STRUCTURAL & MULTIDISCIPLINARY OPTIMIZATION

HW2: OPTIMALITY CRITERIA

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Academic year 2020-2021

HOMEWORK 2
OC FOR THREE BAR TRUSS PROBLEM

Project 2: Optimality criteria and duality

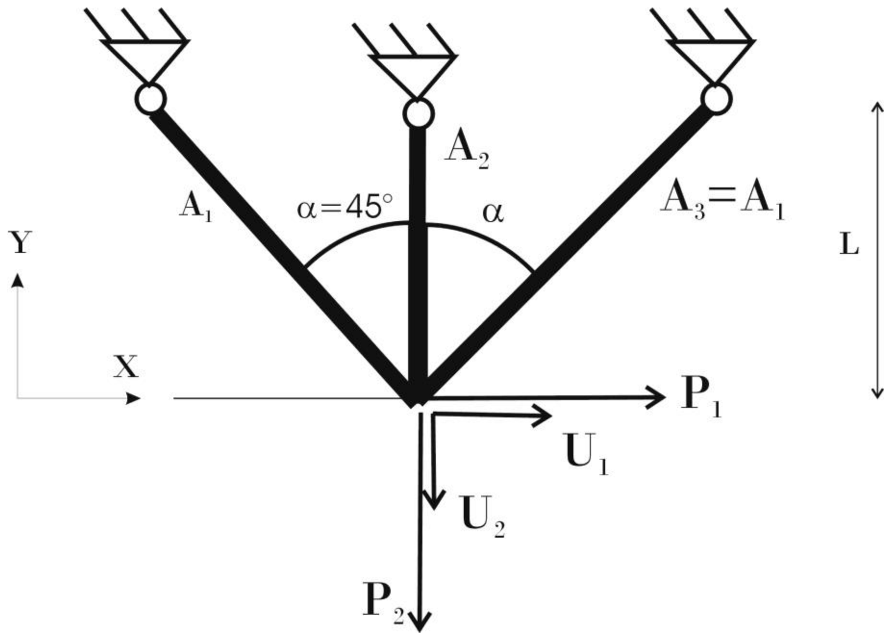
- Write your optimality criteria code to solve truss problems

- Goal: solve efficiently your optimization problem
  - Solve analytically a truss optimization problem using OC
  - Implement your OC in your favorite programming language and to create your software tool to solve truss mass minimization problems

- Evaluation:
  - Individual analytical and computer work
  - Report (max 20 pages)
  - Provide all computer codes
  - Deadline December 11, 2020 (12:00 AM)

THREE BAR TRUSS

- Let's revisit the famous three-bar truss



$E = 70000\ MPa \qquad \nu = 0.3$

$\rho = 2800\ kg/m^3$

$\bar{\sigma} = 500MPa \qquad \underline{\sigma} = -250MPa$

$L = 5000$ mm $\qquad \alpha = 45°$

$P_1 = 10000\ N \quad P_2 = 10000\ N$

$x_1 = A_1 = A_3 \qquad x_2 = A_2$

$\underline{x}_1 = \underline{x}_2 = 0.1\ mm^2$

$\bar{U} = 0.050\ \text{m} = 50\ \text{mm}$

- Solve optimization problem using Optimality Criteria techniques

THREE BAR TRUSS

- Question 2.1: Solve the minimum weight design subject to stress constraint

$$\min \quad W = \sum_{i=1}^{3} \rho_i l_i x_i$$

$$\textbf{s.t.}: \quad -\bar{\sigma} \leq \frac{N_i}{x_i} \leq \bar{\sigma}_i \quad i = 1,3$$

$$0 \leq x_i \quad i = 1,3$$

  - Write the FSD algorithm to solve this problem
  - Write your code to solve numerically the problem
  - Report the iteration history (see comment after)
    - Perform at least 50 iterations
  - Analyze the results. Check with your analytical solution.

THREE BAR TRUSS

- Question 2.1: Solve the minimum weight design subject to stress constraint

  - Iteration history: for each iteration provide a table with the following information

it.	x_1	x_2	W	σ_1	σ_2	σ_3	N_1	N_2	N_3	$x_1^\star$	$x_2^\star$
1	50.	50.									

  - Draw the plot of the values of the cross-section values along the iteration numbers
  - Draw the plot of the stress values along the iteration numbers.
  - Draw the trajectory of the different iteration points (A_1, A_2) in the design space (reuse the MATLAB function provided in Homework #1)

THREE BAR TRUSS

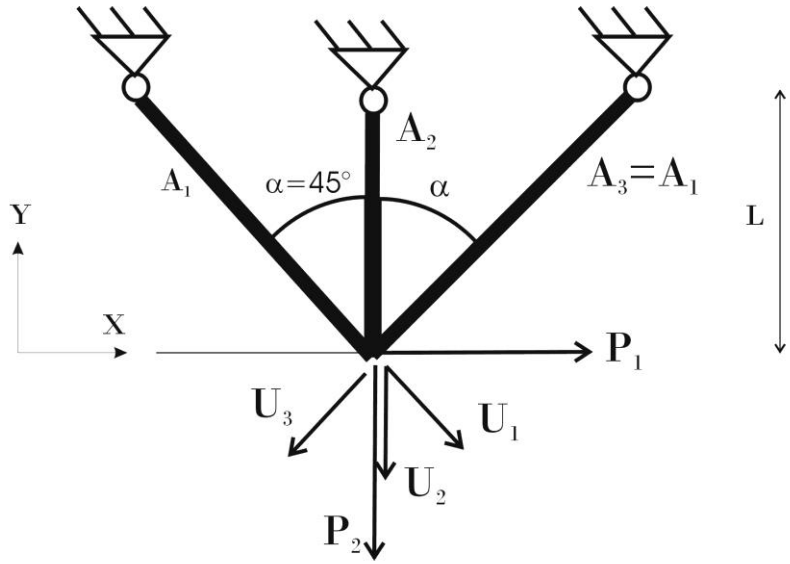
- Question 2.2: Solve the minimum weight design subject to a single displacement constraint

$$\min \quad W = \sum_{i=1}^{3} \rho_i l_i x_i$$

$$\textbf{s.t.}: \quad U_i \leq \bar{U}_i \quad i = 1,2$$

$$\underline{x}_i \leq x_i \quad i = 1,2$$

$$\bar{U} = 0.050\,\text{m} = 50\,\text{mm}$$



- The considered displacement constraints restrict the <u>displacements along the bar directions</u> to 50 mm.

THREE BAR TRUSS

- Question 2.2: Solve the minimum weight design subject to a single displacement constraint

$$
\begin{aligned}
\min \quad & W = \sum_{i=1}^{3} \rho_i l_i x_i \\
\textbf{s.t.}: \quad & U_i \leq \bar{U}_i \quad i = 1,2 \\
& \underline{x}_i \leq x_i \quad i = 1,2
\end{aligned}
$$

  - Give the analytical expression of three displacements at free node using unit load cases
  - Transform your original design problem into a single displacement problem. Justify the assumption you are doing?
  - Write the Berke's approximation of the displacement constraint (dummy load approach)
  - Write the OC for a single displacement constraint

THREE BAR TRUSS

- Question 2.2: Solve the minimum weight design subject to a single displacement constraint

$$\min \quad W = \sum_{i=1}^{3} \rho_i l_i x_i$$

$$\textbf{s.t.}: \quad U_i \le \bar{U}_i \quad i = 1, 2$$

$$\underline{x}_i \le x_i \quad i = 1, 2$$

  - Implement your OC algorithm in your favorite programming language (for instance MATLAB, Excell, Python, C…)
  - Using your code, solve numerically the problem (Perform at least 10 iterations)
  - Report the iteration history
  - Analyze the results.
    - Compare to the FSD results obtained in Question 2.1!

THREE BAR TRUSS

- Question 2.2 Solve the minimum weight design subject to a single displacement constraint
  - Iteration history: for each iteration provide a table with the following information

it.	x_1	x_2	W	U_i	λ	N_1	N_2	N_3	$x_1^\star$	$x_2^\star$
1	50.	50.								

  - Draw the plot of the values of the cross-section values along the iteration numbers
  - Draw the plot of the displacement values along the iteration numbers.
  - Draw the trajectory of the different design points (A_1, A_2) in the design space

THREE BAR TRUSS

- Question 2.3: Solve the minimum weight design subject to stress and two displacement constraints

$$\min \quad W = \sum_{i=1}^{3} \rho_i l_i x_i$$

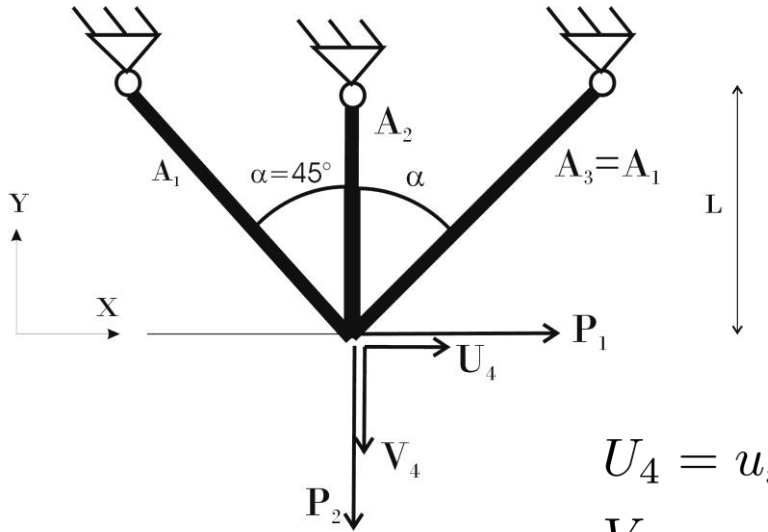
$$\textbf{s.t.}: \quad U_4 \leq \bar{U}$$

$$V_4 \leq \bar{U}$$

$$-\bar{\sigma} \leq \frac{N_i}{x_i} \leq \bar{\sigma}_i \quad i = 1, 3$$

$$\underline{x}_i \leq x_i \quad i = 1, 2$$

$$\bar{U} = 0.020\ \mathrm{m} = 20\ \mathrm{mm}$$



$$U_4 = u_x$$

$$V_4 = -u_y$$

- We now consider two displacement restrictions respectively along the horizontal and vertical directions.
- The displacement bounds are now smaller: 20 mm

THREE BAR TRUSS

- Question 2.3: Solve the minimum weight design subject to stress and two displacement constraints
  - If you have to consider simultaneously stress and multiple displacement constraints, how would you do?
  - Write the Berke's approximation of the two displacement restrictions
  - Write an OC algorithm accounting simultaneously for the stress and the two displacement constraints
  - Try to solve it numerically?
    - What are the a priori set of active and passive variables? How can you determine the Lagrange multipliers?
  - Analyze and comment your results

THREE BAR TRUSS

- Question 2.3 Solve the minimum weight design subject to stress and two displacement constraints

  - Iteration history: for each iteration provide a table with the following information

it.	x_1	x_2	W	U_i	λ	σ_1	σ_2	σ_3	N_1	N_2	N_3	$x_1^\star$	$x_2^\star$
1	50.	50.											

  - Draw the plot of the values of the cross-section values along the iteration numbers
  - Draw the plot of the displacement and stress values along the iteration numbers.
  - Draw the trajectory of the different design points (A_1,A_2) in the design space

MODALITIES

MODALITIES

- It is requested to solve the following exercises, which illustrate and are related to the lectures of Structural Optimization devoted to Optimality Criteria, Finite Elements and Structural Optimization, Dual Methods.

- The exercises will be solved individually (no group).

- Questions and discussions should be preferably formulated during Supervised Work sessions either in presence or using LIFESIZE. They should also be reported using Discussion Forum on e-Campus to share the information with your classmates.

MODALITIES

- Reports have to be in pdf format and should entitled Name_Firstname_HW2.pdf.
- Reports must be written using word processor (e.g. Word, Latex…).
- Reports are bounded to max 20 pages. Appendices (not counted in the 20 pages) include all computer codes and optimization history.
- They have to be posted by emails to Pierre Duysinx (p.duysinx@uliege.be), to Pablo Alarcon (palarcon@uliege.be), and to Denis Trillet (dtrillet@uliege.be) by December 11th, 2020 at 12:00 at the latest.