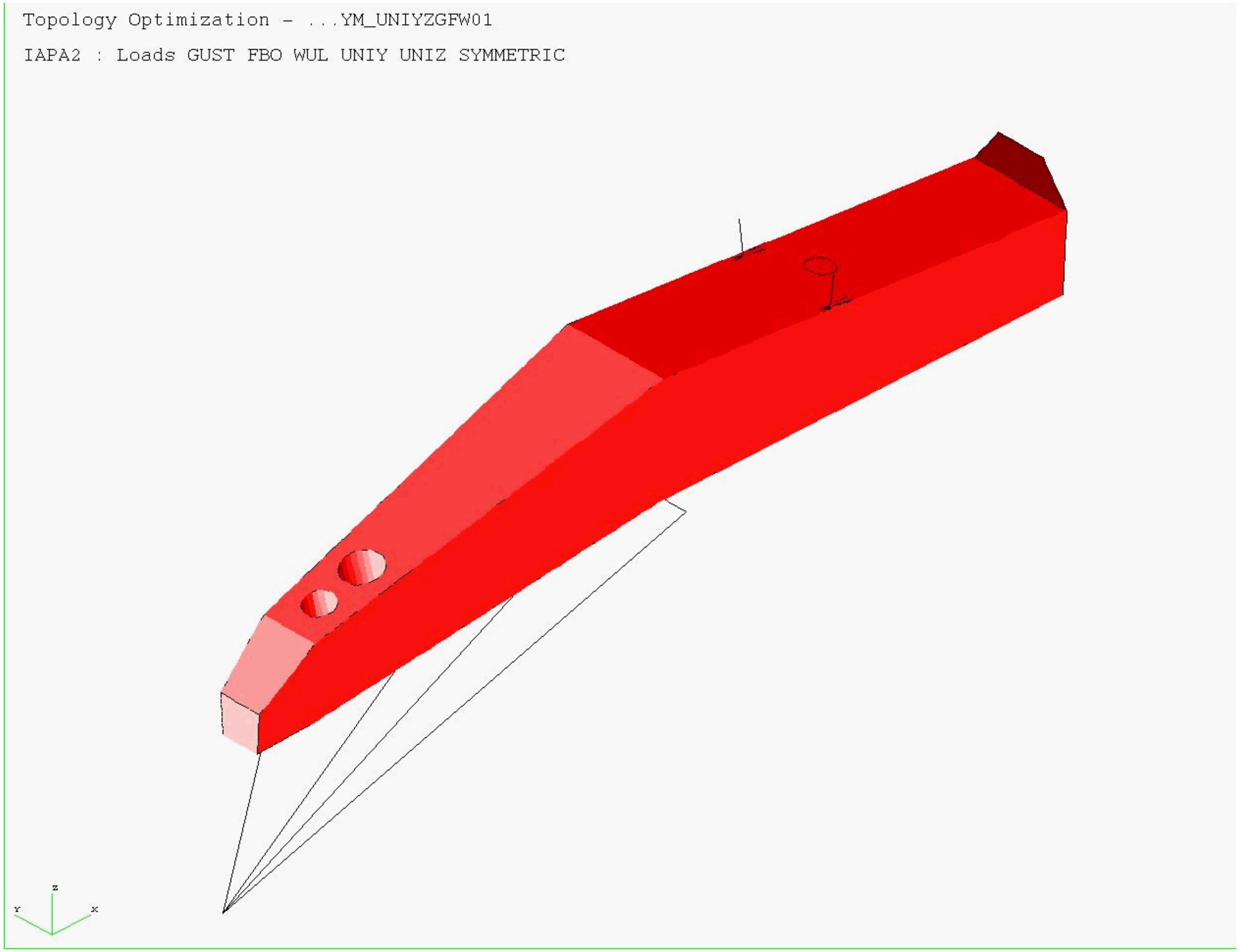
An industrial application: Airbus engine pylon

- Application
  - carried out by SAMTECH and ordered by AIRBUS
- Engine pylon
  = structure fixing engines to the wing
- Initial Model
  - CATIA V5 import → Samcef Model
  - BC's: through shell and beam FE
  - 10 load cases:
    - GUSTS
    - FBO (Fan blade out)
    - WUL (Without undercarriage landing)



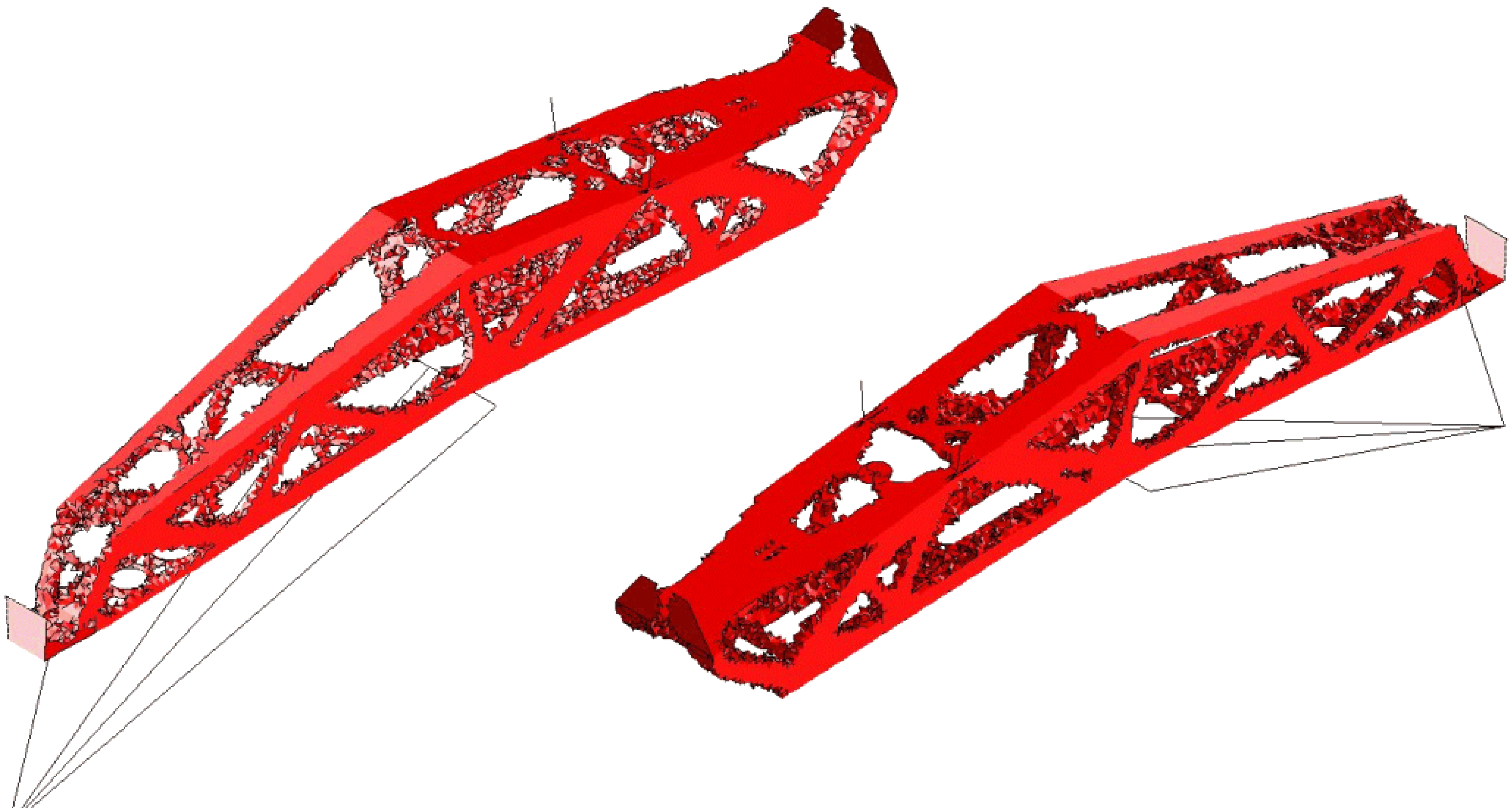
Over 250.000 tetraedral FE

Airbus engine pylon



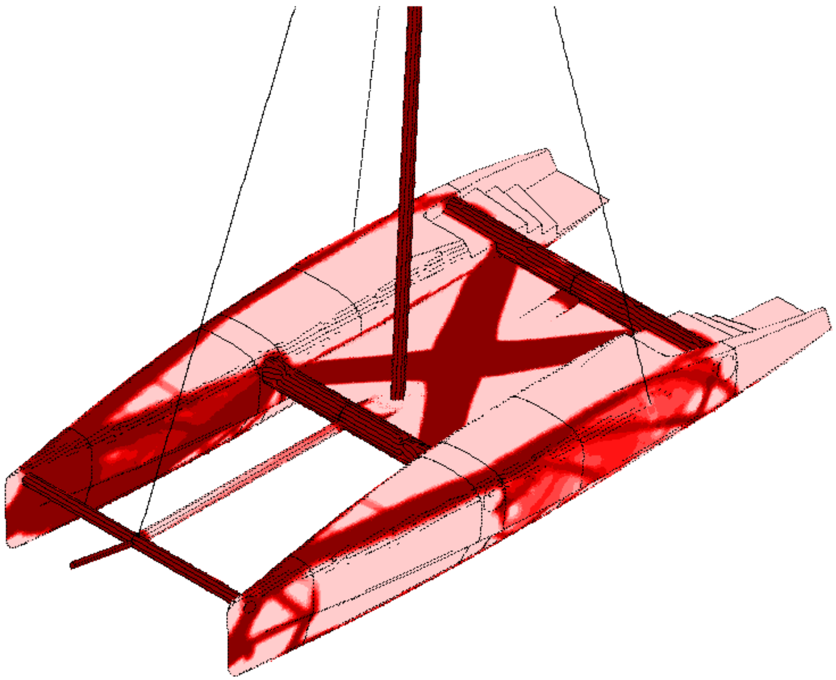
With courtesy by Samtech and Airbus Industries

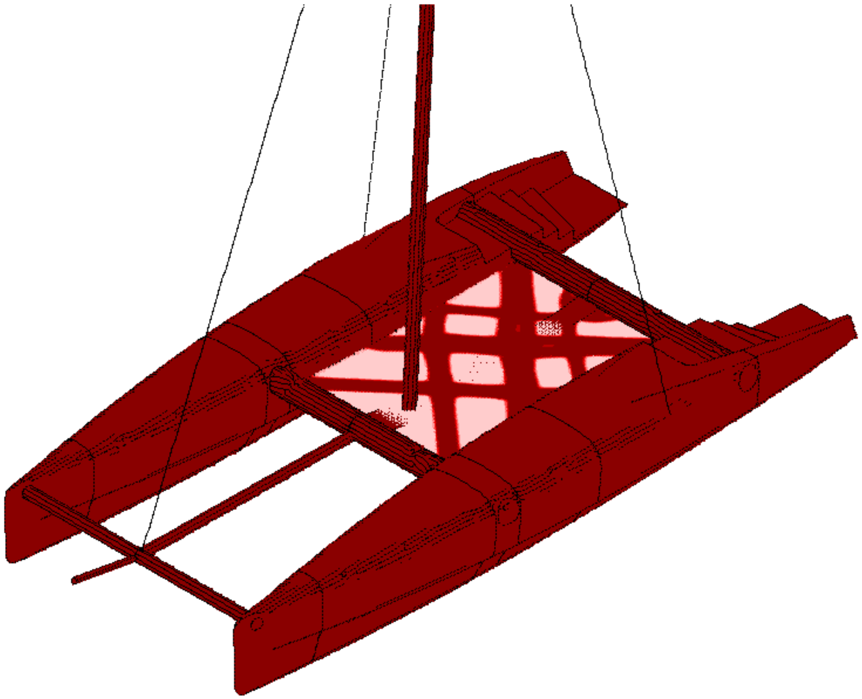
An industrial application: Airbus engine pylon



Topology optimization of catamaran hull

- Hole structure optimization
- Floor structure optimization



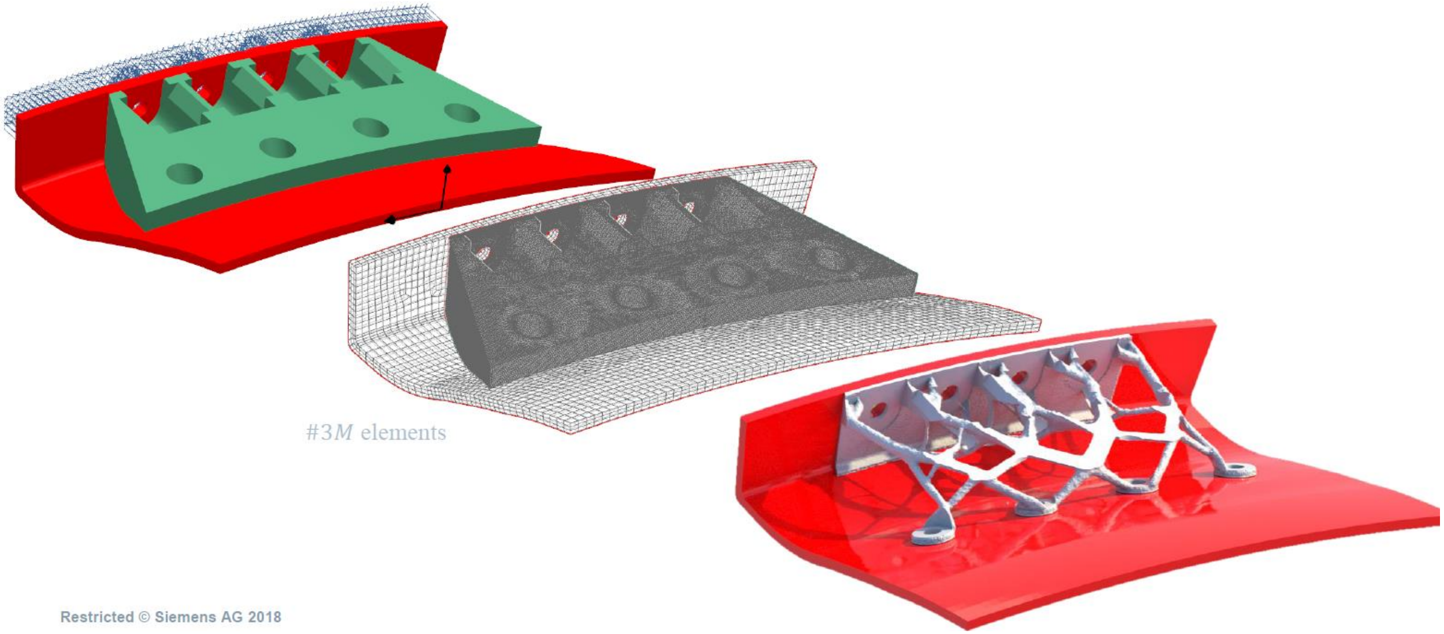


Load cases: Flexion, torsion, and local forces with the same magnitude

With courtesy by Samtech

Topology optimization and additive manufacturing

- Development validated by industrial end-users in AERO+ project



Courtesy of Siemens-SAMTECH and SAFRAN