INTRODUCTION TO SHAPE OPTIMIZATION

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LTAS – Automotive Engineering
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LAYOUT

- Introduction
  - Shape description approaches

- Parametric CAD description
  - Parametric design
  - Sensitivity analysis & velocity field problem
  - Example: The torque arm problem
  - Shape optimisation and FE error control
  - Boss Quattro system

LAYOUT

- SHAPE OPTIMIZATION USING LEVEL SET DESCRIPTION
  - Level Set Description
  - XFEM
  - Sensitivity analysis
  - Examples

- SHAPE OPTIMIZATION OF FLEXIBLE COMPONENTS IN MULTIBODY SYSTEMS
  - Shape description
  - Sensitivity analysis
  - Examples

- TOPOLOGY AND SHAPE OPTIMIZATION
  - Case study of the mass minimization of differential casing

INTRODUCTION AND MOTIVATION

SIZING, SHAPE, TOPOLOGY OPTIMIZATION

- Jog, Haber and Bendsoe (1996) have defined 3 types of structural optimization problems:

  a/ Sizing

  – Cross section, thickness, Young modulus...

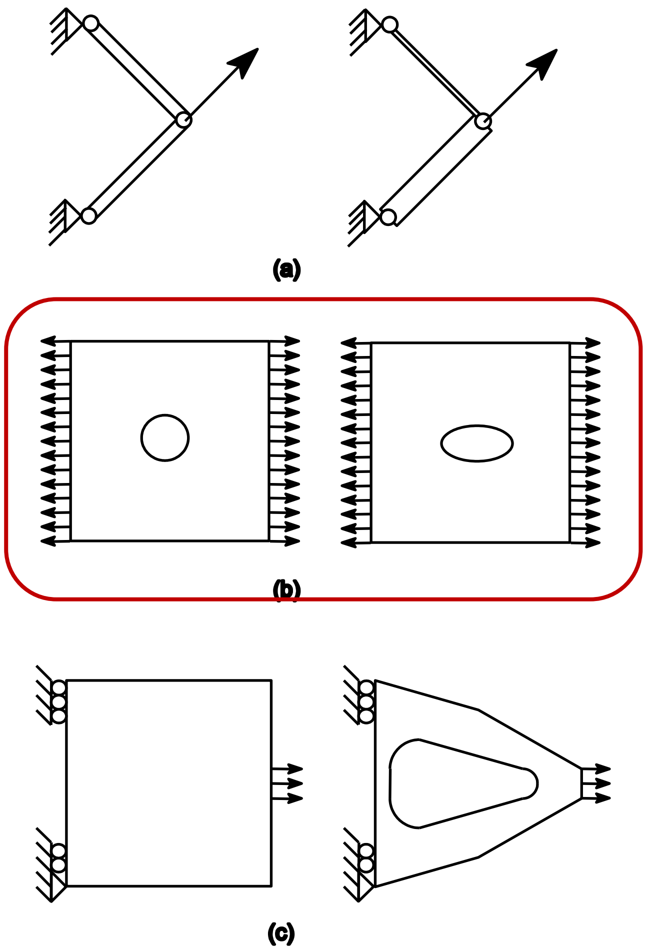
  b/ Shape

  – Parameters of geometrical features: Lengths, angles, control point positions…

  c/ Topology

  – Presence or absence of holes,

  – Connectivity of members and joints...



SHAPE REPRESENTATIONS

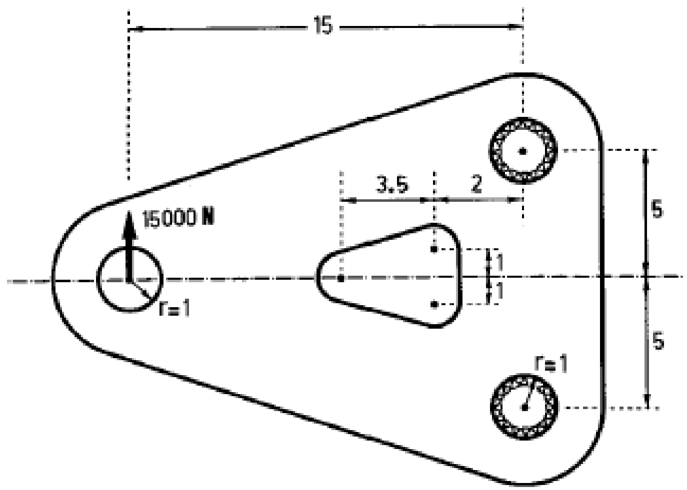
SHAPE DESCRIPTION

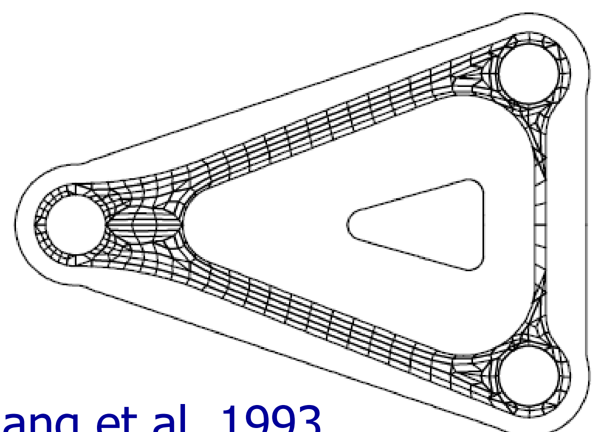
- One can distinguish different approaches to represent shapes:

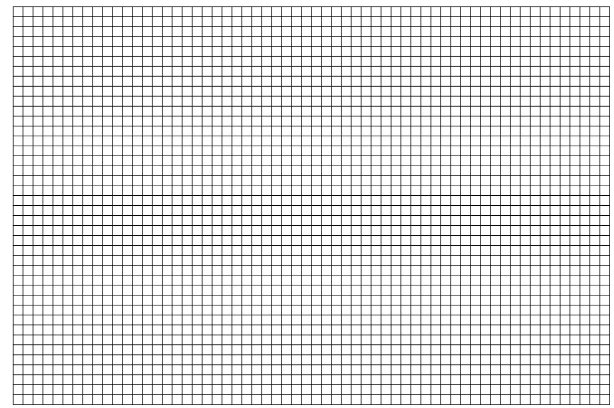
- Lagrangian approach
  - Parametric boundary description: Explicit description
    - Define the boundaries using explicit parametric curves
    - Boundaries define a contour
    - Component domain is inside the contour

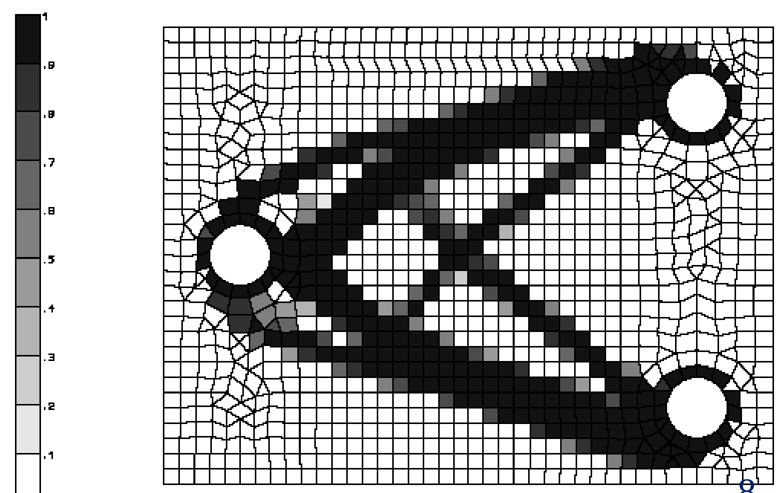
- Eulerian approach: Implicit description
  - Level Set Method
  - Indicator function: Material description
  - Define the domain on a fixed grid

GEOMETRICAL DESCRIPTION









Zhang et al. 1993

BOUNDARY DESCRIPTION

- Boundary description
  - Decompose complex shapes into geometrical features
  - Geometrical features can include parameters that can adjusted
  - For instance, plane cubic lines can be written as:

$$\mathbf{p}(u) = \mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0$$

$$\mathbf{p}(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} \mathbf{a}_3 \\ \mathbf{a}_2 \\ \mathbf{a}_1 \\ \mathbf{a}_0 \end{bmatrix} = \mathbf{UA}$$

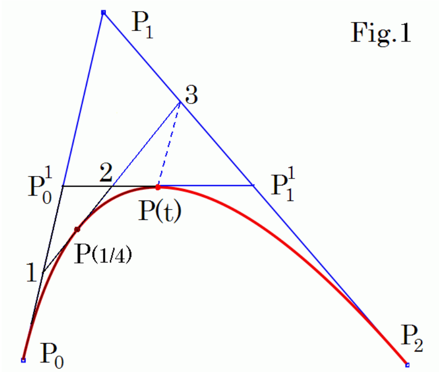
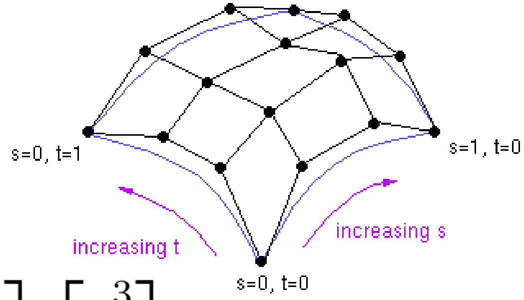


Fig.1

- Where u is the parametric coordinate in [0,1], a_i [a_{xi}, a_{yi}] are the algebraic coefficients of the curve

BOUNDARY DESCRIPTION

– Similarly a cubic spatial parametric surface can be represented as



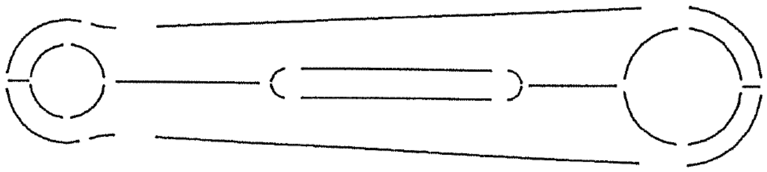
$$\mathbf{p}(u) = \sum_{i,j=0}^{3} a_{ij}\, u^i\, v^j$$

$$\mathbf{p}(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} \mathbf{a}_{33} & \mathbf{a}_{32} & \mathbf{a}_{31} & \mathbf{a}_{30} \\ \mathbf{a}_{23} & \mathbf{a}_{22} & \mathbf{a}_{21} & \mathbf{a}_{20} \\ \mathbf{a}_{13} & \mathbf{a}_{12} & \mathbf{a}_{11} & \mathbf{a}_{10} \\ \mathbf{a}_{03} & \mathbf{a}_{02} & \mathbf{a}_{01} & \mathbf{a}_{00} \end{bmatrix} \begin{bmatrix} v^3 \\ v^2 \\ v \\ 1 \end{bmatrix} = \mathbf{UAV}^T$$

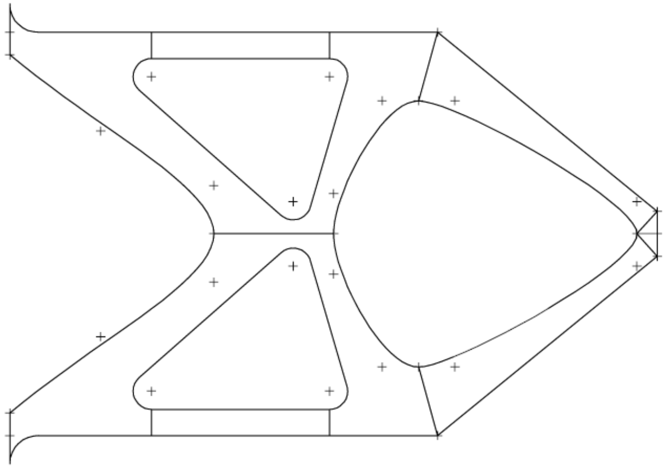
- Where u, v are the parametric coordinates and a_{ij} are the algebraic coefficients of the surface.

BOUNDARY DESCRIPTION

- Parametric curves or pieces of surfaces can be linked together into usual geometrical features with predefined shapes such as circles, ellipses.



- One can create a library of usual elements by interconnecting basic parametric geometric entities and defining the type of the geometric feature.

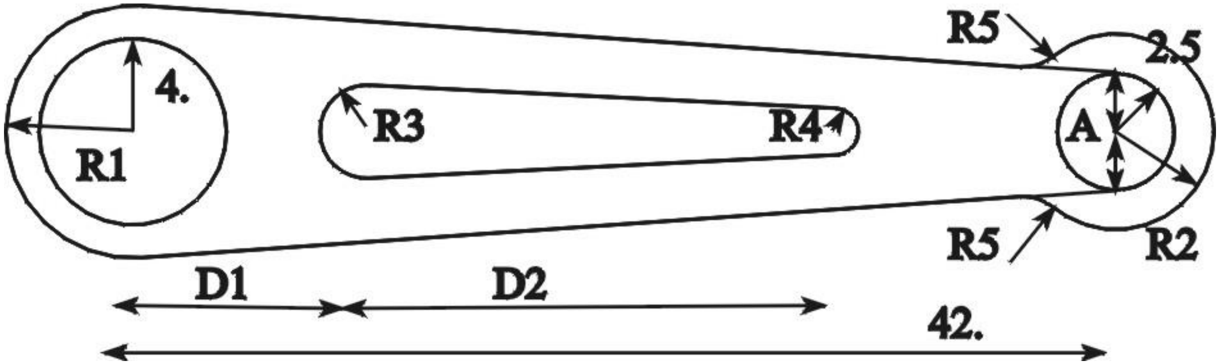


- Many different parametric features can then be combined to form a complex component description

BOUNDARY DESCRIPTION

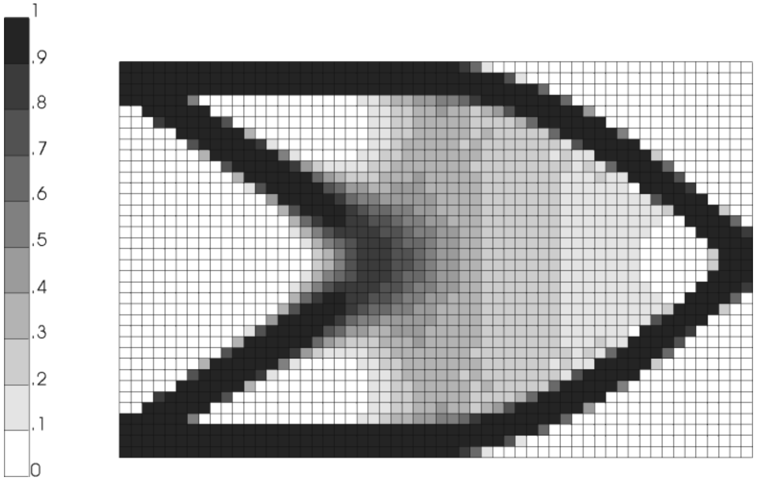
- Shape optimization can be formulated using parametric geometric parameters defining its constituting shape description.

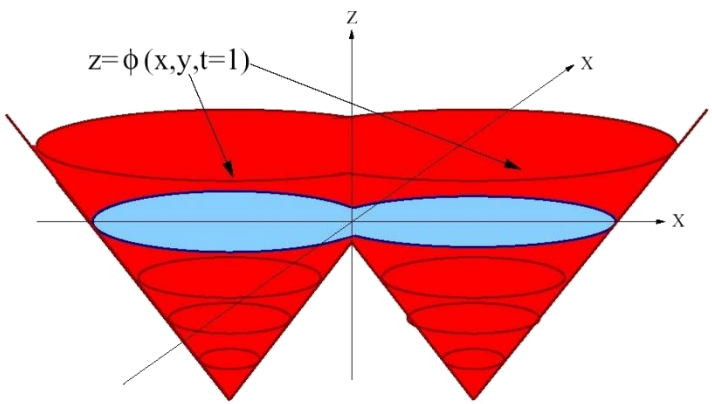
- The design variables are thus the numerical values of the geometric parameters.



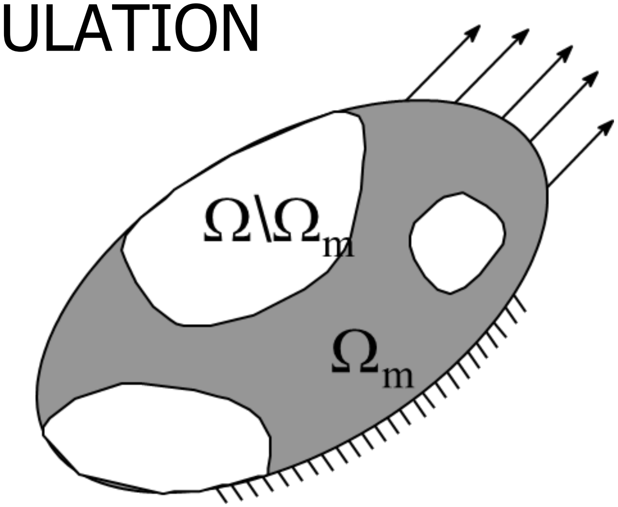
LAGRANGIAN APPROACH: DOMAIN INDICATOR FUNCTION

- Several approaches to determine the indicator function
- Material density function
  - Binary
  - Continuous approximation
    - Porous cellular material ➔ Homogenization
    - Interpolation function: SIMP, RAMP…
- Implicit boundary description
  - Level set description
    - Hamilton-Jacobi function
    - Parametric functions and math programming
    - Nodal values of Level Set
  - Phase Field Description
  - …





MATERIAL DISTRIBUTION FORMULATION



- Abandon CAD model description based on boundary description
- Optimal topology is given by an optimal material distribution problem
- Search for the indicator function of the domain occupied by the material

$$\chi(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega^m \\ 0 & \text{if } \mathbf{x} \in \Omega \setminus \Omega^m \end{cases}$$

- The physical properties write

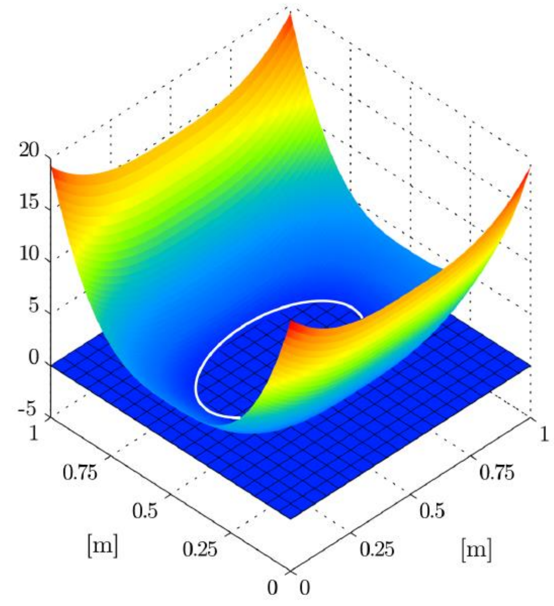
$$E_{ijkl}(\mathbf{x}) = \chi(\mathbf{x})\, E^0_{ijkl}$$
$$\rho(\mathbf{x}) = \chi(\mathbf{x})\, \rho^0$$

- The problem is intrinsically a binary 0-1 problem ➔ solution is extremely difficult to solve

$$\chi \in \{0, 1\}$$

LEVEL SET DESCRIPTION

- LEVEL SET METHOD [Sethian, 1999]
  - Alternative description to parametric description of curves
  - Implicit representation of the geometry
  - Add the dimensionality by one



- The parametric description of the curve

$$\Gamma = \{(\mathbf{x}, \mathbf{p}) \in D \times \mathbb{R}^{n_v} : \phi(\mathbf{x}, \mathbf{p}) = 0\}$$

- Is replaced an implicit description

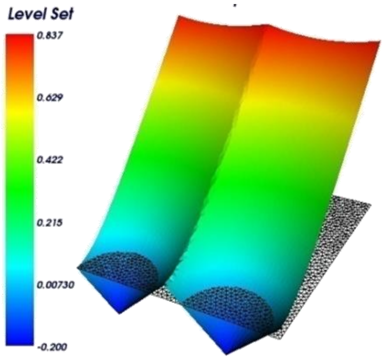
$$\begin{cases} \phi(\mathbf{x}, \mathbf{p}) > 0, & \Leftrightarrow \mathbf{x} \in \Omega \text{ (material)} \\ \phi(\mathbf{x}, \mathbf{p}) < 0, & \Leftrightarrow \mathbf{x} \in D \backslash \Omega \text{ (void)} \\ \phi(\mathbf{x}, \mathbf{p}) = 0, & \Leftrightarrow \mathbf{x} \in \partial\Omega \text{ (interface)}. \end{cases}$$

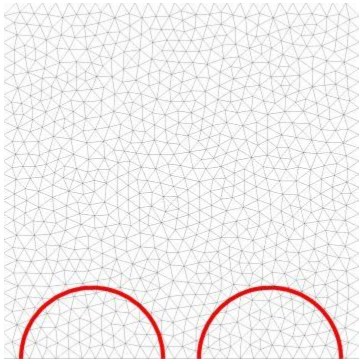
- There are many ways to define the level set corresponding to known shape. For instance the signed distance function

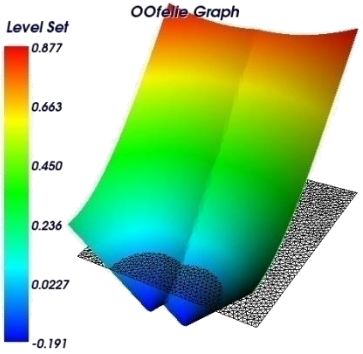
$$d(\mathbf{x}) = \min_{\mathbf{x}_\Gamma \in \Gamma} \|\mathbf{x} - \mathbf{x}_\Gamma\|, \quad \forall \mathbf{x} \in D$$

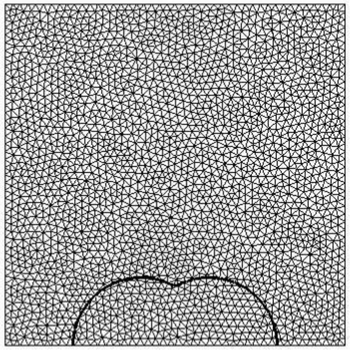
GEOMETRICAL DESCRIPTION USING LEVEL SETS

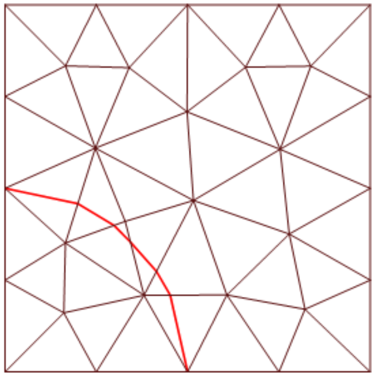
- Advantages :
  - Same definition in 2D and 3D
  - Combination of entities (min, max)
  - Removing entities
  - Separating entities
  - Merging entities
- Drawbacks :
  - Construction (available tools, analytical functions)
  - Mesh refinement necessary

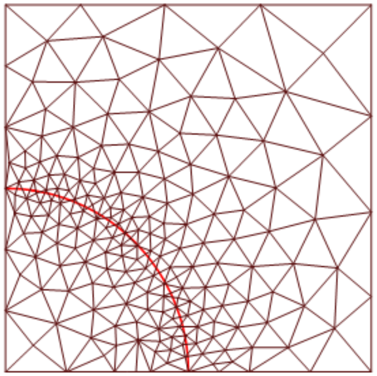






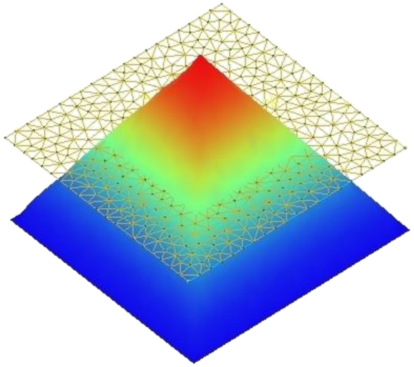


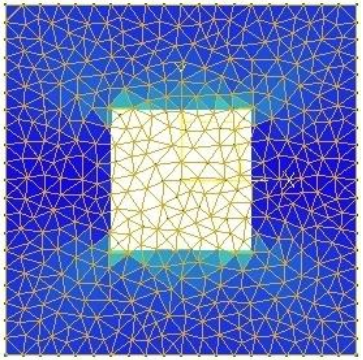




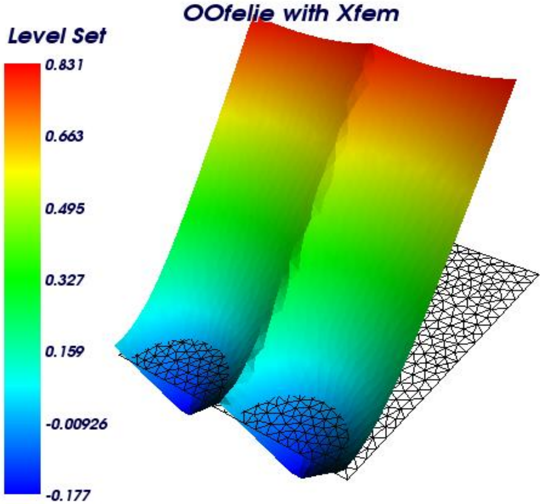
GEOMETRICAL DESCRIPTION USING LEVEL SETS

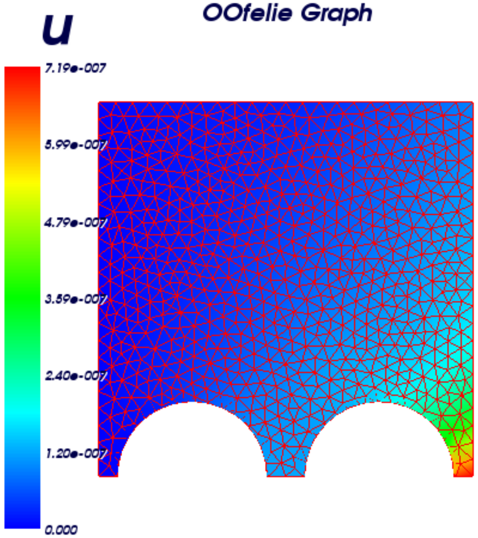
- Level Set of a square hole





- Combination of two holes





GEOMETRICAL DESCRIPTION USING LEVEL SETS

- In XFEM framework: discretization of the level set,
  - Each node has a Level Set dof
  - Interpolation using classical shape functions

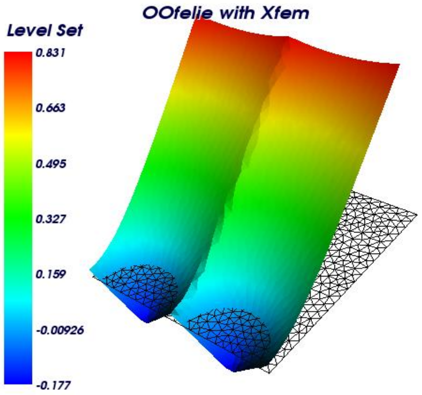
$$\Psi(x, s) = \sum_{i=1}^{n} N_i(\mathbf{x})\, \Psi_i$$

  - Material assigned to a part of the Level Set (positive or negative)

CONSTRUCTIVE GEOMETRY USING LEVEL SETS

- Constructive geometry approach
  - Elaborate complex geometries using Level Sets:
    - Primitive shapes with dimension parameters

$$\Psi = \Psi(\mathbf{x}, s)$$



    - Linear combinations of basic functions

$$\Psi(x, s) = \sum_{i=1}^{n} s_i \, \chi_i(\mathbf{x})$$

  - Library of graphic primitives and features
    - Lines, circles, ellipses, rectangles, triangles
    - NURBS
- Combine the basic levels sets using logic and Boolean operations ➔ constructive geometry

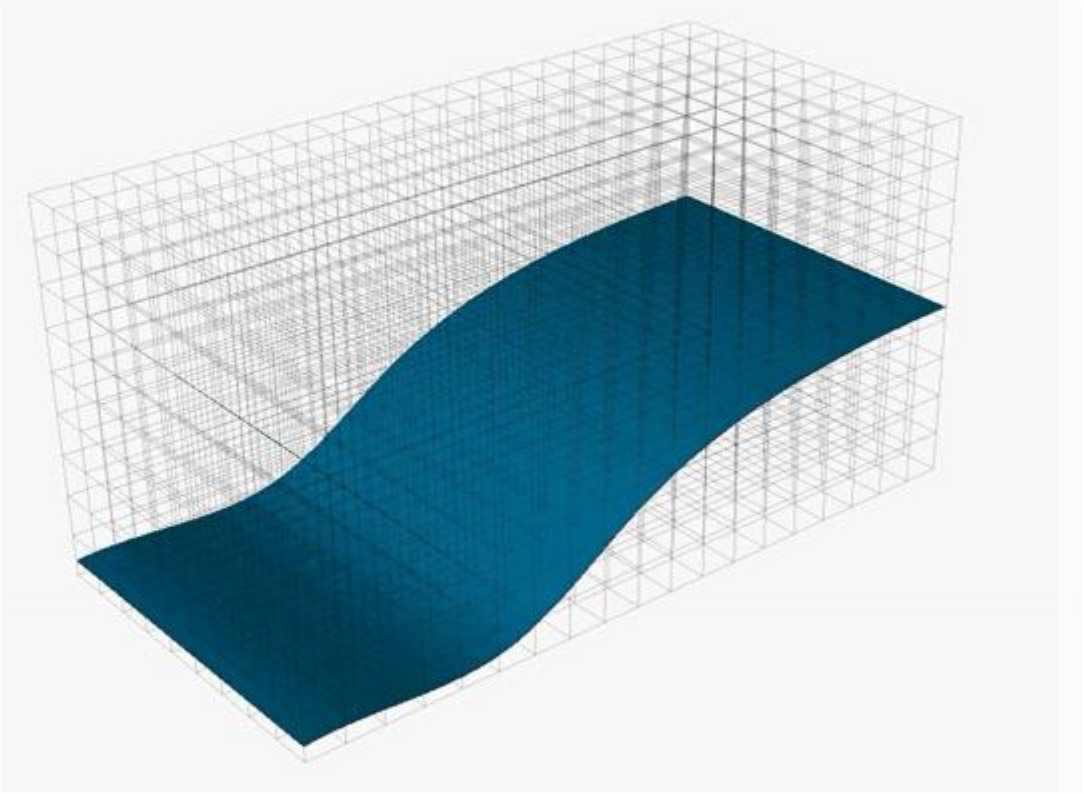
LEVEL SET BASED CONSTRUCTION SOLID GEOMETRY

- To represent complex geometries with Level Set
  - Introduction of Constructive Solid Geometry (CSG) based on Level Set [Chen et al. 2007]
  - CSG = build complex geometries by combining simple solid object called primitives using Boolean operators
    - ➔ Development of "Level Set geometrical modeler"

- Geometrical primitives are represented with Level Set (analytical, geometrical, CAD based, predefined compound Level Sets)
- Use Boolean operators on Level Set primitives

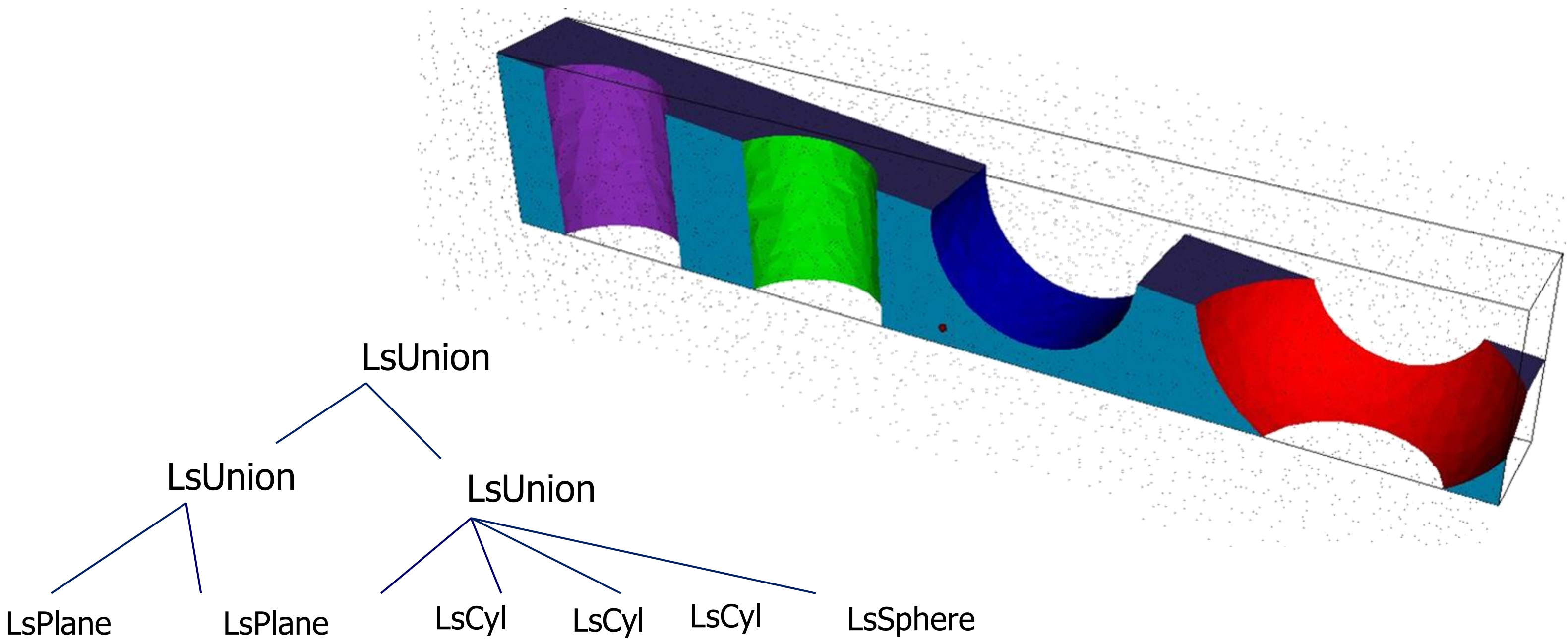
LEVEL SET BASED CONSTRUCTION SOLID GEOMETRY

- Example of complex geometry with CSG Level Set
  - Two cylinders
  - One oblong hole
  - One external oblong surface
  - 3 NURBS surfaces



GEOMETRICAL DESCRIPTION USING LEVEL SETS

- The Level Set geometry is organized as a tree
  - Where :
    - Each leaf is a basic level set
    - Each node is an operator
  - Each sub cell is classified after all cut as inside/outside or boundary



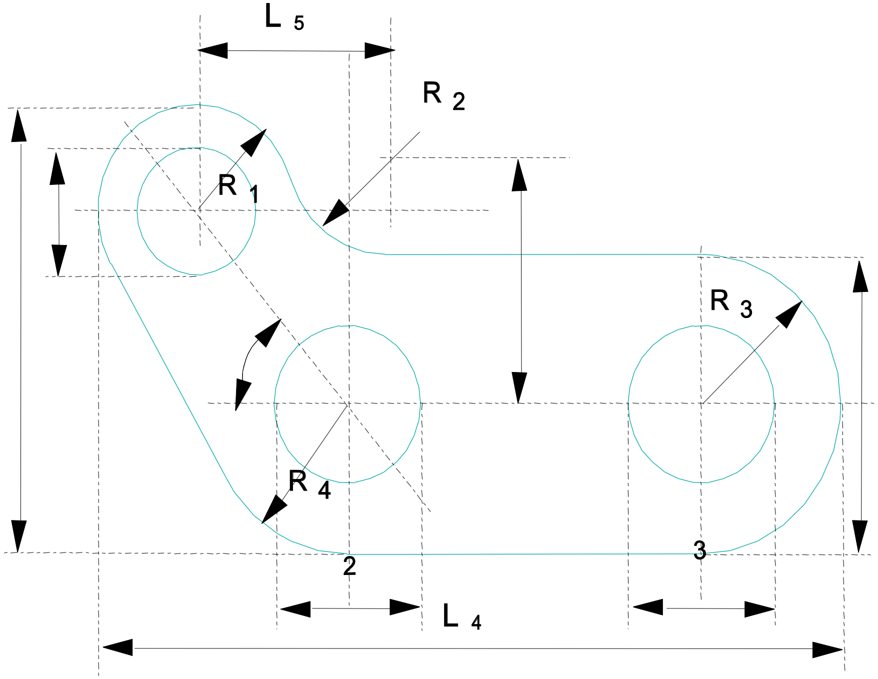
SHAPE OPTIMIZATION USING PARAMETRIC BOUNDARY DESCRIPTION

PARAMETRIC BOUNDARY DESCRIPTION

- Modification of external or inner boundaries

- Key issue: definition of a consistent parametric CAD model
  - Geometrical constraints (tangency, linking of points)
  - Geometrical features: straight lines, circles, NURBS, surfaces, etc.

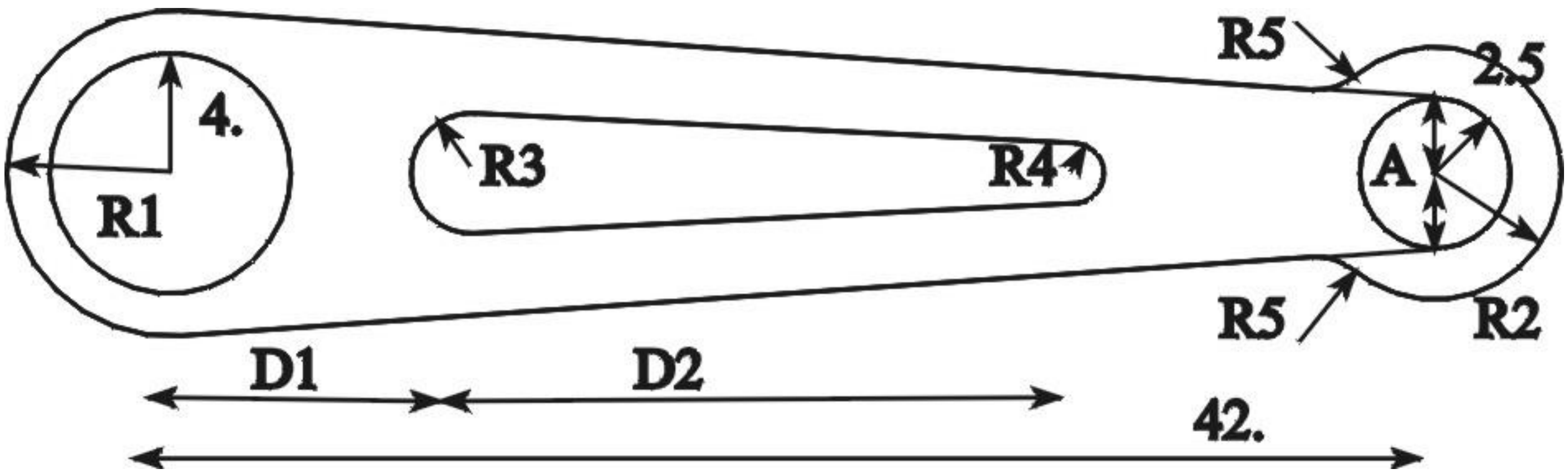
- Implementation issue: API to and from CAD systems (CATIA, Pro E, etc.)



Zhang, Duysinx, Fleury (1993)

- Design variables = a set of independent CAD model parameters

A CAD MODEL WITH PARAMETERISATION



- <u>Regular curves and surfaces</u>:
  - Straight lines, arcs of circles, splines, NURBS
  - Planes, spherical, spline and NURBS surfaces

- <u>Model Parameterisation</u>:

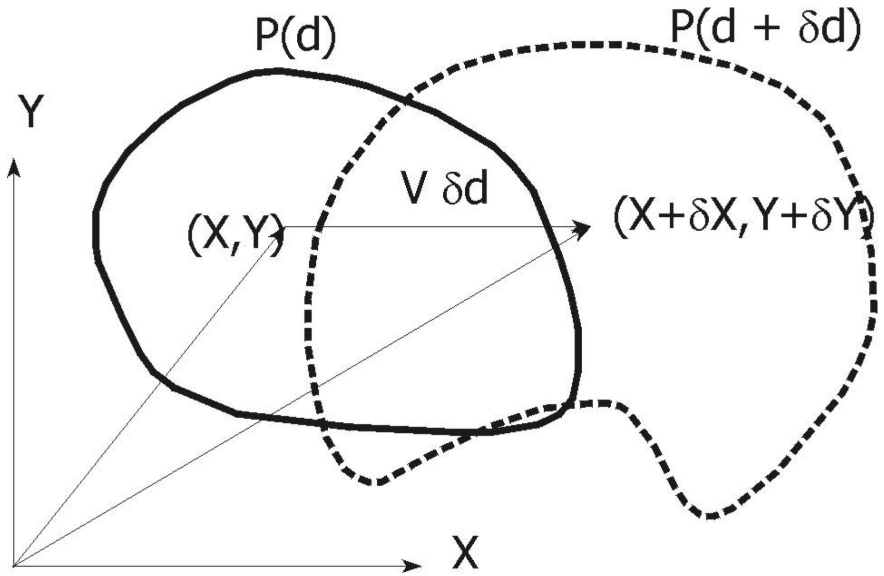
  Definition of the model with a set of independent parameters

SENSITIVITY ANALYSIS AND VELOCITY FIELD

- Position of a point after a perturbation of the design variable d_i

$$\mathbf{X}(d_i + \delta d_i) = \mathbf{X}(d_i) + \mathbf{V}_i \, \delta d_i$$

$$\text{with } \mathbf{V}_i = \frac{\partial \mathbf{X}}{\partial d_i}$$



- Derivative of a response in a given point:

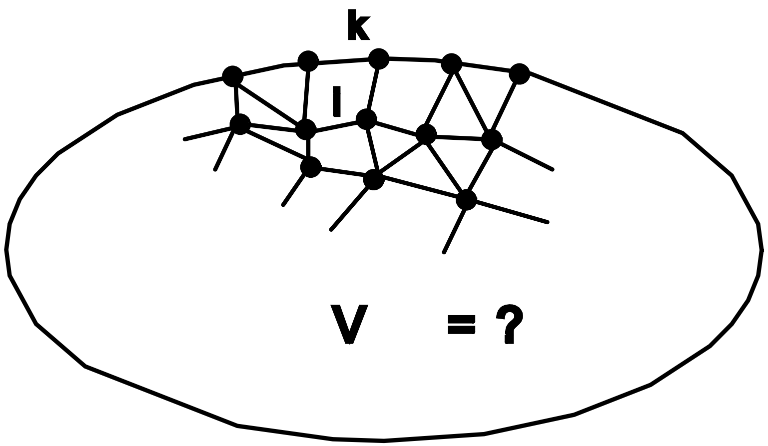
$$\frac{DR}{Dd_i} = \frac{\partial R}{\partial d_i} + \sum_k \frac{\partial R}{\partial \mathbf{X}_k} \frac{\partial \mathbf{X}_k}{\partial d_i}$$

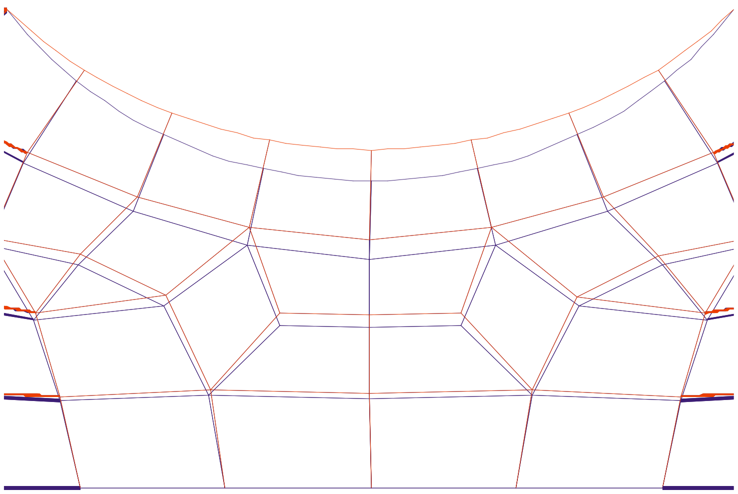
$$= \frac{\partial R}{\partial d_i} + \mathbf{V}_i \, \nabla R$$

- Conclusion: determine the velocity field at first

VELOCITY FIELD PROBLEM

- Key issue: Velocity field
- Practical calculation of velocity field
  - Boundary velocity field ➔ CAD model
  - Inner field ➔ Velocity law





Duysinx, Zhang, Fleury (1993)

- Inner field:
  - Transfinite mapping
  - Natural / mechanical approach
  - Laplacian smoothing
  - Relocation schemes

VELOCITY FIELD DETERMINATION: THE MESH RELOCATION TECHNIQUE

- On the boundaries:

  The velocity field is uniquely determined by the parametric equations of the contour curves and surfaces

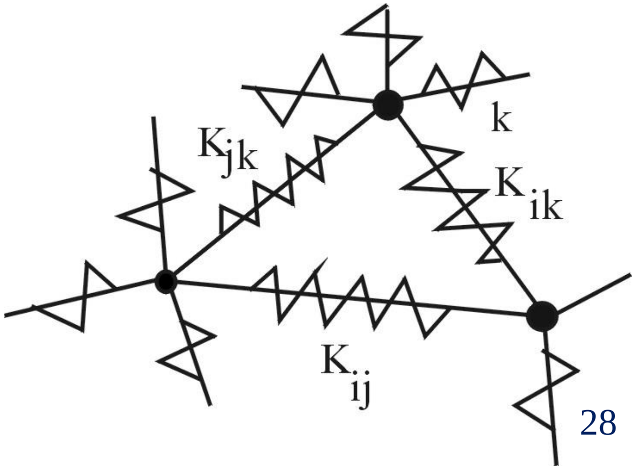
$$\Psi = \sum_i W_i(s)\, d_i \qquad \mathbf{V}_\Gamma = \left[\frac{\partial \Psi}{\partial d_i}\right] = [W_i(s)]$$

- Inside of domain

  The velocity field is determined with a node relocation technique

  Link the perturbations of neighbouring nodes with stiffness:

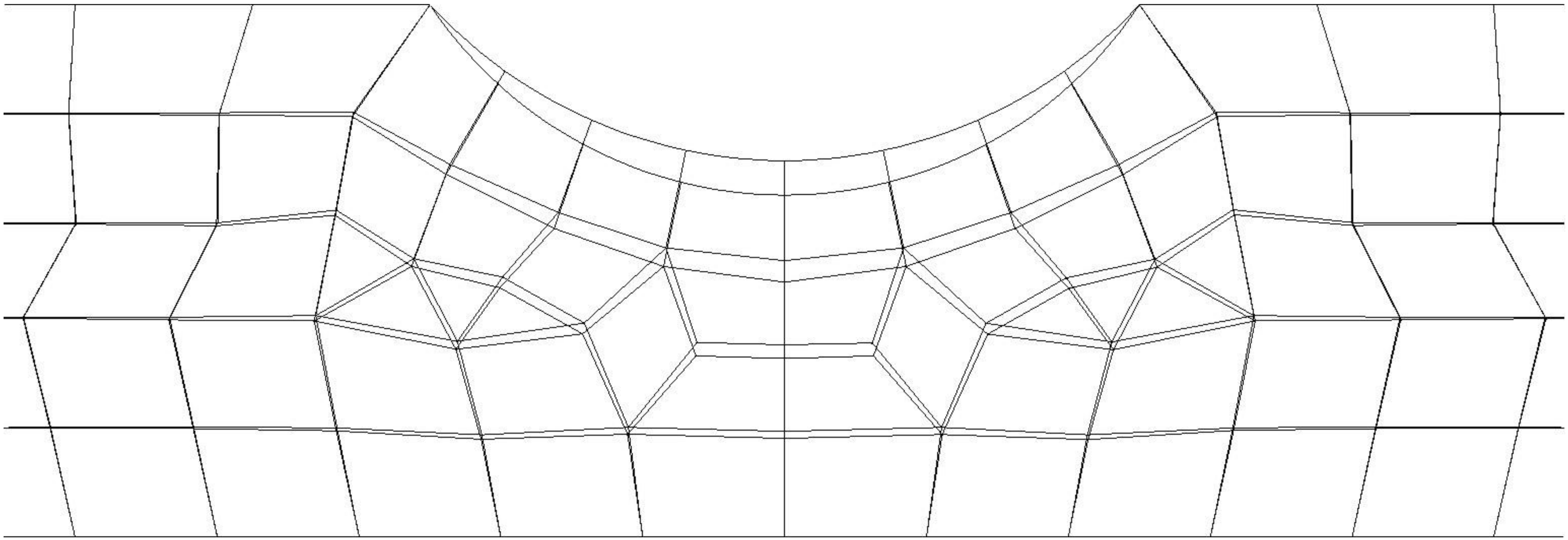
$$\sum_{i \neq j} k_{ij} \left(V_k|_{(i)} - V_k|_{(j)} \right) = 0$$



Advantages:

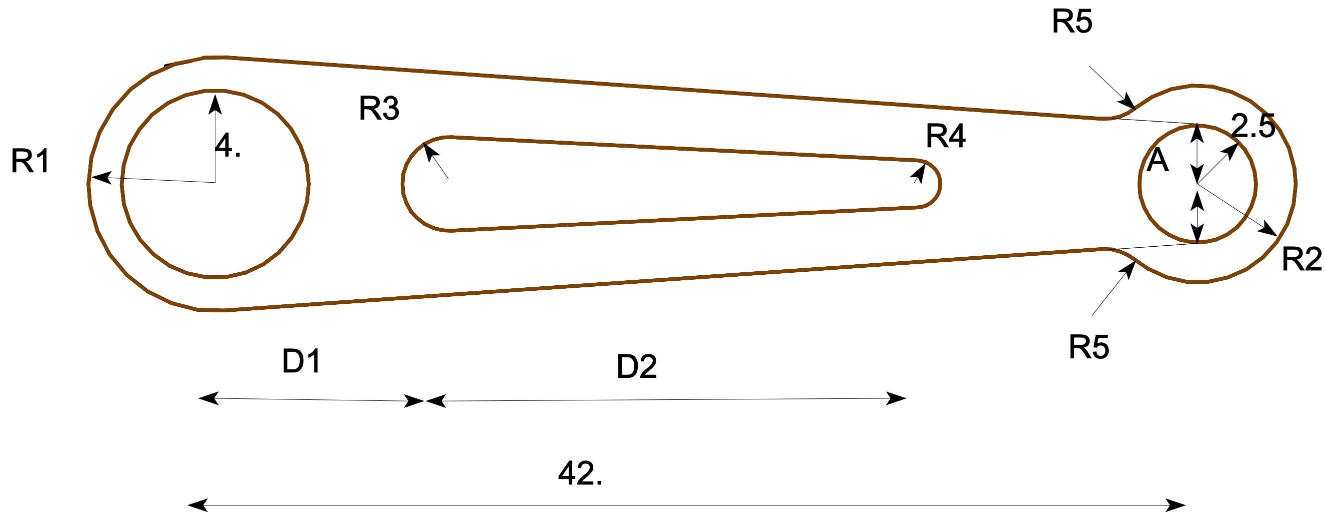
- low computation cost
- possible extension to volume structures and shells

Example of velocity field determined by node relocation



Velocity field relative to a modification of the radius of the notch

Shape Optimisation of a Torque Arm



- Statement of the design problem:

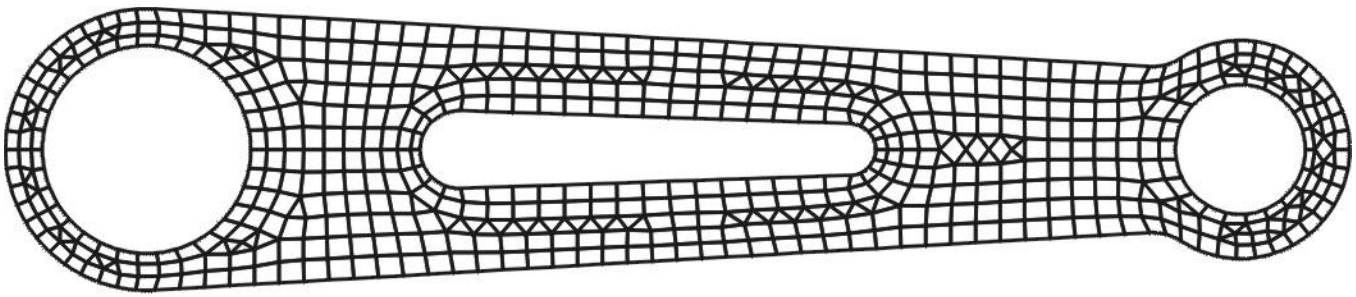
  Minimise Weight

  8 design parameters

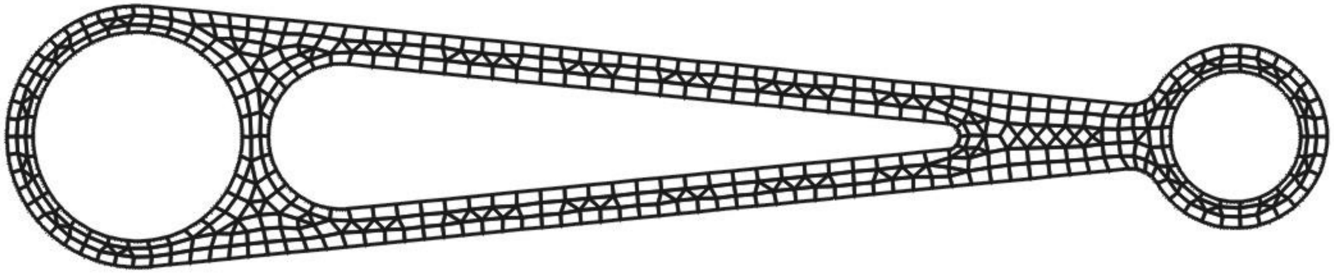
  s.t. Von Mises equivalent stress under 80000 N/mm²

  Geometry constraints (thickness of members > 1 cm)

Shape Optimisation of Torque Arm

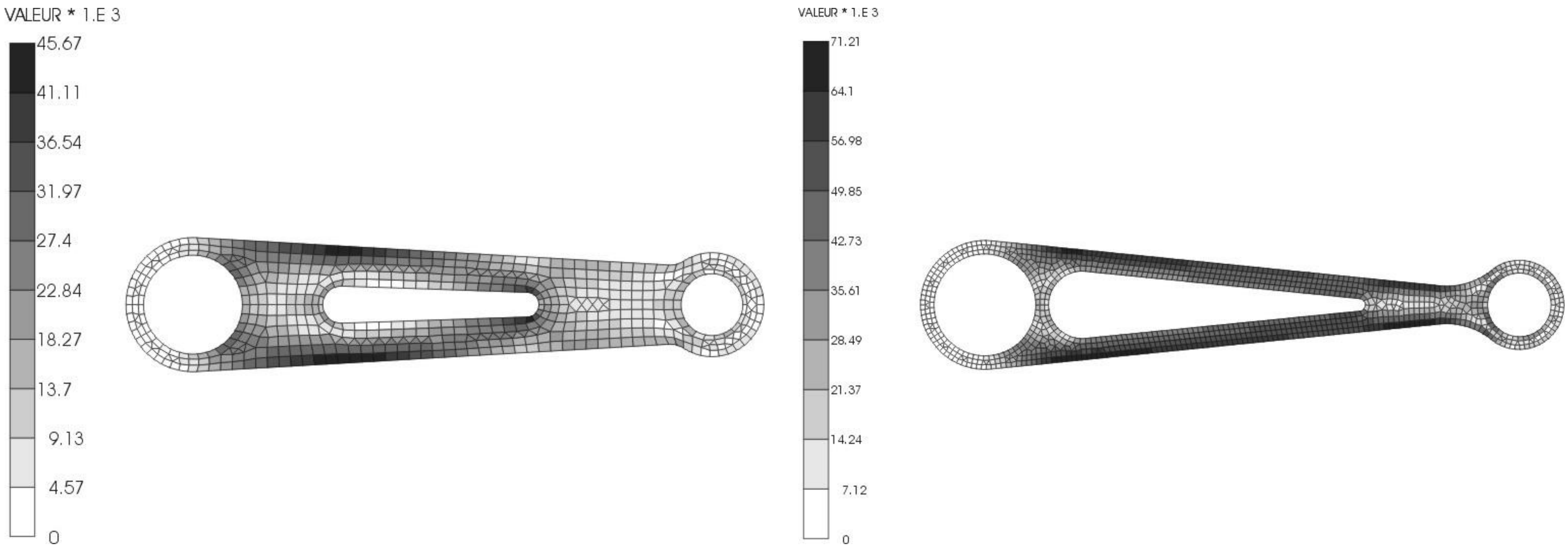


Iteration 0



Iteration 12

Shape Optimisation of Torque Arm

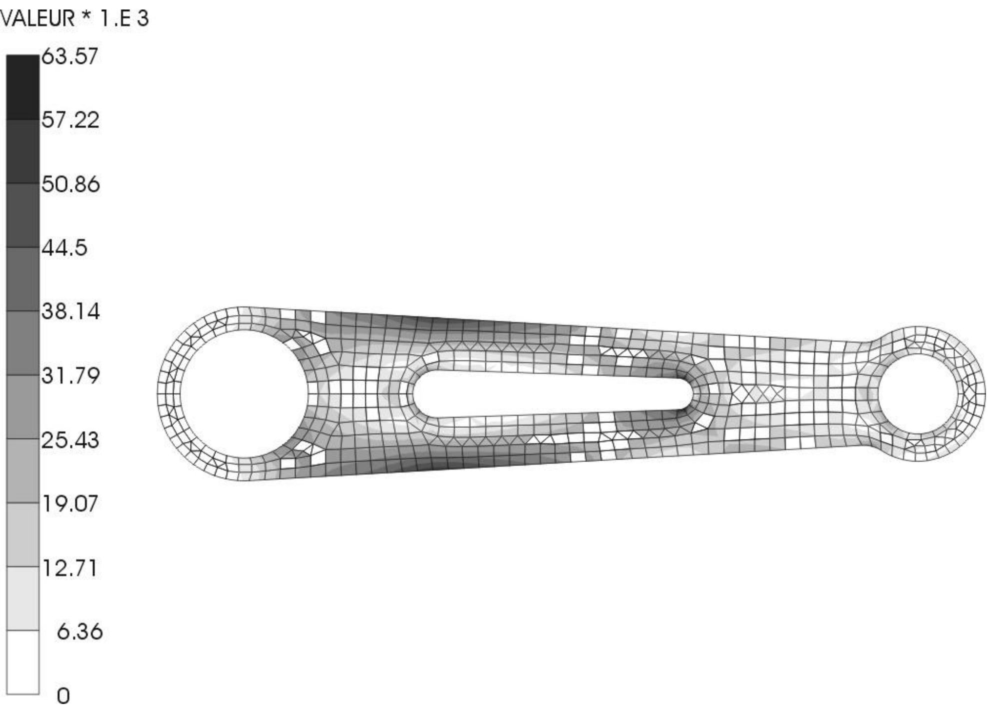


Iteration 0

Iteration 12

Von Mises stress : average values per finite element

Shape Optimisation of Torque Arm



Iteration 0

Iteration 12

Von Mises stress at Gauss Points

SHAPE OPTIMISATION AND F.E. ERROR CONTROL

- Shape modifications due to optimisation process can lead to important mesh distortions
- The optimisation results are strongly dependent on the quality of the analysis (especially the stresses)

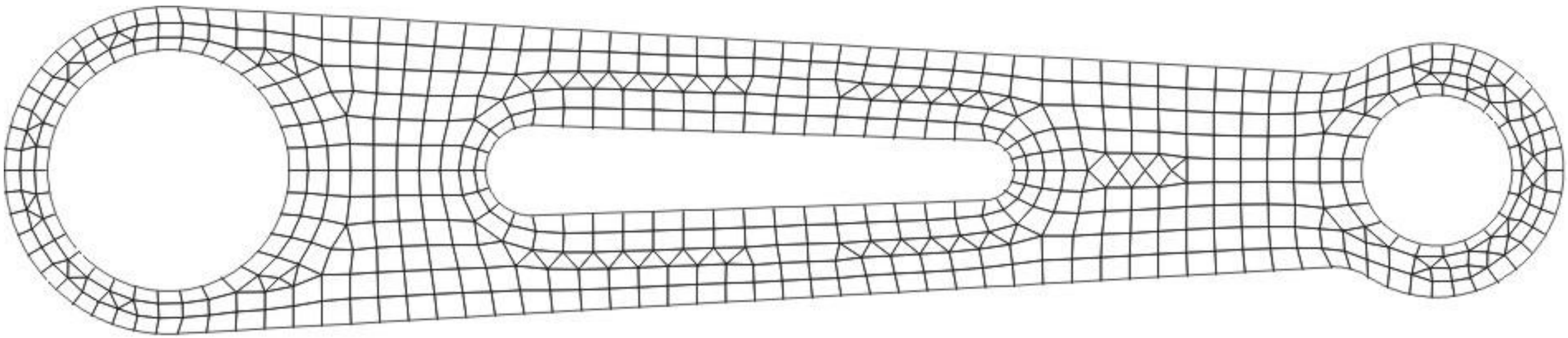
ONE ALWAYS OPTIMISES THE MODEL

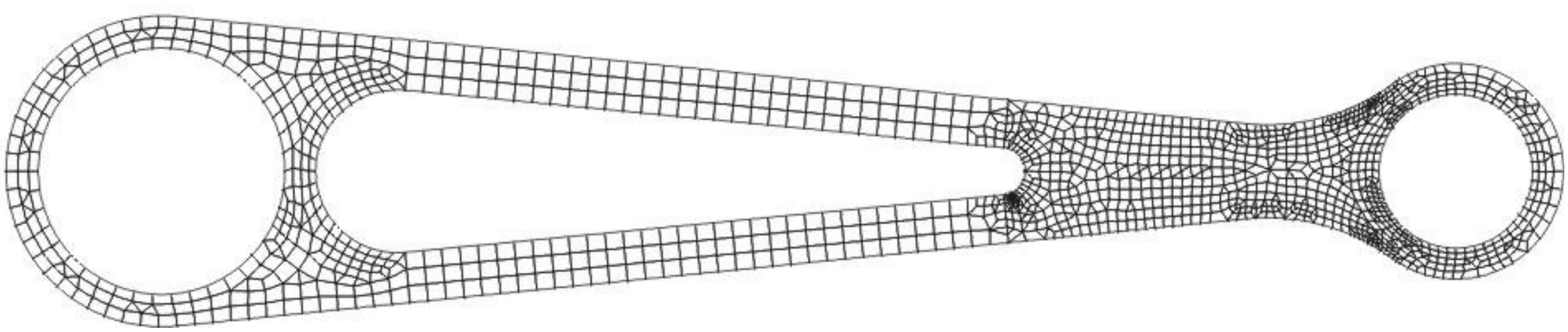
- To have relevant and meaningful results

CONTROL THE ERROR LEVEL OF THE ANALYSIS

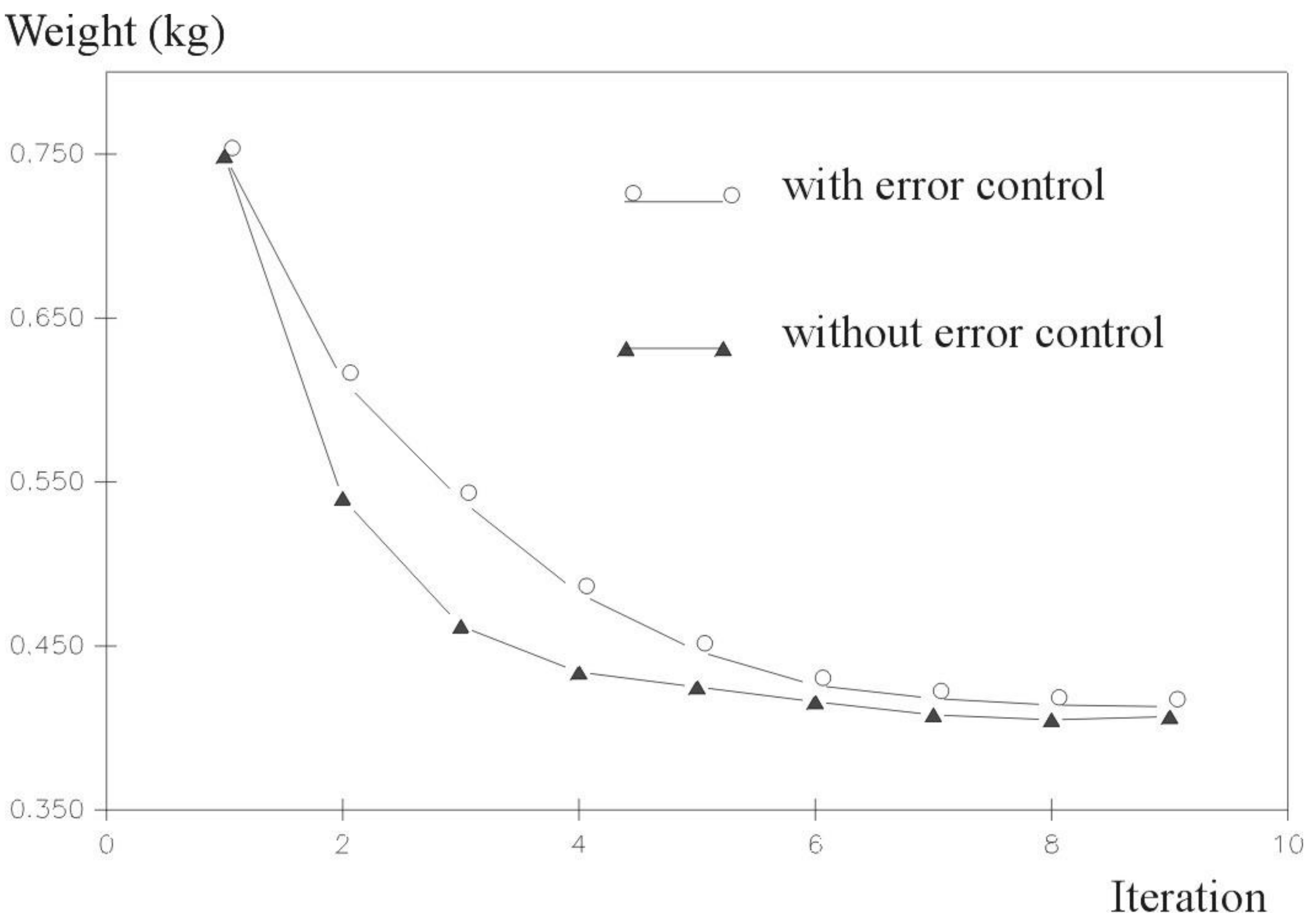
- Integration of an error estimation procedure and of a mesh adaptation tool into the optimisation loop

Shape Optimisation of Torque Arm





Shape Optimisation of Torque Arm

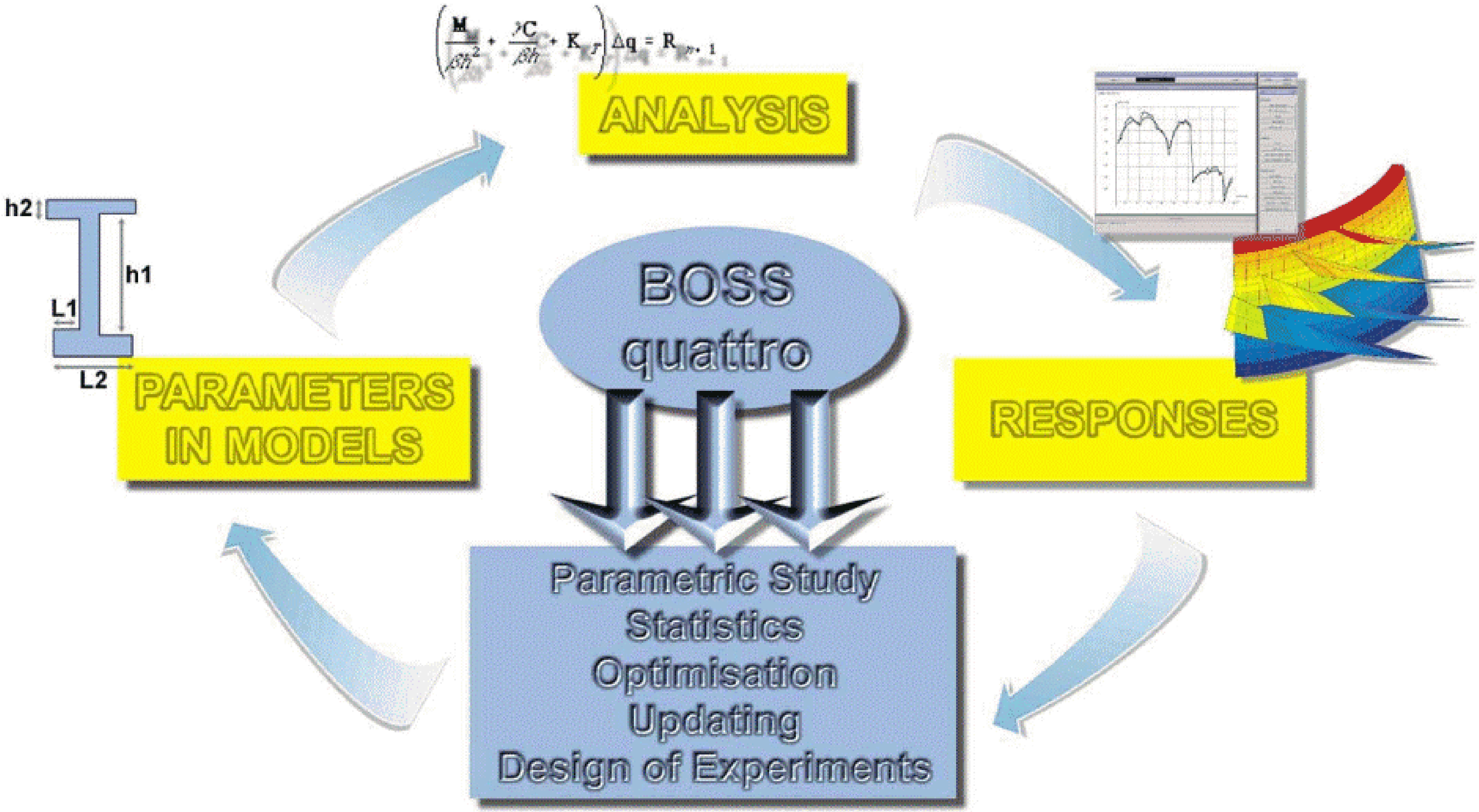


BOSS-Quattro

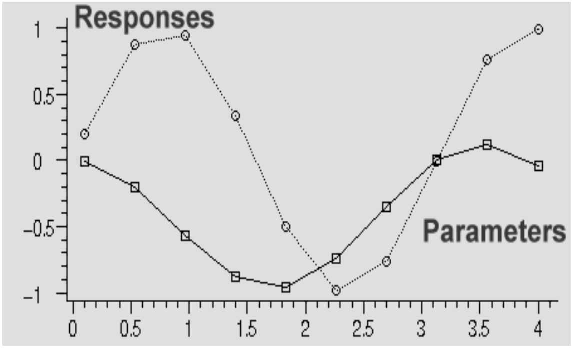
- These Concepts have been implemented in a commercialised tool BOSS-Quattro developed by SAMTECH in partnership with LTAS (Ulg)

- Optimisation of parametric models
- Open system
- A design environment for multi-model / multidisciplinary problems
- Object oriented code
- Optimisation algorithms
- Application manager (more than a task manager)
- Model manager (update, perturbations, etc.)

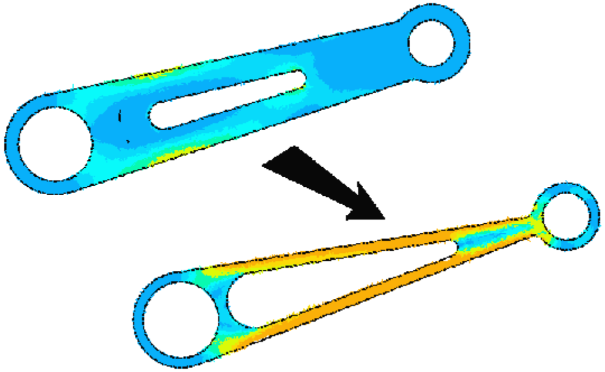
Boss Quattro philosophy



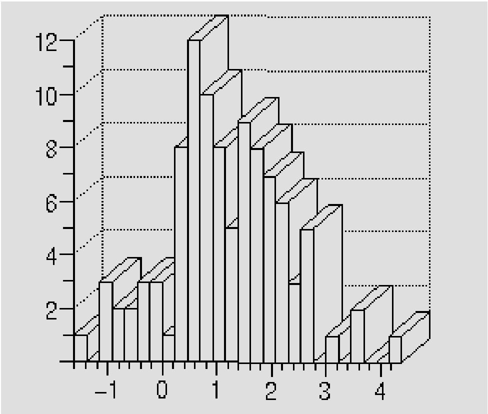
Boss Quattro generic engines



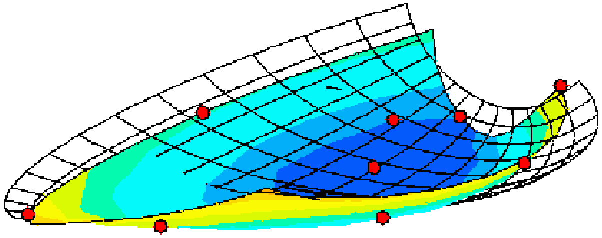
Parametric Study



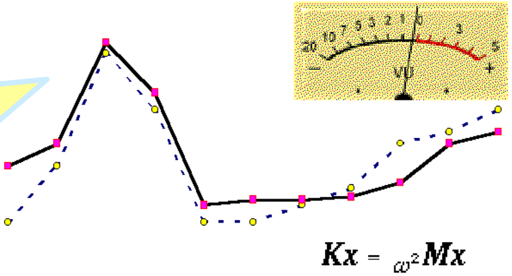
Gradient Optimization
Genetic Algorithms



Monte Carlo



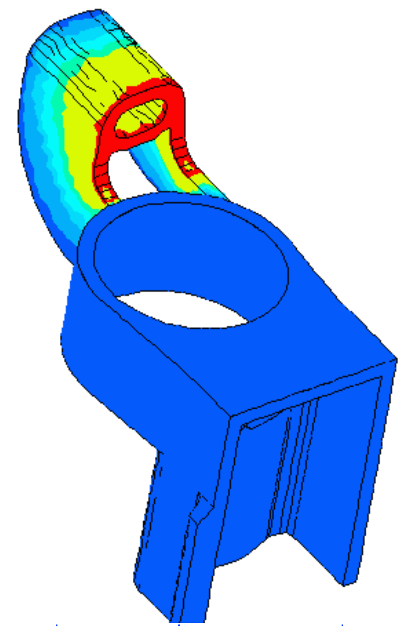
Design of Experiments
Response Surfaces
Predictors (RBF…)

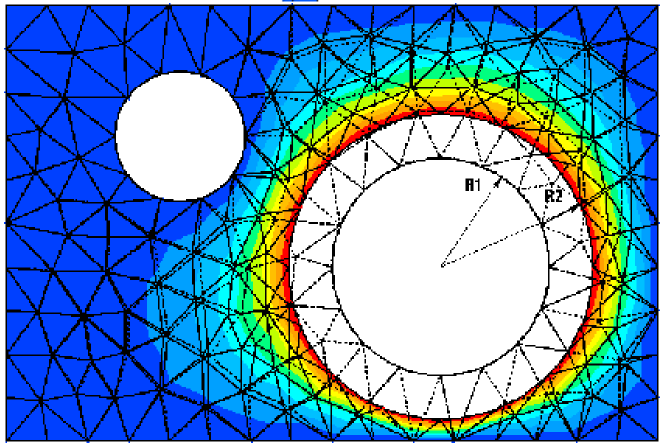


Updating
(what if study)

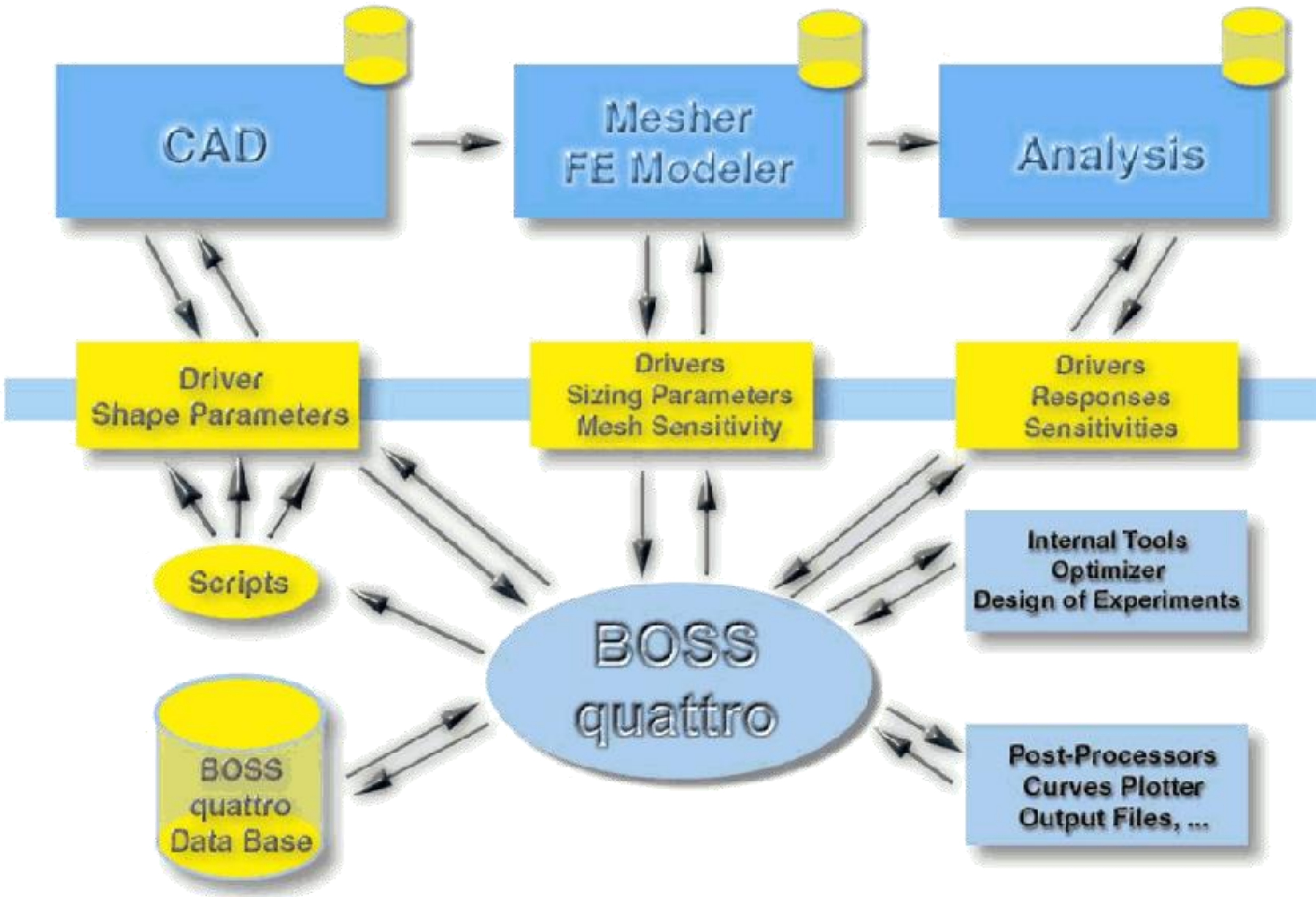
Sensitivity analysis in Boss Quattro

- Sensitivity (derivative) of response with respect to a design variable d
- Sensitivities are either:
  - Computed by finite-differences
  - Computed semi-analytically and read from SAMCEF, NASTRAN Sol200, NEUTRAL, Excell…
- Finite Difference scheme: OK!
- Semi analytical properties: requires a first order mesh perturbation law: mesh relocation technique

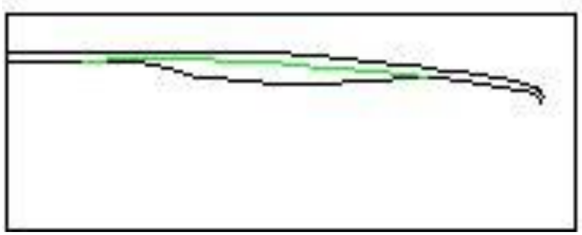




CAD-FEM coupling in shape otpimization



Boss Quattro + Think3 package



Aesthetic Character of a product

(described by Terms in natural language : "sporty", "aggressive", "dynamic"....)

Mapping of Aesthetic Character on objectively describable Properties

Aesthetic Properties as carrier of Aesthetic Character

Simulation of reflection line flow

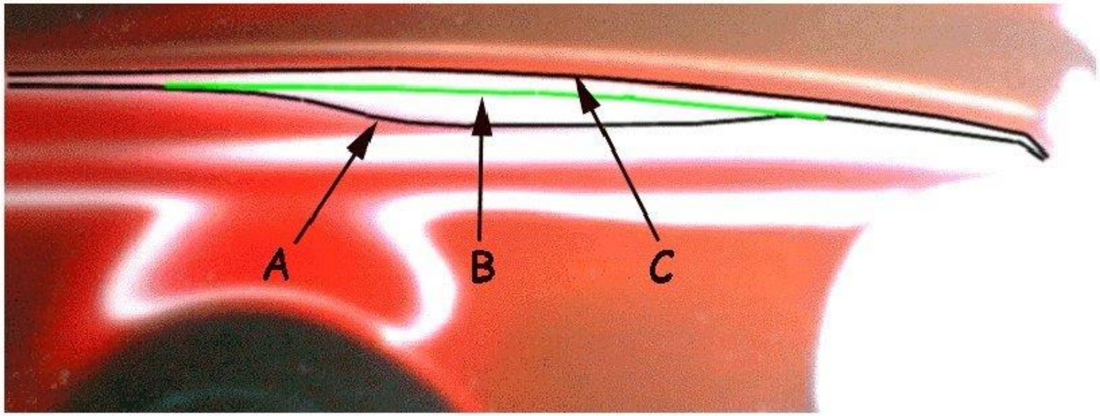
Extract

Elementary Aesthetic Properties

Reflection lines given as geometric curve to calculate curvature, inflection lines,...

Boss Quattro + Think3 package

Simulated reflection lines on the CAx model of a car's side panel.



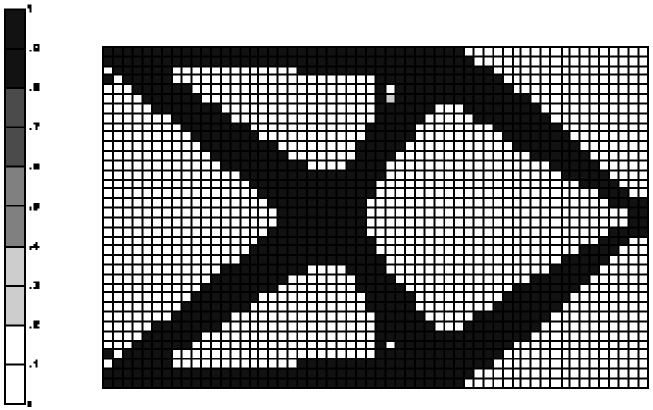
The designer modifies the reflection line A to (target line) B
which will be parallel to line C.

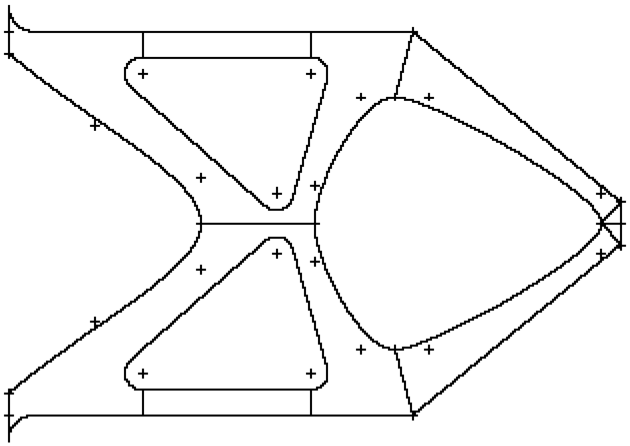
SHAPE OPTIMIZATION USING PARAMETRIC LEVEL SET DESCRIPTION

GENERALIZED SHAPE OPTIMIZATION WITH XFEM

- Topology optimization:
  - Fixed grid approach
  - Image like description
  - Limited control over regularity of geometry
- Shape optimization
  - CAD approach
  - Good control of geometrical characteristics
  - Complex machinery to handle mesh modifications, distortion, etc.
- There is some room for another approach!

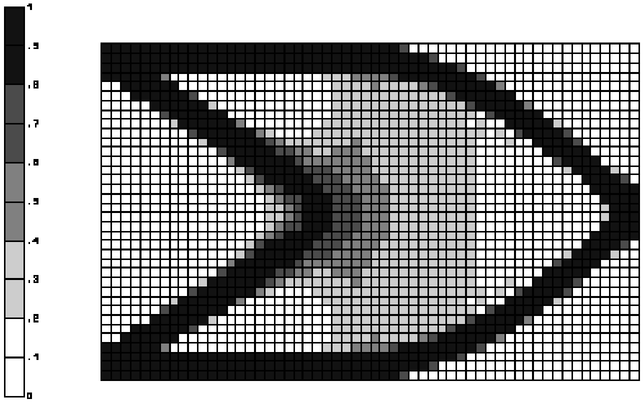
  ➔ Level Set description





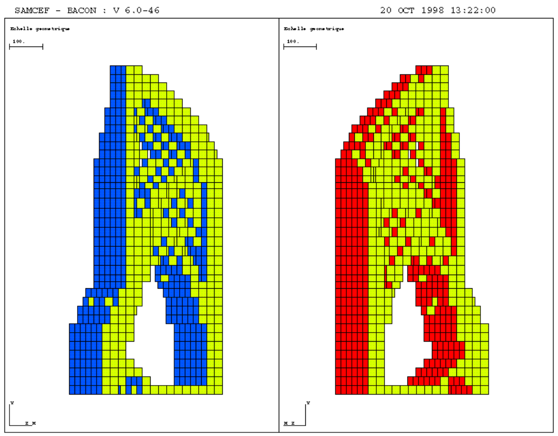
GENERALIZED SHAPE OPTIMIZATION WITH XFEM

- Topology optimization:
  - Variable material density ➔ interpolation of material properties
  - Large scale optimization problem
  - Unclear image (grey material, no shape boundaries, chattering boundaries)



- Shape optimization
  - Smooth boundaries
  - A small number of parameters is necessary to describe the shape
- There is some room for another approach!
  - Reduced work to transfer results to detailed design models

➔ XFEM



GENERALIZED SHAPE OPTIMIZATION WITH XFEM

- LEVEL SET METHOD
  - Alternative description to parametric description of curves
  - Constructive geometry using parametric level sets

- EXTENDED FINITE ELEMENT METHOD (XFEM)
  - Alternative to remeshing methods
  - Alternative to homogenization: void is void!

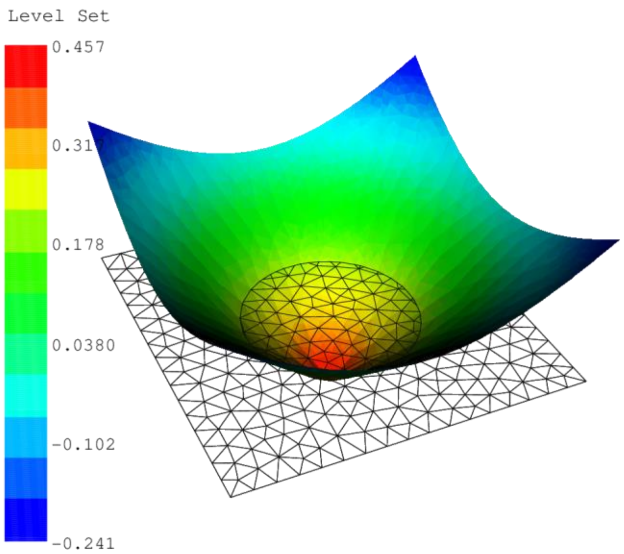
- XFEM + LEVEL SET METHODS
  - Efficient treatment of problem involving discontinuities and propagations
  - Early applications in structural optimisation Belytschko et al. (2003), Wang et al. (2003), Allaire et al. (2004)
  - Problem formulation:
    - Global and local constraints
    - Limited number of design variables

EXTENDED FINITE ELEMENT METHOD

- Early motivation :
  - Study of propagating crack in mechanical structures → avoid the remeshing procedure (Moës *et al* IJNME Vol 46).
    - *Allow discontinuities inside the element*
      - *nonconforming the mesh*
- Principle :
  - Allow the model to handle discontinuities that are nonconforming with the mesh
  - Introduce additional shape functions :
    - To model a discontinuous behavior inside the element
    - To model a non polynomial response (Enrich the shape functions space)
  - Applications : cracks, holes, multi-material, multi-phases, …

Geometrical description using Level Sets

- Principle of Level Set Description (Sethian & Osher, 1988):
  - Eulerian representation
  - The interface is represented implicitly using a scalar function f(x)
  - Interface = the zero level of f(x)



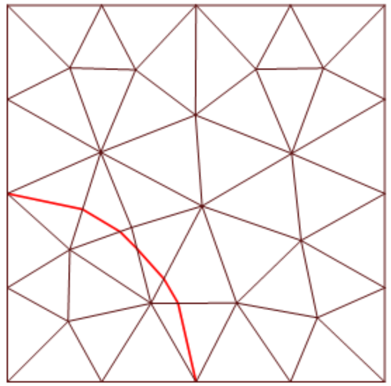
$$
\begin{aligned}
\phi(\mathbf{x}, r) &> 0 \quad \text{if} \quad \mathbf{x} \in \text{structure} \\
\phi(\mathbf{x}, r) &= 0 \quad \text{if} \quad \mathbf{x} \in \text{structure boundary} \\
\phi(\mathbf{x}, r) &< 0 \quad \text{if} \quad \mathbf{x} \in \text{void}
\end{aligned}
$$

- Level Set is used to represent the structural geometry
- Shape parameter *r* ➔ Parametric Level Set

GEOMETRICAL DESCRIPTION USING LEVEL SETS

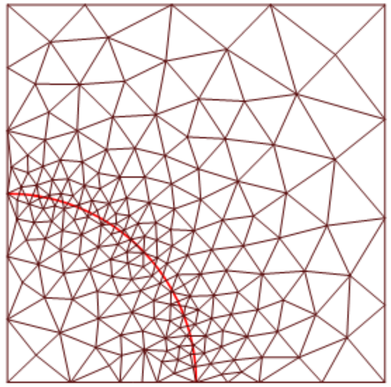
- Practical LS construction : a signed distance function:

$$\phi(\mathbf{x},\Gamma) = sign(\mathbf{n}\cdot(\mathbf{x}-\mathbf{x}_\Gamma))\cdot \min_{\mathbf{x}_\Gamma\in\Gamma} \|\mathbf{x}-\mathbf{x}_\Gamma\| \quad \forall \mathbf{x}\in\Omega$$



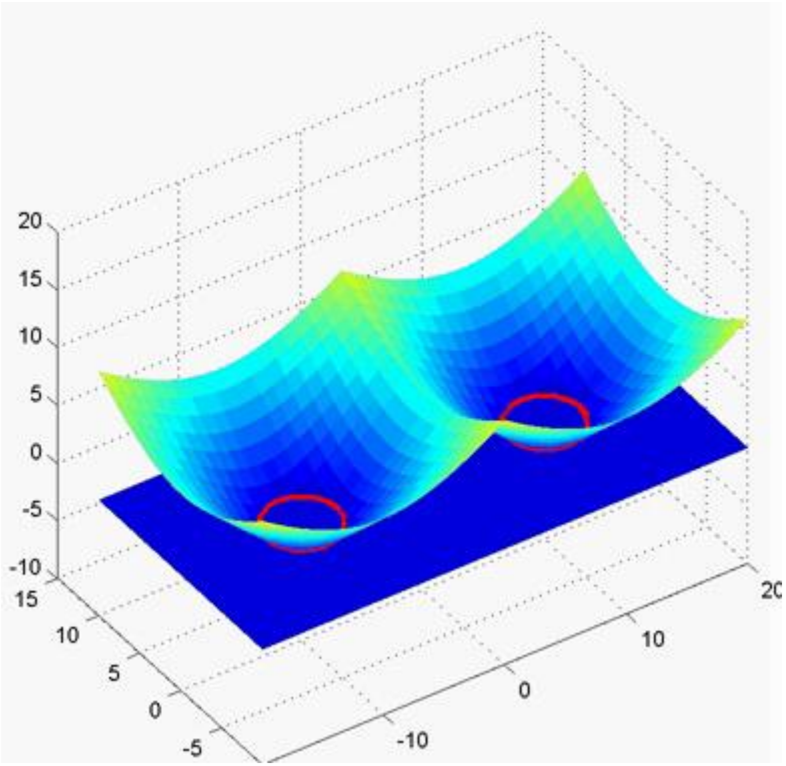
- In XFEM framework: discretization of the level set,
  - Each node has a Level Set dof
  - Interpolation using classical shape functions

$$\phi(\mathbf{x},r) = \sum_{i=1}^{N} \phi_i(\mathbf{x},r)$$



- To obtain a Level Set, a first mesh is needed. Mesh refinement can be necessary

GEOMETRICAL DESCRIPTION USING LEVEL SETS



<u>Advantages :</u>

- Same definition in 2D and 3D
- Combination of basic level sets is possible (union, intersection)
- Close to image processing
- Topological modifications are naturally handled

  No modification of model definition is needed when topology change

- Example: Two overlapping circles

$$\phi_1(x,y,b,r) = \sqrt{(x-b)^2+y^2} - r$$
$$\phi_2(x,y,b,r) = \sqrt{(x+b)^2+y^2} - r$$

$$\boxed{\phi(x,y,b,r) = \min(\phi_1,\phi_2)}$$

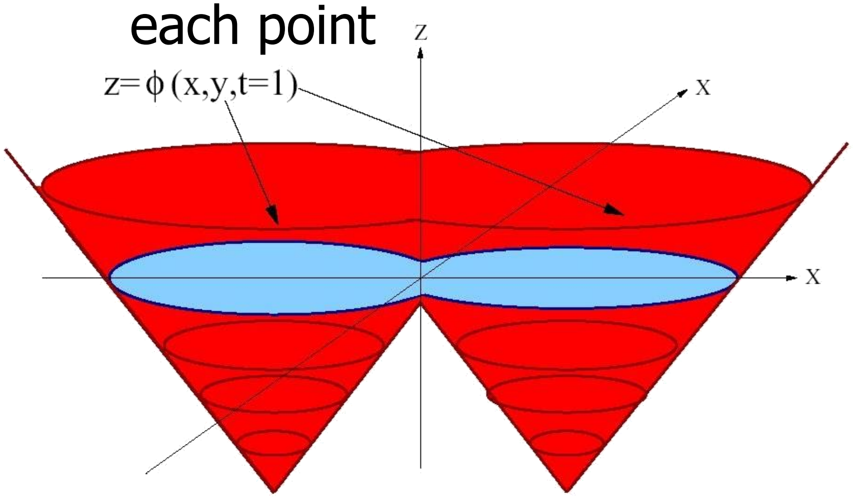
THE LEVEL SET METHOD

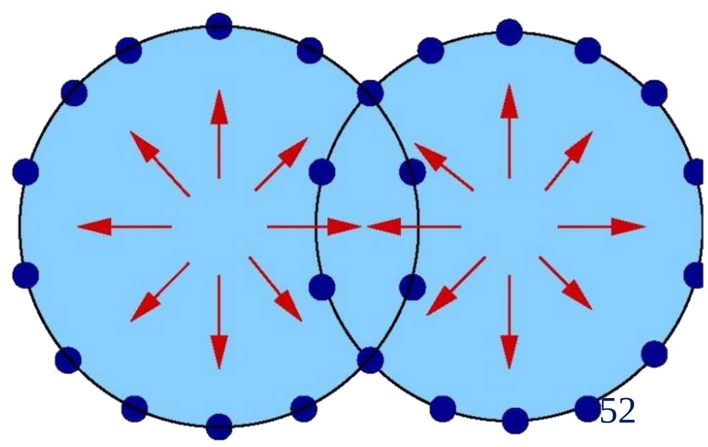
- Evolution of interface is ruled by the Hamilton Jacobi equation [Allaire et al. 2003, Wang et al. 2003]

$$\frac{\partial \psi}{\partial t} + V\,\|\nabla \psi\| = 0$$
$$\psi(\mathbf{x}, t) = 0 \text{ given}$$

Very difficult to use in practical implementation!

  – V: velocity function of Γ in the outward normal direction to interface and is given by the sensitivity of the level set in each point





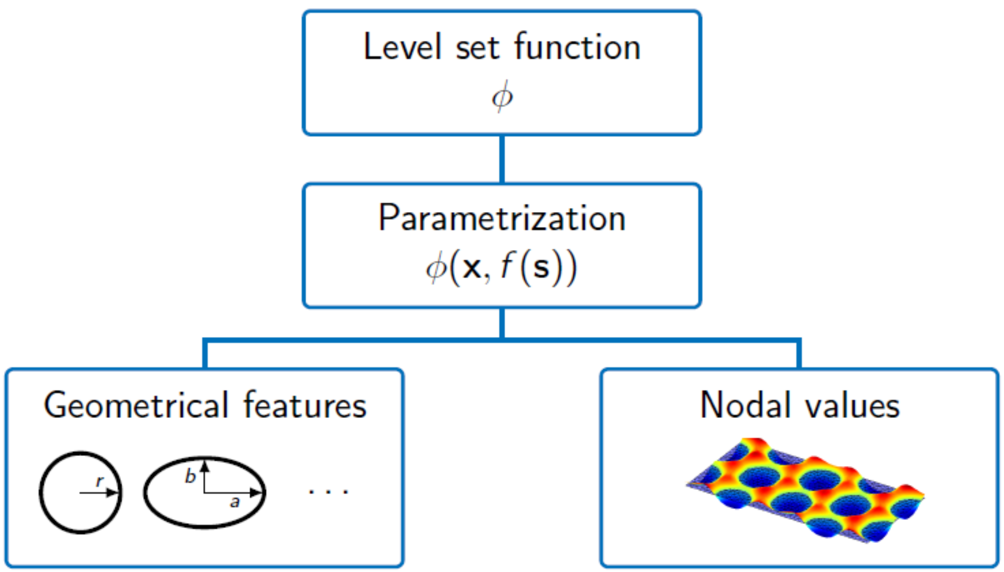
DESIGN VARIABLES WITH LEVEL SET DESCRIPTION

- In structural optimization, the design variables can be either:
  - The nodal values of the Level Set Ψi
  - Parameters of the elementary graphical features of the level set

$$\phi(\mathbf{x}) = \left|\frac{(x-c_x)}{a}\right|^{\xi} + \left|\frac{(y-c_y)}{b}\right|^{\eta} - 1.$$

More shape optimization

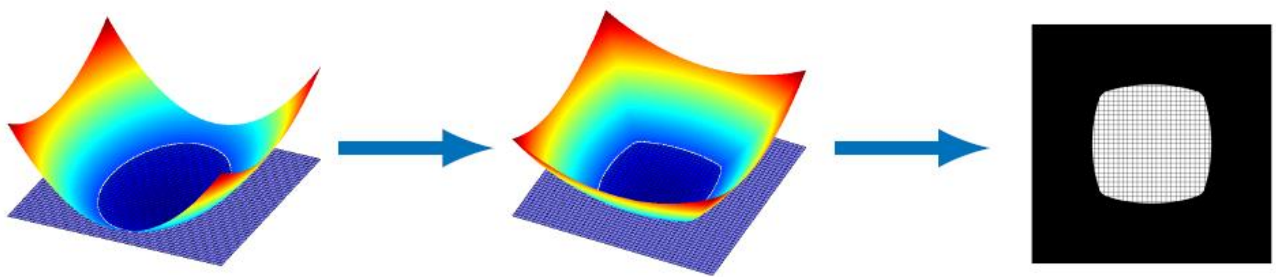
More topology optimization



DESIGN VARIABLES WITH LEVEL SET DESCRIPTION

- Geometric shapes ➔ "shape optimization"
  - Level set function is constructed using parametric CAD entities
  - Geometrical parameters are used as design variables
  - Complex geometry: build a global level set function applying boolean operations :

$$\phi = \max_i \phi_i^{\text{shape}}(\mathbf{x}, s_i).$$



- Advantages / Drawbacks:
  - (+) Simple and compact parametrization, manufacturable designs.
  - (-) Limited freedom in the design.

DESIGN VARIABLES WITH LEVEL SET DESCRIPTION

- Nodal design variables : ➔ "topology optimization"
- A design variable is associated to each mesh node

  ➔ yields local sensitivities slowing down the convergence.
- Use a linear filter by [Kreissl and Maute (2012)] :

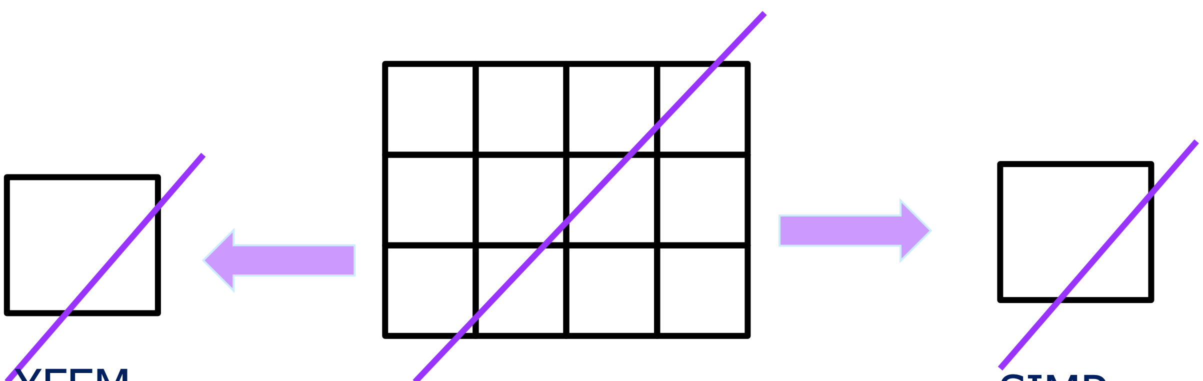
$$\phi_i = \left(\sum_j w_{ij} \right)^{-1} \sum_j w_{ij} \; s_j, \qquad w_{ij} = \max\left(0, \left(r - \|\mathbf{x}_i - \mathbf{x}_j\|\right)\right),$$

- Filter does provide control over feature size but it does not guarantee convergence with mesh refinement
- Perimeter penalization is also beneficial for smoothness of solution

- Advantages / Drawbacks:
  - (+) More freedom in the design.
  - (-) More design variables.

LEVEL SET AND FINITE ELEMENTS

- Two main approaches to combine the Level Set description and the finite elements.
- Two strategies to deal with the Finite Elements that are crossed by the boundary
  - Use XFEM, GFEM etc. new finite elements that can deal with nonconforming meshes
  - Use Ersatz material approach, similar to material density



XFEM

SIMP:

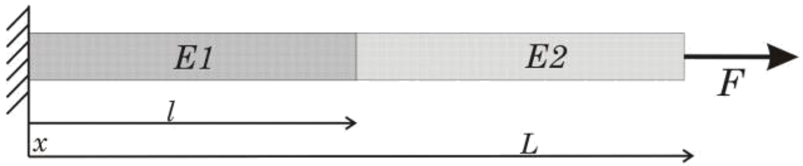
$$E_e = x^p \, E^0$$

$$\rho = x \, \rho^0$$

$$x = \frac{A}{A_0}$$

EXTENDED FINITE ELEMENT METHOD

- Bi material example :



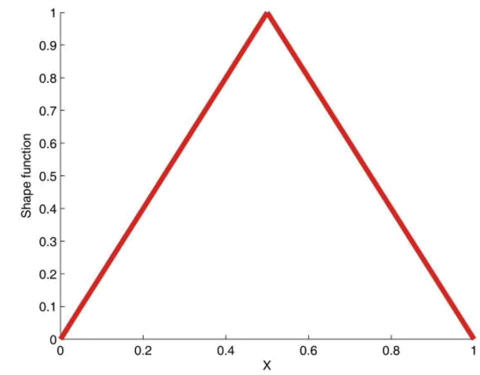
$$u(x) = \underbrace{N_1 u_1 + N_2 u_2}_{FEM} + \underbrace{N_1 \phi a_1 + N_2 \phi a_2}_{Enrichment}$$

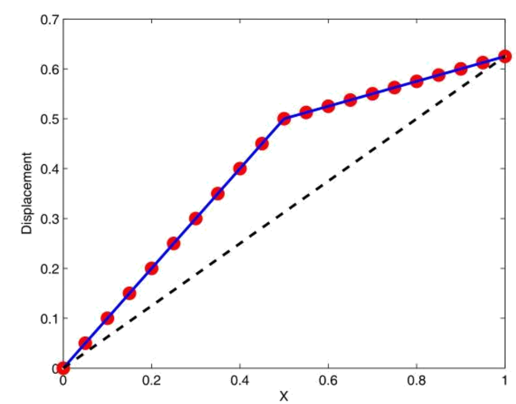
- Discontinuous

$$\phi(x) = \sum |\psi_i N_i| - \sum |\psi_i| N_i$$

- Solve extended system

$$K \cdot u = f \Leftrightarrow \begin{bmatrix} K_{uu} & K_{ua} \\ K_{au} & K_{aa} \end{bmatrix} \begin{bmatrix} u \\ a \end{bmatrix} = \begin{bmatrix} f_u^{ext} \\ f_a^{ext} \end{bmatrix}$$



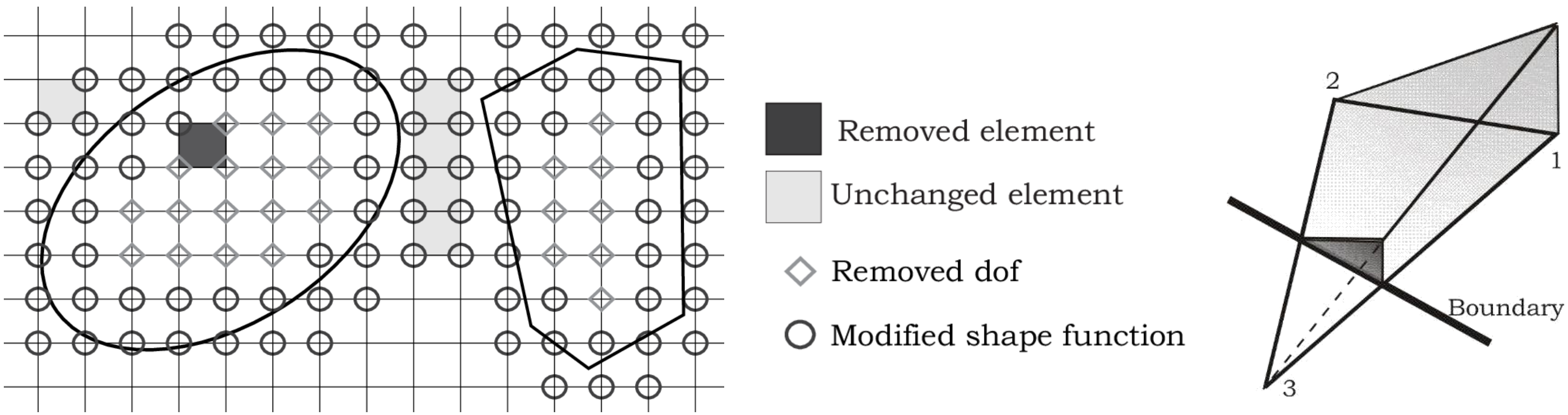


XFEM for void-solid structures

- X-FEM are used for Material – Void interfaces
  - No additional shape function ➔ no additional DOF
  - The displacement discretization is multiplied by a Heaviside function

$$u(x) = \sum_i u_i N_i(x) H(\phi(x))$$

(Sukumar et al. 2001)

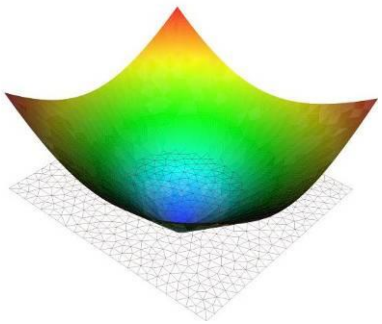


- The shape function has a zero value in the void

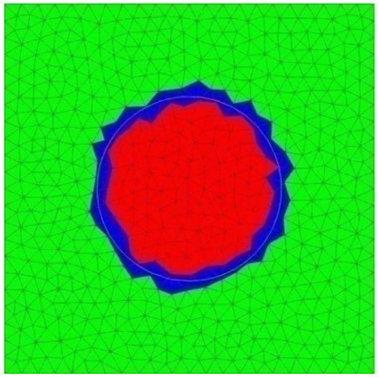
  ➔ Assembly only element with active DOFs

XFEM Procedure

- Build the mesh for the domain and build the Level Set

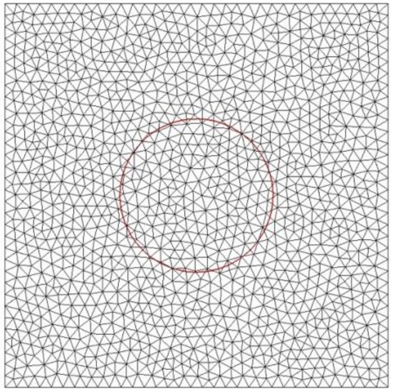


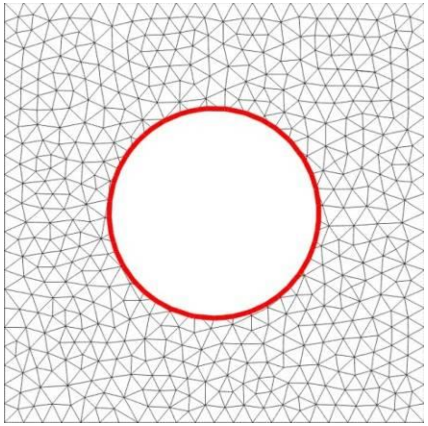
- Detect element type
  - Green = FEM
  - Red = void
  - Blue = X-FEM

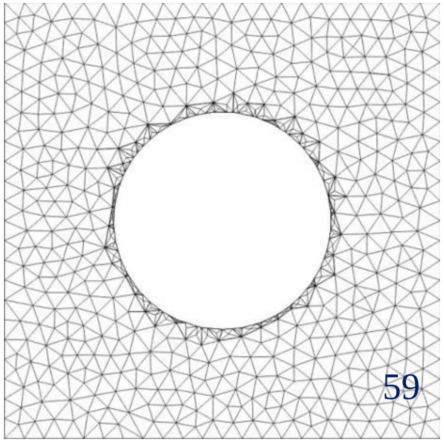


- Cut the mesh
  - Search for the intersection

- Create sub domain for integration







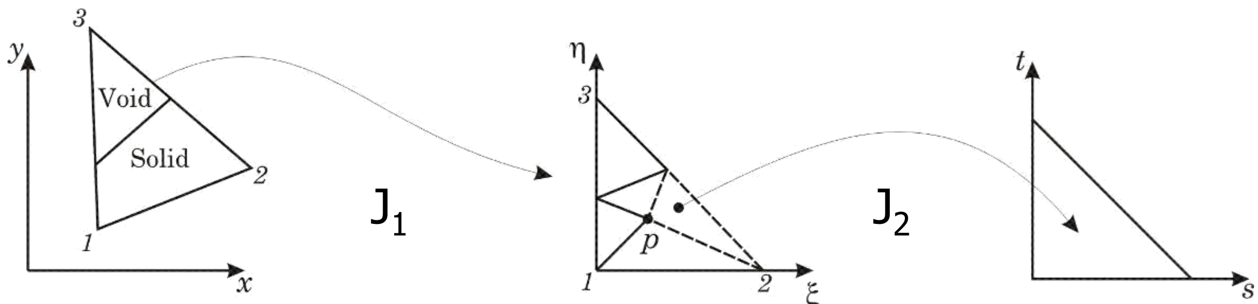
XFEM: Numerical Integration

- Take into account of discontinuous behavior of shape functions (Bi material, void-solid boundary…)
- Integrate over solid domain (no integration in void) or over every material subdomains
  - In FEM :

$$K = \int_{-1}^{1}\int_{-1}^{1} \hat{B}^T H \hat{B} \,|J|\, d\eta d\xi = \sum_{gp}^{N_{gp}} w_{gp} B(\xi_{gp}, \eta_{gp})^T H B(\xi_{gp}, \eta_{gp}) \,|J_{gp}|\, t$$

  - In X-FEM :
    - Introduction of a cascade of two mappings
    - Subdivision into triangles

$$K = \int_{\Omega_{Solid}} B^T H B dV = \sum_{\Delta} K_{\Delta} = \sum_{\Delta} \int_{\Delta} \hat{B}^T H \hat{B} \,|J_1|\,|J_2|\, ds dt$$



XFEM: Numerical Integration

- 2D stiffness matrix :

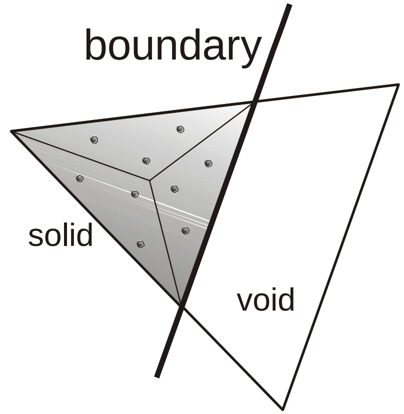
$$K_{\Delta} = \sum_{gp}^{N_{gp}} w_{gp} B(s_{gp}, t_{gp})^T H B(s_{gp}, t_{gp}) \, |J_1| \, |J_2| \, e$$

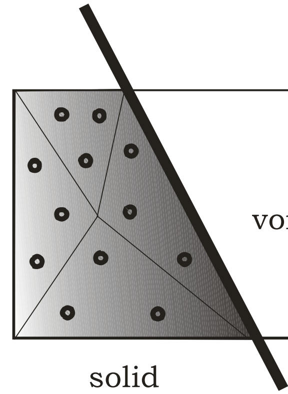
- 2D mass Matrix :

$$M_{\Delta} = \rho \sum_{gp}^{N_{gp}} w_{gp} N(s_{gp}, t_{gp})^T N(s_{gp}, t_{gp}) \, |J_1| \, |J_2| \, e$$

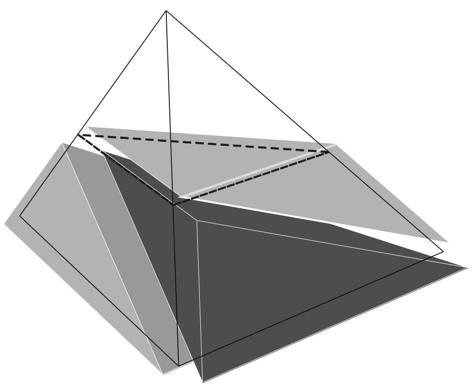
XFEM: Numerical Integration: 3D case

- Integration:
  - 2D:

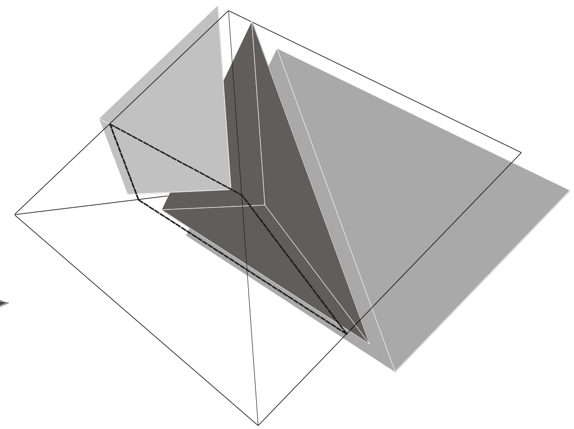




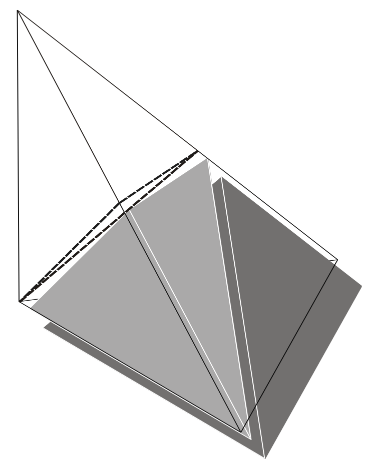
  - 3D element: Sub-division into tetrahedra



1 negative node



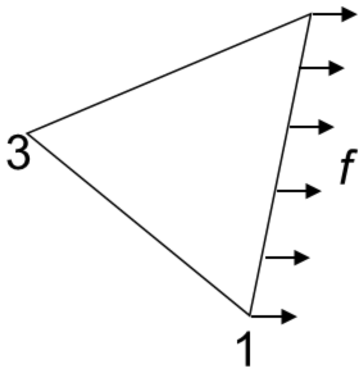
2 negative nodes



1 negative node & 1 zero node

XFEM: loads

- FEM formulation of loads :



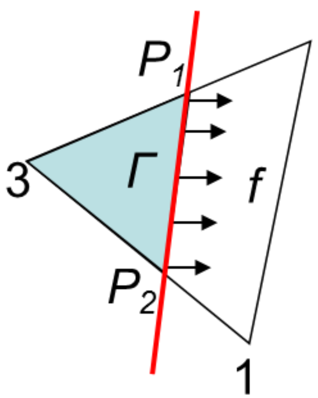
In practice : $\underline{F} = \int_{\Gamma(x,y)} f N^T d\Gamma(x,y) = \int_{\Gamma(s)} f N^T |J_1| d\Gamma(s)$

• Creation of line pressure element with nodes 1 and 2

• Integration of shape function on the Γ curve as N_3=0 on Γ

• If f = cst, $\underline{F} = \begin{bmatrix} \frac{fL}{2} & \frac{fL}{2} & 0 \end{bmatrix}$ $N^T = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$

- X-FEM formulation of loads : $\underline{F} = \int_{\Gamma(x,y)} f N^T d\Gamma(x,y) = \int_{\Gamma(\xi,\eta)} f N^T |J_1| d\Gamma(\xi,\eta)$



In practice : $= \int_{\Gamma(s)} f N^T |J_1| |J_2| d\Gamma(s)$

• Creation of integration scheme with points P_1 and P_2

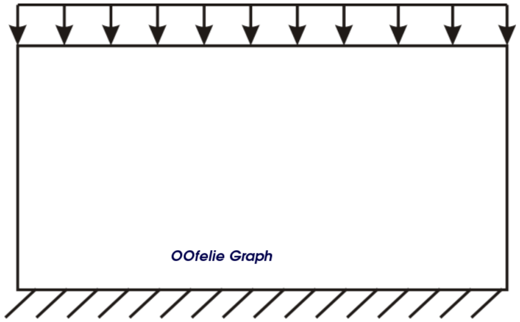
• Get gauss Points in (s), transform it into (ξ,η) to evaluate N_i

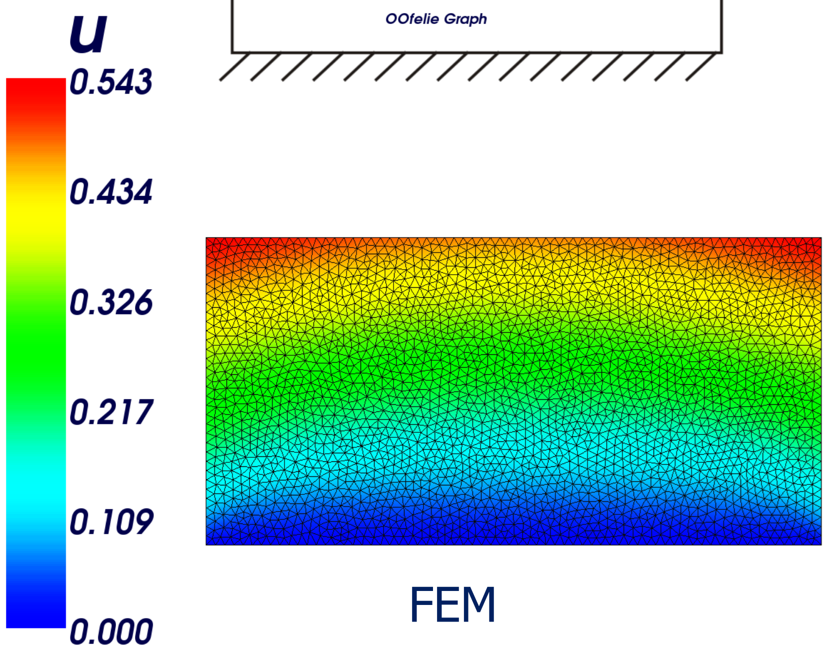
• Integration of 1st degree shape function of the triangle on the curve Γ

• Ni ≠ 0 ➔ $\underline{F} = \begin{bmatrix} a & b & c \end{bmatrix}$

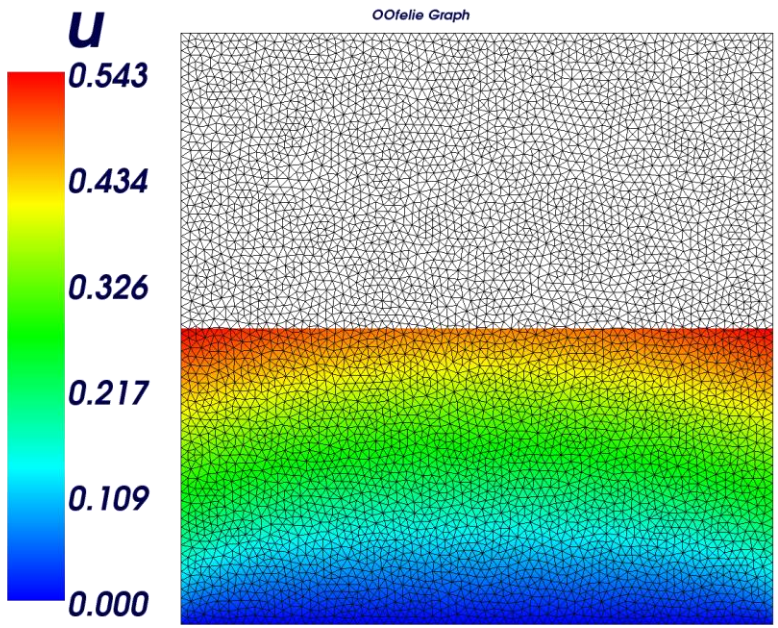
XFEM: loads

- Validation of energy consistent loads in XFEM





FEM



XFEM

DESIGN SENSITIVITY ANALYSIS

- Four Methods are available:

  – Finite Difference: 1 analysis per variable ➔ time consuming

  – Automatic code differentiation = automatic generation of function derivative in the computed code ➔ code maintenance problems, black box

  – Semi-analytic approach: usual approach

  – Analytic approach: best approach, most difficult

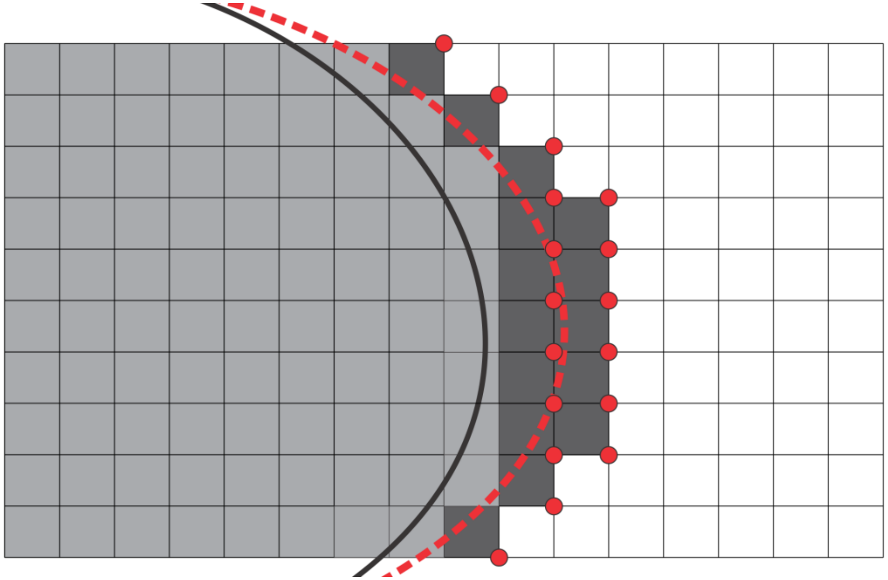
SEMI-ANALYTIC METHOD

Standard approach for sensitivity analysis in industrial codes

- Discretized equilibrium equation: $$\boldsymbol{K}\boldsymbol{q} = \boldsymbol{f}$$
- Generalized displacement derivative: $$\frac{\partial \boldsymbol{q}}{\partial z} = \boldsymbol{K}^{-1}\left(\frac{\partial \boldsymbol{f}}{\partial z} - \frac{\partial \boldsymbol{K}}{\partial z}\boldsymbol{q}\right)$$
- Structural matrices and load derivatives computed by finite differences
$$\frac{\partial \boldsymbol{K}}{\partial z} \simeq \frac{\boldsymbol{K}(z+\delta z) - \boldsymbol{K}(z)}{\delta z}$$
$$\frac{\partial \boldsymbol{f}}{\partial z} \simeq \frac{\boldsymbol{f}(z+\delta z) - \boldsymbol{f}(z)}{\delta z}$$
- Compliance sensitivity: $$\frac{\partial C}{\partial z} = -\boldsymbol{q}^T \frac{\partial \boldsymbol{K}}{\partial z}\boldsymbol{q} + 2\boldsymbol{q}^T \frac{\partial \boldsymbol{f}}{\partial z}$$

SEMI-ANALYTIC METHOD

- Because of fixed grid approach:
  - Sensitivity is computed only on the element modified by the perturbation
  - The perturbation can introduce new DOFs ➔ structural matrix dimensions can change



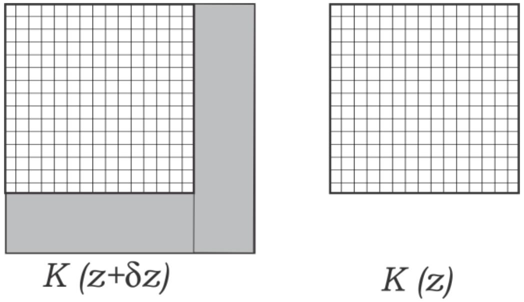
Element present at step z

Element present at step $z + \delta z$

New DOFs at step $z + \delta z$

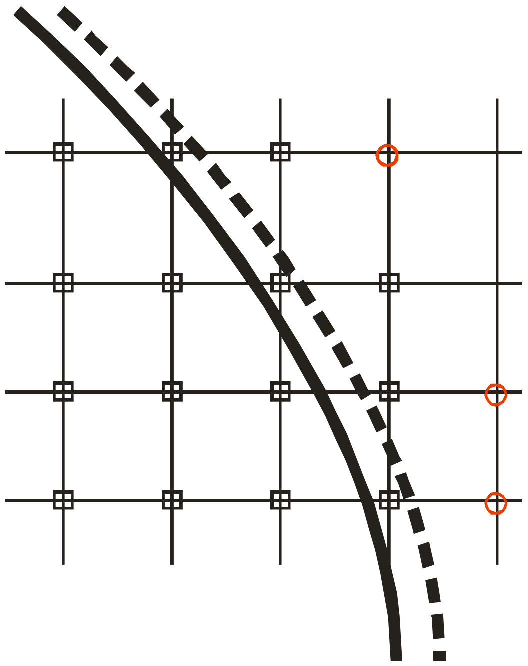
$$\frac{\partial \boldsymbol{q}}{\partial z} = \boldsymbol{K}^{-1}\left(\frac{\partial \boldsymbol{f}}{\partial z} - \boxed{\frac{\partial \boldsymbol{K}}{\partial z}}\boldsymbol{q}\right)$$

$$\frac{\partial \boldsymbol{K}}{\partial z} \simeq \frac{\boxed{\boldsymbol{K}(z+\delta z)} - \boxed{\boldsymbol{K}(z)}}{\delta z}$$



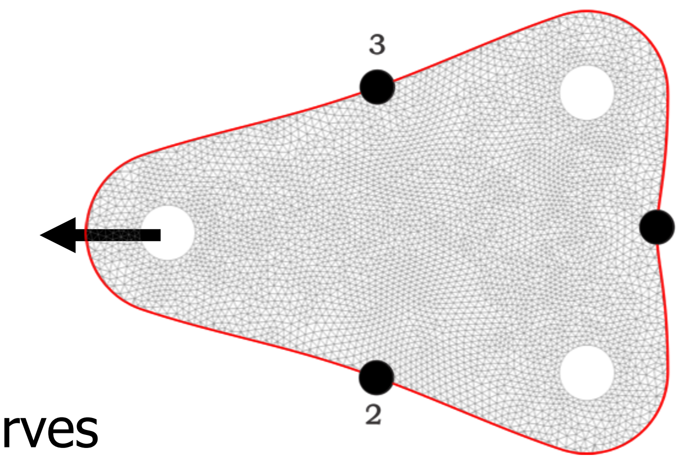
SEMI-ANALYTIC METHOD

- Strategies to freeze the number of dof
  - What happens if perturbed level sets comes into new FE?
  - Ignore the new elements that become solid or partly solid
    - small errors, but minor contributions
    - practically, no problem observed
    - efficiency and simplicity
    - validated on benchmarks



SEMI-ANALYTIC METHOD

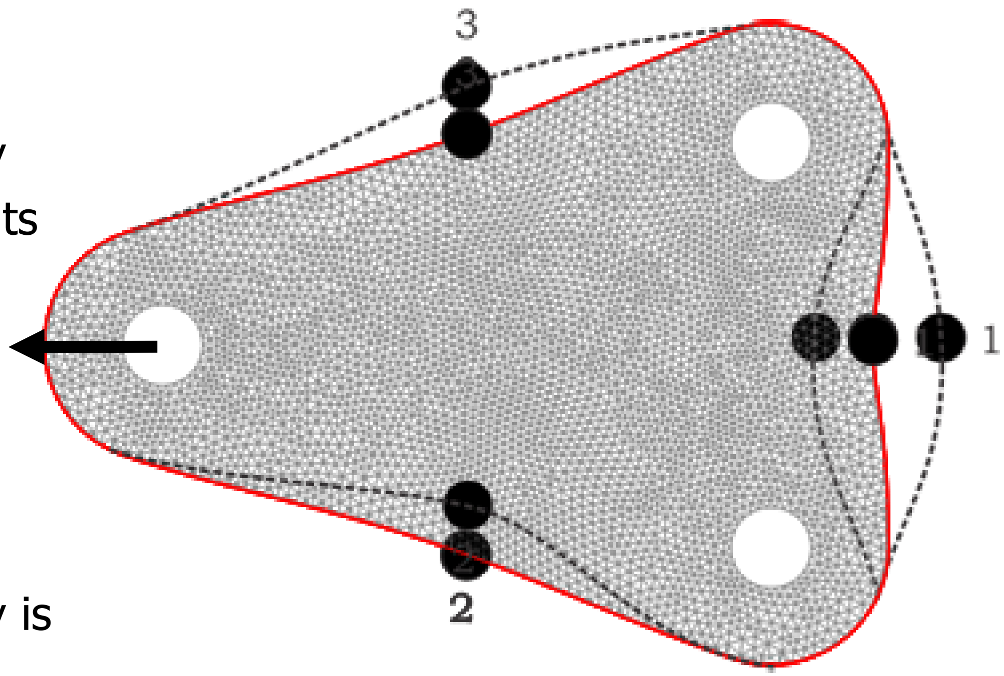
- Ignore the new elements that introduce new DOFs because (Van Miegroet 2005) :
  - Creation of DOFs does not often occur
  - Creation of material is generally very limited compared to the number of modified cut elements
  - Practically, no problem observed on several test cases



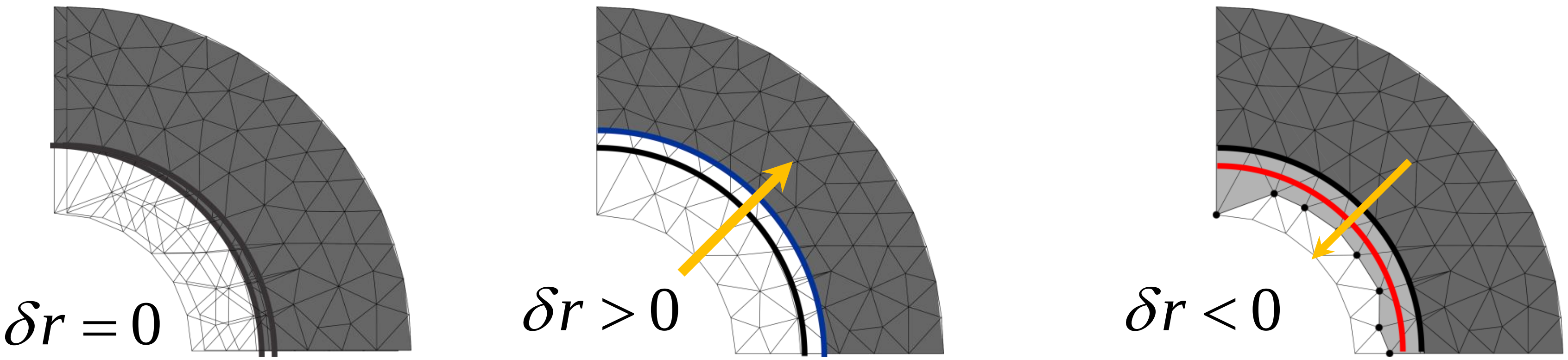
- Illustration:
  - Symmetric structure and loading
  - Boundary represented with NURBS curves
  - Design variables: control points *P1x, P2y, P3y*
  - Fct = compliance

SEMI-ANALYTIC METHOD

- Ignore the new elements
- Point 1:
  - In both cases, the sensitivity is computed on **454** elements
  - +dz sensitivity = -0.128 (**2** elements created)
  - -dz sensitivity = -0.128 (no element created)
- Point 2/3:
  - In both cases, the sensitivity is computed on **632** elements
  - Point 3: +dz sensitivity = -37.497 (**4** elements are created)
  - Point 2: +dz sensitivity = -37.504
  - Relative difference= 0.02%



SEMI-ANALYTIC METHOD



Compliance sensitivity				
Method	r	δr	*Value*	*Relative error (%)*
FD	0.6	$10^{-4}r$	1.677E-5	–
FD	0.6	$-10^{-4}r$	1.620E-5	3.3%
SA	0.6	$10^{-4}r$	1.676E-5	$\boxed{0.025\%}$
SA	0.6	$-10^{-4}r$	2.794E-6	$\boxed{83\% \text{ (16 new elements)}}$

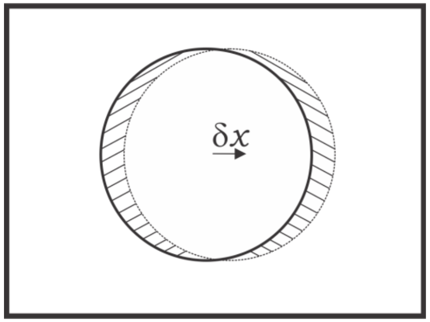
- With $\delta r < 0$, the perturbation introduces new DOFs

➔ A perturbation step going inward is safer but does not guarantee correct sensitivity

Semi-analytic method – pathologic case

- Imagine that the parameter move: the circle to the right ➔ Impossible to prevent from creation of DOFs

- Strategies to circumvent DOFs creations:



  – Added soft material in the void domain (constant number of DOFs)
  ➔ Small modification of the initial problem

  – Compute elementary sensitivity rather than model sensitivity and adapt perturbation step
  ➔ Round off error may occur if perturbation step is small

  – Analytical derivatives

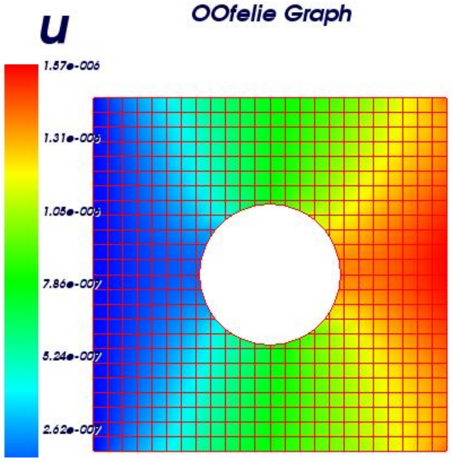
IMPLEMENTATION

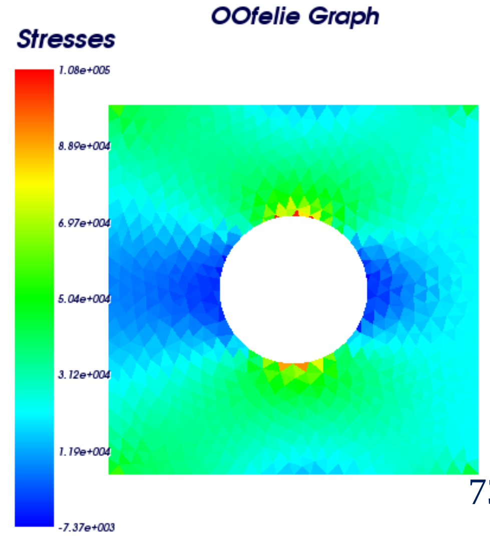
- Implementation in a multiphysic finite element code in C++ (OOFELIE from Open Engineering www.open-engineering.com)

- XFEM library:
  - 2D : library of quadrangles and triangles.
  - 3D : libray of tetraedra
  - Void/solid; bimaterial

- Available results for optimization:
  - Compliance, Displacements, Energy density
  - Strains, Stresses
  - Eigenfrequencies
  - Electrostatic
  - Electromechanical (in work in progress)

- Visualization:
  - Level Sets
  - Results

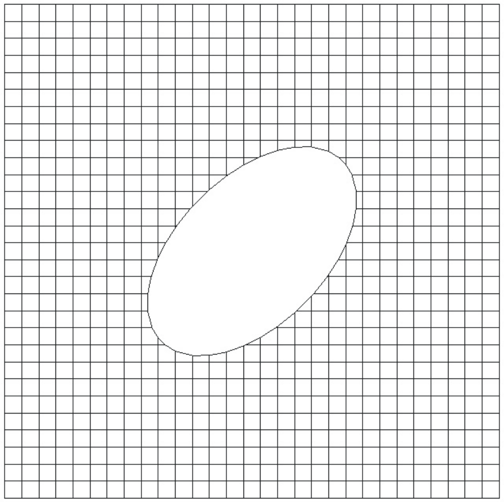


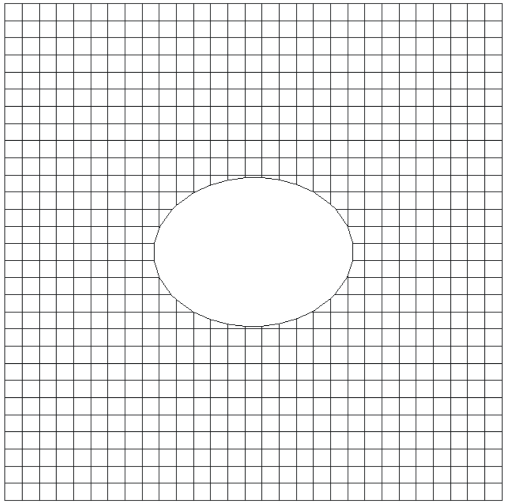


MECHANICAL AND MANUFACTURING CONSTRAINTS

- With the Level Set approach, one has access to:
  - All local stress constraints with high precision
  - Easier to evaluate manufacturing constraints: e.g. unmolding direction, maximum size, minimum size, etc. [Michailids et al. 2015]

APPLICATIONS





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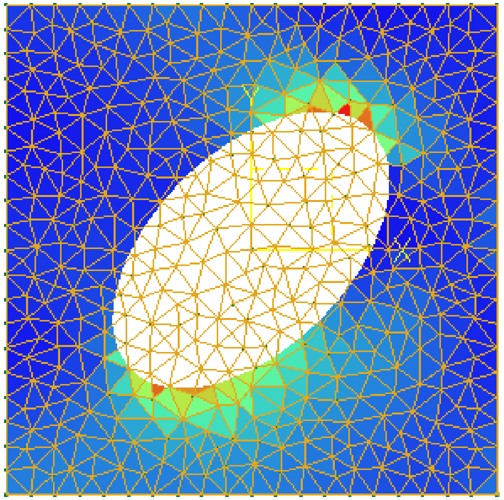
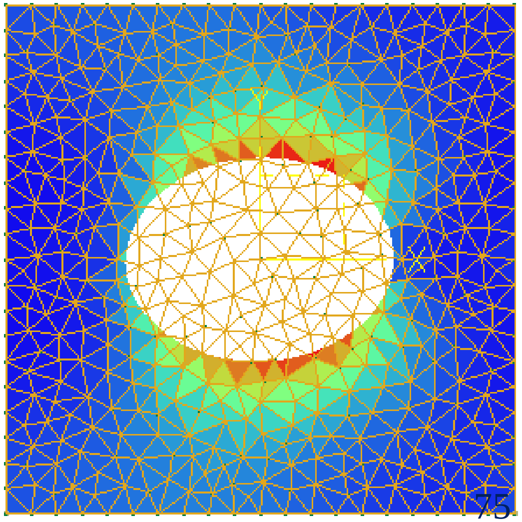
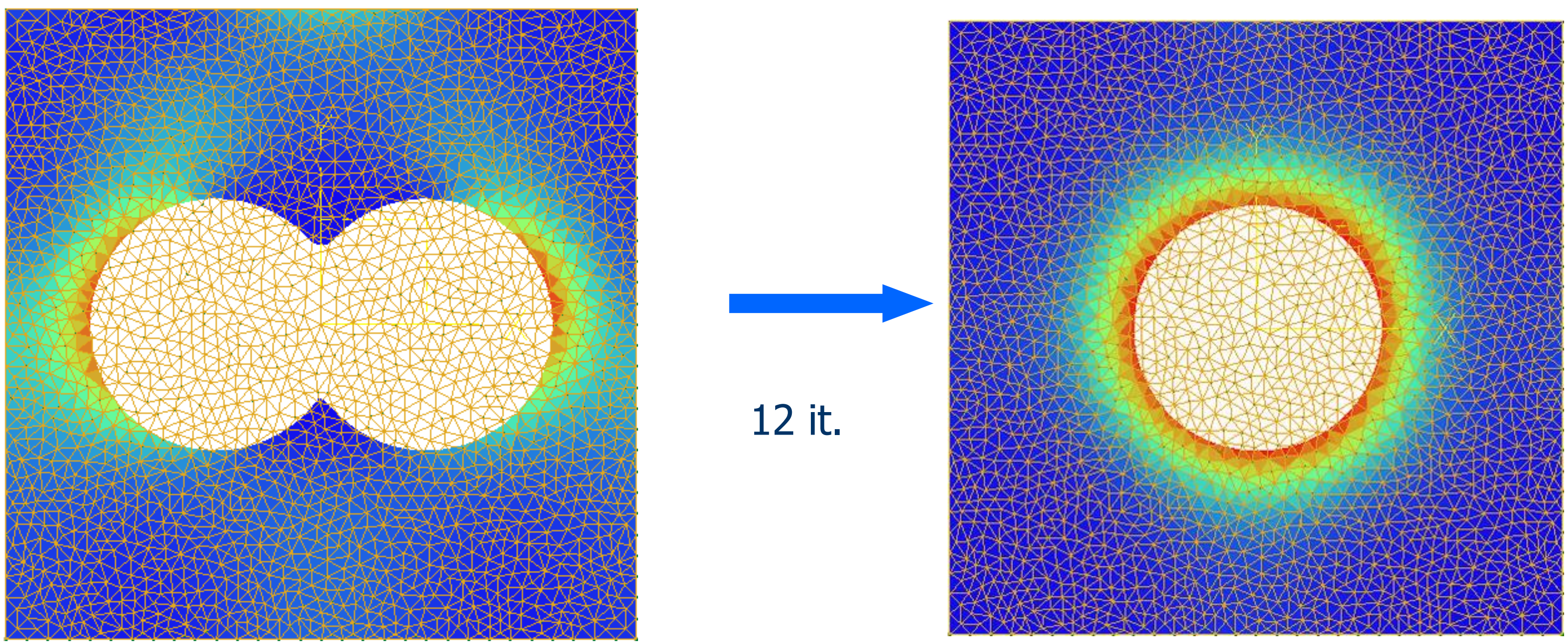


Min Compliance
s.t. Volume constraint



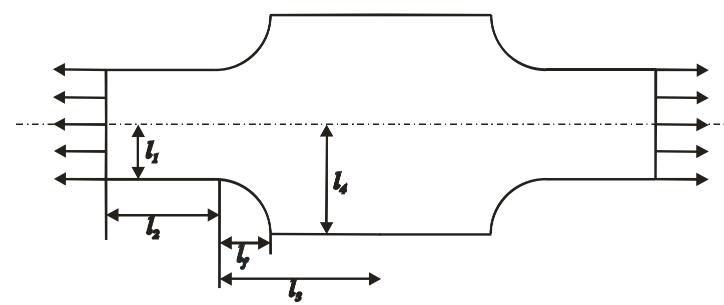
APPLICATIONS

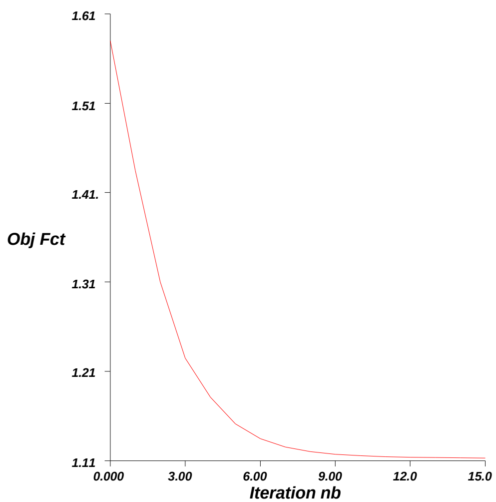
- Topology modification during optimization
  - Two variables : *center* x_1, *center* x_2
  - Min. potential energy under a surface constraint
  - Uniform Biaxial loading : $\sigma_x = \sigma_0$, $\sigma_y = \sigma_0$

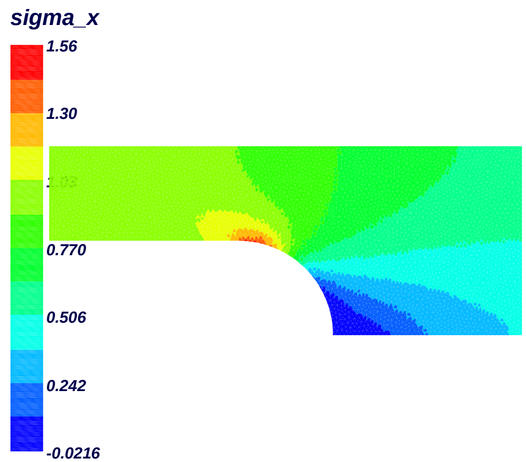


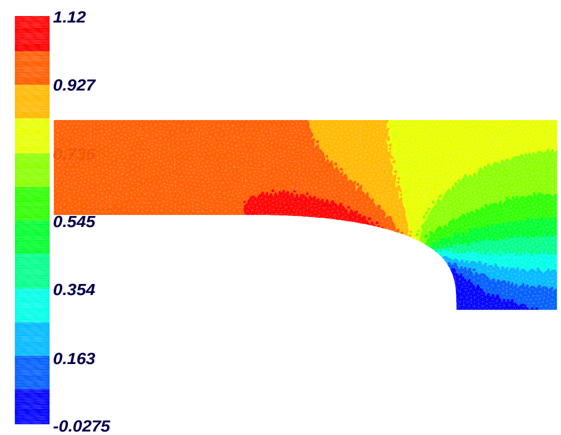
Applications – 2D fillet in tension

- Shape of the fillet : generalized super ellipse $\frac{|x|^{\alpha}}{|a|} + \frac{|y|^{\eta}}{|b|} = r$
  - Parameters : a, η, α
  - Objective: min (max Stress) $\sigma_x = \sigma_0$
  - No Constraint
  - Uni-axial Load:
  - Solution: stress reduction of 30%









Van Miegroet & Duysinx, SMO, 2007

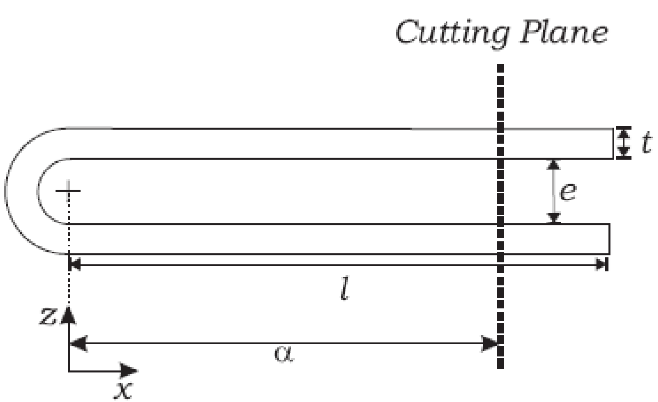
APPLICATION: Tuning fork

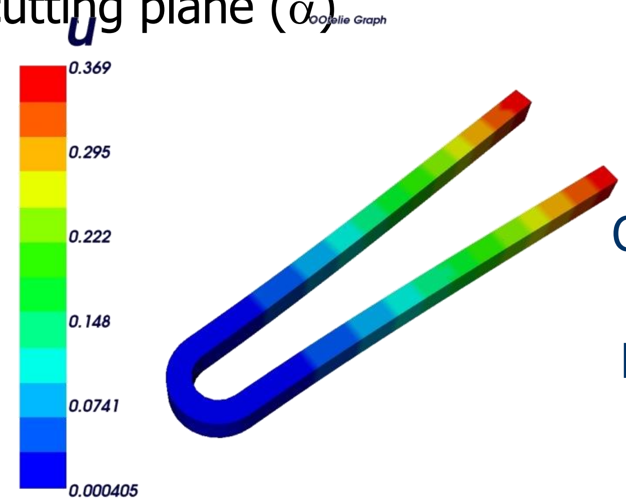
- Goal: tune the fork at prescribed frequency of a A-440 Hz
- Initial design
  - l=130 mm; t= 3.9 mm; e= 8.3mm; thick.= 5.5mm
  - Frequency : 182 Hz
- Optimization Problem to tune the length:

  Min Volume

  s.t. Freq <440 Hz

  Variable : x position of the cutting plane (α)

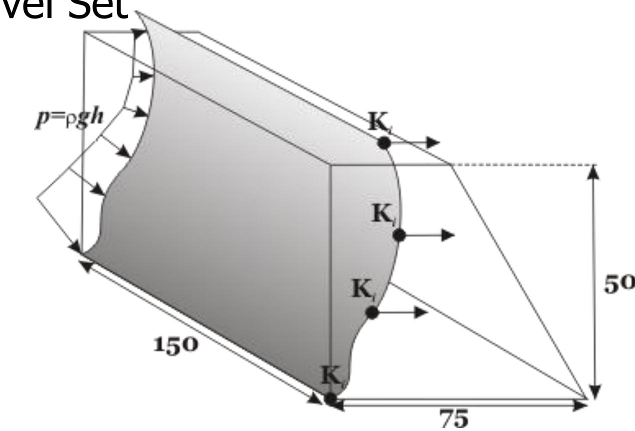


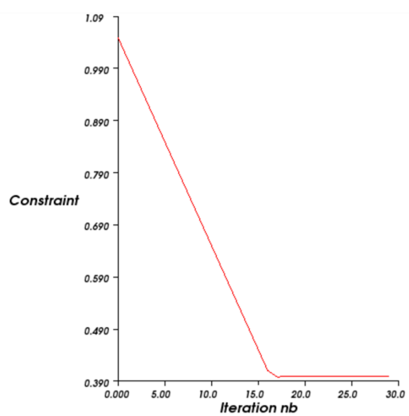


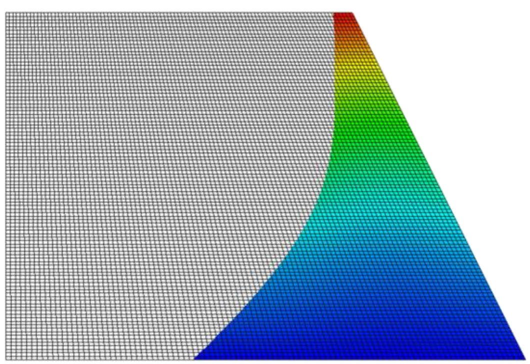
Optimum solution
α =80.27 mm
Freq =439.79 Hz

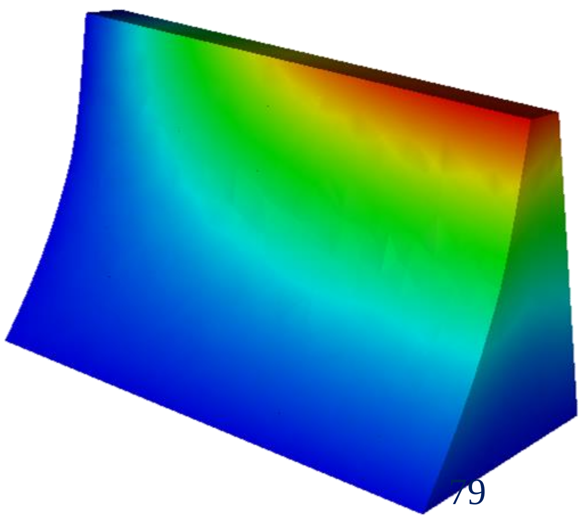
APPLICATIONS: Dam cross section optimization

- External boundary : 1 Surface Level Set defined by a Nurbs curve
  - Hydrostatic pressure f(h), normal to the iso zero Level Set
  - Parameters : K_i control points
  - Objective function : min Compliance
  - Constraints : Volume <0.3 V_i
  - Variables : Mvt. of K_i along x axis
  - Solution after 30 iterations
  - Due to move limits on variable
  - Extruded Level Set ➔ profile update



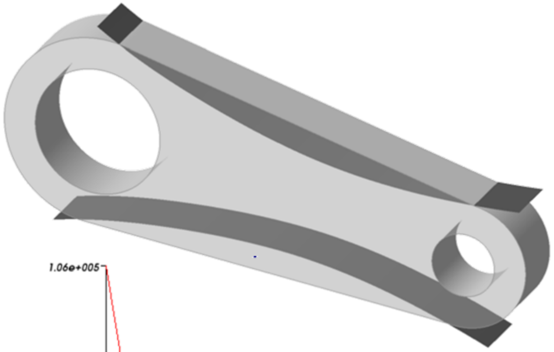


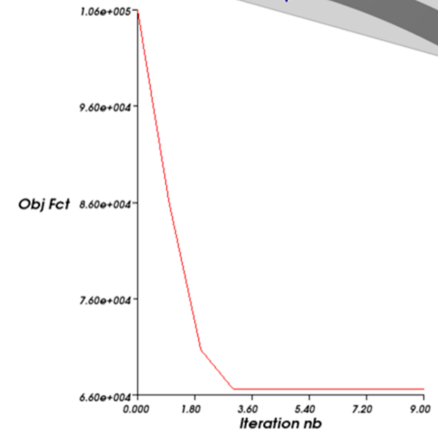


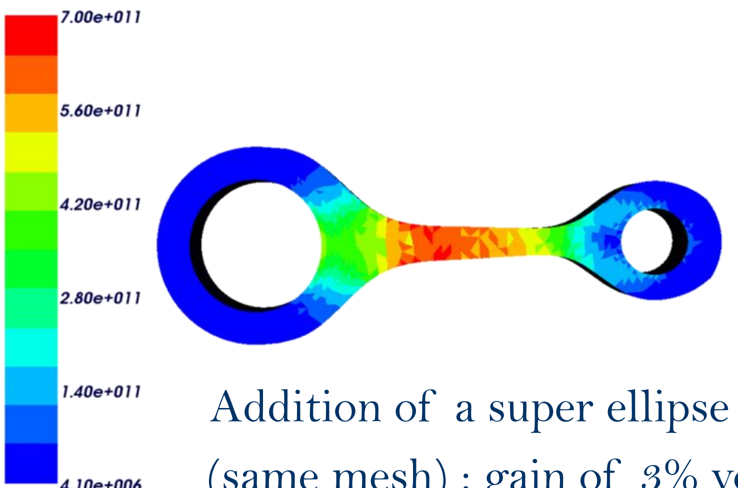


Generalized Shape Optimization With XFEM (Van Miegroet et al., 2007)

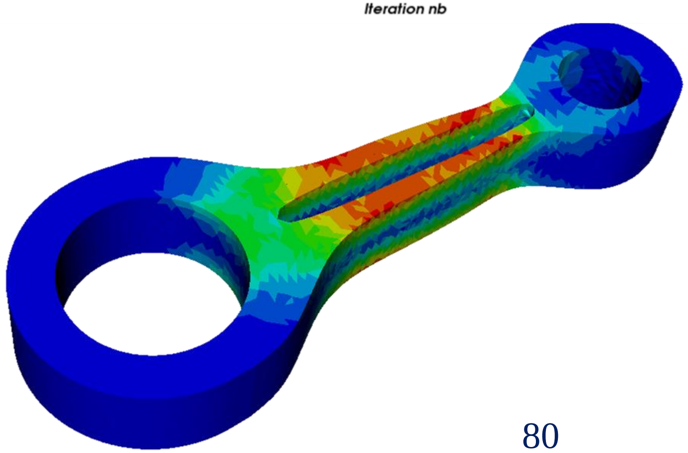
- Connecting rod problem :
  - 2 Level Set 3D surface defined by Nurbs curves
  - Parameters : Control points of the Nurbs
    - Variables (12) : Mvt. of K_i along y axis
  - Objective function : min Volume
  - S.t. Constraints : Von Mises<70 Mpa
    - 65000 Elements ~ 30000 constraints
  - Volume reduction ~ 50%





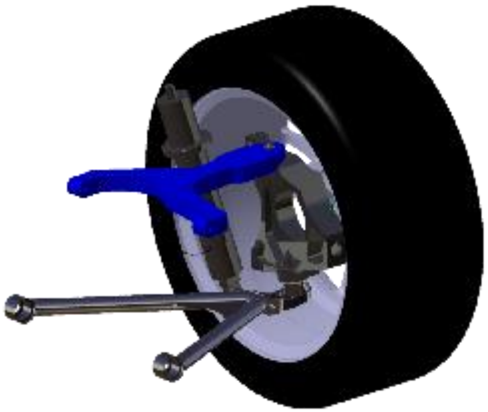


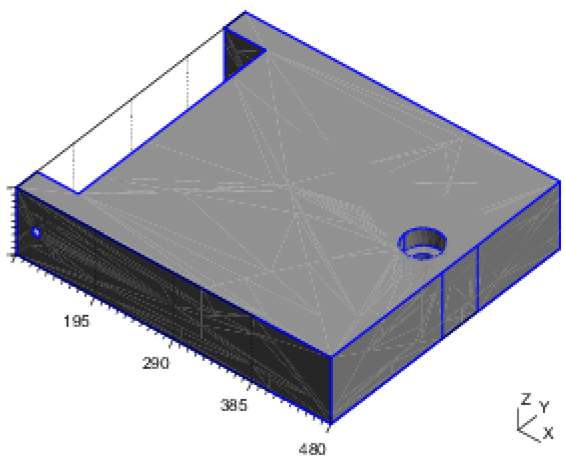
Addition of a super ellipse at center (same mesh) : gain of 3% volume ➔

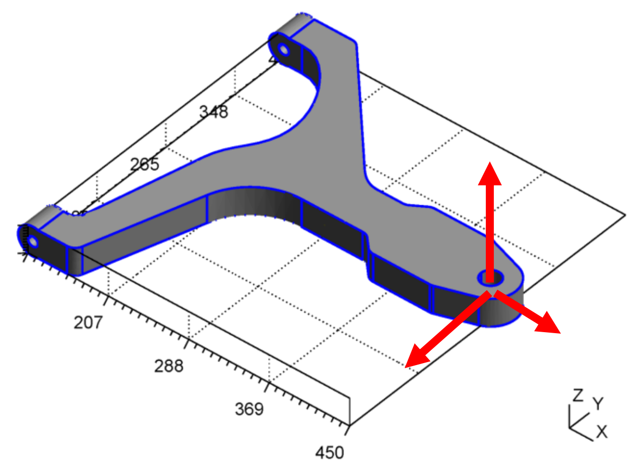


Applications: 3D suspension

- Given a fixed geometry for fixation :
- Design a new suspension triangle with same weight and a higher rigidity
- Definition of design domain from a bounding box
- Conforming surfaces for fixations and loads





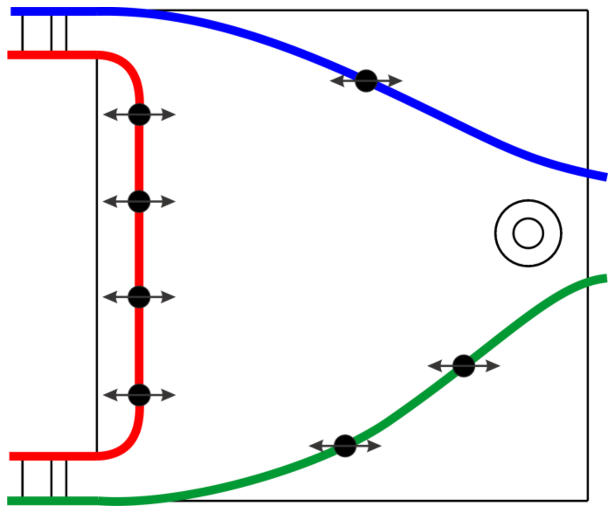


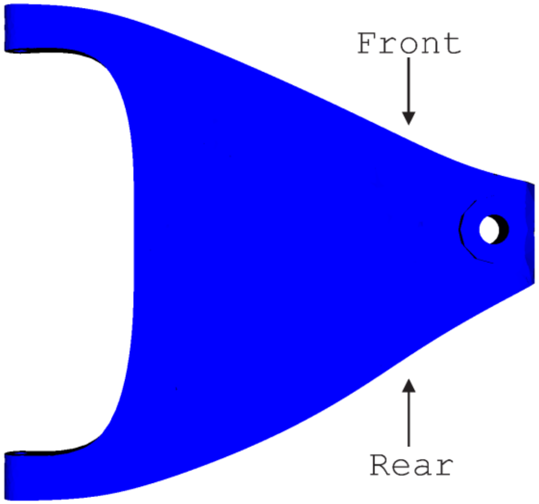
- Fx=100 kN, Fy=-28kN, Fz=62.5 kN

Applications: 3D suspension

- 3 NURBS curve to build 3 Level Sets, 7 variables

Top View

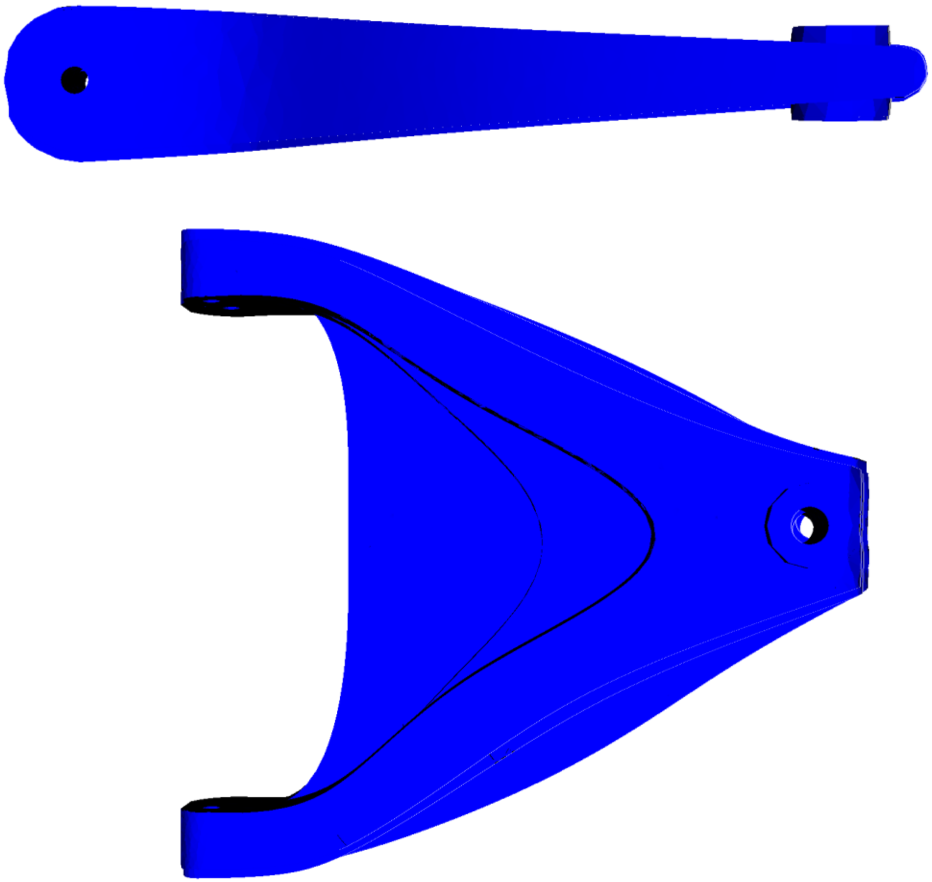




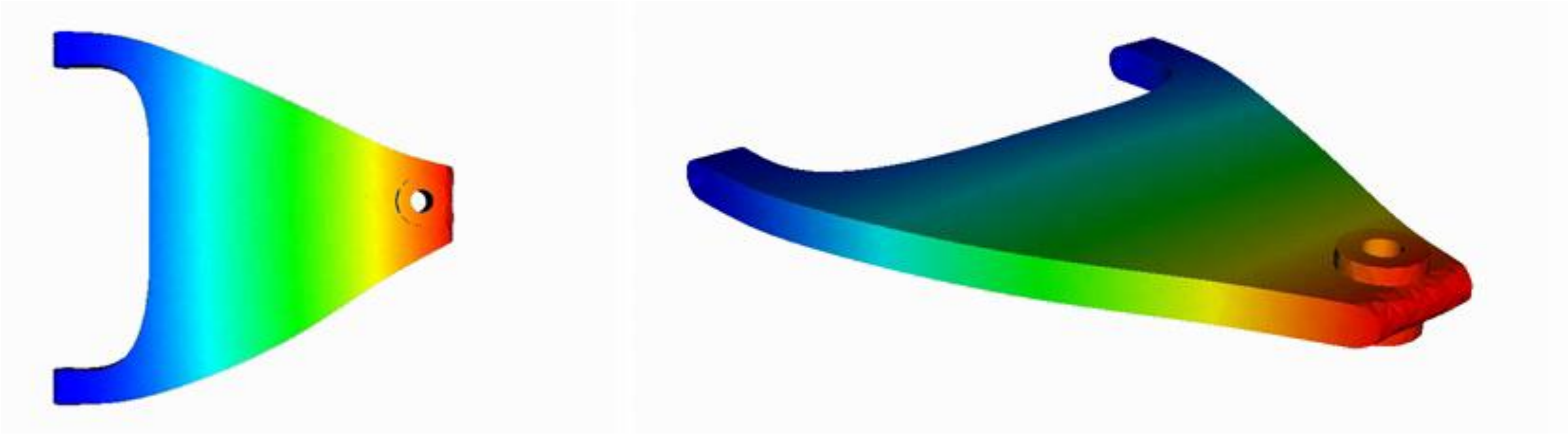
- 1 Level Set with 2 variables:

Side View

SUSPENSION ARM OPTIMIZATION



Applications: 3D suspension

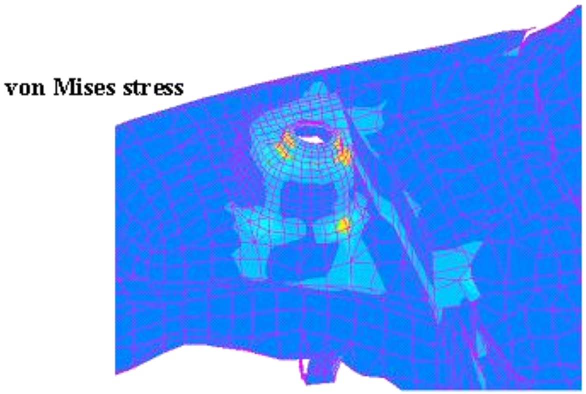


30 iterations, 42 % stiffer than initial design
with the same weight

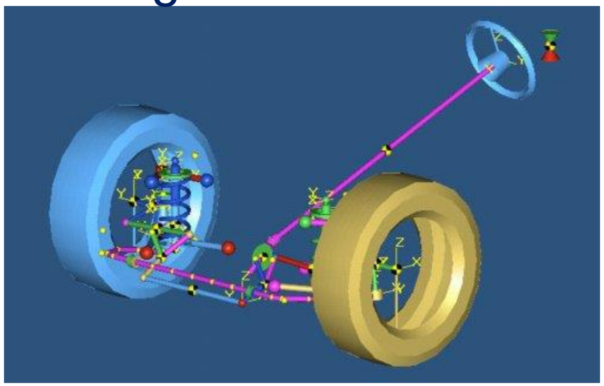
OPTIMIZATION OF STRUCTURAL COMPONENTS IN MULTIBODY SYSTEMS DYNAMICS

EVOLUTION OF FINITE ELEMENT IN AUTOMOTIVE

FE: structural analysis of component



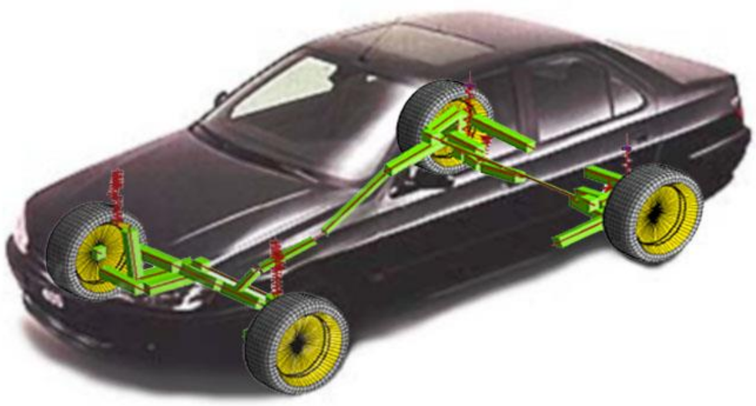
Multibody system: mechanism of rigid bodies



Flexible Multibody systems:

System approach (MBS)

& structural dynamics (FEM)



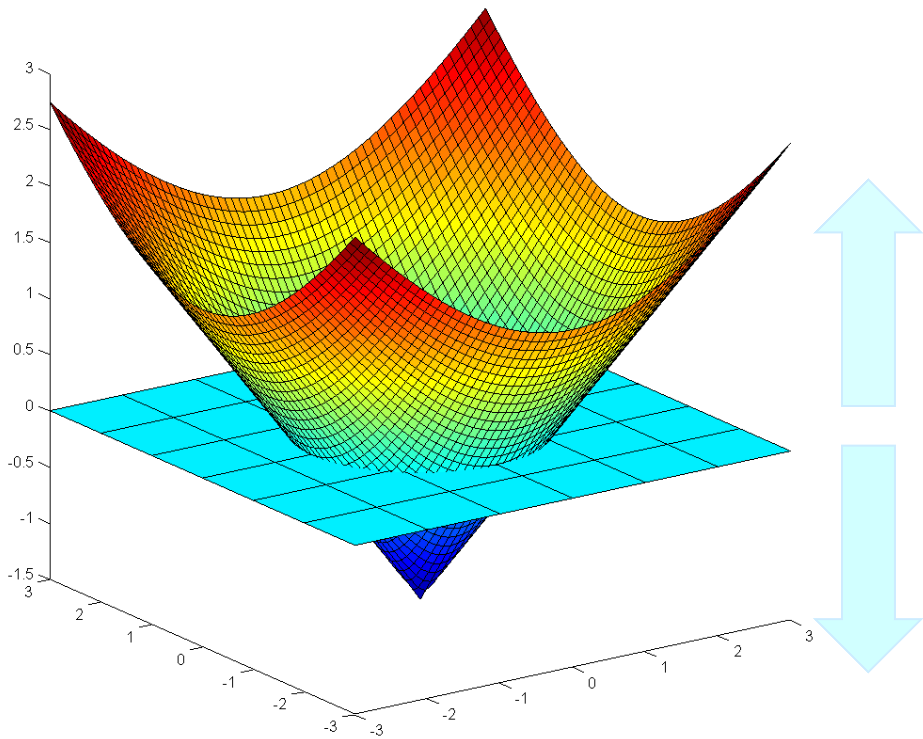
The method: Square plate with a hole

- Mesh definition (fixed during all the process) + Level Set definition:

  Fixed mesh grid: 6*6 elements

  Level Set: a cone

- No element is removed to create the hole but the properties of elements are modified: the density and the Young modulus.



Positive value of the level set

Negative value of the level set

The method: Square plate with a hole

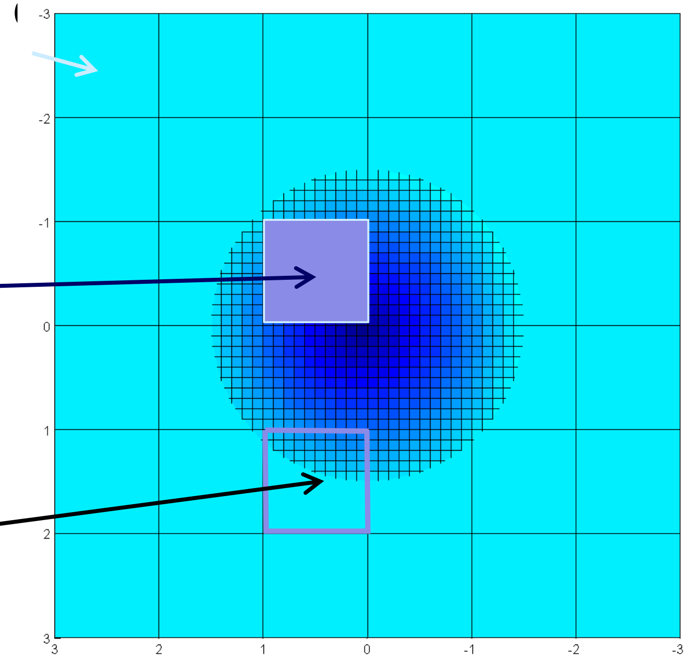
- For each node: Computation of the level set value.
- Different possibilities can happen for each
  - 4 positive nodal values: Solid material

$$\rho = \rho_0 \text{ and } E = E_0$$

  - 4 negative nodal values: void

$$\rho = 10^{-3}\rho_0 \text{ and } E = 10^{-9}E_0$$

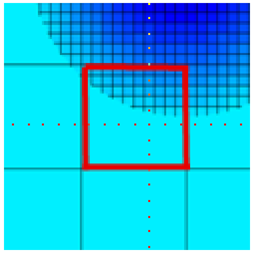
  - Positive and negative nodal values = boundary element



The method: Square plate with a hole

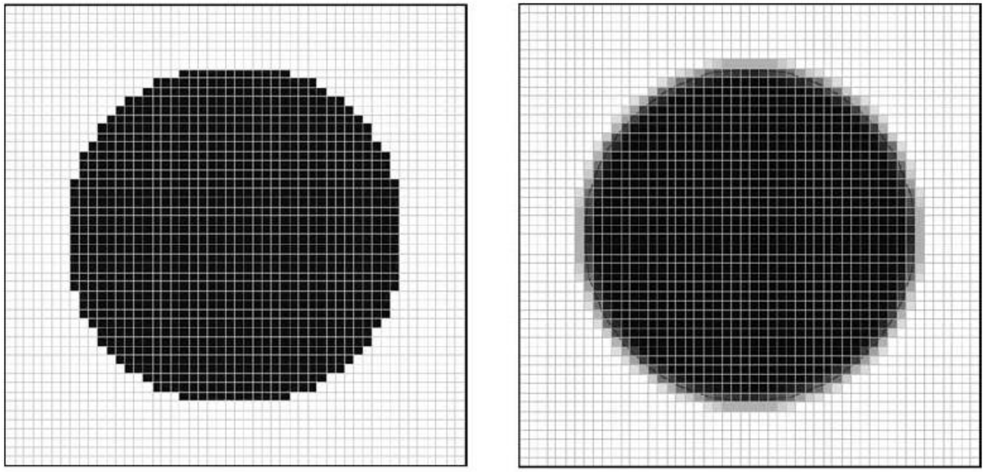
- For the boundary elements ➔ SIMP law
  - Introduction of a pseudo-density

$$\mu = \frac{\text{Volume of material}}{\text{Volume of the element}}$$



  - SIMP law $\rho = \mu \rho_0$ and $E = \mu^3 E_0$

- Consequence:



(a) (b)

Equation of FEM-MBS dynamics

- Motion of the flexible body (FEM) is represented by absolute nodal coordinates $\mathbf{q}$ (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{\text{ext}} - \mathbf{g}^{\text{int}}$$

- Subject to kinematic constraints of the motion

$$\mathbf{\Phi}(\mathbf{q}, t) = 0$$

- Solution based on an augmented Lagrangian approach of total energy

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}^{T}(k\lambda + p\mathbf{\Phi}) = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ k\mathbf{\Phi}(\mathbf{q}, t) = 0 \end{cases} \qquad \mathbf{B} = \frac{\partial \mathbf{\Phi}}{\partial \mathbf{q}}$$

$$\mathbf{q}'(0) = \mathbf{q}'_{0} \text{ and } \dot{\mathbf{q}}'(0) = \dot{\mathbf{q}}_{0}$$

Time Integration

- The set of nonlinear DAE solved using the generalized-α method by Chung and Hulbert (1993)
- Define pseudo acceleration **a**:

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n$$

- Newmark integration formulae

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_{n+1} + h^2(1/2-\beta)\mathbf{a}_n + h\beta\mathbf{a}_{n+1}$$

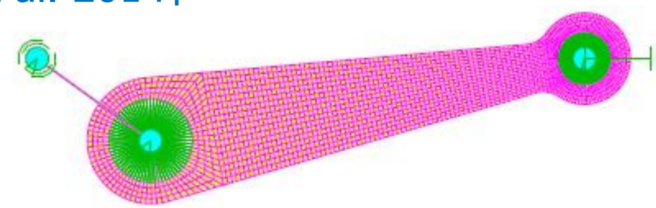
- Solve iteratively the dynamic equation system (Newton-Raphson)

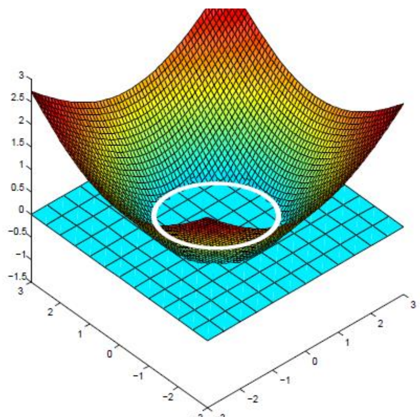
$$\begin{cases} \mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{B}^T\Delta\boldsymbol{\lambda} = \Delta\mathbf{r} \\ \mathbf{B} = \mathbf{0} \end{cases} \qquad \mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} - \mathbf{g} + \mathbf{B}^T\boldsymbol{\lambda}$$

Shape optimization and level set description

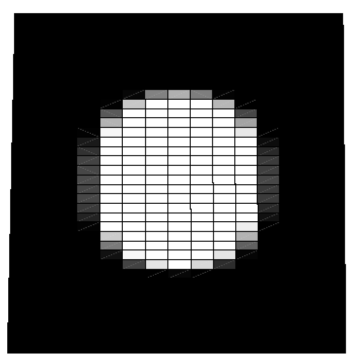
- Novel approach for shape optimization of flexible components based on level set description [Tromme et al. 2014]

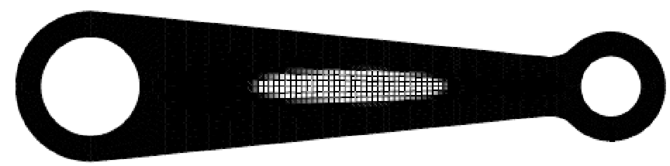
$$\phi(\mathbf{x}) \;=\; \sqrt{(\frac{x-c_x}{a})^m + (\frac{y-c_y}{b})^n} - r$$

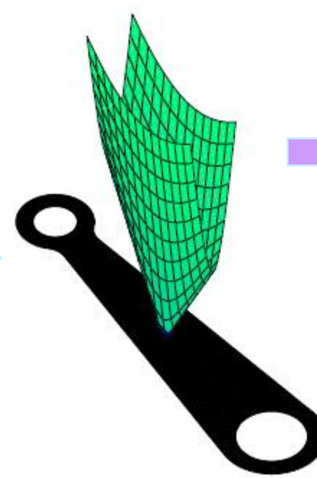




$$\begin{aligned} \rho_e &= \mu_e * \rho^0 \\ E_e &= \mu_e * E^0 \end{aligned}$$

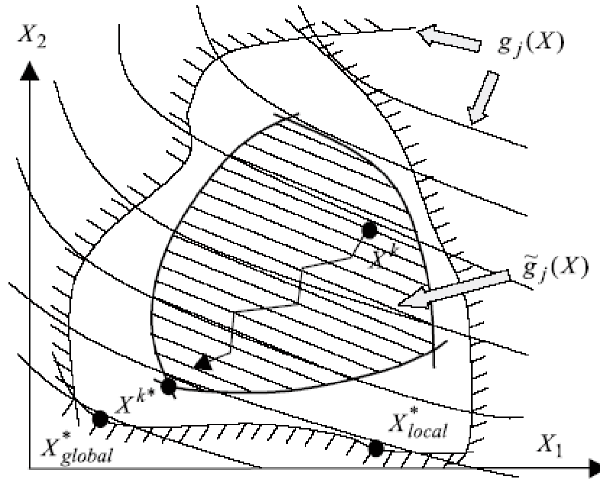






General form of the optimization problem

- Design problem is cast into a mathematical programming problem



$$\min_{\mathbf{x}} g_0\left(\mathbf{x}\right)$$

$$s.t. \begin{cases} g_j\left(\mathbf{x}\right) \leq \overline{g}_j, & j = 1, \ldots, m \\ \underline{x}_i \leq x_i \leq \overline{x}_i, & i = 1, \ldots, n \end{cases}$$

- Provides a general and robust framework to the solution procedure

- Efficient solver :
  – Sequential Convex Programming (Gradient based algorithm)

    ➔GCM (Bruyneel et al. 2002)

Sensitivity analysis

- Gradient-based optimization methods require the first order derivatives of the responses

- Finite differences $$\frac{\partial \mathrm{f}}{\partial x} \approx \frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}$$

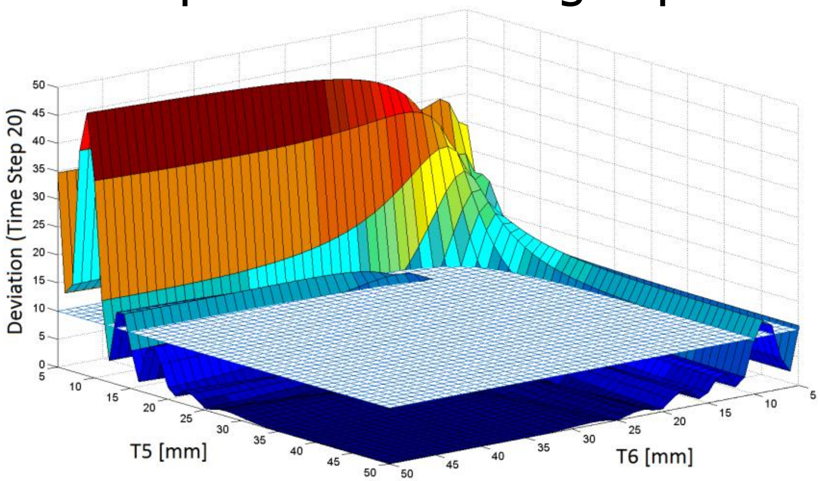
  Perturbation of design variable
  ➔ Additional call to MBS code

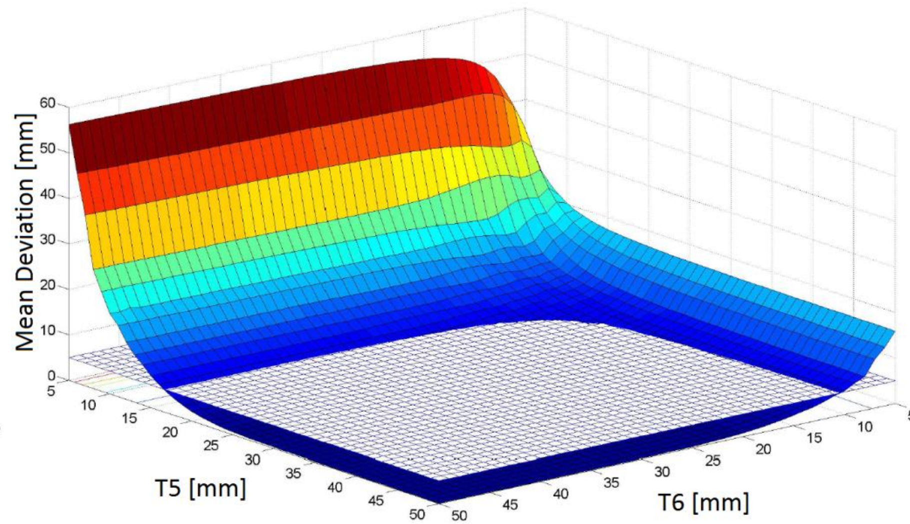
- Semi-analytical approach (Not yet developed)

$$\frac{\partial \mathbf{r}}{\partial x} \approx \frac{\mathbf{r}(x+\delta x)-\mathbf{r}(x)}{\delta x} \qquad \frac{\partial \mathbf{\Phi}}{\partial x} \approx \frac{\mathbf{\Phi}(x+\delta x)-\mathbf{\Phi}(x)}{\delta x}$$

The formulation

- The formulation is a key point for this type of problems:
  Very complex nonlinear behavior
- Impact on the design space

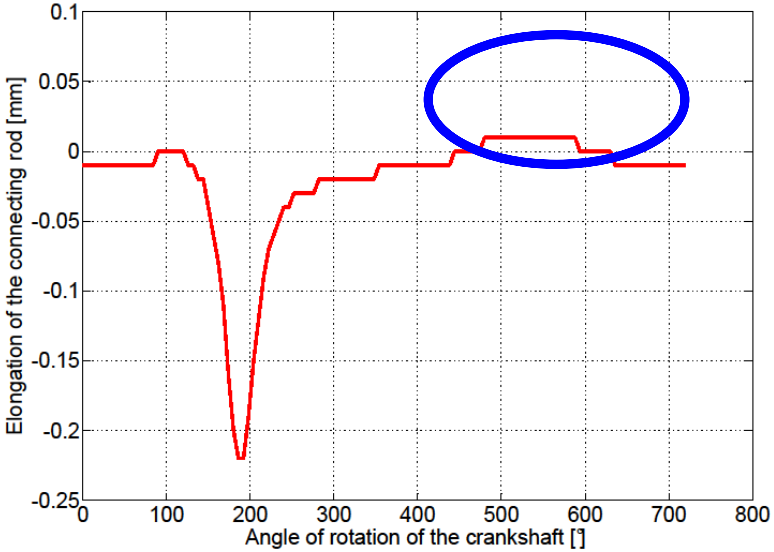


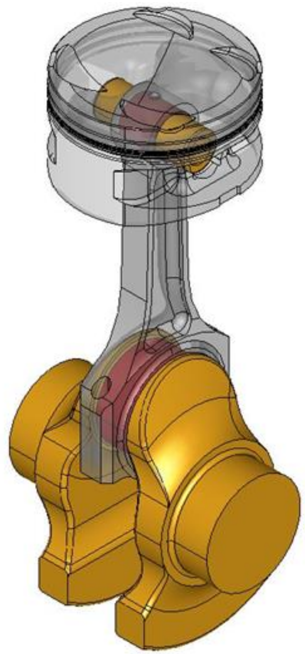


- Extremely important for gradient based algorithm
- Genetic algorithms
  – Do not necessary give better results
  – Computation time much more important

Connecting rod optimization

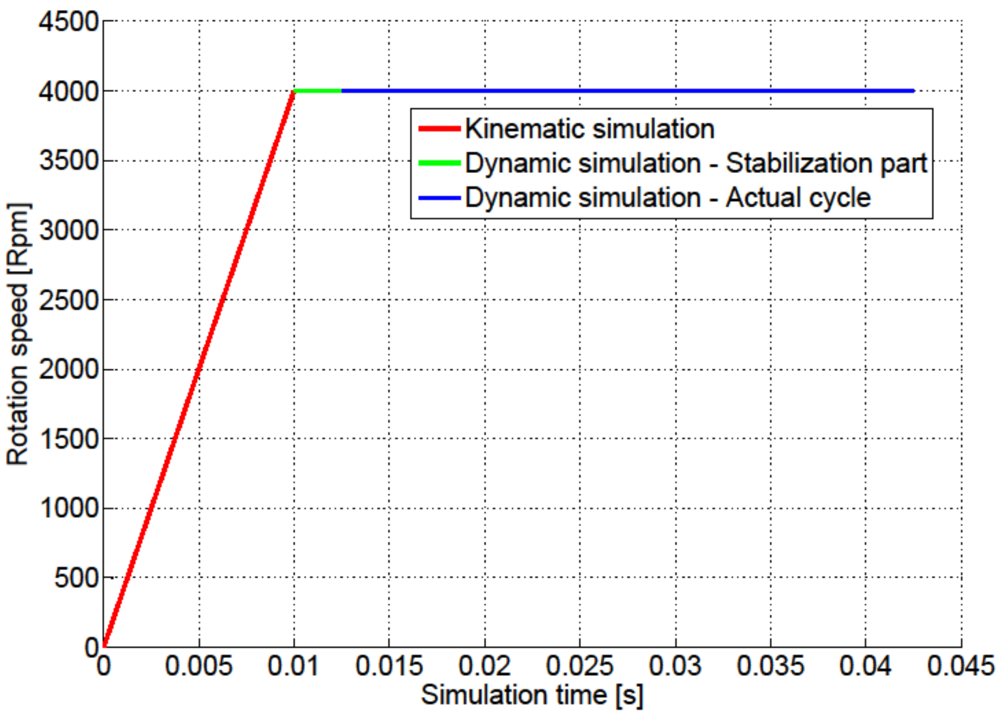
- The link between the piston and the crankshaft in a combustion engine.
- During the exhaust phase, the connecting rod elongates which can destroy the engine.
  - ➔Collision between the piston and the valves.
- Minimization of the elongation





Modeling of the connecting rod

- Simulation of a single complete cycle as the behavior is cyclic (720°)
- Rotation speed 4000 Rpm
- Gas pressure taken into account.



Local formulation

$$\min_{\mathbf{x}} m\left(\mathbf{x}\right)$$

$$s.t. \;\; k\left(\Delta l\left(\mathbf{x}, t_i\right) \leq \Delta l_{max}\right)$$

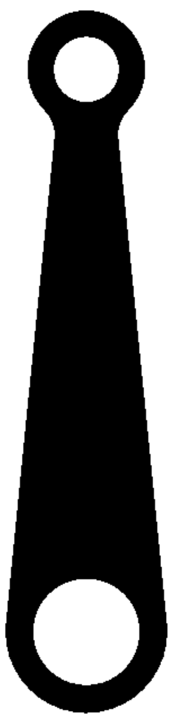
$$\text{with } i = 1, \ldots, \text{nbr time step}$$

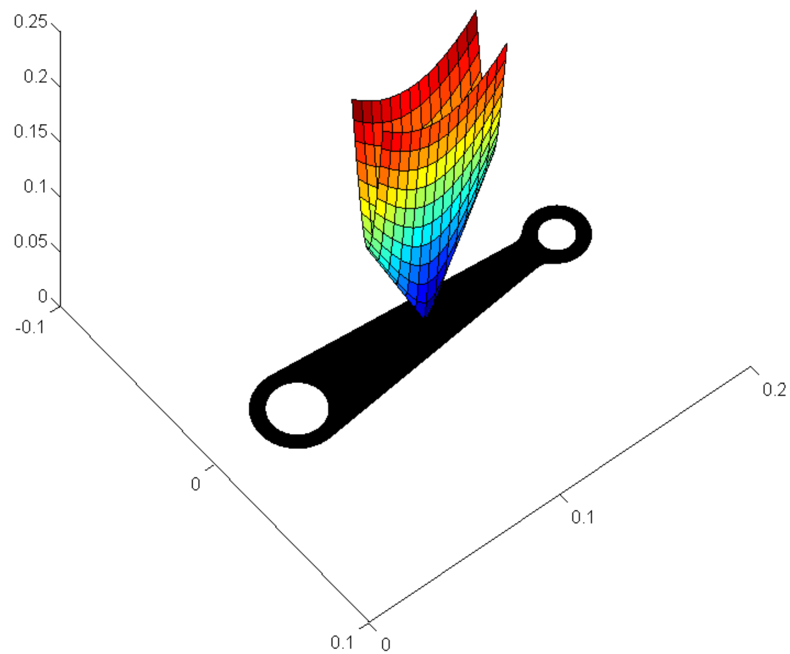
- The constraint on the elongation $\Delta l\left(\mathbf{x}, t_i\right)$ is considered at each time step.

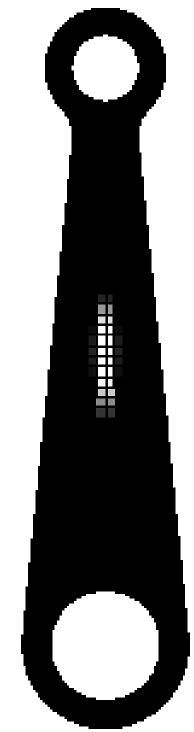
First application – 1 level set

- The level set is defined in order to have an ellipse as interface.
- 3 different design variables :a, b, d. Here only c is chosen.

$$\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0$$

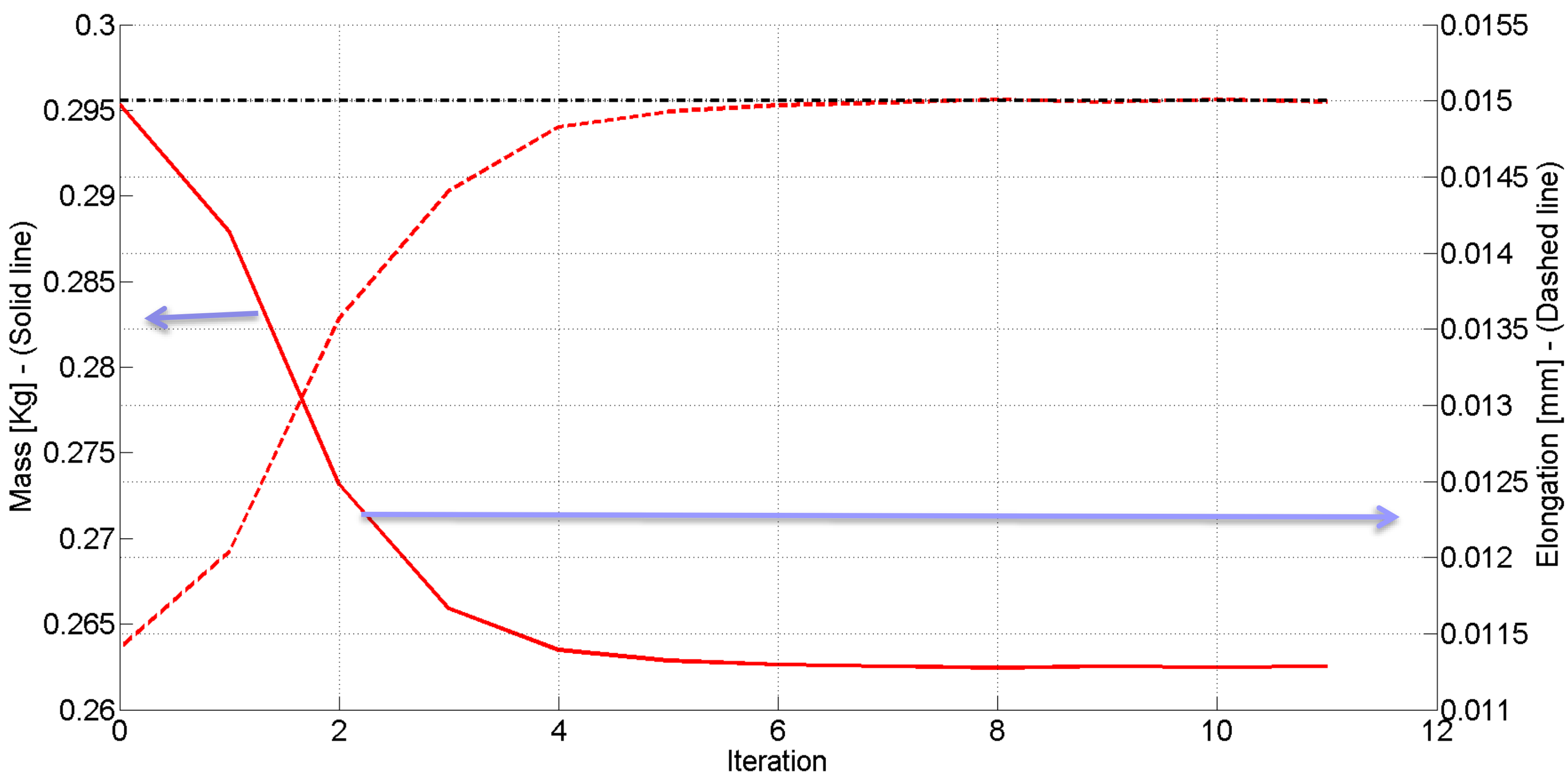






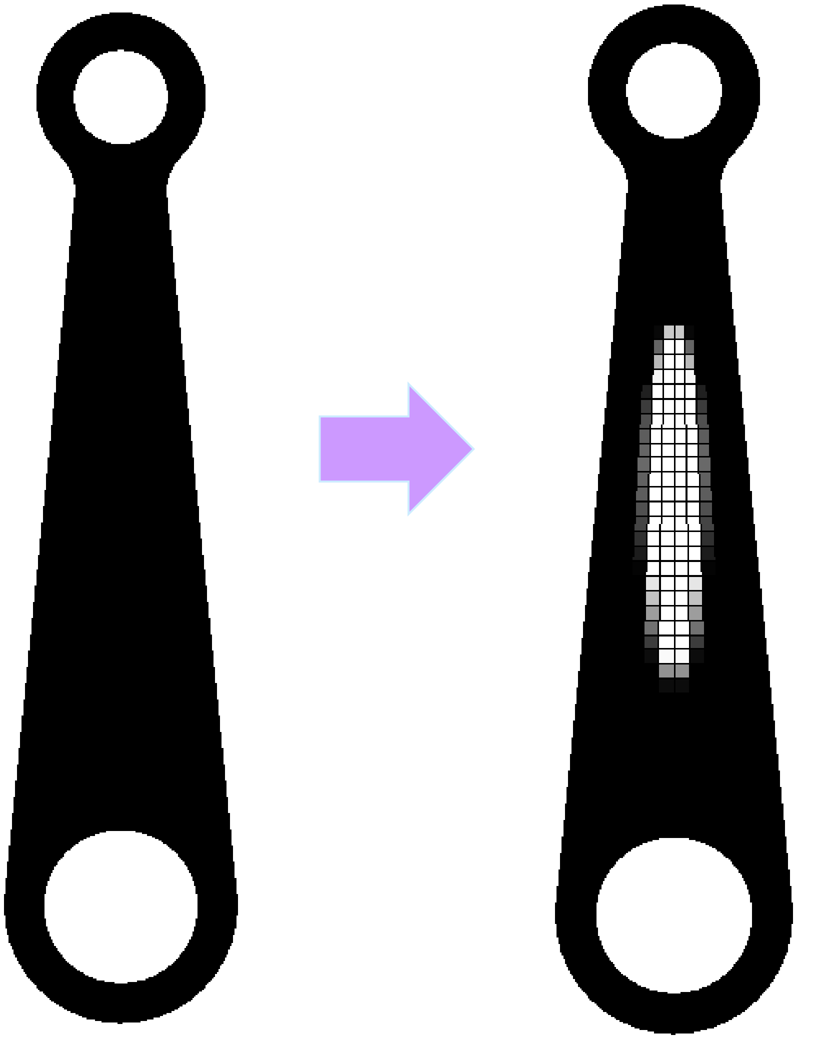
Results

- Convergence obtained after 12 iterations
- Monotonous behavior of the optimization process



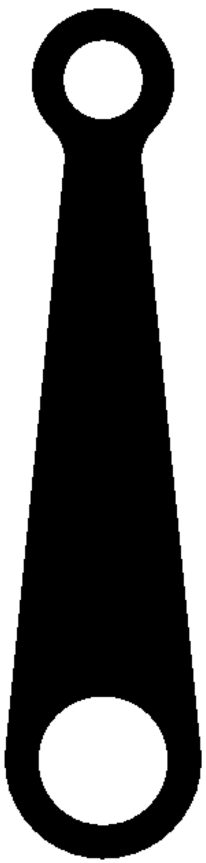
Results – Optimal design

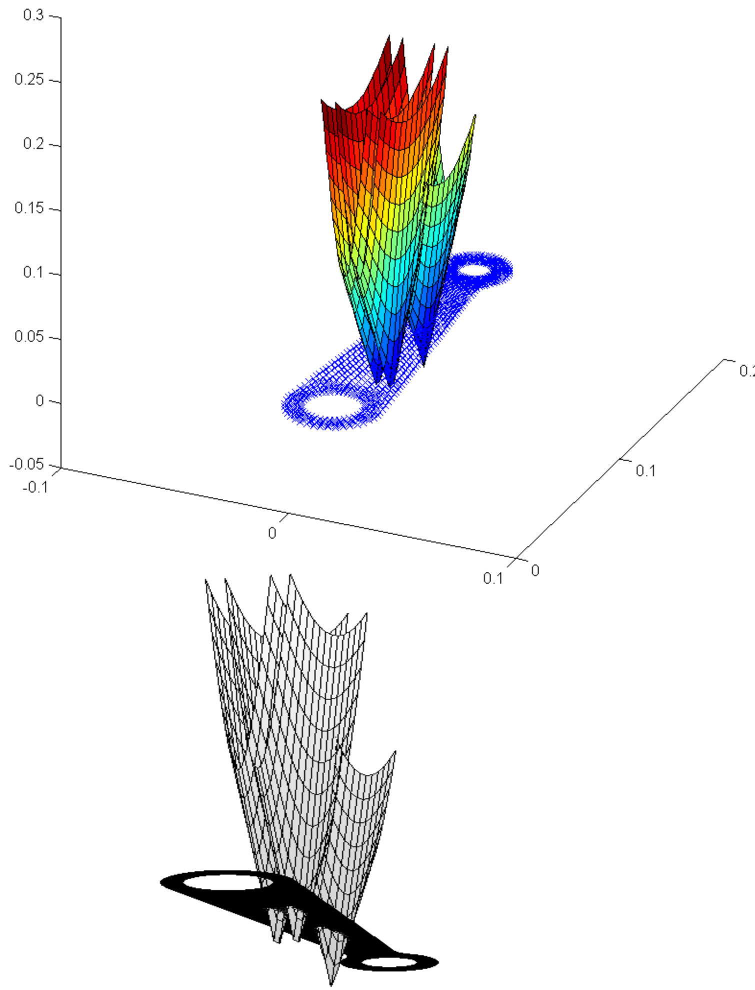
- Even if the boundary of
  the hole is not clear
  on the mesh, the boundary
  is defined by a CAD entity
  and the connecting
  rod can then be manufactured
  without any post processing.

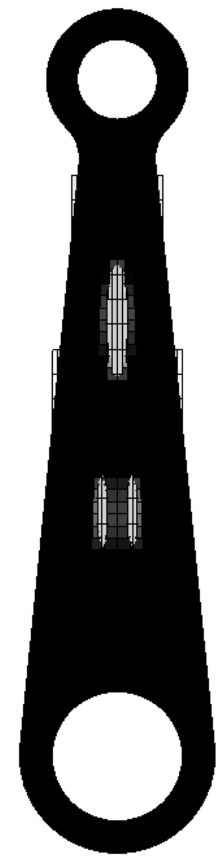


Second application – 3 level sets

- 3 ellipses are defined. $\Phi(x,y) = \dfrac{(x-c_x)^2}{a^2} + \dfrac{(y-c_y)^2}{b^2} - c = 0$

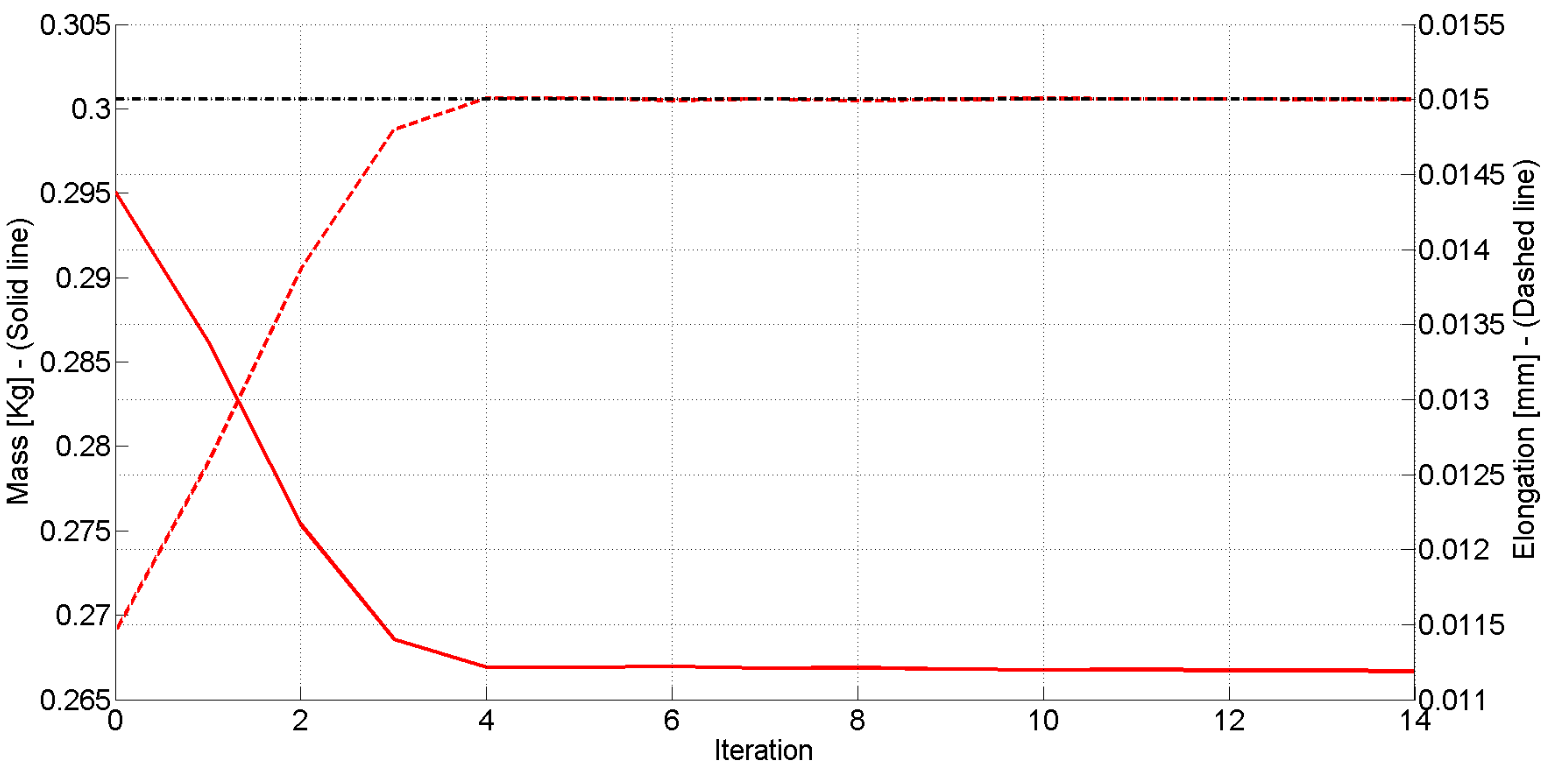






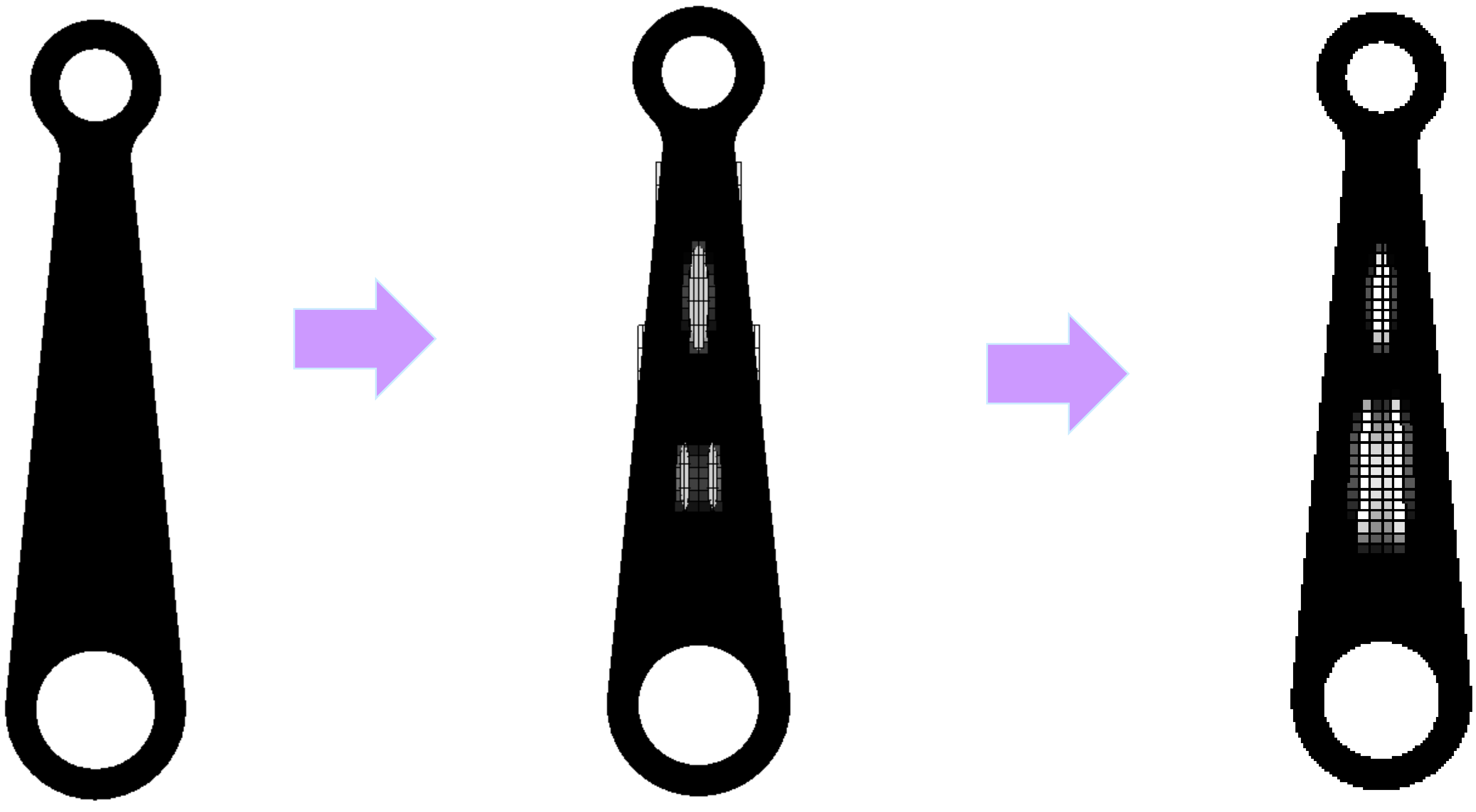
Results

- Convergence obtained after 15 iterations
- Monotonous behavior of the optimization process
- Even better than the simpler case



Results – Optimal design

- Modification of the topology



SUMMARY OF LEVEL SET APPROACH

- Develop an intermediate approach between shape and topology optimization
- Presenting ideally the advantages of both methods

FEM ➔ X-FEM :

- Eulerian Method: work on fixed mesh
- No mesh perturbation and remeshing required ➔ Less time spent in mesh (re)generation
- Alternative to homogenization/SIMP: void is void!

CAD model ➔ Level Set:

- Topology can be changed as entities can merge or separate >< Shape
- Smooth curve description of boundaries >< Topology
- Convenient to use with X-FEM

Shape and Topology Optimization of Lightweight Automobile Transmission Components

P. Duysinx, G. Virlez, S. Bauduin, E. Tromme
LTAS - Aerospace and Mechanical Engineering Department - University of Liège

N. Poulet
JTEKT Torsen Europe, Belgium

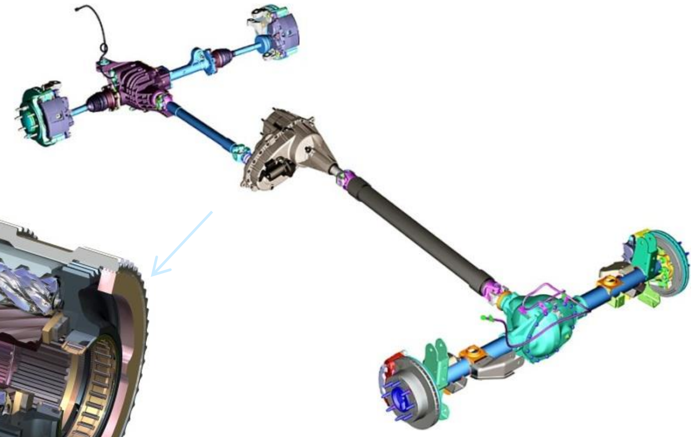
OUTLINE

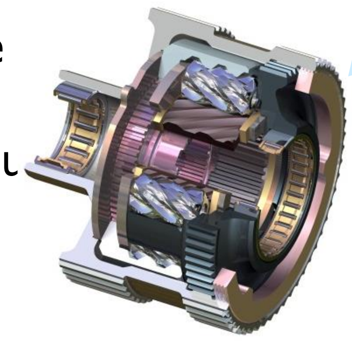
- Introduction & motivation
- Modelling of Torsen Differential
- Design approach using combined topology and shape optimization
- Topology optimization
- Shape optimization
  - 2D shape
  - 3D shape
- Conclusion & Perspectives

MODELLING OF JTEKT DIFFERENTIAL

SIMULATION OF DRIVELINE COMPONENTS

- JTEKT TORSEN Central Differential (Type C)
  - Central differential (4 wheels drive vehicles)
  - Non symmetric distribution of torqu (42/58)



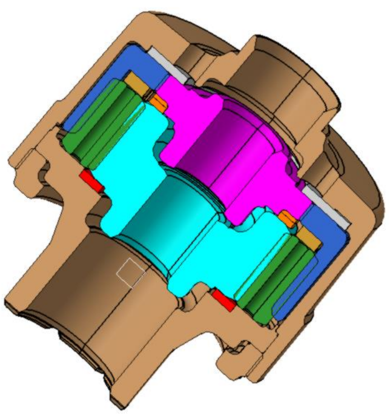


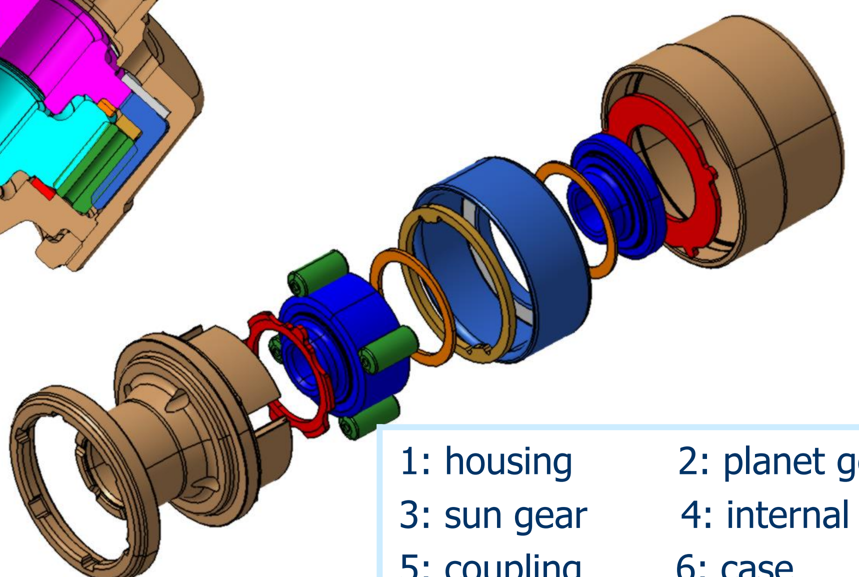
Torsen differential

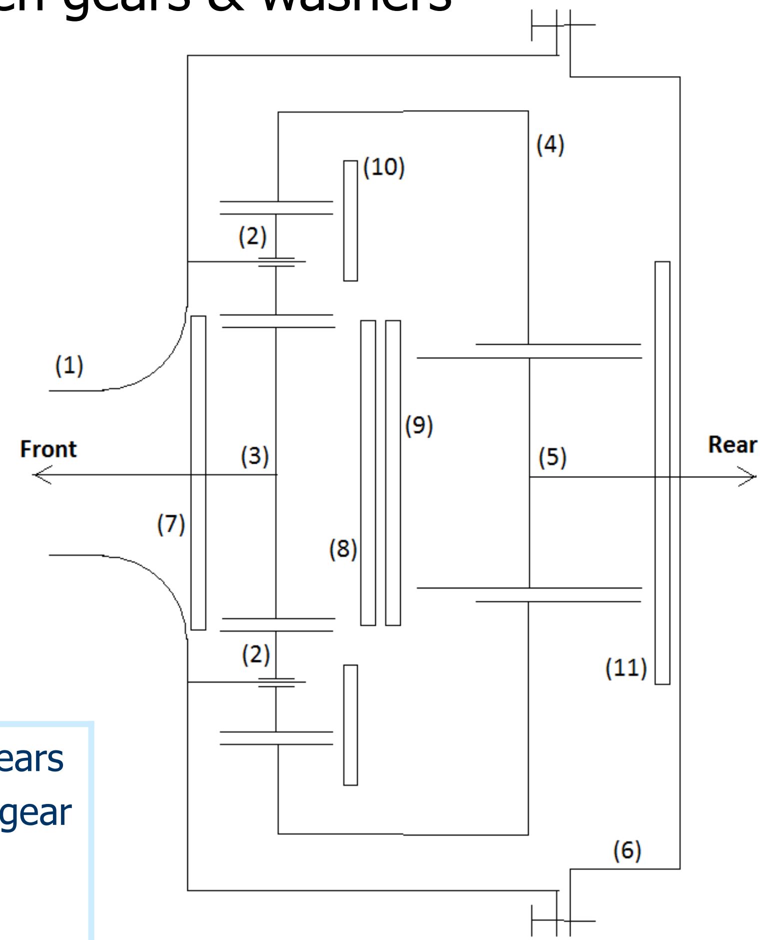
Courtesy of JTEKT

TYPE C TORSEN DIFFERENTIAL

- Composed of gear pairs and thrust washers
- Locking due to relative friction between gears & washers
- 4 working modes







1: housing
2: planet gears
3: sun gear
4: internal gear
5: coupling
6: case
7,8,9,10,11: thrust washers

OPTIMIZATION OF DIFFERENTIAL HOUSING

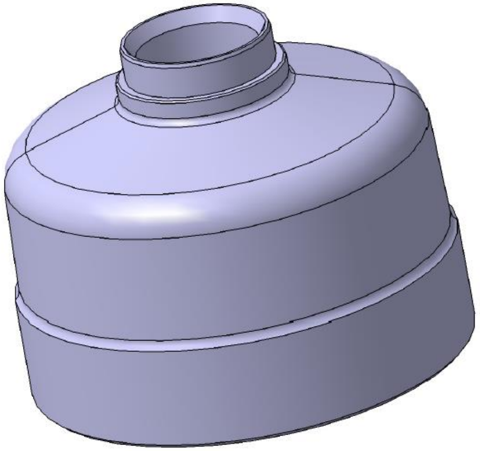
- The goal of the work is to propose and validate a design methodology of transmission components including topology optimization and shape optimization

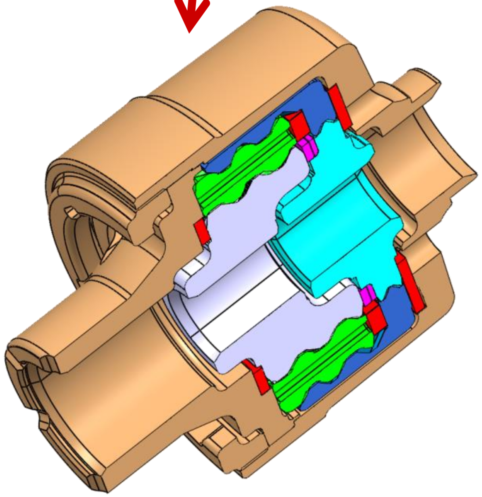
- The methodology will be validated on the optimization of the housing of the type-C Torsen differential

- Different steps will be carried out:
  - Specifications
  - Modelling
  - Topology optimization
    - 2D / 3D
  - Shape optimization:
    - 2D / 3D / dynamic loading

OPTIMIZATION OF DIFFERENTIAL CASING

- Housing is a heavy component that has not been properly optimized w.r.t. weight





OPTIMIZATION OF TRANSMISSION COMPONENTS A HIERARCHICAL APPROACH

A HIERARCHICAL APPROACH

- STEP 1: SPECIFICATIONS
  - Boundary conditions
  - Material data
  - Design specifications: stiffness, displacement constraints, allowable stress limits, etc.

- STEP 2: TOPOLOGY OPTIMIZATION
  - Determine optimal material distribution to minimize the housing mass s.t. a set of fundamental constraints
  - Use a subset of relevant constraints
    - Compliance
    - Displacement constraints : perpendicularity or parallelism restrictions

A HIERARCHICAL APPROACH

- STEP 3: CAD model construction
  - Interpretation of optimal material distribution
  - Construction of CAD model
  - Parametric design model
  - Introduction of manufacturing and technological restrictions

- STEP 4: SHAPE & PARAMETRIC OPTIMIZATION
  - Determine optimal set of parameters of the model
  - Detailed analysis and design model
  - Consider constraints including local constraints
    - Compliance
    - Displacement constraints : perpendicularity or parallelism restrictions
    - Stress constraints

A HIERARCHICAL APPROACH

- STEP 5: DETAILLED VERIFICATION
  – Detailed verification of the optimized model using non linear analysis
  – Adaption to manufacturing constraints

- STEP 6: EXPERIMENTAL TESTING AND VALIDATION
  – Build prototype
  – Experimental testing

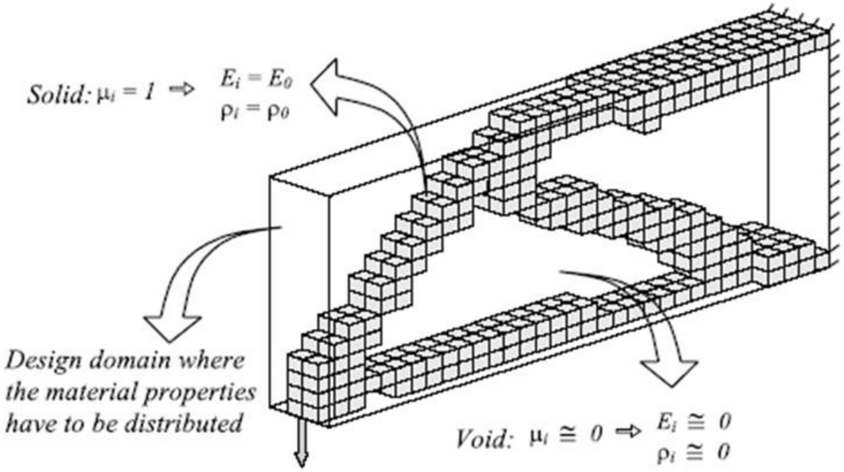
TOPOLOGY OPTIMIZATION

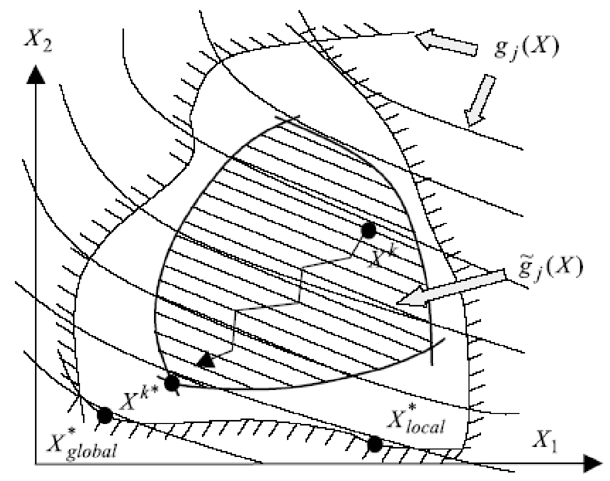
TOPOLOGY OPTIMIZATION

- Topology optimization is formulated as an optimal material distribution
- Finite element discretization of the design domain
- Interpolation of material properties between void and solid: SIMP law

$$E = x^3 E_0 \text{ and } \rho = x\rho_0$$

- Filtering densities (Sigmund)
- Efficient solution of optimization problem based on sensitivity analysis and mathematical programming algorithms (CONLIN)



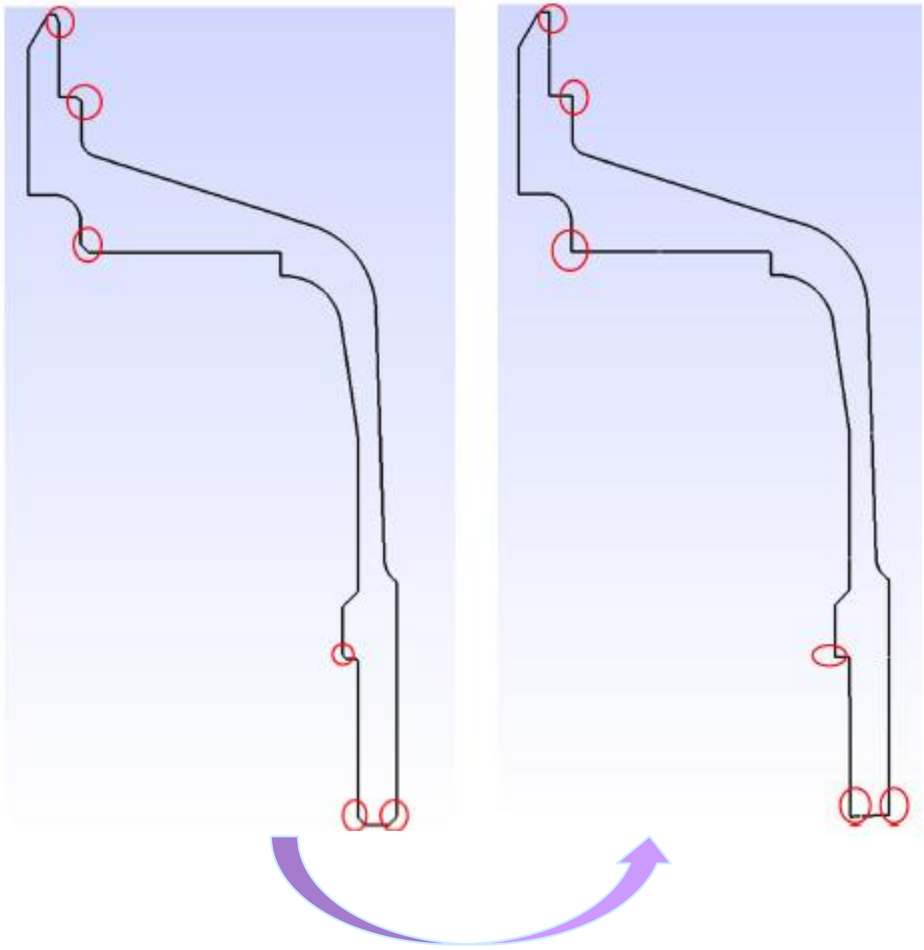


TOPOLOGY OPTIMIZATION

- Simplified geometrical model
  - Remove unnecessary local details e.g. small rounded shapes

    ➔ use regular mesh with rectangular elements
  - Cover with adapted mesh size.

- 2D models are preferred because 3D models are time consuming and do not bring sufficient information for modest meshes



Remove unnecessary geometrical details

TOPOLOGY OPTIMIZATION

- Support: axial support
- Applied loads:
  - Rotation speed

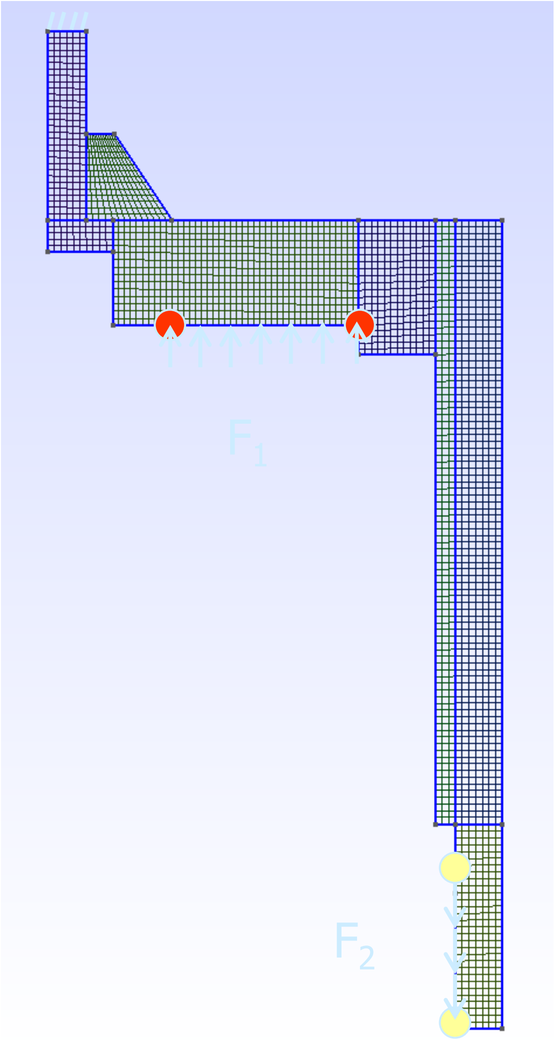
    Rpm 3500 (engine) →

    (6th 0.614 and axle ration 3.563)

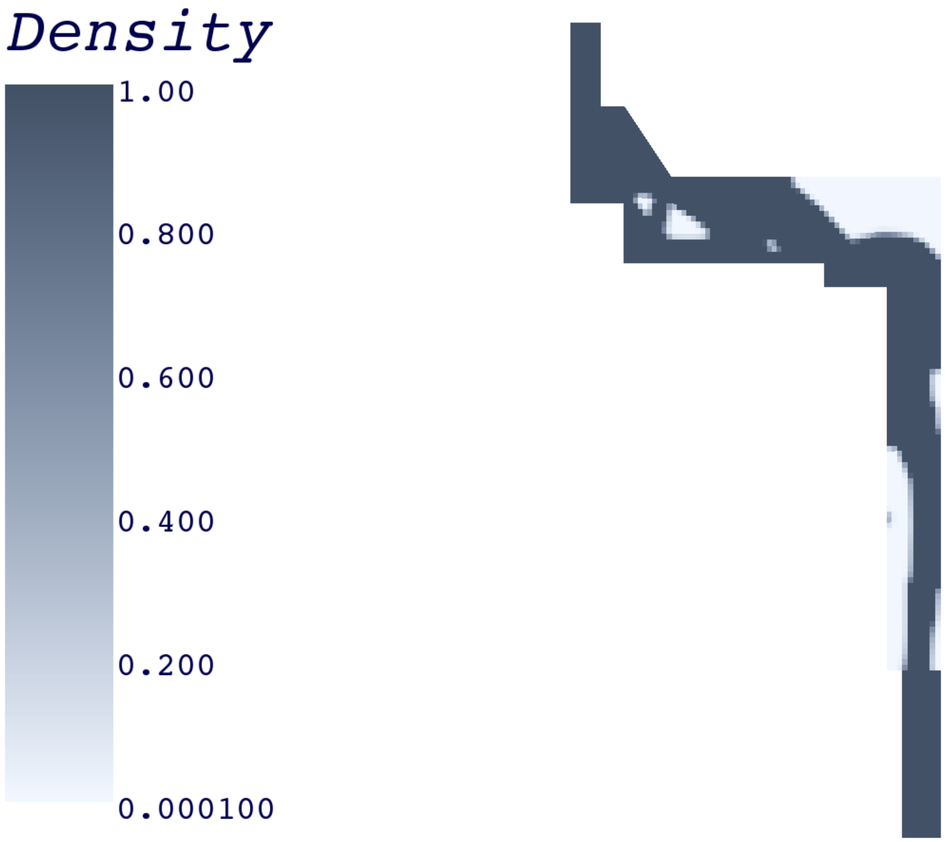
    → 2000 rpm at Housing
  - Loads F1 & F2: axial pressure of bearing and axial reaction of filet

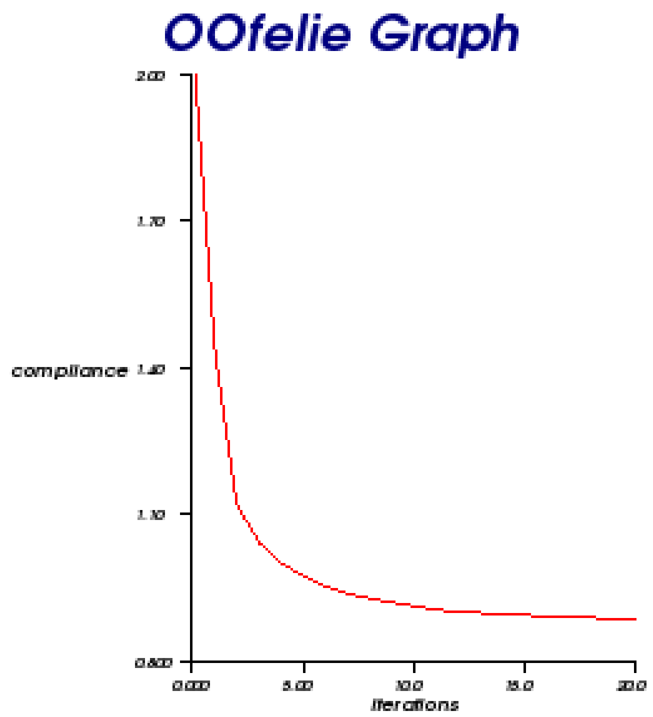
    Based on the application of a 1000 [Nm] at the engine input (D-IG)

    - F1 = IG end surface = 10147 [N]
    - F2 = SG end surface = 8055 [N]
- Perpendicularity and parallelism constraints: 1/1000
- Max radial deformation: 1.E-5



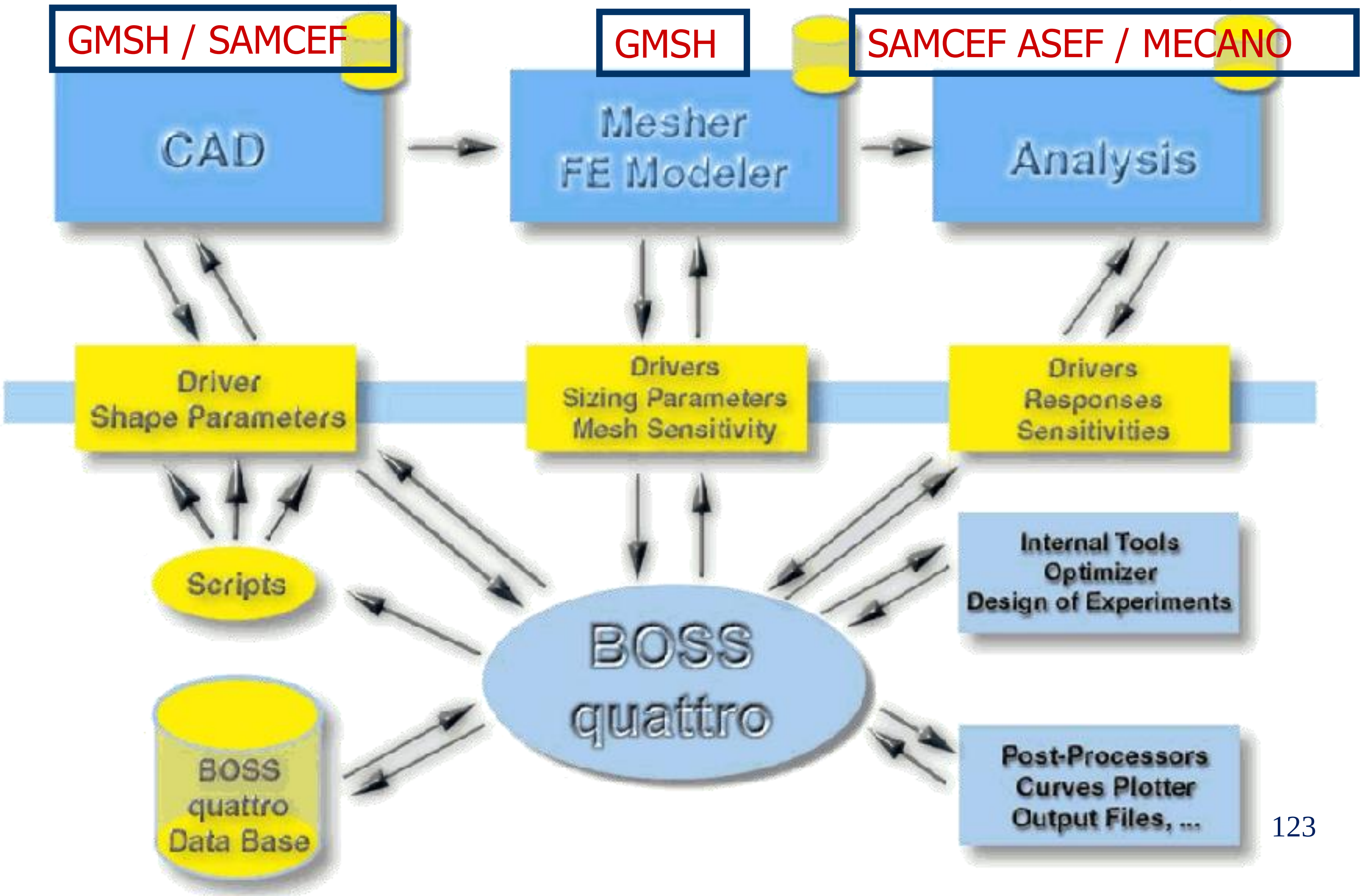
TOPOLOGY OPTIMIZATION





2D SHAPE OPTIMIZATION

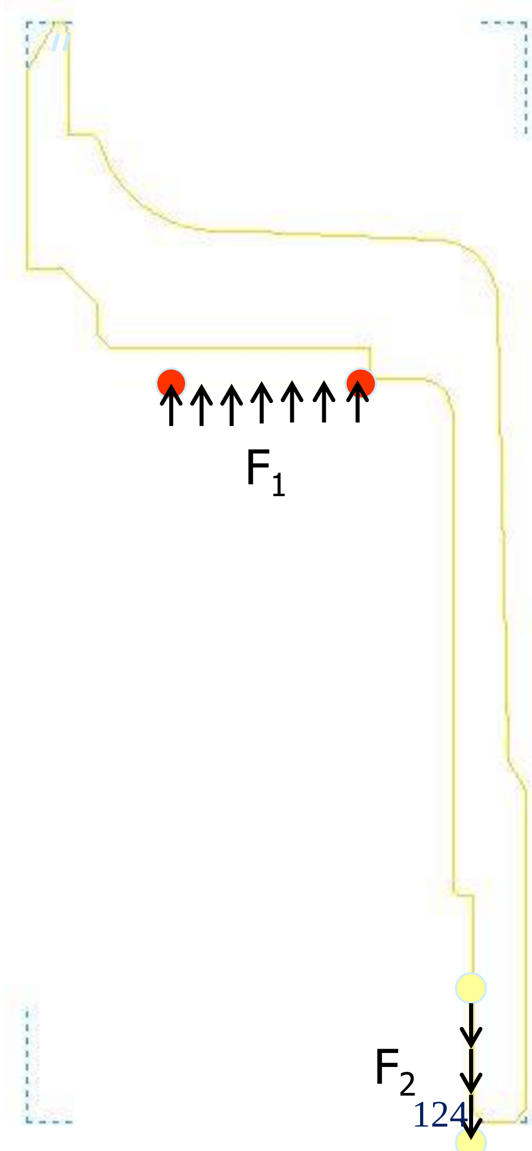
CAD-FEM coupling in shape otpimization



2D shape optimization

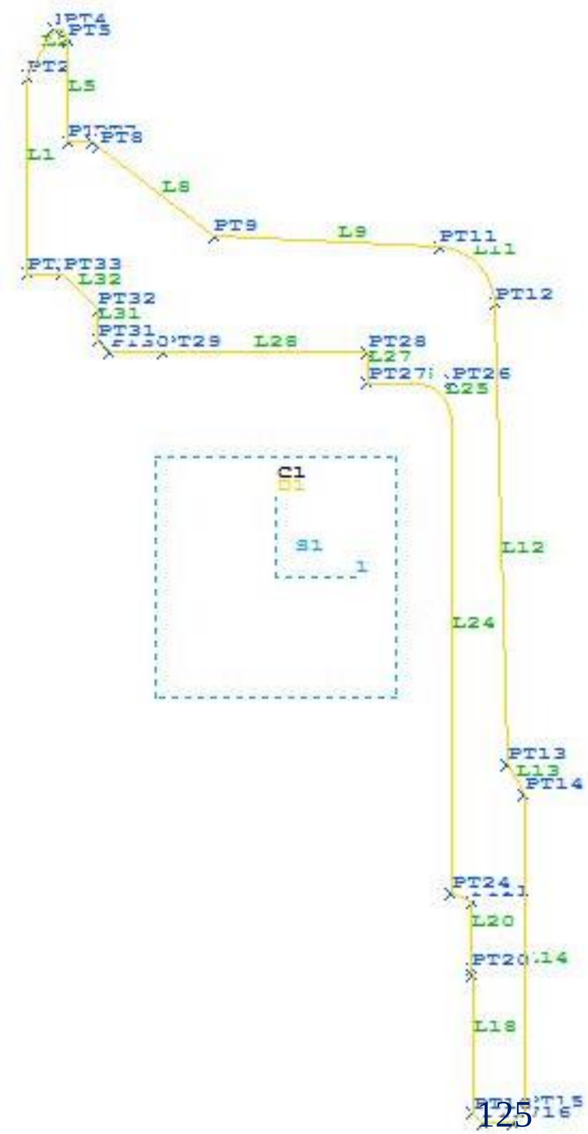
- Accurate geometrical model
- Rotation speed + loads
- Perpendicularity and parallelism
  displacement constraints
- Max radial deformation

- Boundary Conditions:
  - Axial fixation
  - Rpm 4000 rm
  - Loads based on **10.000** [Nm] D-IG
    - F1 = IG end surface = 101470 [N]
    - F2 = SG end surface = 80550 [N]
  - Perp. and parall. restrictions: 20/1000
  - Max radial def = 1E-4



2D shape optimization: parameterization

- Design variables: control points and curve parameters (here 9 dv)
  - X_11_12r
  - Y11
  - X9
  - Y9
  - Cr
  - X24
  - Y24
  - X25
  - Y12_14
  - +filet



2D shape optimization

- No modification of inner design of housing

	Initial model	Optimal Model	
Mass [kg]	1,456	1,370	Min
Stress by element [MPa]	611	620	620
Stress by nodes [MPa]	687	709	/
Perp Wash	5,07/1000	3,42/1000	20/1000
Perp Thread	3,78/1000	3,79/1000	20/1000
Max radial def [µm]	33,54	33,43	100

-5,9%

- Modification of inner housing allowed

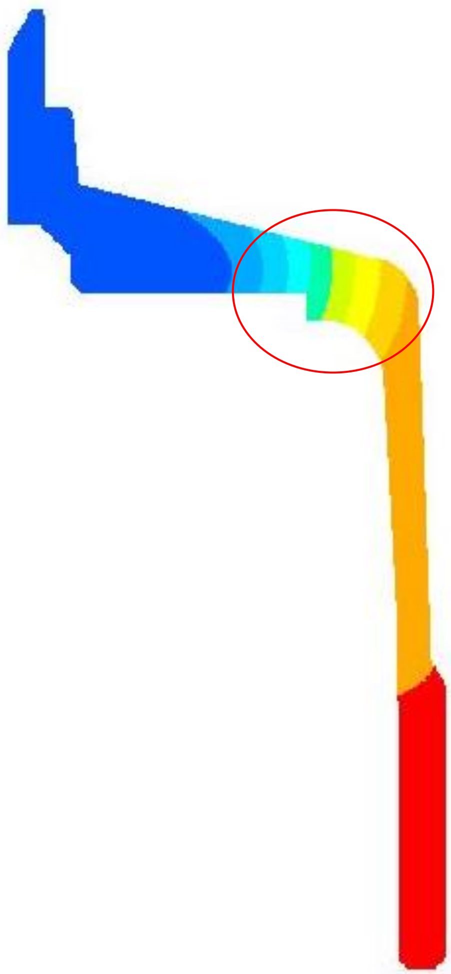
	Initial model	Optimal Model	
Mass [kg]	1,456	1,062	Min
Stress by element [MPa]	611	619	620
Stress by nodes [MPa]	687	671	/
Perp Wash	5,07/1000	5,83/1000	20/1000
Perp Thread	3,78/1000	4,15/1000	20/1000
Max radial def [µm]	33,54	39,07	100

-27%

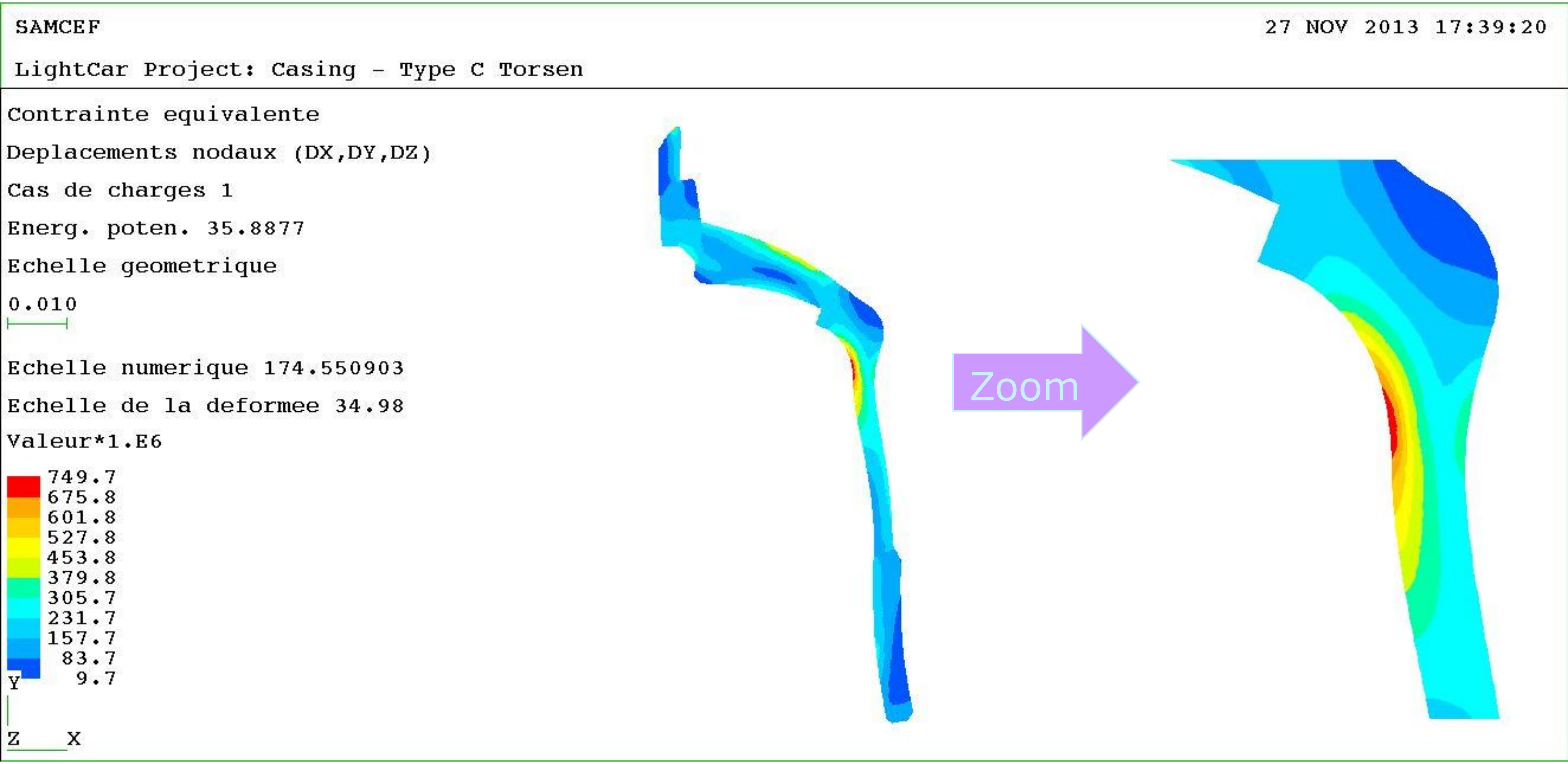
2D shape optimization: Optimal shape

Initial shape design

Optimal shape design



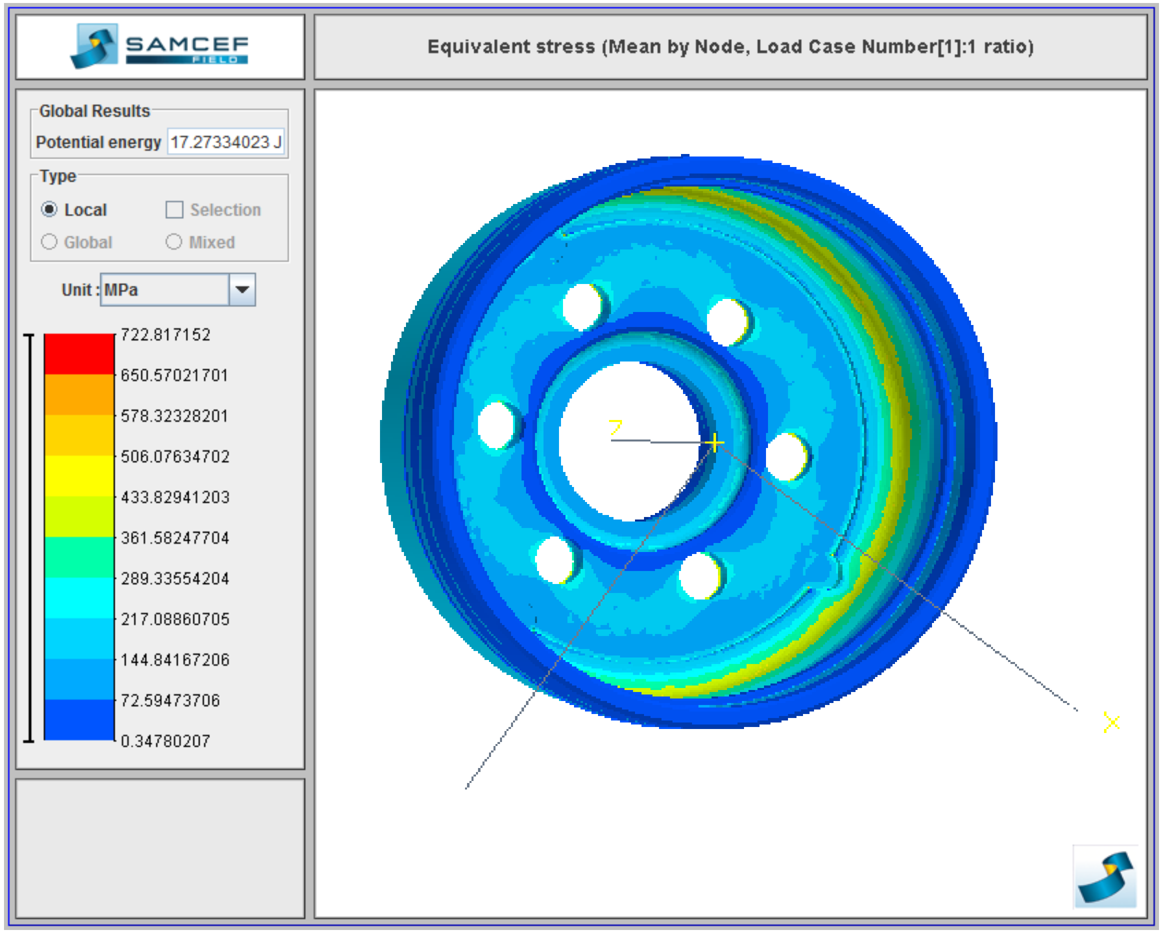
2D shape optimization: assessment of stress



- Optimized shape: mesh refinement

2D shape optimization

- Validation of stress level of the optimized shape with a 3D-model of optimized shape

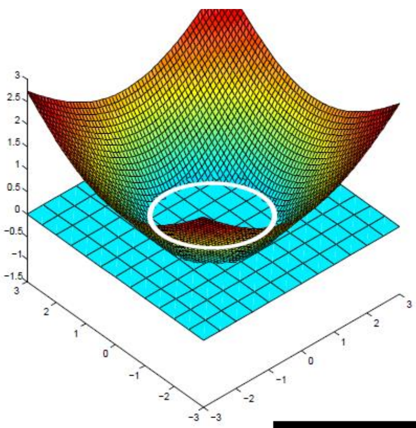


3D SHAPE OPTIMIZATION

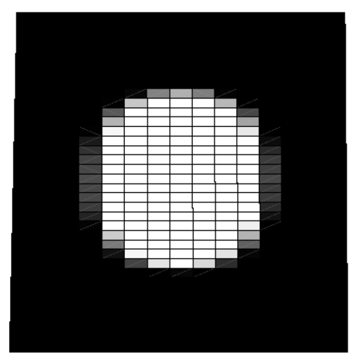
3D shape optimization: level set description

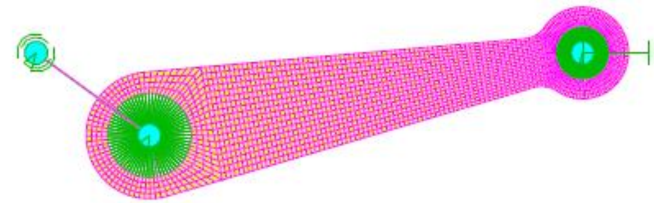
- Novel approach for shape optimization of flexible components based on level set description (Tromme et al. 2014)

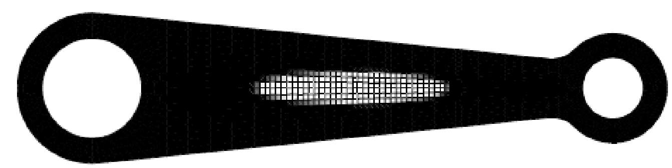
$$\phi(\mathbf{x}) = \sqrt{(\frac{x-c_x}{a})^m + (\frac{y-c_y}{b})^n} - r$$

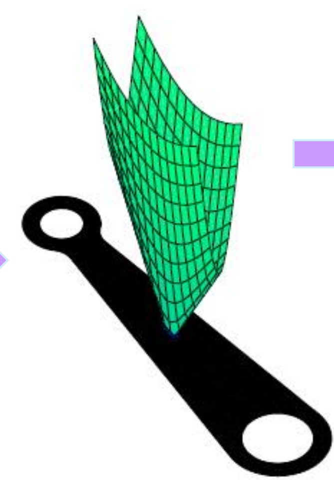


$$\rho_e = \mu_e * \rho^0$$
$$E_e = \mu_e * E^0$$







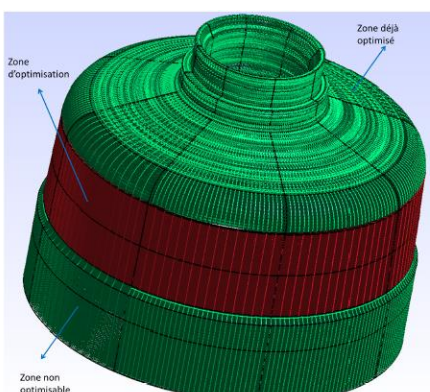


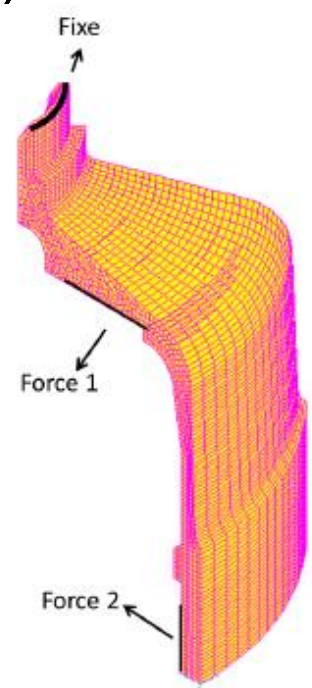
3D shape optimization

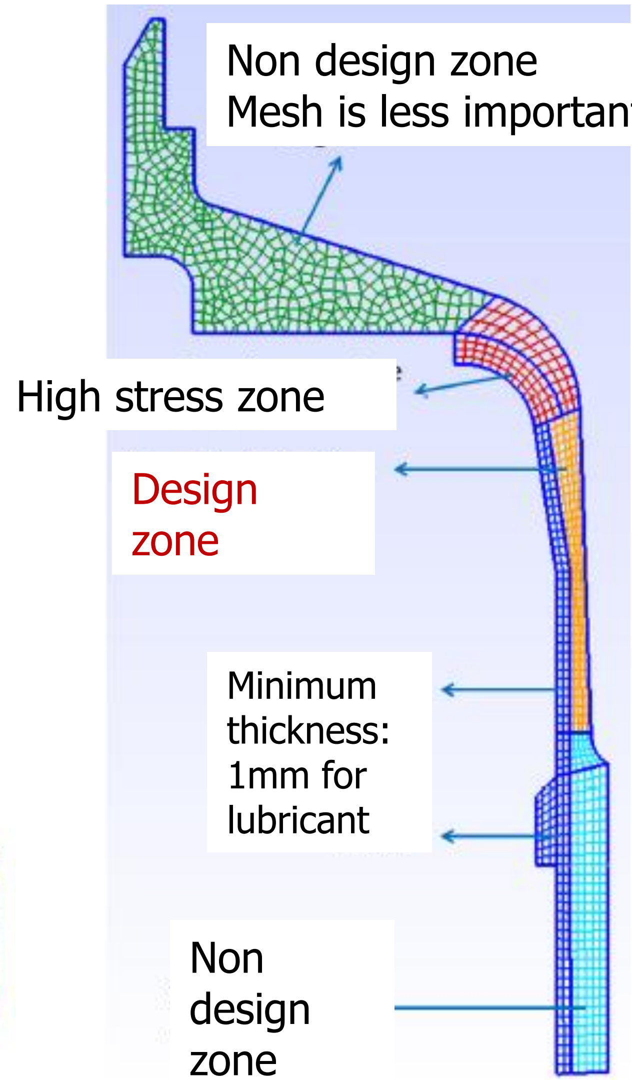
- Design zone is mostly located in the upper skin flange.

- One has to preserve a minimum thickness of 1 mm on the inner side (to maintain the lubricant in the differential)

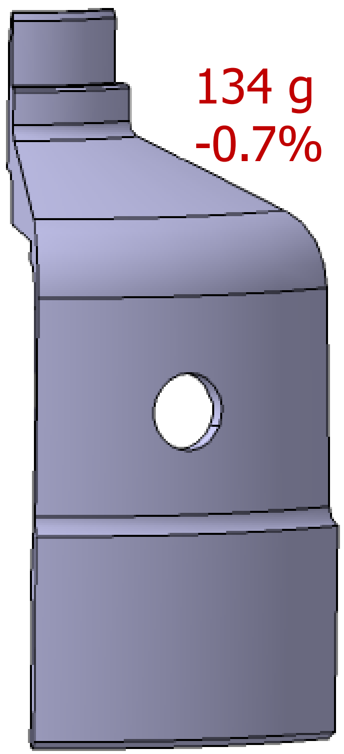
- 3D model: use cyclic periodicity (1/8= 45°)



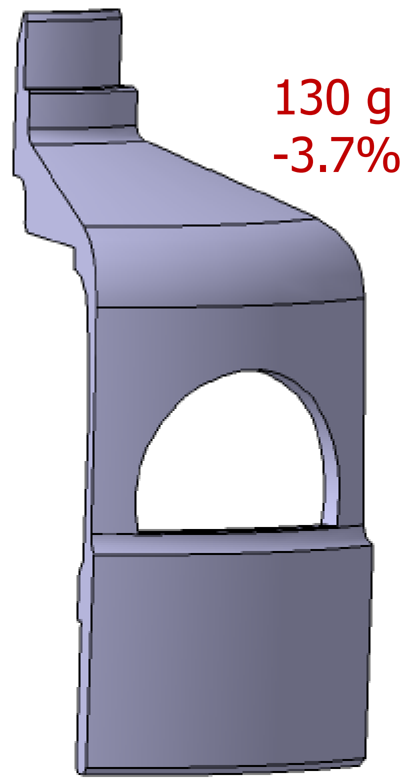




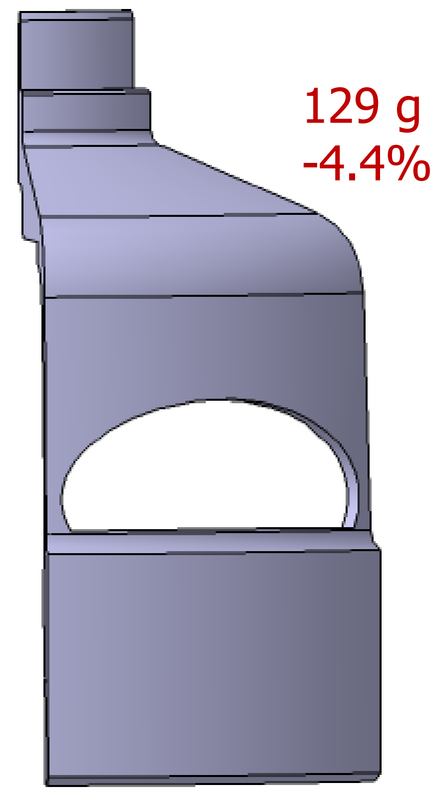
3D shape optimization



$$\phi(\mathbf{x}) = \sqrt{(\frac{x-c_x}{a})^2 + (\frac{y-c_y}{b})^2} - r$$



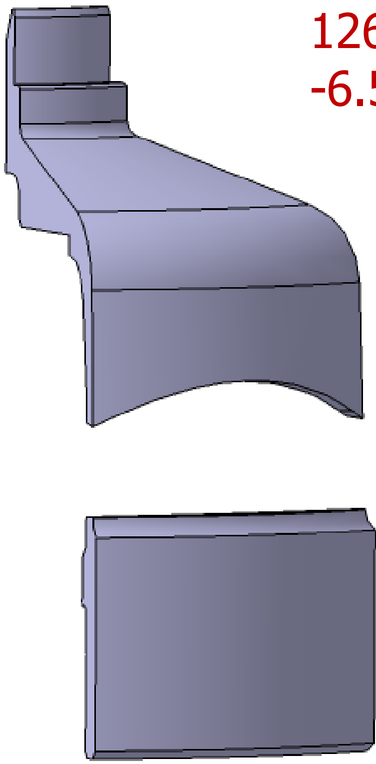
$$\phi(\mathbf{x}) = \sqrt{(\frac{x-c_x}{a})^2 + (\frac{y-c_y}{b})^2} - r$$



$$\phi(\mathbf{x}) = \sqrt{(\frac{x-c_x}{a})^2 + (\frac{y-c_y}{b})^2} - r$$

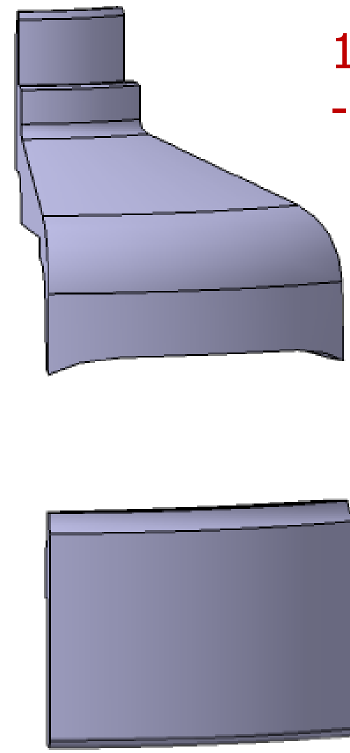
- Single hole per 8th of the housing: different parameterizations

3D shape optimization



126 g
-6.5%

$$\phi(\mathbf{x}) = \left(\frac{\mathbf{x}-\mathbf{c_x}}{\mathbf{a}}\right)^{\mathbf{m}} + \left(\frac{\mathbf{y}-\mathbf{c_y}}{\mathbf{b}}\right)^{\mathbf{n}} - \mathbf{r}$$



125 g
-7.5%

$$\phi_{1,2,3,4}(\mathbf{x}) = \left(\frac{\mathbf{x}-\mathbf{c_x}}{\mathbf{a}}\right)^{\mathbf{m}} + \left(\frac{\mathbf{y}-\mathbf{c_y}}{\mathbf{b}}\right)^{\mathbf{n}} - \mathbf{r}$$

- Single hole and four holes (super elliptical shape with 5 parameters each) per 8th of the housing

CONCLUSIONS & PERSPECTIVES

CONCLUSIONS

- Great interest of industrial designers in using structural optimization to weight reduction in automotive components
- Successful application of topology and shape optimization to design cycle of driveline components.
- Approach validated on several components from real automotive sector(JTEKT TORSEN and TOYOTA MOTOR)
- One major output of optimization is also to be able to find innovative and feasible solutions in complex problems
- Nice and flexible approach of level set in solving shape optimization on real life / industrial problems including 3D models.
  - Especially great interest in optimizing dead geometrical models (not necessary to have the parametric model)