MECA0063 : Suspensions
Part 1: Design

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OUTLINE

- THE SUSPENSION : PRINCIPLES
  - Introduction
    - Functional analysis and specifications of suspension systems
  - General principles of suspension design
  - Geometrical parameters of rolling gears
  - Suspension characteristics
  - Kinematics and elasto-kinematic approaches
  - Summary of planar mechanism properties
  - Tailoring the characteristics of suspension mechanisms
  - Kinematic performance curves
  - Appendix: Quarter car model

INTRODUCTION

Suspension function

- Enable the wheels to follow the road uneven profile
  - Prevent the bending and torsion of the car body and, so minimize the resulting stresses

- Allow the wheel to keep in contact with the ground to enable the maximum force generation by the tires and to minimize the tyre force variations around their nominal values
  - Avoid any loss of contact between the tire and the ground
  - Insure a good pressure distribution in the contact patch
  - Maintain the desired geometrical configuration of the wheels: steering and camber angles…
  - → Maintain good adherence force generation
  - → Keep a good control of the trajectory

Operating conditions

- Keep the contact pressure within the contact patch within a small range around its nominal value despite the road irregularities

- Maintain the wheels in an adequate geometrical configuration
  - Variation of steering and camber angles lags lateral force perturbations
  - Variations of the track width leads to lifting efforts and axle jacking motion
  - Modifications of steering and camber angles yield perturbation of the lateral loads
  - Modification of the wheelbase alters the vehicle dynamic behaviour

Operating conditions

- Withstand and transfer the loads between the car body and the ground
- Sustain the vehicle weight (vertical forces)
- Develop reaction forces against longitudinal/lateral forces in the tire contact patches:
  - Longitudinal forces: acceleration and braking
  - Lateral forces: cornering forces
  - Traction and braking torques

Operating conditions

- Ensure a good comfort level of the passengers and of the payload

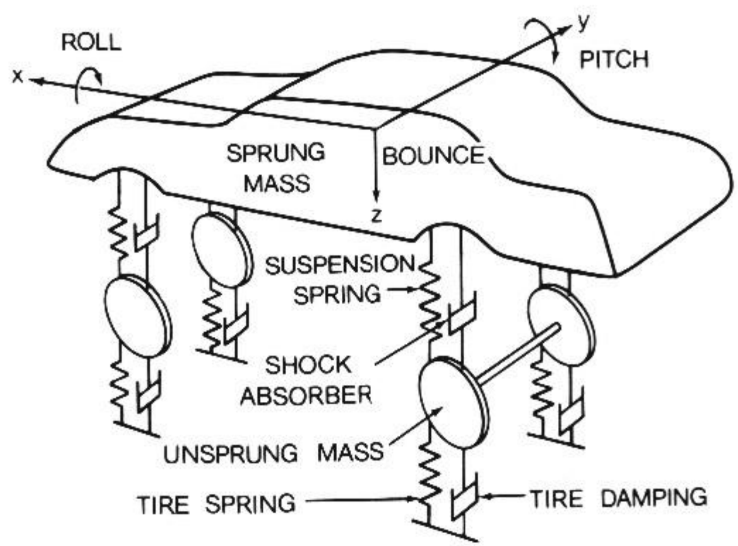
- Filter the vibrations and the shocks generated by the road roughness and surface irregularities
  - Improve the comfort of the passengers and the freight
  - Filter the vibrations and jerks from the road

- Limit and constrain the body movements in roll and pitch within some admissible ranges for the passengers, the freight without altering the road holding performance of the vehicle.
  - Resist against roll motion of the chassis while cornering
  - Resist against pitch attitude during braking (anti dive) and acceleration (anti squat) manoeuvre

Operating conditions

- Keep a constant ground clearance between the body and the road, even during weight modification
  - Aerodynamic conditions
- Manufacturing: minimum cost and tends to constructive simplicity

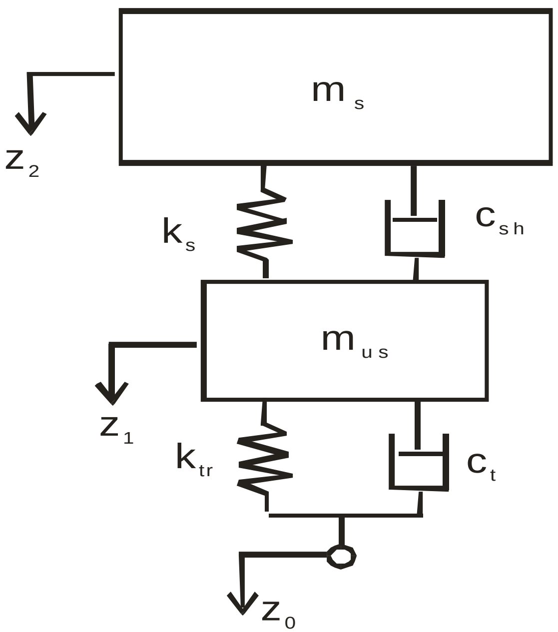
Suspended and non suspended masses



- Suspended masses (sprung mass) = all masses of solids that are located above the elastic elements
  - The body
  - The engine / motor
  - The passengers and the freight
- Non suspended masses (unsprung mass)= all masses that lie between the elastic elements and the road
  - The wheels
  - The axle
  - The brakes if they are outboard
  - The differential is attached to the axle

Conclusions from the Quarter Car Model

- The unsprung mass must be as small as possible compared to the suspended mass to reach a good comfort level (filtering), to minimize the suspension travel and to achieve a high level of road holding and adherence.
- The damping of the shock absorber must be intermediate (~0,4) with respect to critical damping
- The suspension stiffness depends of the prominent criteria
  - Soft to enhance the comfort
  - Stiff to have the best road holding



GENERAL PRINCIPLES OF SUSPENSION DESIGN

General principles of suspension design

- The suspension system includes the following components:
  - A mechanism (often called suspension) allowing for the wheel motion to follow the road roughness and irregularities while insuring a high level of control and guidance
    - The mechanism is made of rigid body elements connected by a set of kinematic joints
  - Elastic and damping elements placed between the sprung mass and the unsprung mass
    - Coil or leaf springs, elastic bumpers, compressed gas of fluids pockets
    - Hydraulic or gas dampers, passive, semi active or active systems.
    - Flexible joints (bushing)

General principles of suspension design

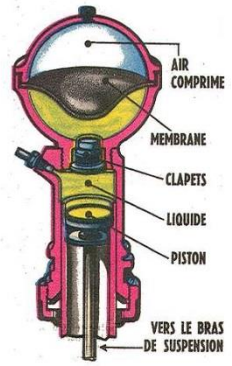
- The suspension = set of components connecting the wheel and the body, excluding the steering systems, the elastic and damping components (shock absorber, springs, etc.)

- Things are not generally so clear in practice, because optimized suspension designs are such that some components exert multiple functions. For instance some elements insure simultaneously guiding and elastic functions.

- The suspension includes guidance elements but also some transmission components (shafts) or braking components.

General principles of suspension design

- Elastic elements:
  - High strength steel elements
    - Work in bending: leaf springs
    - Work in torsion: helical coil spring, torsion bars, anti-roll bars
  - Compressible gas systems
    - Work in compression
  - Rubber elastic blocks
    - Rubber bumpers exhibiting a porous structure
  - Anti roll bars
- Stiffness characteristics
  - Linear
  - Non-linear



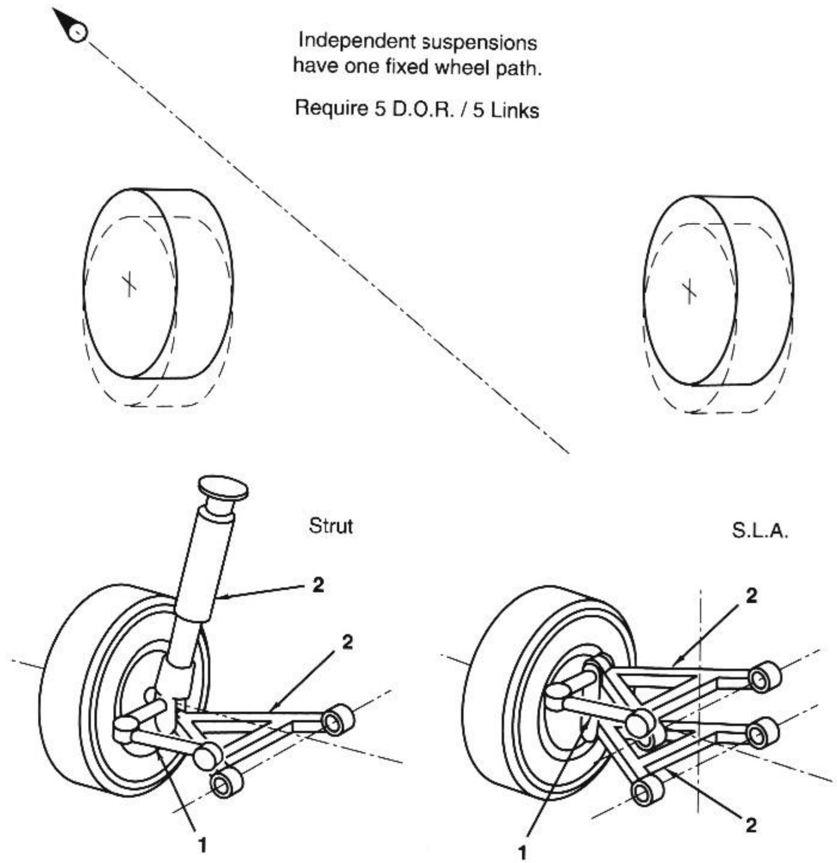
General principles of suspension design

- One should distinguish <u>differents types of suspensions</u>
  - Rigid axles
    - The two wheels are rigidly connected
    - Single shaft
    - Rigid rear axle can be equipped with leaf springs or coil springs
  - Independent suspensions
    - The motion of one wheel has no influence on the motion of the opposite one
    - Each half axle is articulated about horizontal, diagonal or transversal hinges
  - Semi-rigid axles
    - The two wheels are mounted on both sides of a shaft whose flexibility (mainly twist) enables a relative independence of the individual wheel motions

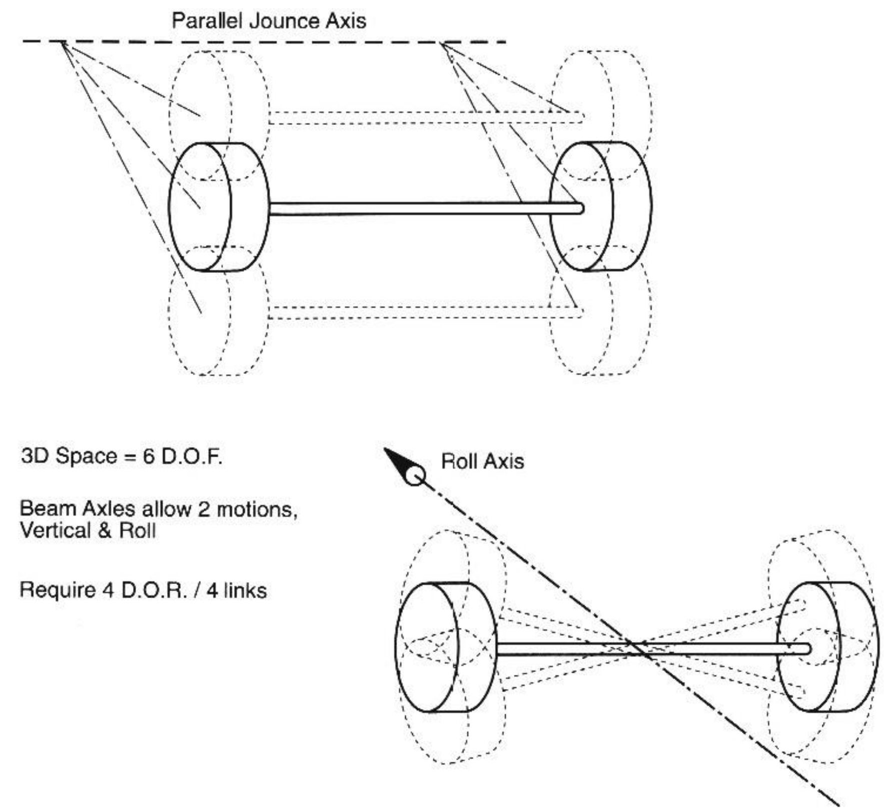
General principles of suspension design

- Mission of the suspension mechanism: to control the motion of the wheel
  - Independent suspension: allow a motion of the rotation axis of the wheel along a desired curve enabling the vertical motion of the wheel
  - Rigid axle: allow a synchronous bound and rebound motion and an anti symmetric motion of the wheels (roll motion)
- Wheel or axle = rigid body has got <u>6 dof</u>
- Conclusion : suspension introduces <u>kinematic constraints to generate a trajectory</u>
  - For independent suspension, one has to introduce 5 kinematic constraints
  - For rigid axles, the suspension blocks 4 dof using 4 kinematic constraints

General principles of suspension design



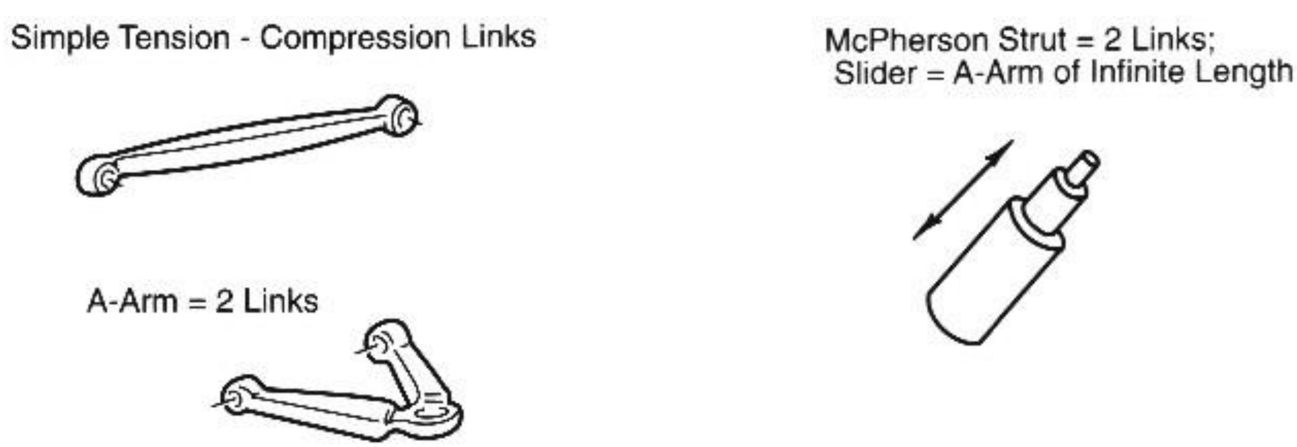
Milliken : Fig 17.3



Milliken : Fig 17.4

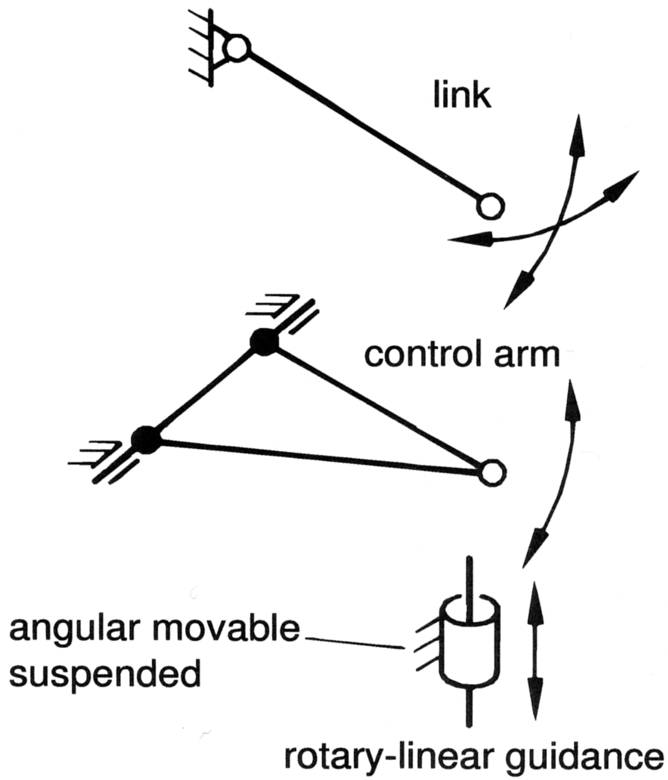
General principles of suspension design

- Linkage components in suspension mechanisms:
  - Bar: acting in traction/ compression → 1 kinematic constraint
  - A arm → 2 kinematic constraints
  - MacPherson strut → 2 kinematic constraints



Milliken Fig 17.2 : Kinematic linkage

General principles of suspension design



Essential of kinematics

- Let's define
  - n_B, the number of bodies
  - n_J, the number of joints
  - n_L, the number of loops
- It is rather easy to show that the number of loops is related to the number of bodies and to the number of joints by the following equation

$$\boxed{n_L = n_J - n_B}$$

- Indeed if the kinematic chain is open, the number of bodies is equal to the number of joints. To close some loops, it is necessary to add a new joint. The number of loops is equal to the number of additional joints with respect to the number of bodies.

The mobility index and the Grübler's formula

- Mobility index M:
  - M is defined as the minimal number of free parameters that are necessary to determine any configuration of the mechanism
- Grübler formula:
  - If
    - n_B, is the number of bodies
    - n_J, is the number of joints
    - n_L, is the number of loops
    - f_j is the number of free dof of joint j, and $c_j = 6-f_j$ is the number of fixed dof, i.e. the class of the joint
  - The <u>number of dof of the kinematic chain</u> is given by:

$$M = 6n_B - \sum_{j=1}^{n_J}(6 - f_j) \qquad M = 6(n_B - n_J) + \sum_{j=1}^{n_J} f_j$$

The mobility index and the Grübler's formula

- Grübler's formula:
  - The number of loops being given by:

$$n_L = n_B - n_J$$

  - The mobility index writes

$$M = \sum_{j=1}^{N_J} f_j - 6n_L$$

The mobility index and the Grübler's formula

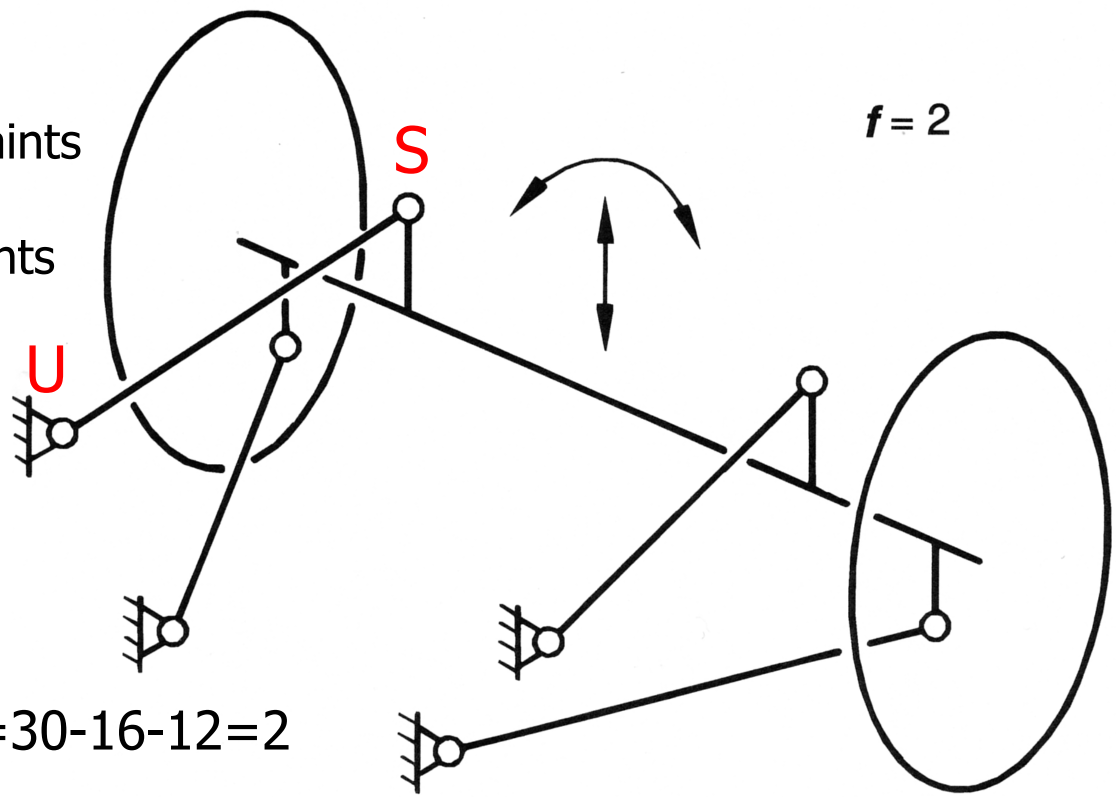
- Important remark
  - For plan or spherical mechanism, each body has only 3 dof so that the number of degrees of freedom of the mechanism is given by:

$$M = 3(n_B - n_J) + \sum_{j=1}^{N_J} f_j \qquad\qquad M = \sum_{j=1}^{N_J} f_j - 3n_L$$

  - Application of Grübbler's formula can produce wrong results in the case of complex mechanisms if the kinematic chain include redundant kinematic constraints
    - Example: when there is a 2D sub-mechanism included in a 3-D mechanism

General principles of suspension design

Spherical joint
→ 3 dof / 3 constraints
Universal joint
→ 2 dof / constraints



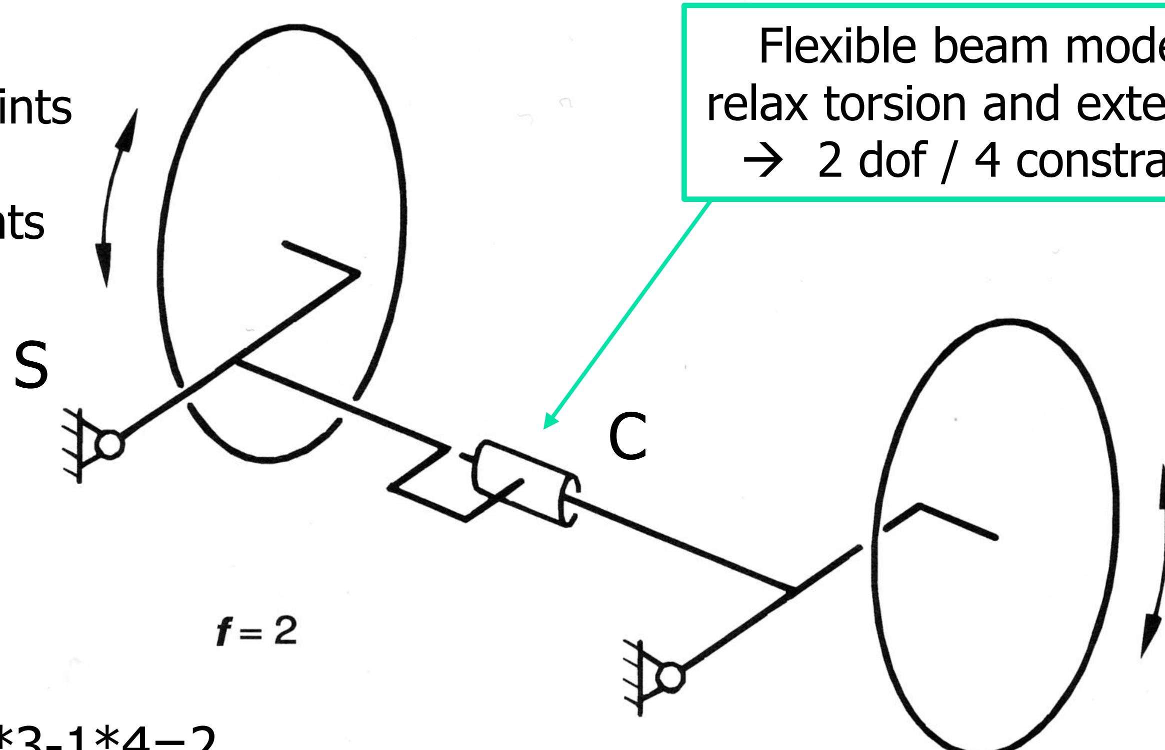
N_B=5
N_J=8
M=5*6-4*4-4*3=30-16-12=2

Rigid axle

General principles of suspension design

Spherical joint
→ 3 dof / 3 constraints
Cylindrical joint
→ 2 dof / constraints

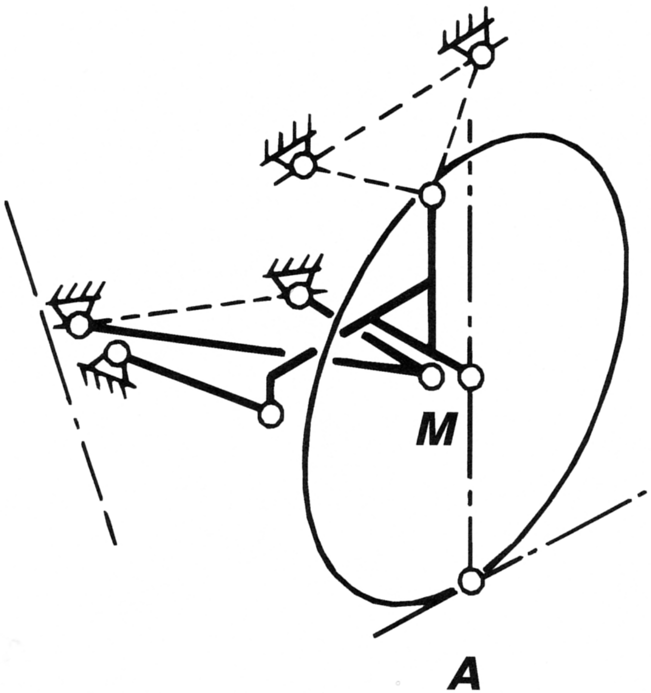
Flexible beam model:
relax torsion and extension
→ 2 dof / 4 constraints



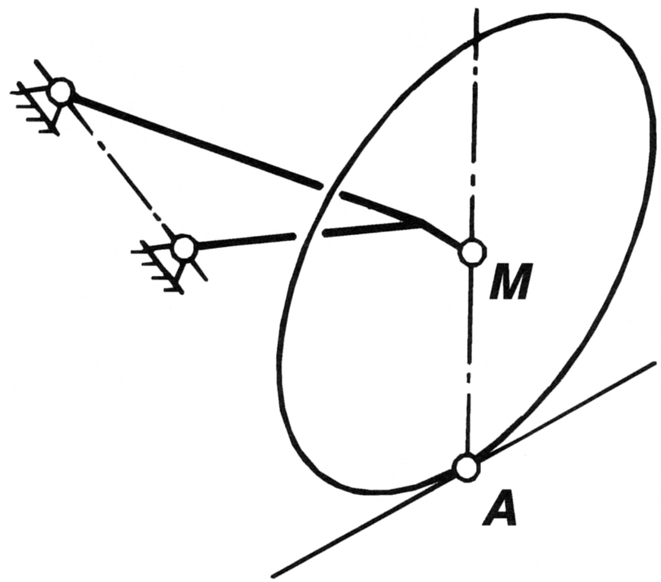
N_B=2
N_J=3
M=2*6-2*3-1*4=2

Semi-rigid suspension

General principles of suspension design



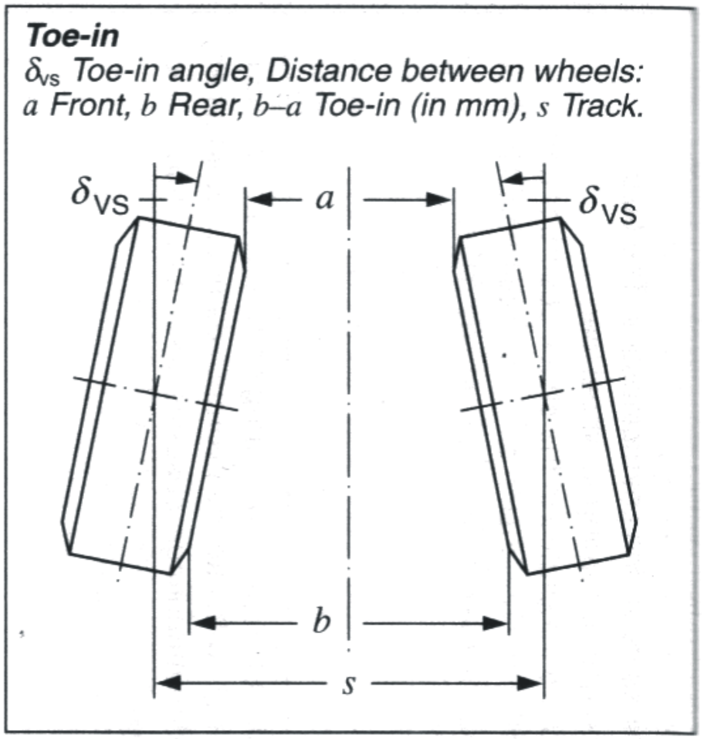
Double triangle



Trailing arm

GEOMETRICAL PARAMETERS OF ROLLING GEARS

Geometry of the rolling gear



Toe definition

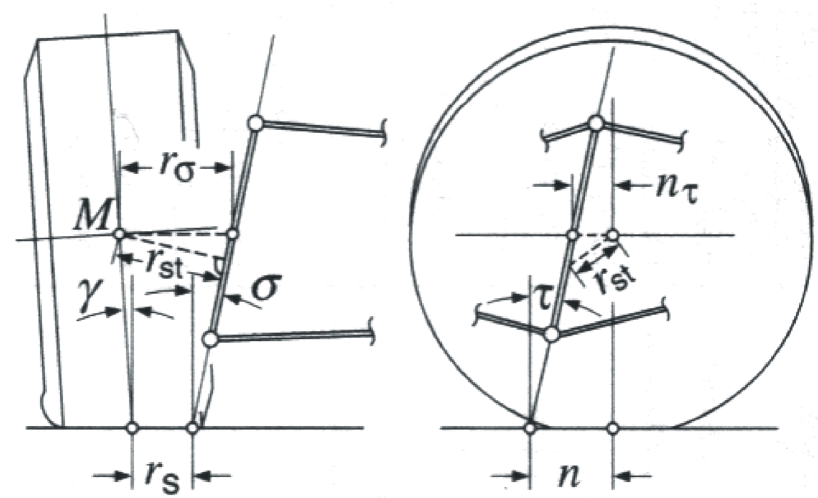
- The toe measures the misalignment of the wheels
  - If the distance between the wheels is smaller in the front direction, we are in presence of toe-in ('pinçage' in French)
  - If the distance is smaller in the rear part, we have toe-out ('ouverture' in French)
- The toe is often present in bounds and rebounds because the steering links connecting the wheel carrier and the body do not necessarily follow a kinematically compatible trajectory with the wheel travel.

Geometry of the rolling gear

- Camber angle γ :
- Kingpin inclination angle σ :
- Kingpin offset r_s :
- Mechanical trail/caster angle τ:
- Mechanical trail/caster n :

Wheel position

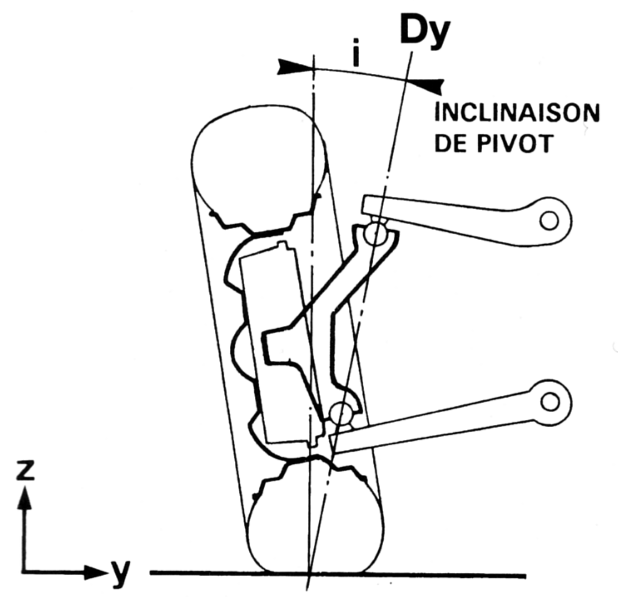
M Center of wheel, r_{st} Radius of deflection leverage, n_τ Caster offset, n Positive caster, τ Caster angle, r_σ Kingpin inclination offset, r_s Kingpin offset, γ Camber angle, σ Kingpin angle.



Geometry of the rolling gear

KINGPIN INCLINATION ANGLE

- Defines the distance between the wheel mid plane and the intersection point of kingpin axis with the road plane
- Reduces the efforts involved in the steering system
- Enables a return force in the steering wheel proportional to the vehicle weight

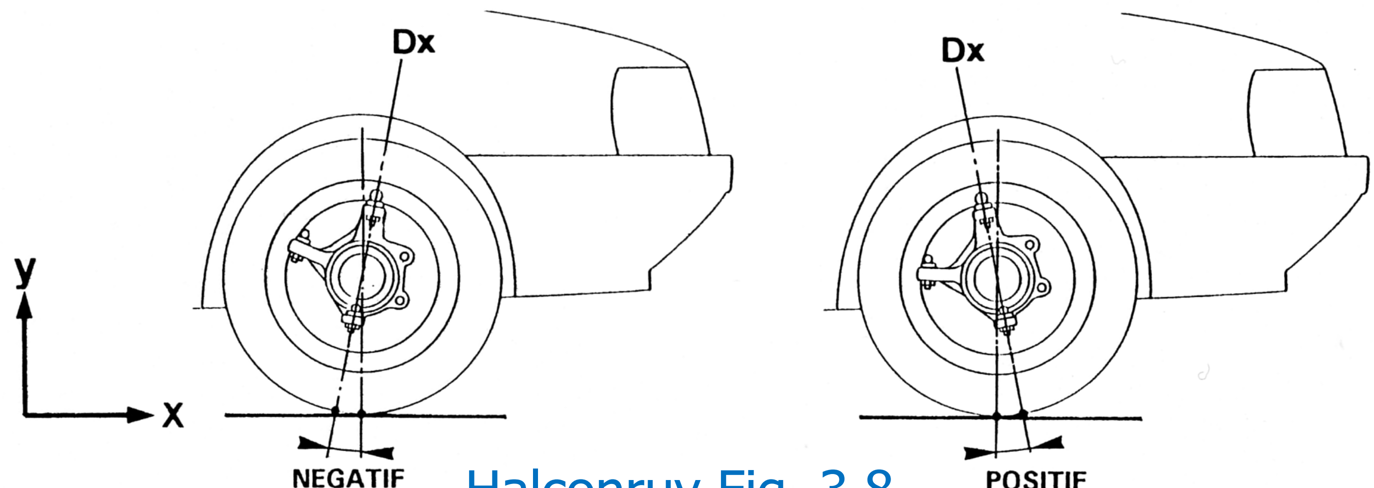


Halconruy Fig 3.7
Kingpin inclination

Geometry of the rolling gear

CASTER ANGLE

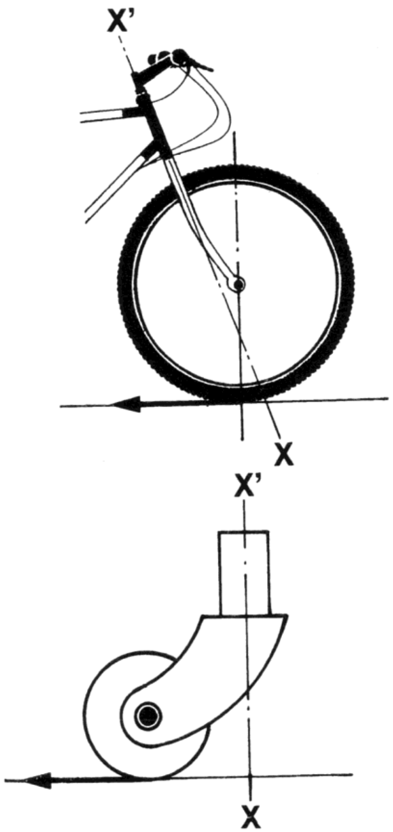
- The lateral forces generated in the contact patch act about the intersection point of the kingpin and the ground plane with a lever arm d
- A positive mechanical trail is able to amplify the return torque to the centring position and so it helps to increase the vehicle stability



Halconruy Fig. 3.8

Geometry of the rolling gear

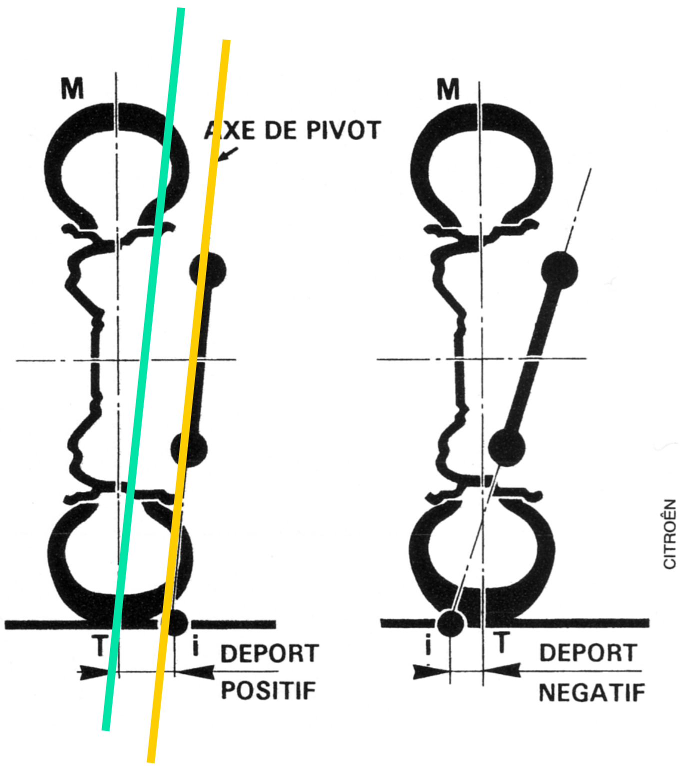
- Using a positive trail (kingpin intersection with road plane in front of the centre of contact patch) has also some drawbacks because it leads to increasing efforts in the steering system.
- The most important is the gradient of the steering torque, because it gives feedback to the driver about the neutral position (straight line).
- A positive trail contributes to this gradient perception.



Halconruy Fig. 3.11 Using a positive trail to improve the vehicle stability

Geometry of the rolling gear

- The kingpin offset is the distance between the intersection of the kingpin axis and the ground and the wheel midplane. It is counted positively if it lies inside w.r.t. the wheel.
- With a non zero offset, shocks are is transferred in the steering system and magnified by the increasing offset.
- This would pledge for a null offset. However there are other advantages to have a non zero offset.



Halconruy Fig. 3.9

Geometry of the rolling gear

- A negative offset is beneficial for braking stability. When braking on a surface with low slip conditions, the <u>negative offset</u> creates a torque about the kingpin yielding a steering rotation that <u>tends to reduce the yaw moment coming from the unequal braking forces in the two wheels</u>.

- The offset has also an impact on comfort.

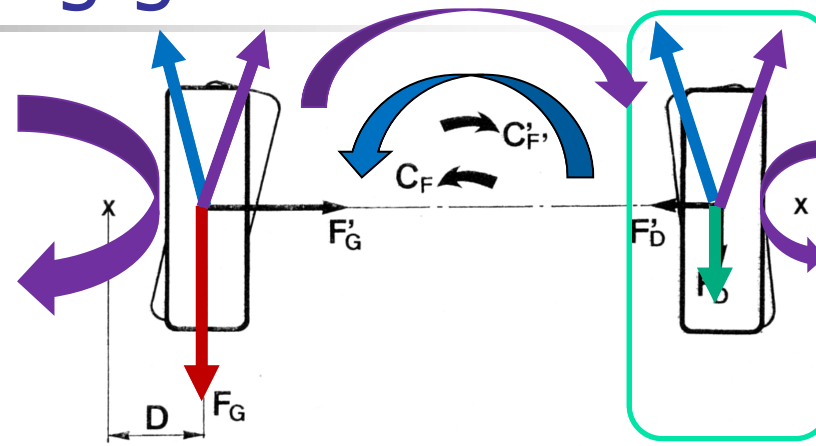


FIG. 3.12 Effet stabilisant du déport négatif.

.a figure représente une vue de dessus d'un essieu avant avec ın déport (D) négatif. L'adhérence est supposée plus importan-e côté gauche que côté droit. Les forces de freinage transmis-;ibles sont proportionnelles à l'adhérence disponible (FG et -D). La présence d'un déport négatif tend à faire tourner les oues vers la droite (F'G "l'emporte" sur F'D) ce qui crée un :ouple C'F' qui stabilise le véhicule, s'opposant au couple CF ṭénéré par la dissymétrie d'adhérence.

Halconruy Fig. 3.12

Geometry of the rolling gear

- The offset also contributes to reduce the torsion load of the tyres during low speed cornering and manoeuvres.
- Indeed the offset allows the tire to roll during steering manoeuvres.
- With a null offset, the tire. torsion can lead to :
  - A flexibility feeling in the steering system.
  - A possible scrubbing torque in the tyres of steering wheels.
  - An increased response time in dynamic manoeuvres.

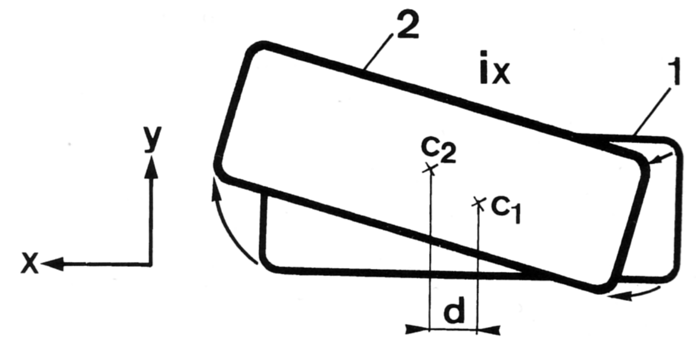


FIG. 3.13 Mouvement au sol d'un pneumatique en cas de déport nul.

L'axe de pivot est supposé vertical pour faciliter la représentation. Il coupe le plan horizontal considéré pour la figure en un point noté I. Si le déport est non nul le braquage de la roue s'accompagne d'un déplacement (d) de celle-ci vers l'avant ou l'arrière : le pneumatique roule sur lui-même lors du braquage.

1. pneumatique avant braquage
2. pneumatique après braquage
C1 : centre de la trace du pneu sur le plan de référence avant braquage
C2 : centre de la trace du pneu sur le plan de référence après braquage

Halconruy Fig. 3.13

Geometry of the rolling gear

FIG. 3.14

Influence de l'inclinaison de pivot sur l'effort de direction.

1. axe fusée (axe de roulement de la roue)

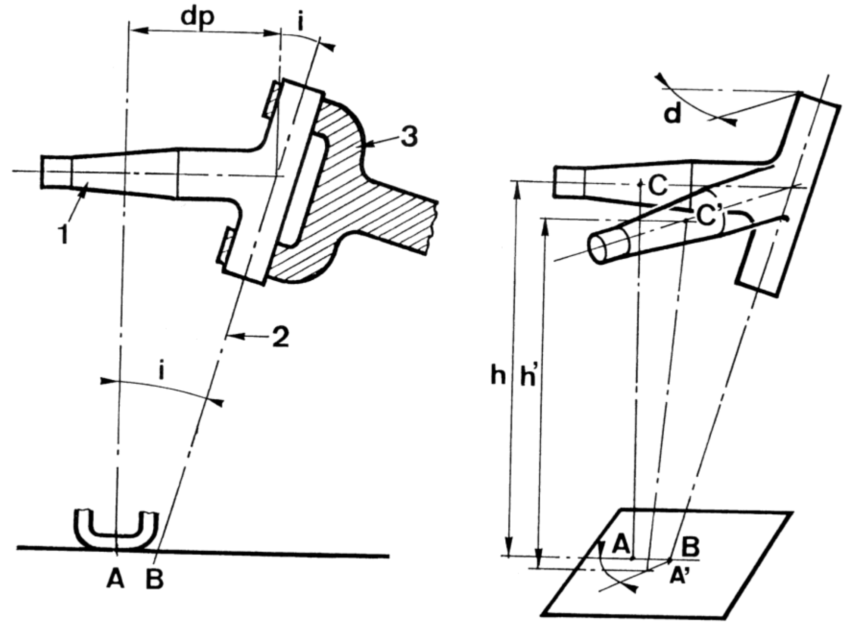
2. axe pivot

3. pivot

d = angle de braquage,

h = hauteur du centre de roue avant braquage,

h' = hauteur cinématique théorique après braquage.

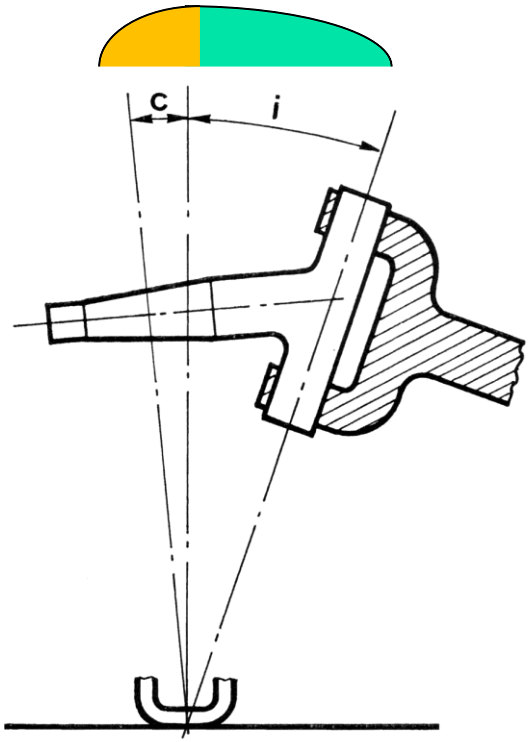


Halconruy Fig. 3.14

- The kingpin inclination also allows to create a return force proportional to the vehicle weight.
- During the wheel steering, the end of the rotation axis takes a 3D bended trajectory moving downwards or upwards. This leads to lift up/down the vehicle body. So the kingpin inclination gives rise to return force proportional to the gravity forces in static and dynamic conditions

Geometry of the rolling gear

- The kingpin axis involves also induced camber angles during the manoeuvres
- The kingpin axis angle and the camber angle are closely related and associated. They shape the included angle (=kingpin + camber).
- One can play with the choice of both angles to optimize the suspension performance.

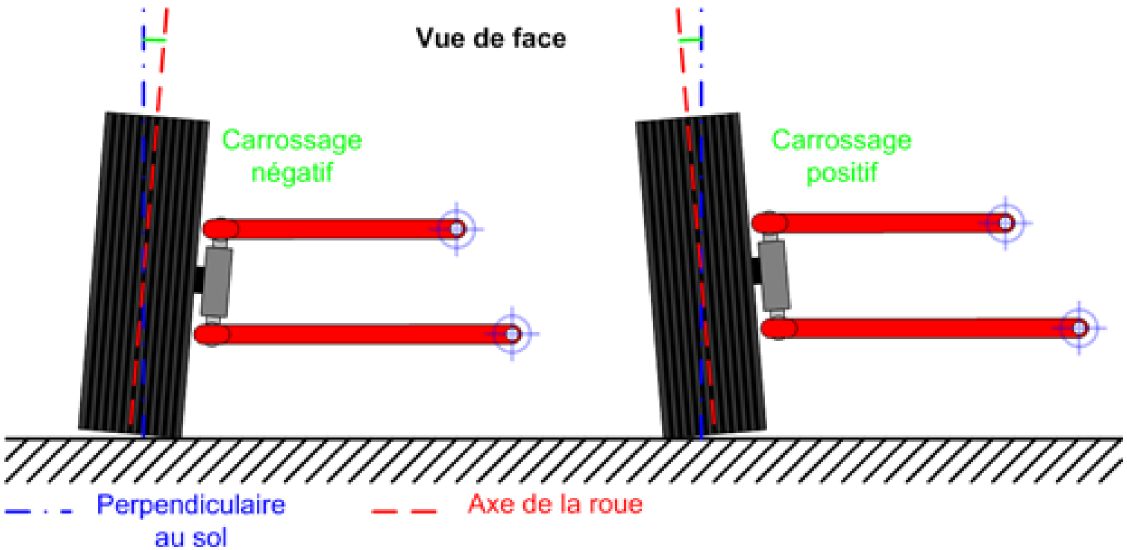


Halconruy Fig. 3.15

Geometry of the rolling gear

CAMBER ANGLE

- The camber angle is defined as the angle between the wheel mid plane axis and the vertical direction.
- Camber is positive is the tire upper part leans outside and negative if it leans inside.



Geometry of the rolling gear

- The camber is generally small (<10°).
- Camber can be positive of negative or null.
- A negative camber increases the track distance.
- A static camber is generally defined to achieve optimized tire geometry in critical cornering conditions (compensating induced camber angles due to suspension compression / rebound)
- Excessive camber angles could involve some problems:
  - A converging or diverging set of forces when driving on a conical road surface.
  - A parasitic steering on curved roads
  - Unusual wear of the tires
  - Additional rolling resistance

SUSPENSION CHARACTERISTICS

Suspension characteristics

- The suspension geometrical characteristics depend on the relative motion of the wheel and the body. This motion defines the position of the wheels with respect to the ground and thus the operating performance of the tires
- The geometry of the wheel is characterized by a series of geometrical parameters: camber, toe, half track distance, wheelbase, caster…
- Designers often resorts to the wheel path drawings to investigate and characterize the evolution of these geometrical parameters with the wheel travel.
- One distinguishes: kinematic investigations, which account only for rigid body kinematics and elasto kinematics that is able to take into account some deformations of elastic elements under the load transferred through the suspension.

Kinematic and elasto kinematic performance

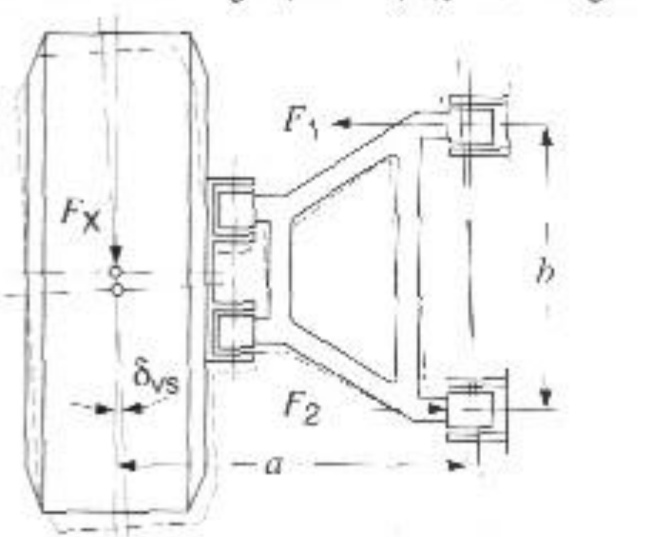
- Kinematics, or « wheel travel » in the DIN form, or suspension geometry, describes the motion of wheel induced by bounds and rebounds and steering actions. In kinematic analysis, one only considers the geometrical characteristics and dimensions of the rigid links as well as their layout in the space.

- Elasto-kinematics is defined as the alteration of the position and angles of the wheel coming from the loads and torques generated at the contact patch between the wheel and the road. Elasto-kinematics also accounts for the longitudinal motion of the wheel due to the compliance of the connection joints (bushing). In this approach, one includes also the concept of dynamics loads applied to the vehicle.

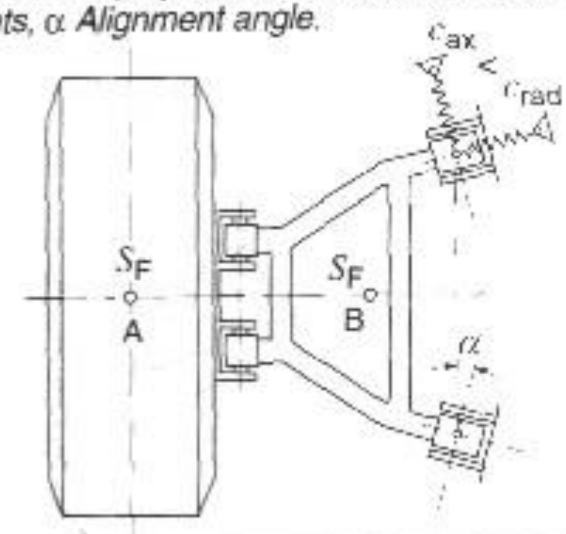
Kinematic and elasto kinematic performance

- Elasto kinematic performance

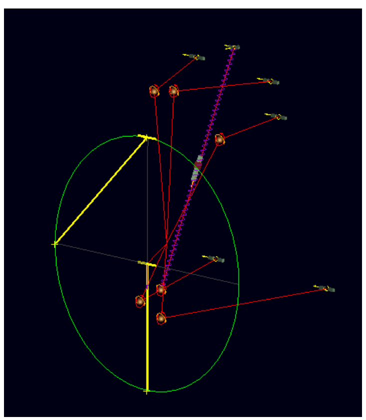
Wheel displacement by longitudinal force F_x due to system flexibility
F_1, F_2 Suspension-arm mounting forces, a Distance between wheel axes and suspension arm, b Suspension arm mounting separation, δ_{vs} Toe-in angle.



Positioning of suspension-arm mountings to counteract steering deflection due to flexibility
S_F Center of gravity (A Reduction, B Complete equalization), c_{ax}, c_{rad} Axial/radial spring constants, α Alignment angle.

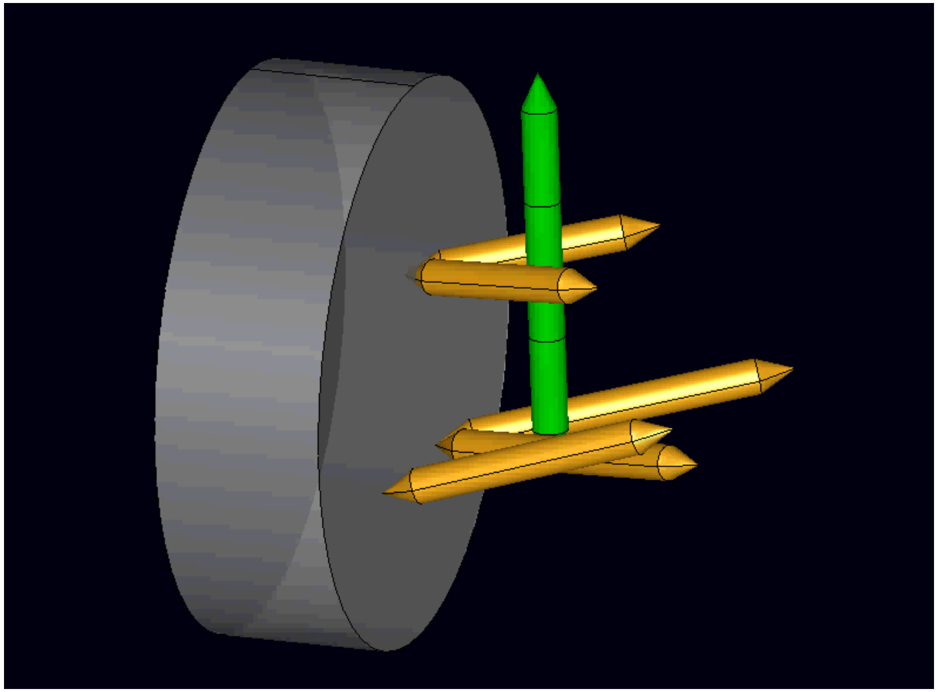


Suspension characteristics



- Modern techniques to investigate the kinematics performance of suspension systems:
  - Measurement systems able to precisely determine the variations of the geometrical parameters as a function of the wheel travel
  - Simulation tools, generally based on the Multibody Systems Dynamics.
- These tools never relieve engineers to have a deep knowledge of the fundamental working principles of suspension mechanisms
  - Basic tools based on 2D and 3D geometry
  - Understand a set of fundamental rules to generate good quality concepts to start projects
  - Predict (and anticipate) the way parameters influence the suspension performance.

Suspension characteristics



Suspension characteristics

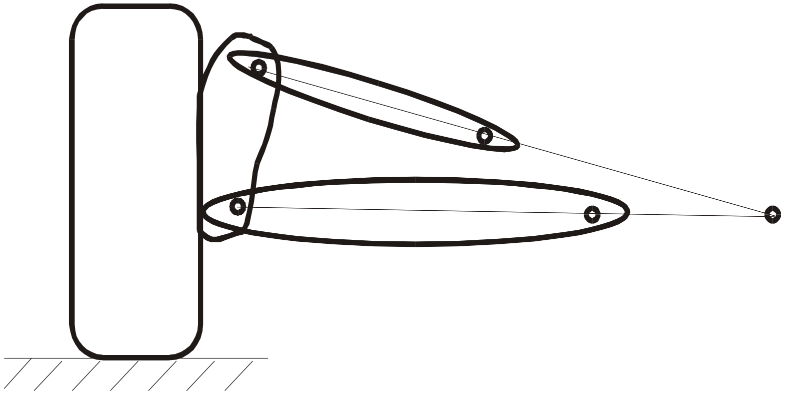
- Study of the suspension geometry is generally performed in two projection planes:
  - Front view is used to investigate mainly the lateral guidance
  - Side view reveals the characteristics of the longitudinal motion

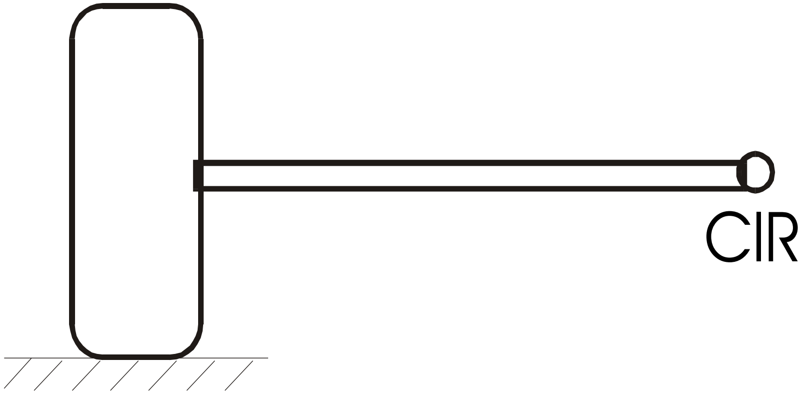
- To determine the kinematic drawings of the suspension, the approach relies on determining the key point that is the roll centre (CRo), which is the instantaneous centre of rotation (CI) of the chassis with respect to the ground (point CS)

- To determine the roll centre, it is necessary to determine first the instant centre (CIR) of rotation of the wheel with respect to the chassis (point RC)

Suspension characteristics

- The concept of instantaneous center of rotation (CIR) is very helpful to determine the kinematic parameters of a suspension mechanism
- At a given time t, the motion is equivalent to a pure rotation about a fictitious hinge located at the CIR
- One could replace the complex suspension mechanism by a rigid bar rotating about the CIR
- When the mechanism undergoes large motions, the CIR is also modified!



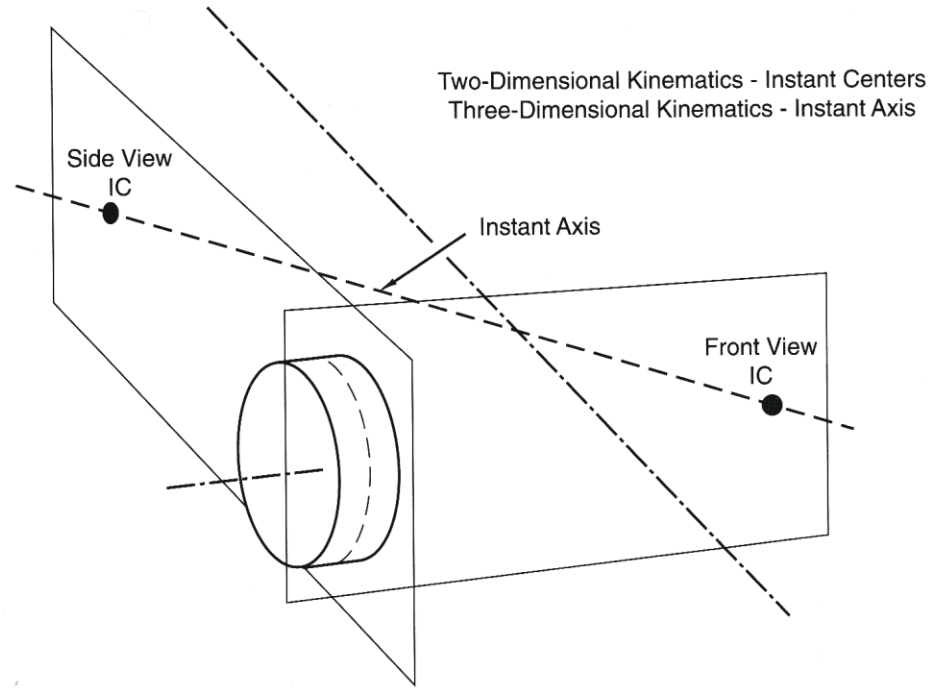


Suspension characteristics

- In 3D, the concept of Instantaneous Centre of rotation CIR is replaced by the concept of Instantaneous Rotation Axis.
- When using the three views, we determine the intersection of the instantaneous rotation axis with the considered projection planes
- Instantaneous axis is the rotation axis about which the wheel moves with respect to the car body
- Because they are instantaneous axis are modified for each configuration

Suspension characteristics

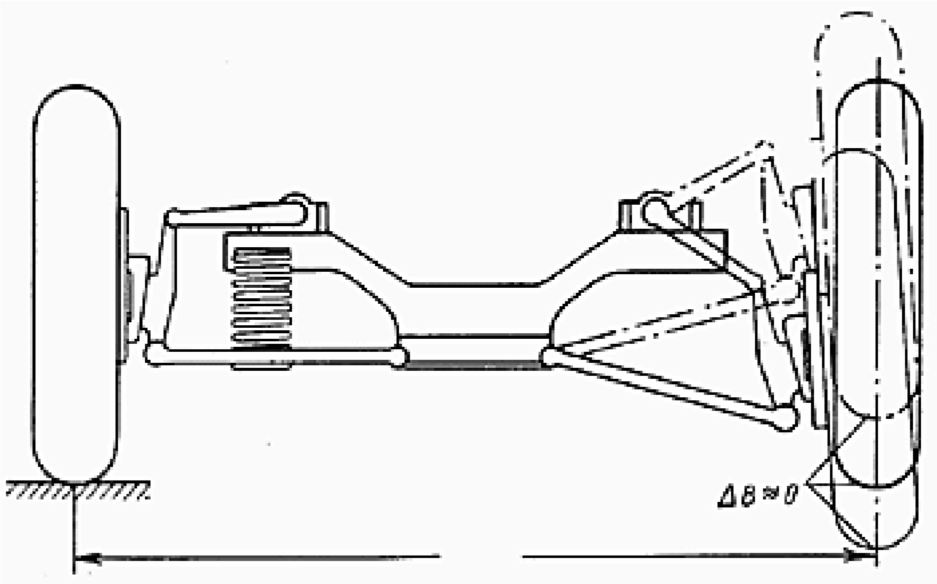
- The concept of instantaneous rotation center is basically two dimensional.
- For 3D problems, one can refer to planar problems by using projections onto particular planes including the wheel center: front view and lateral view.



Milliken Fig 17.6 : CI in the front and lateral view planes

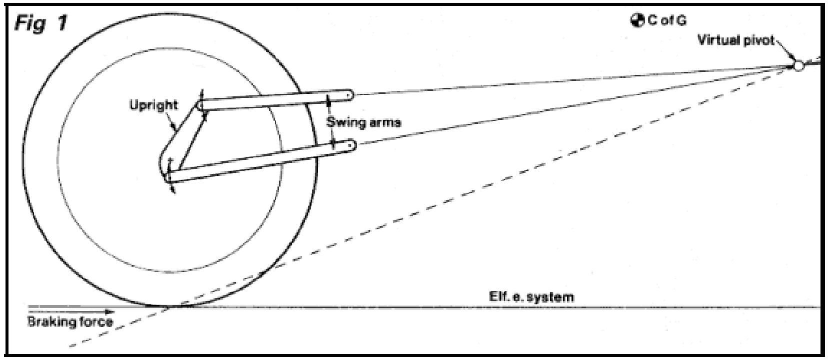
Suspension characteristics

- In the front view, the IC provides information about:
  - The rate of change of the camber angle
  - A partial information about the roll centre
  - The scrub motion of the wheels due to the modification of the half track distance
  - The rate of change of the camber and kingpin axis



Suspension characteristics

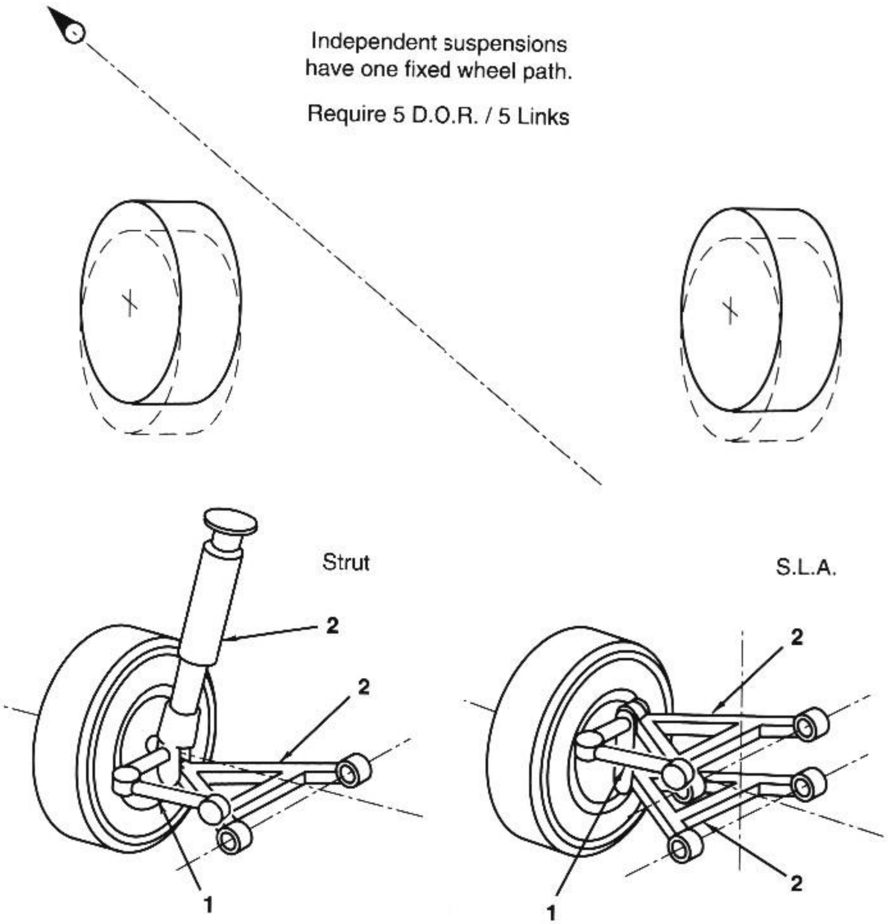
- In the side plane view, the IC defines :
  - The wheel travel in forward and backward direction, and so also the rate of change of the wheel base
  - The behaviour in pitch motion: anti squat and anti dive behaviour
  - The rate of change of the trail



- In the top view
  - Little information can be extracted since it is perpendicular to the wheel trajectory
  - The change of steering angle during wheel travel

Suspension characteristics

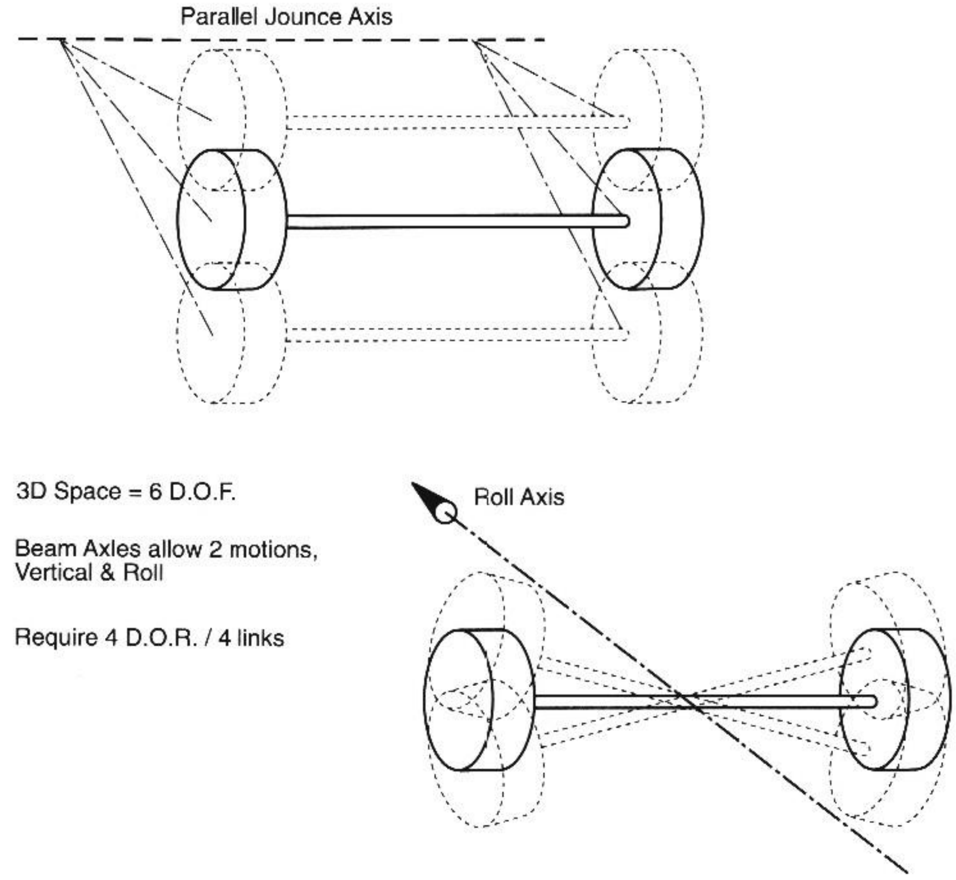
- For independent suspension, there is one IC axis



Milliken Fig 17.3

Suspension characteristics

- For rigid suspensions, we have two instantaneous axes of rotation, one for the jounce (symmetric motion) and one for the roll (anti symmetric motion)



Milliken Fig 17.4

Roll Center

- From a kinematic point of view, the roll center (RC) is the instantaneous center of rotation of the body with respect to the road

- Static definition: the roll center (RC) is the point at which the lateral forces are transferred from the axles to the sprung mass

- The roll center can also be regarded as the point on the body at which applied lateral forces produce no roll angle.

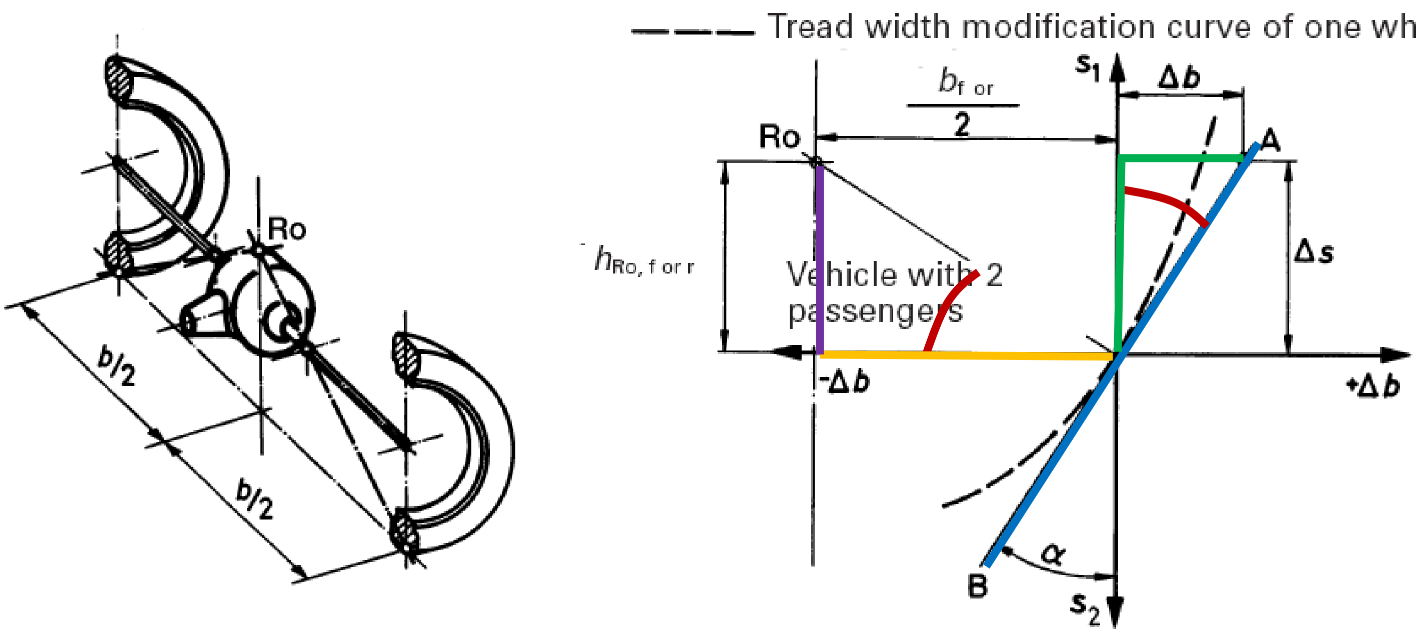
- The roll center is also the point around which the axle rolls when subjected to a pure roll moment

Roll Center

- How can we calculate / determine the roll centre?
  - By using the methods of instantaneous rotation centers and making use of the Kennedy theorem
    - *The CIR of 3 bodies undergoing relative motions are aligned*
  - By using the principle of virtual work
  - By using the curve of half track modification with the wheel travel

Roll Center

- Determining the roll center using the curve of half track modification with the wheel travel (Reimpel, p 165)



$$h_{Ro,f} = \frac{\Delta b}{\Delta s} \times \frac{b_f}{2}$$

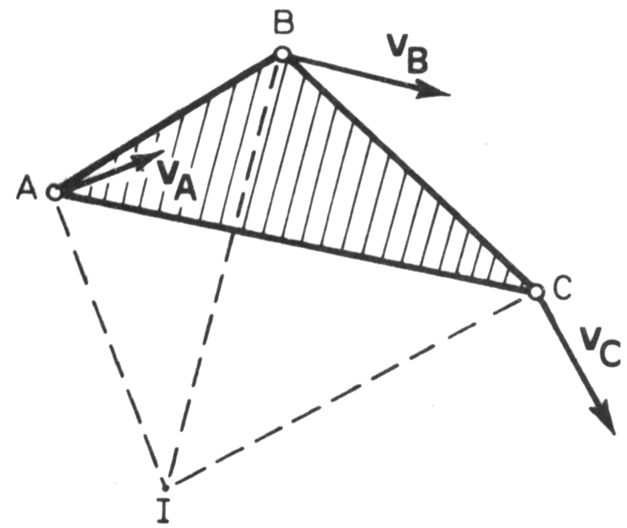
$$h_{Ro,r} = \frac{\Delta b}{\Delta s} \times \frac{b_r}{2}$$

Fig. 3.22 The height $h_{Ro,f\ or\ r}$ of the body roll centre can be determined using a tangent from the measured track alteration curve in the respective load condition.

Summary of useful properties of 2D mechanisms

Planar mechanisms

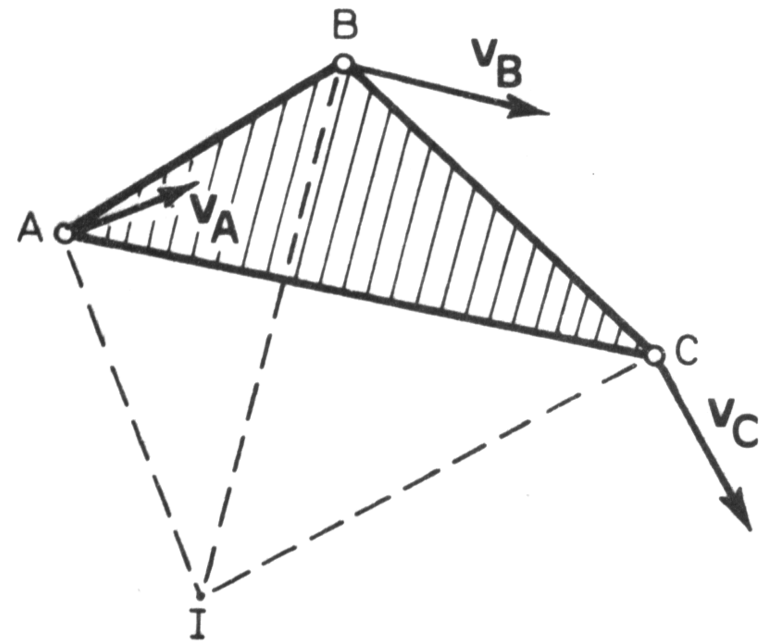
- Let’s consider a rigid body ABC undergoing a rotation motion in the plane
- The trajectories and the velocities of the A, B and C are supposed to be known.
- The velocities of point A, B and C have the following properties



Planar mechanisms

Existence of an instantaneous rotation centre (C.I.R.)

- The perpendicular lines to the velocity vector are convergent to a single point I called instantaneous centre of rotation (C.I.R.)
- Let ω the instantaneous angular velocity of the solid about point I
- We can write the velocities in point A, B, and C:



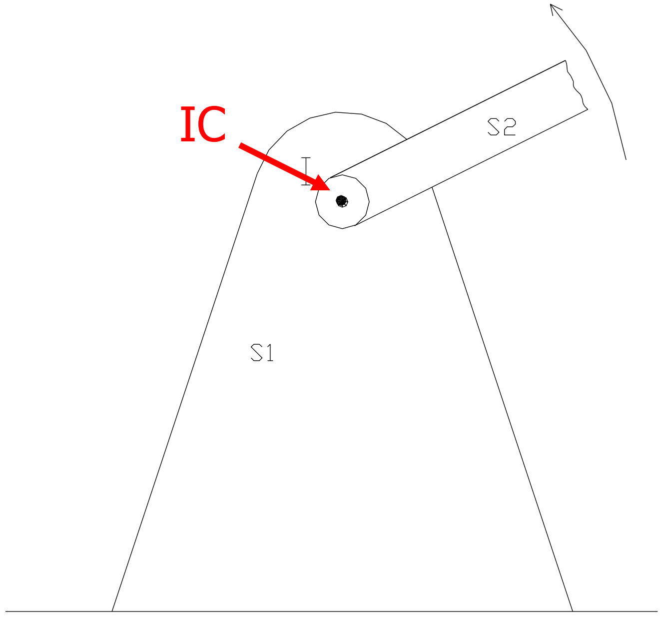
$$\vec{v}_A = \vec{\omega} \wedge \overrightarrow{IA} \qquad v_A = \omega \; IA$$

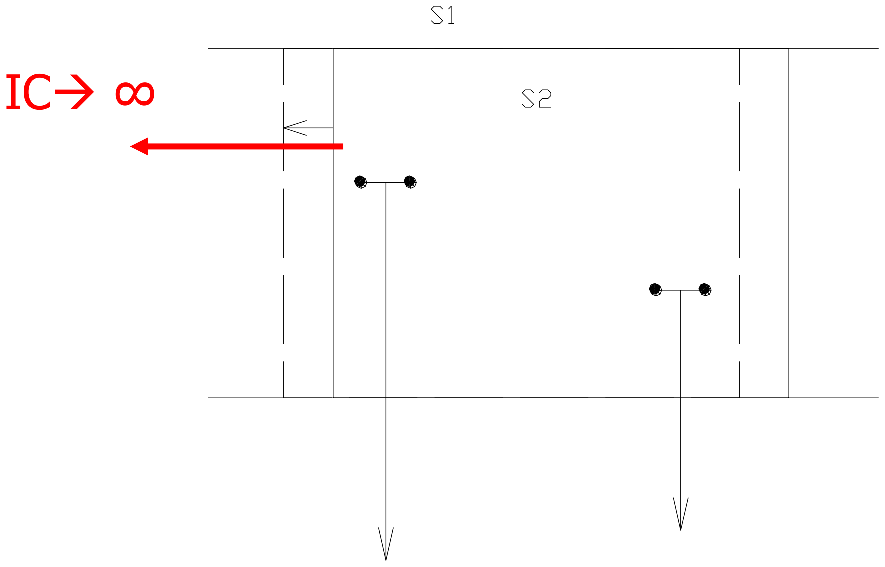
$$\vec{v}_B = \vec{\omega} \wedge \overrightarrow{IB} \qquad v_B = \omega \; IB$$

$$\vec{v}_C = \vec{\omega} \wedge \overrightarrow{IC} \qquad v_C = \omega \; IC$$

Planar mechanisms

Some basic ICs



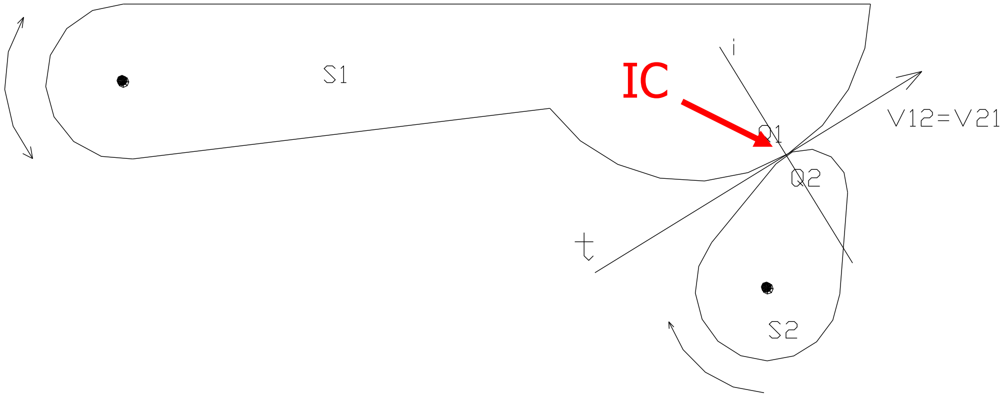


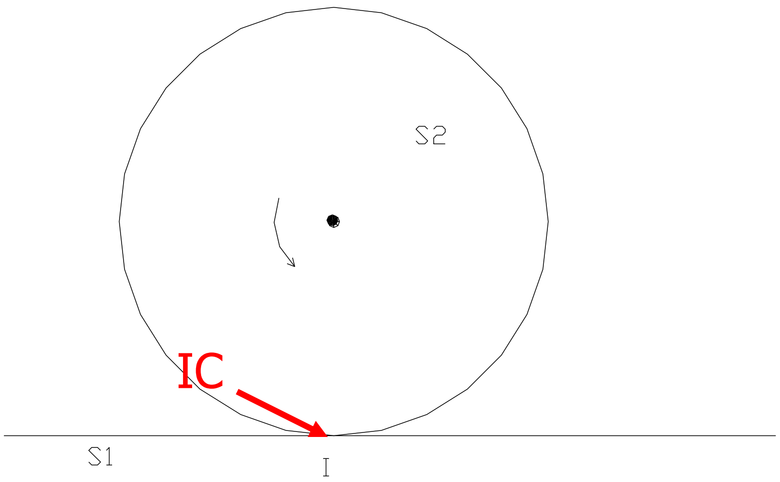
Hinge: CIR is on the joint

Prismatic joint: CIR at infinity in the perpendicular direction to the hinge sliding direction

Planar mechanisms

Some basic ICs





Cam: CIR is located on the perpendicular line to the common tangent line

Wheel with non-slip rotation: CIR is at the contact point

Planar mechanisms

Theorem of velocity composition

- The velocity vector in any point of a rigid body can be written:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

  - Velocity in A
  - Velocity in B with respect to A

- The relative velocity of B with respect to A

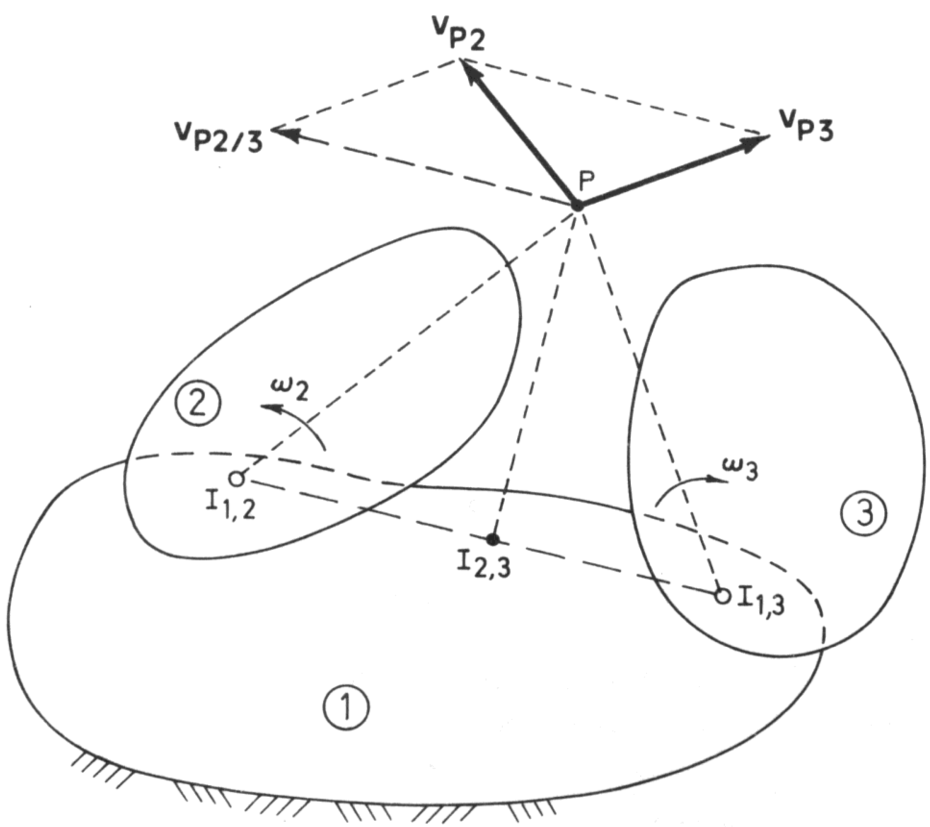
$$\begin{aligned}\vec{v}_B &= \vec{\omega} \wedge \overrightarrow{IB} \\ &= \vec{\omega} \wedge (\overrightarrow{IA} + \overrightarrow{AB}) \\ &= \vec{v}_A \ + \ \vec{\omega} \wedge \overrightarrow{AB}\end{aligned}$$

$$\vec{v}_{B/A} = \vec{\omega} \wedge \overrightarrow{AB}$$

- So

$$\boxed{\vec{v}_B = \vec{v}_A + \vec{\omega} \wedge \overrightarrow{AB}}$$

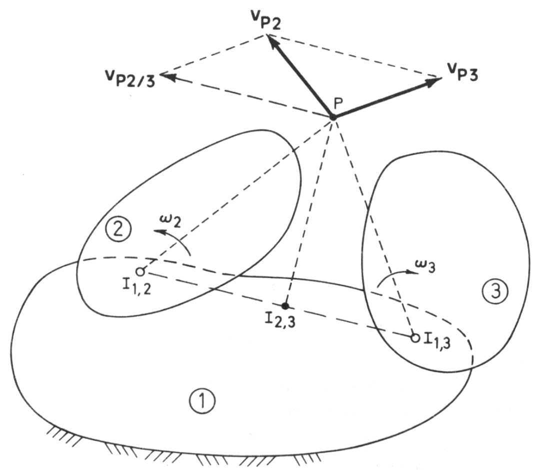
Kennedy Theorem



- Let’s consider 3 bodies undergoing relative motions
- Let’s denote by $I_{1,2}$ the CIR of the motion of body 2 with respect to body 1
- Let’s denote by $I_{2,3}$ the CIR of the motion of body 2 and 3
- Let’s denote by $I_{1,3}$ the CIR of the motion of body 1 and 3
- These three CIR exhibit the property to be inline all together as demonstrated by the Kennedy Theorem

Kennedy Theorem

- Theorem: *The IC of three bodies undergoing relative motions are in line*
- Proof



If point P belongs to body 2, its velocity is

$$\vec{v}_{P2} = \omega_2 \, \vec{e}_z \wedge \overrightarrow{I_{1,2}P}$$

If point P belongs to body 3, its velocity writes

$$\vec{v}_{P3} = \omega_3 \, \vec{e}_z \wedge \overrightarrow{I_{1,3}P}$$

If P is the instantaneous center IC of rotation of the motion of body 2 with respect to body 3, it comes

$$\vec{v}_{P2/3} = \vec{v}_{P2} - \vec{v}_{P3} = 0$$

Kennedy Theorem

- This means that both vectors have the same magnitude and the same direction
- Same direction means

$$\overrightarrow{I_{1,2}P} \;//\; \overrightarrow{I_{1,3}P}$$

so if P= $I_{2,3}$

$$\overrightarrow{I_{2,3}} = \lambda\,\overrightarrow{I_{1,2}} + (1-\lambda)\,\overrightarrow{I_{1,3}}$$

- Same magnitude

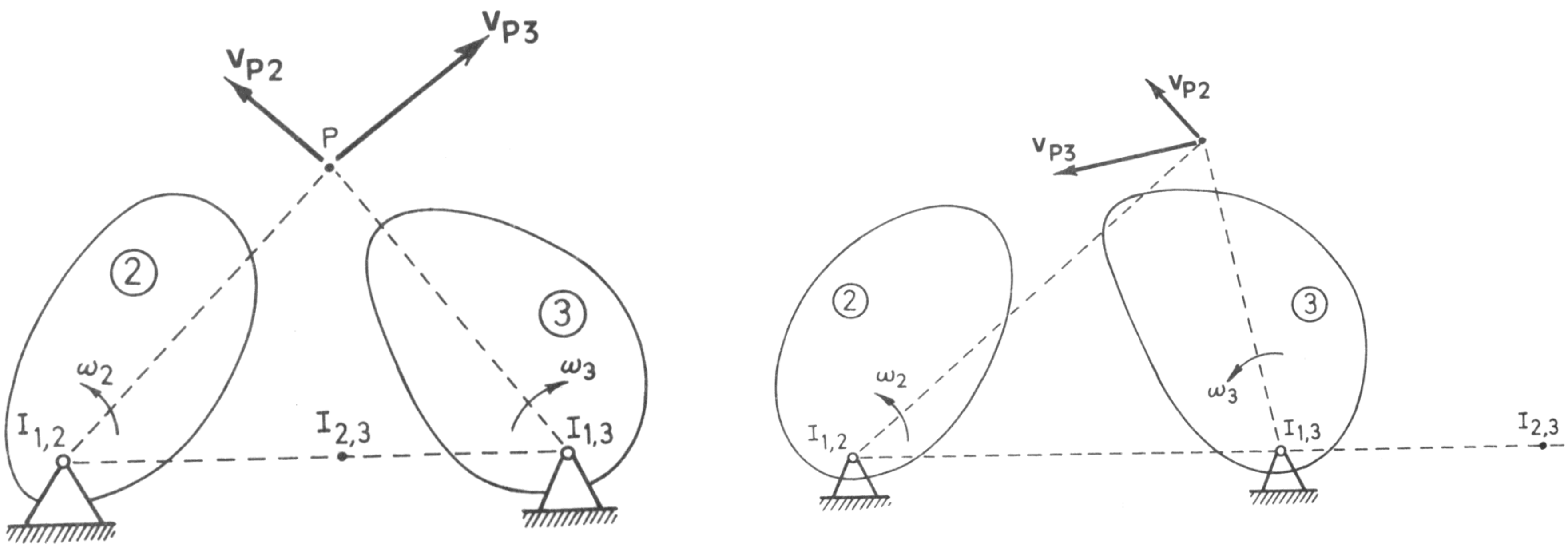
$$v_{P2} = \omega_2\, I_{1,2}P = v_{P3} = \omega_3\, I_{1,3}P$$

$$\frac{I_{1,2}P}{I_{1,3}P} = \frac{\omega_3}{\omega_2} = \frac{I_{1,2}I_{2,3}}{I_{1,3}I_{2,3}}$$

Kennedy Theorem

- If $\omega_2.\omega_3<0$ then the point I_{23} is located in between I_{12} and I_{13}.
- If $\omega_2.\omega_3>0$ then the point I_{23} is located outside the segment I_{12} and I_{13} in the side of the body having the biggest angular velocity.
- If $\omega_2=\omega_3$ then if I_{12} is different from I_{13}, the sole option is to have the point I_{23} at infinity (case of bodies in relative translation).

Kennedy Theorem



Applications of Kennedy Theorem

- The Kennedy theorem allows to determining the IC of two bodies when one knows the IC of these two bodies with respect to a third one.
- In a recursive approach, one can determine the IC of a set of n bodies.
- For 2D suspension mechanisms, the Kennedy Theorem allows determining the IC of the suspended mass (body) with respect to the ground.

Applications of Kennedy Theorem

TO DETERMINE THE CIR OF N BODIES IN RELATIVE MOTIONS

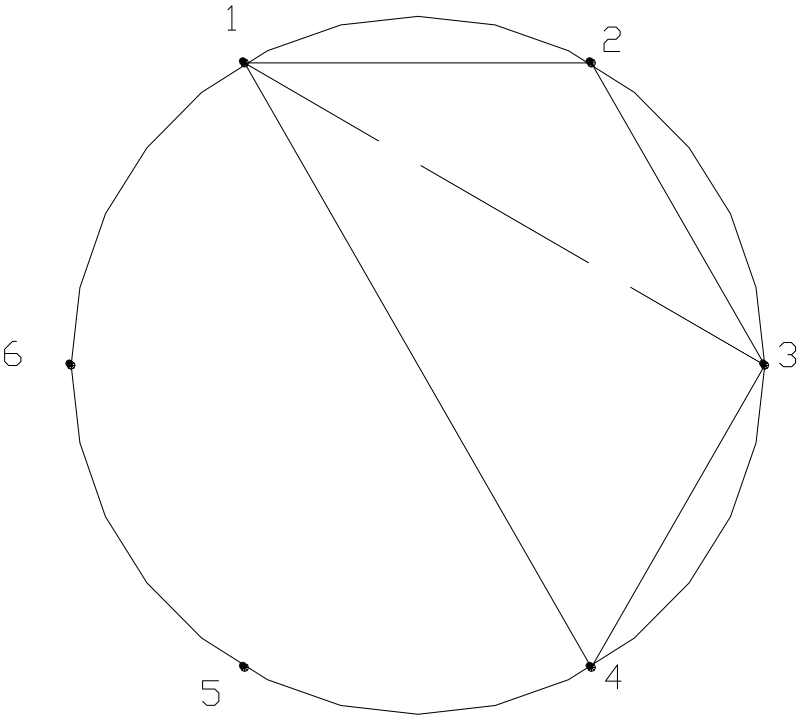
- Total numbre of CIR
  - #CIR = n*(n-1)/2
- For simple cases:
  - Determine the total number of CIR
  - List the CIR
  - Determine as many CIR as possible using simple inspection strategy (hinge, slider, wheel joints)
  - Determine the missing CIR using Kennedy Theorem

Applications of Kennedy Theorem

TO DETERMINE THE CIR OF N BODIES IN RELATIVE MOTIONS

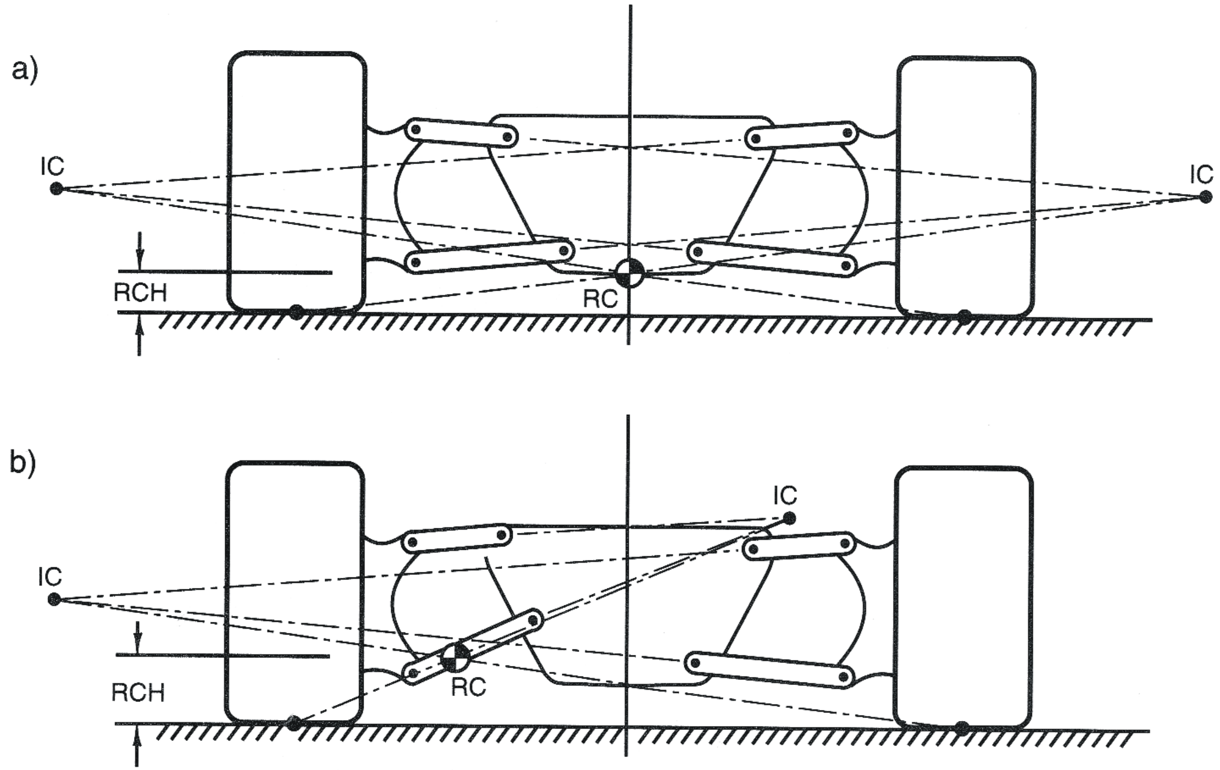
- Method for complex problems:
  - Determine the total number of CIR
  - Draw on a circle as many points as the different bodies
  - Determine a maximum number of CIR by simple inspection of the mechanisms
  - Connect the points on the circles corresponding to the known CIR
  - Determine the missing CIR using Kennedy theorem
    - Find two sets of three bodies including body under investigation
    - The CIR is located at the intersection of the lines holding the CIR
    - For instance, the CIR of 1 and 3 is located on the intersection of the line I_{12} I_{23} and I_{14} I_{34}.
  - Repeat the procedure untill all CIR are found

Applications of Kennedy Theorem



Roll Center

- Graphical determination of the roll center using the methods of instantaneous roll centers



Roll Center

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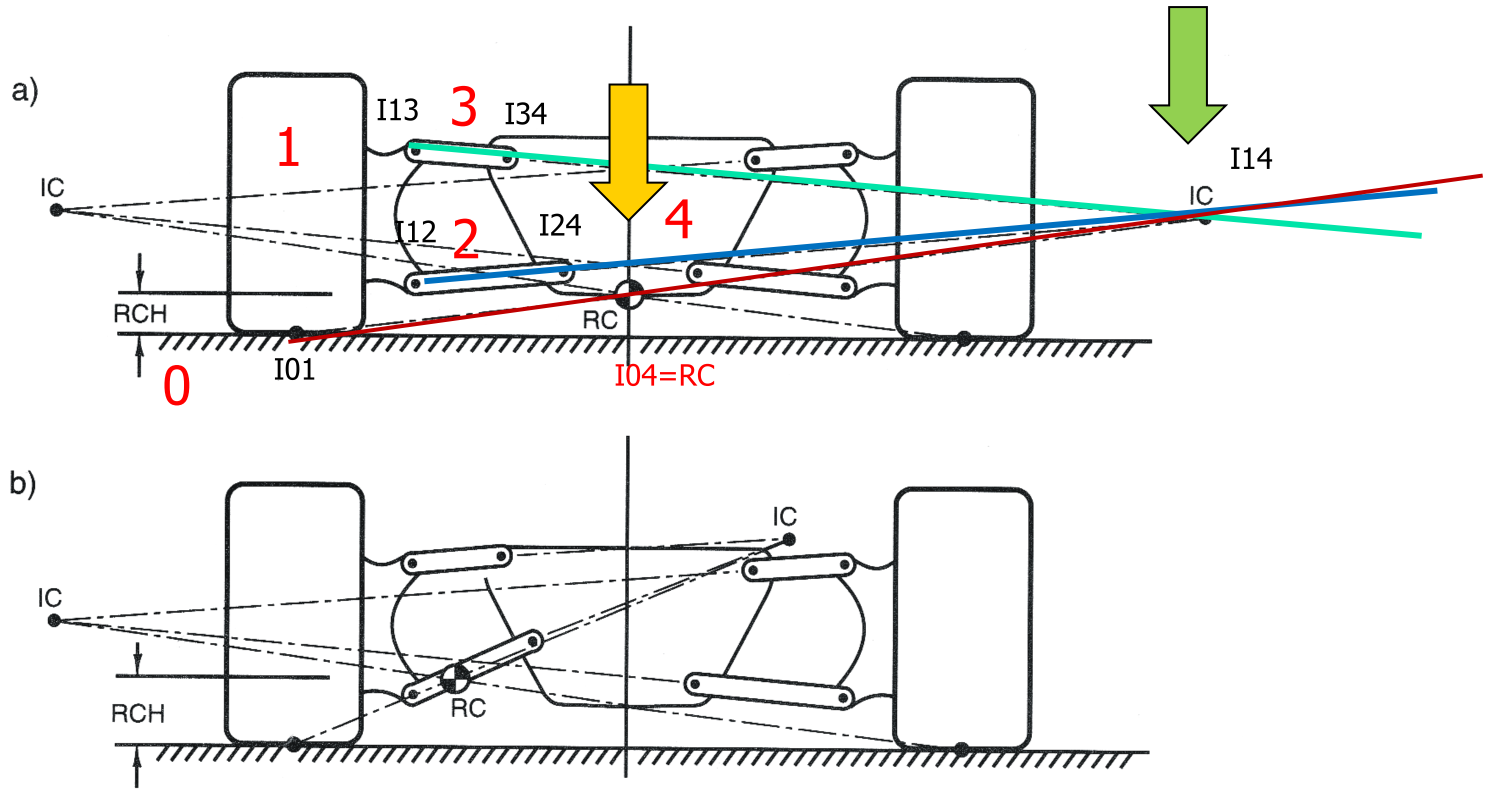
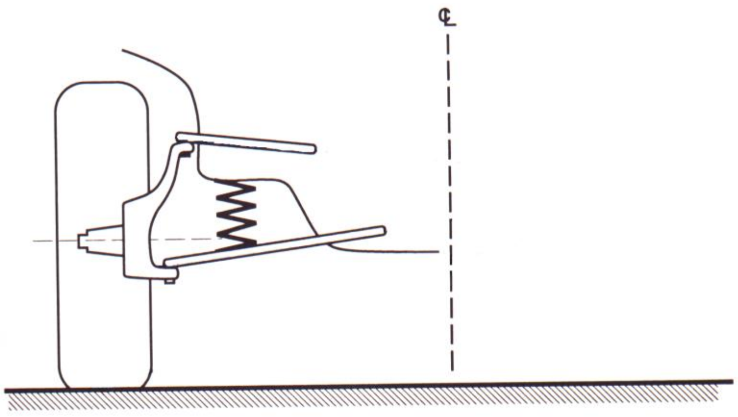
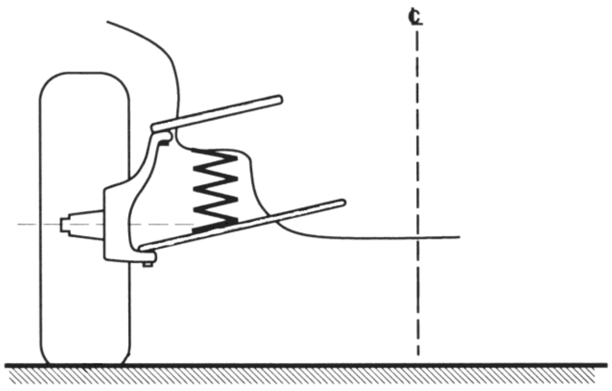


Exercices:

Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanisms





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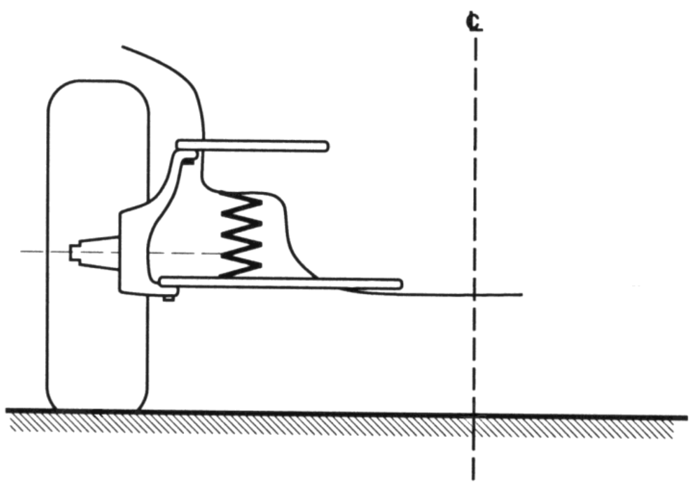

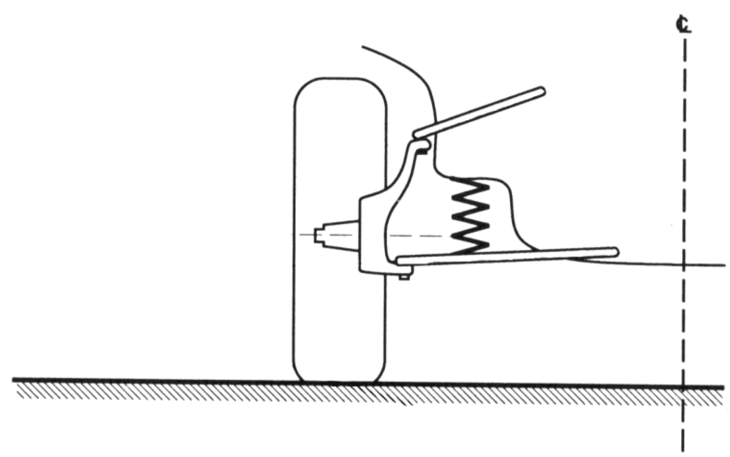

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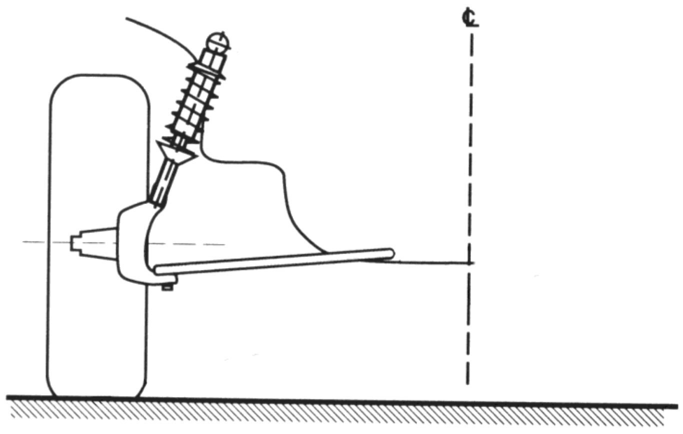
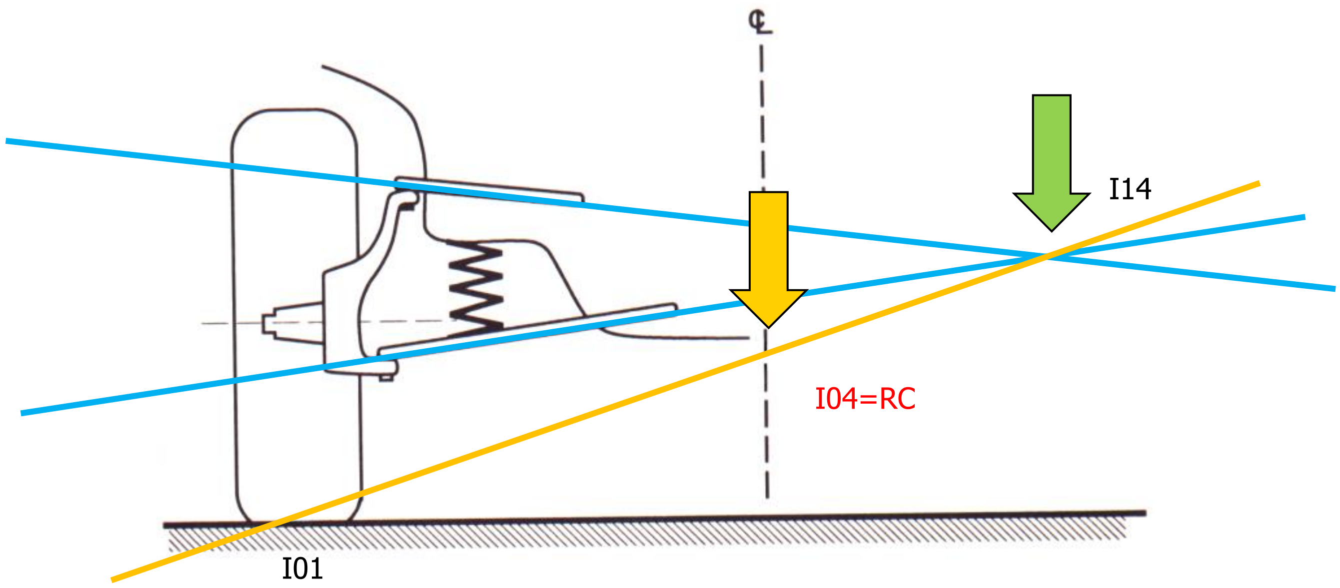
Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanism



Exercices: Solution

Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanisms



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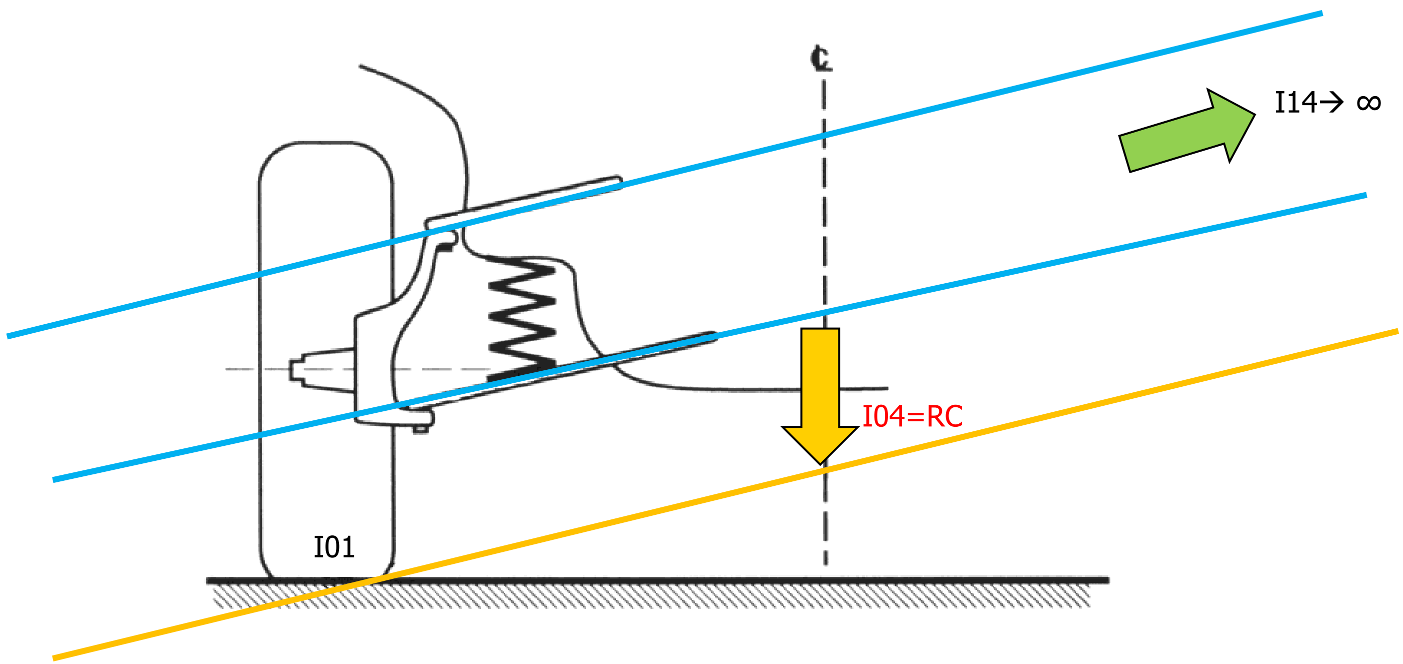
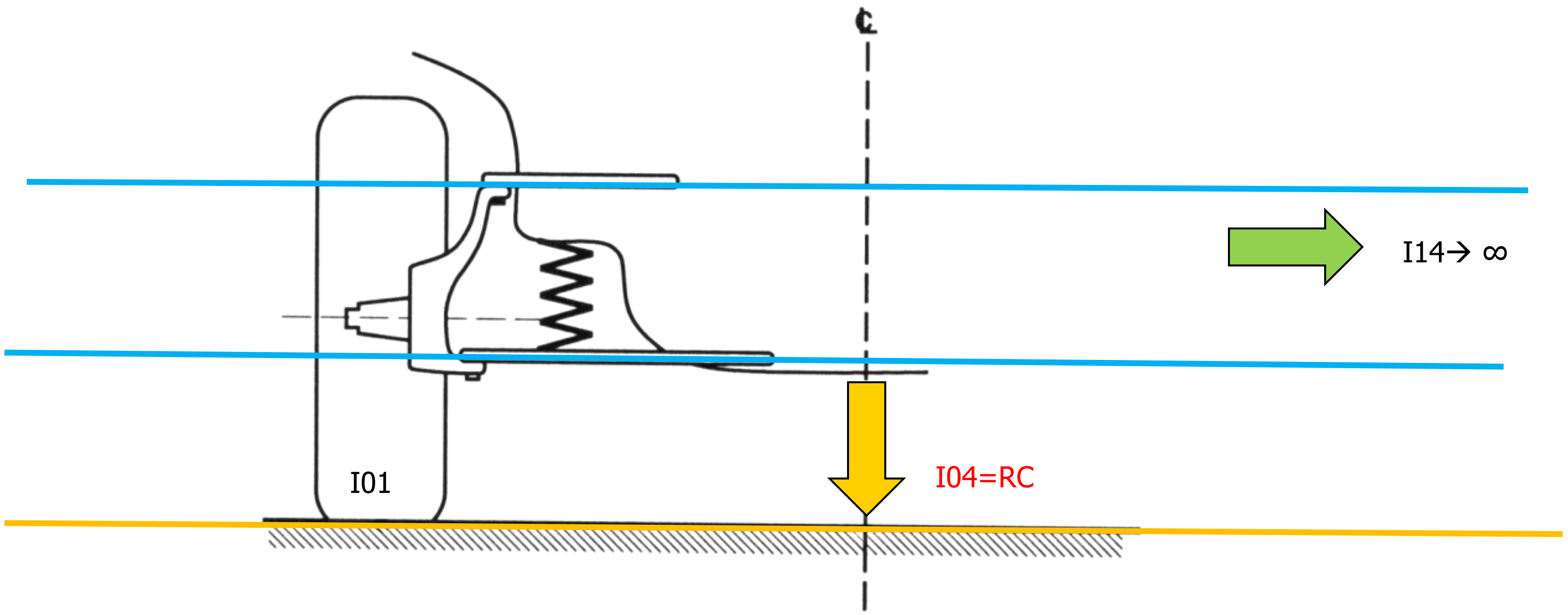
Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanisms



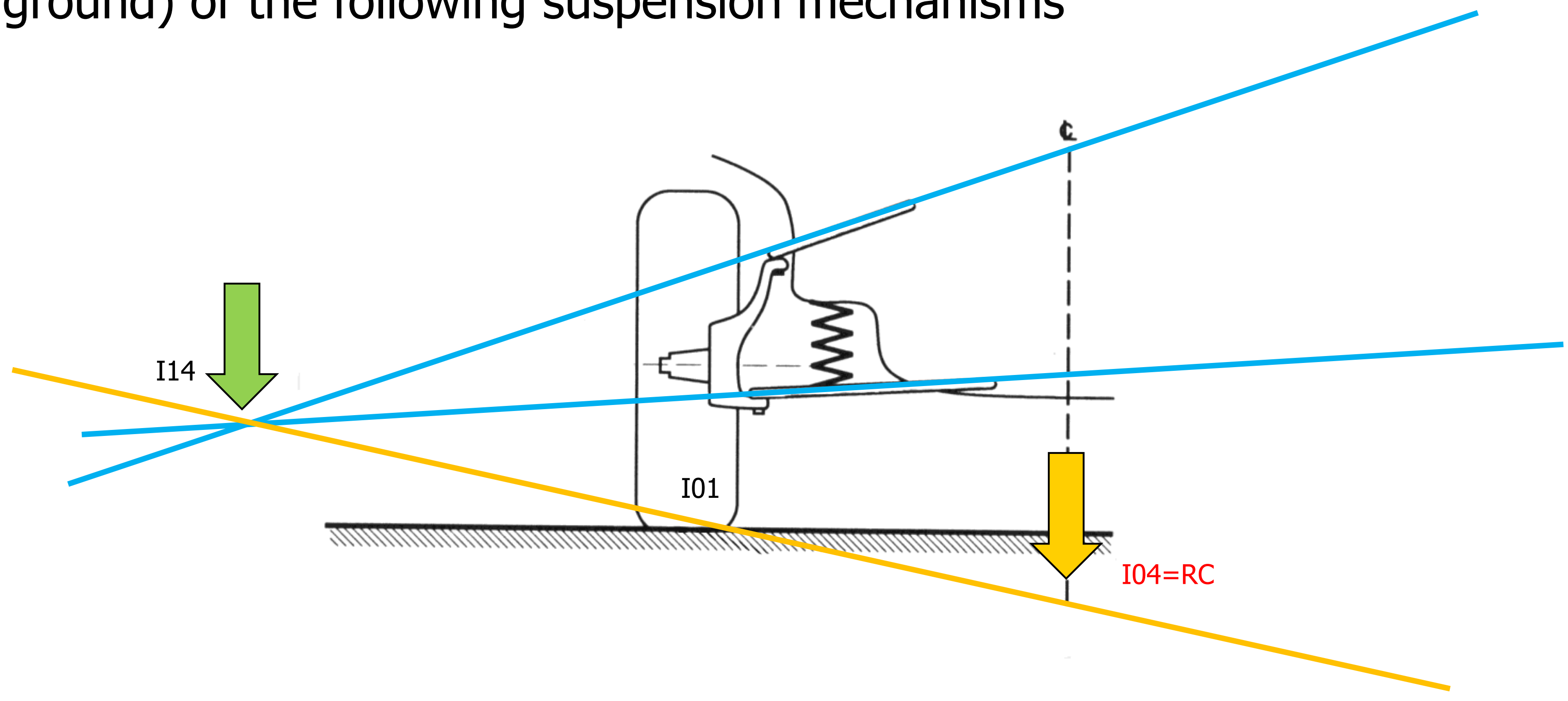
Exercices: Solution

Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanisms



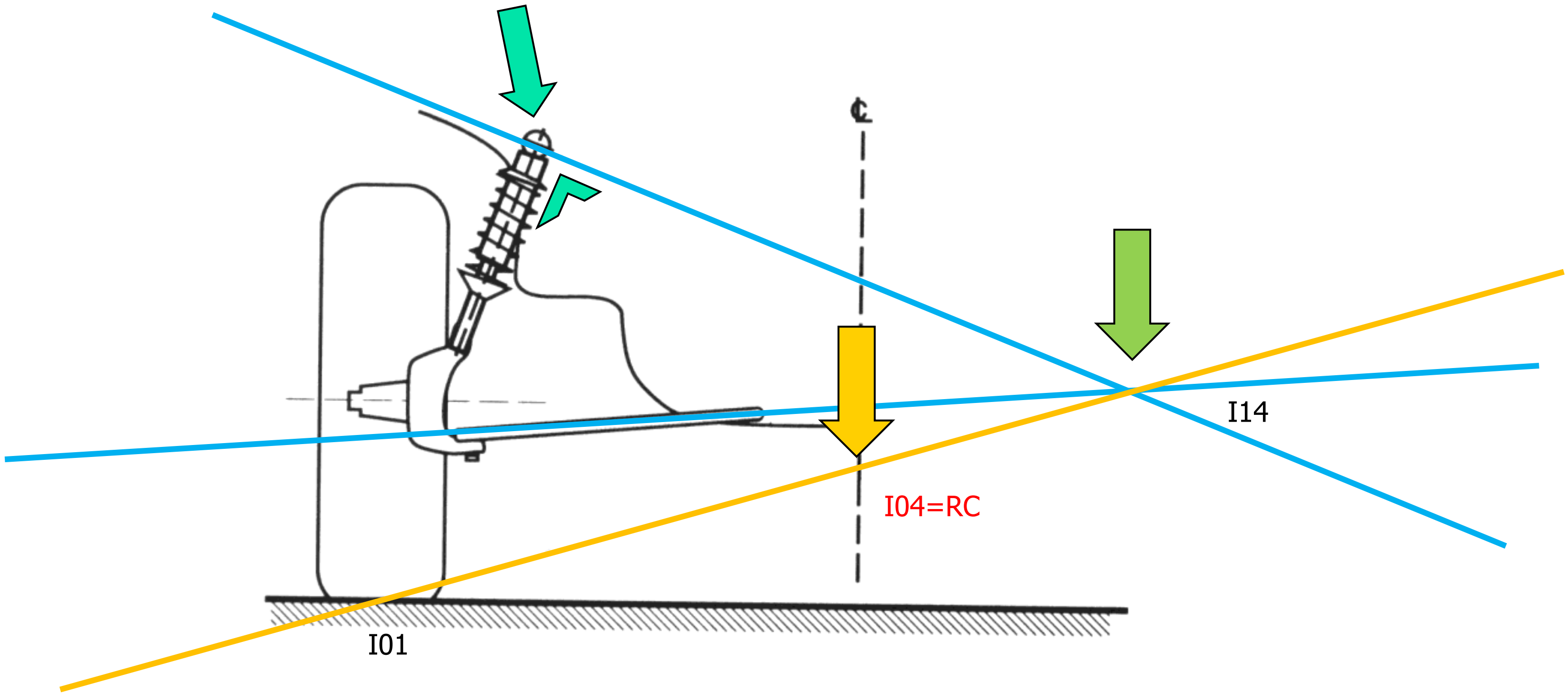
Exercices: Solution

Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanisms



Exercices: Solution

Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanisms



Exercise:

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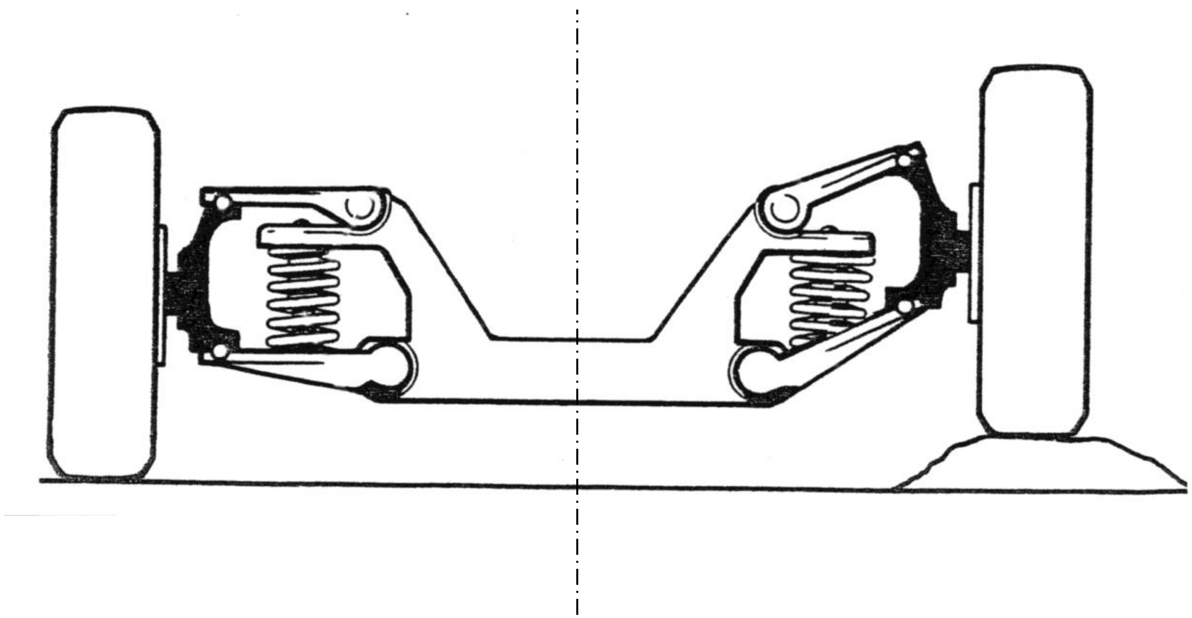
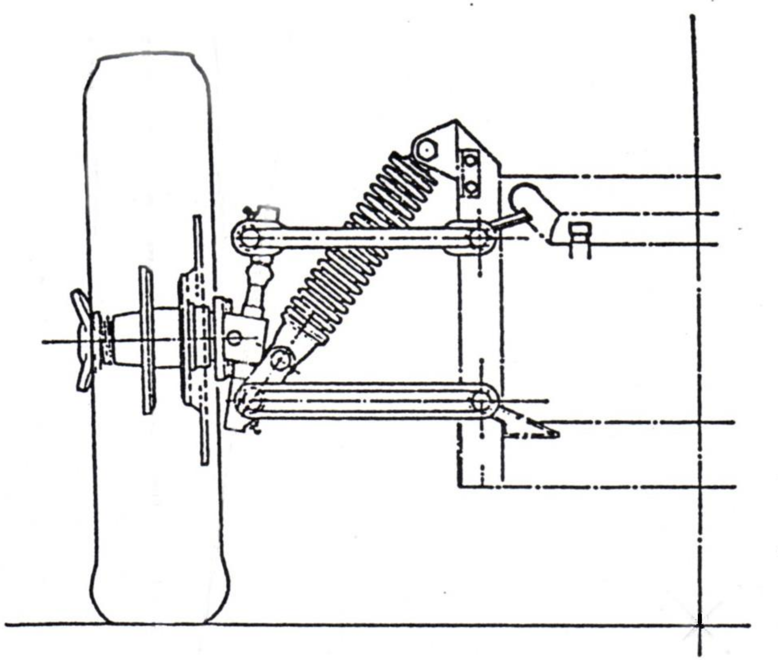


Exercise:

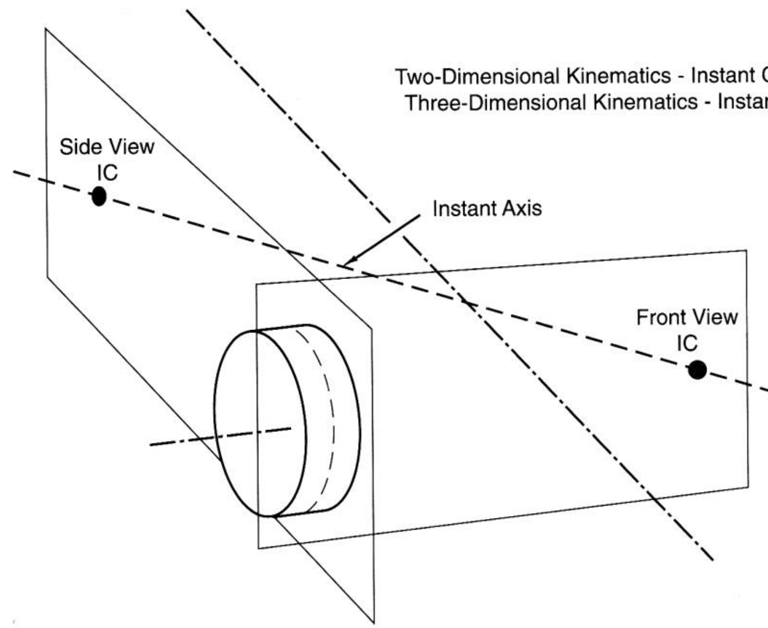
Determine the roll centers RC (i.e. IC of the body with respect to the ground) of the following suspension mechanism



TAILORING THE GEOMETRICAL CHARACTERISTICS OF SUSPENSIONS

Suspension characteristics

- In frontal view, the position of CIR is related to the control of the motion and of the forces and torques due to the lateral forces and accelerations
- In the side view, the CIR is related to the forces and longitudinal accelerations/ decelerations



Suspension characteristics

GEOMETRY OF SUSPENSION ARMS IN FRONTAL VIEW

- In front view, the length and the position of CIR of the wheels with respect to the body controls:
  - The elevation of the roll center
  - The rate of change of camber
  - The lateral scrubbing of the tire

Detailed explanation → see after

- The CIR can be
  - Inside or outside of the track (wheel)
  - Above or below the ground level
- The localization of the CIR has to be tailored to reach the expected performance

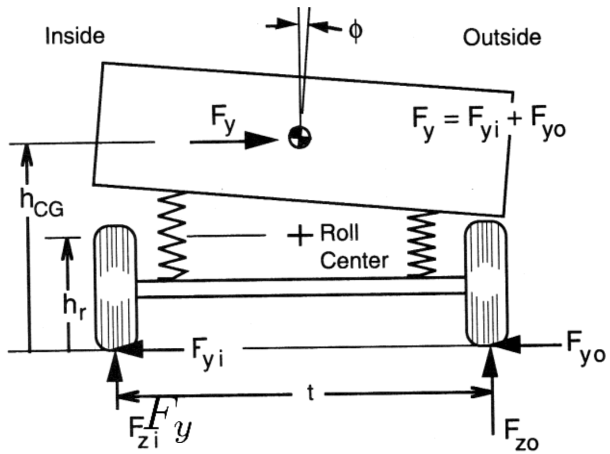
Suspension characteristics

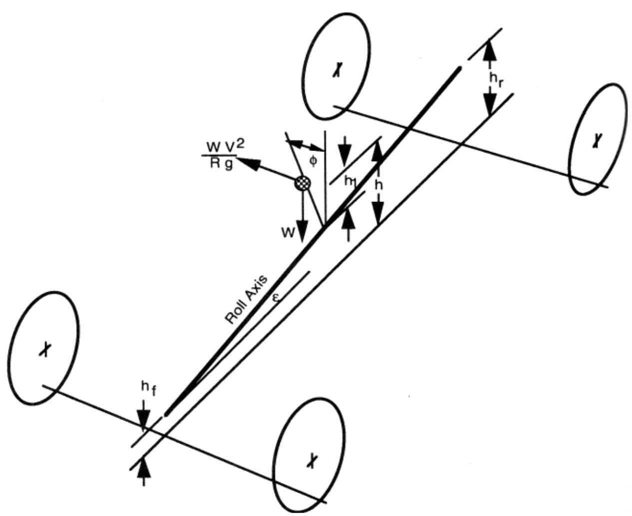
ELEVATION OF THE ROLL CENTER

- The vertical position of the roll center is controlled by the elevation of the CIR of the suspension

- If the roll center is high and close to the center of mass of the suspended mass, one has a smaller moment under the action of the centrifugal loads, and so smaller spring compression / extension. However the lateral load transfer will be large.
- Conversely, a roll center close to the ground gives rise to high roll moment in the springs but a low lateral load transfer.
- Conclusion: the roll center position results from a compromise!

Reminder: Load transfer





- Lateral load transfer is a elevation of the Roll centre h_r:

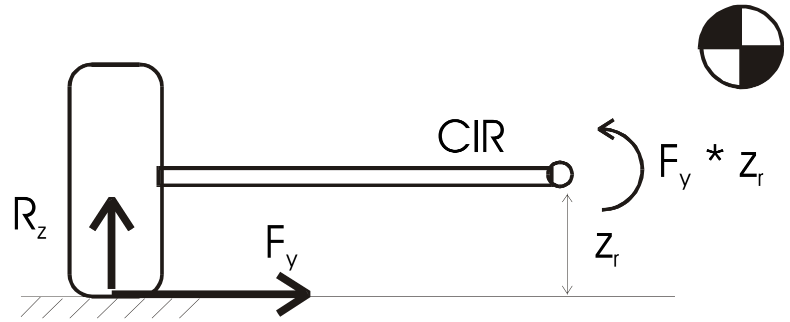
$$\boxed{F_{zo} - F_{zi} = 2\,F_y\,\frac{h_r}{t} + 2\,K_\phi\,\frac{\phi}{t} = 2\,\Delta F_z}$$

- Body roll depends of the distance h_1 between the CG and the roll axis

$$\boxed{\phi = \frac{W h_1 \frac{V^2}{gR}}{(K_{\phi f} + K_{\phi r}) - W h_1}}$$

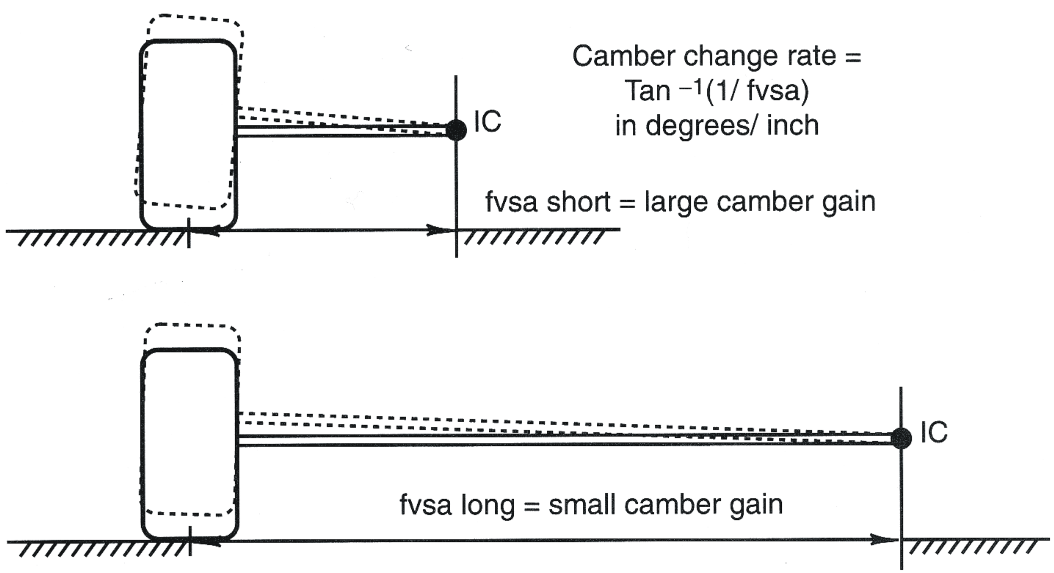
Suspension characteristics

- The elevation of the roll centre has also an influence upon the coupling between horizontal and vertical motion (jacking phenomenon)
  - If the roll centre is above the ground, the lateral forces developed by the tires, yield moments that tend to lift up the body
  - To cancel the phenomenon, it is requested to put the roll centre at the ground level.
  - A roll centre below the ground tends to push the body down to the ground.



Suspension characteristics

- The rate of change of the wheel camber
  - Is a function of the length of front view swing arm (fvsa) defined as the distance between the IC and the wheel
  - Camber change rate = $tg^{-1}(1/fvsa)$
  - If the length of the swing arm is small, this increases the camber change

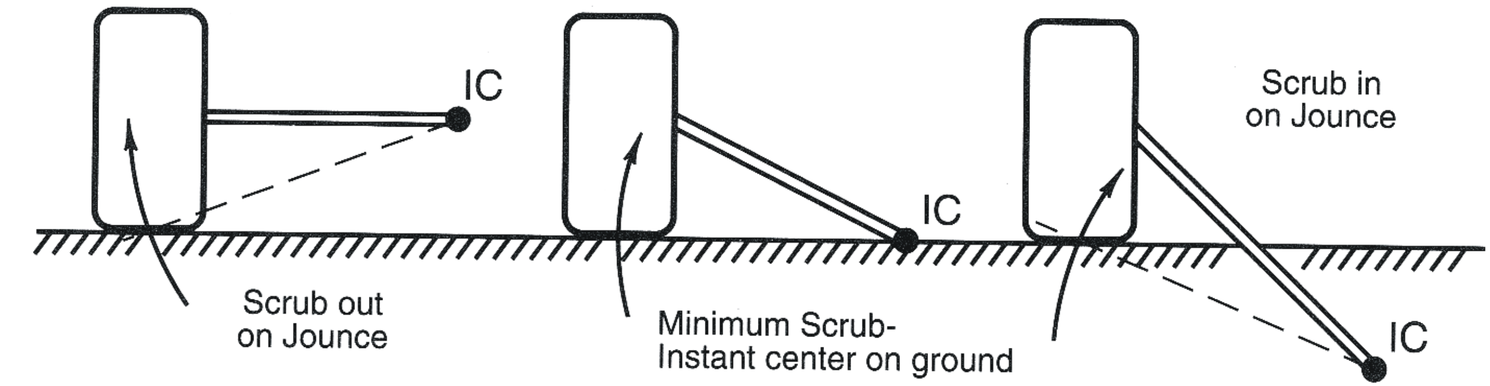


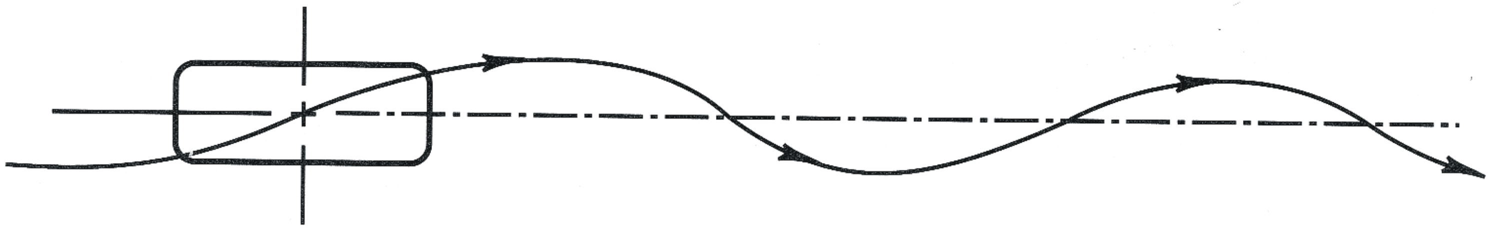
Miliken
Fig. 17.9

Suspension characteristics

- The lateral scrubbing
  - The scrubbing is the relative motion between the tire and the ground → dry friction and wear between the tire and the ground
  - The tire scrubbing is a <u>function of the absolute and relative lengths of the swing arm and of the height of the CIR above the ground</u>.
  - When the CIR is not on the ground, the tire scrubbing increases.
  - When the CIR is above the ground level, the tire swings towards the outer direction when going up.
  - The scrubbing is such the tire trajectory is not a straight lines but includes a <u>lateral motion</u> when bounds and rebounds.
  - The lateral speed creates a slip angle, that yields <u>parasitic cornering forces</u>.
  - Lateral motion increases also a lot the <u>tire wear</u> and gives rise to some <u>damping</u> of the vertical motion.

Suspension characteristics





Miliken Fig 17.10: tire scrubbing as a function of the IC center
& Fig 17.11: wheel path in presence of scrubbing

Suspension characteristics

GEOMETRY OF THE ARMS IN THE SIDE VIEW PLANE

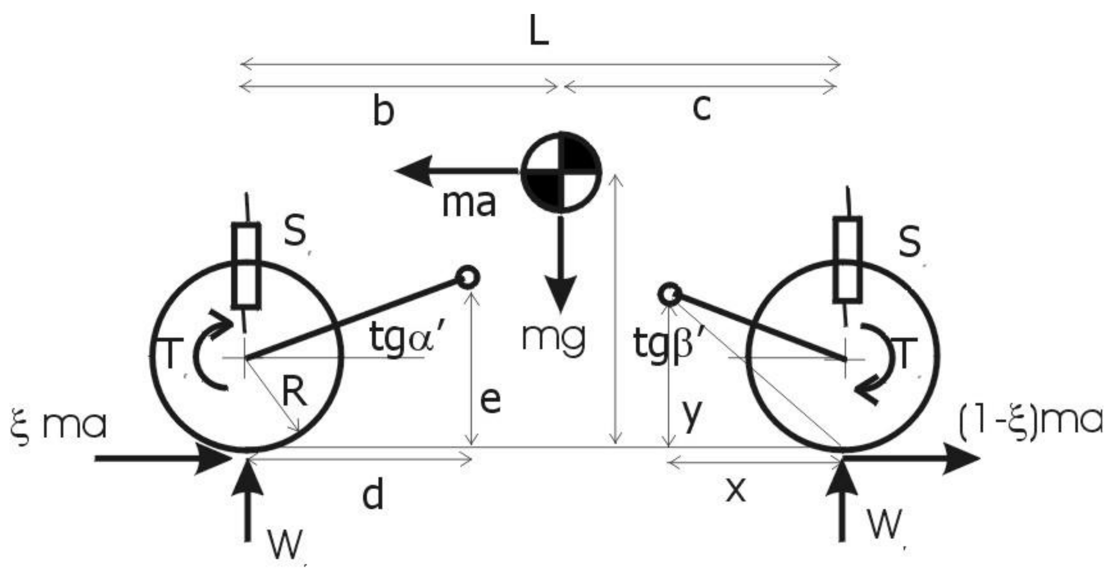
- The characteristics of the CIR and of the swing arm in the side view plane control the motion and the forces / accelerations in the longitudinal direction.

- The following parameters are typically controlled in the side view projection:
  - Anti dive
  - Anti squat
  - Wheel path and wheelbase variation (L)

Suspension characteristics

GEOMETRY OF THE ARMS IN THE SIDE VIEW PLANE

- The position of the CIR in the side view can be located before or after / below of above the wheel center, for front or either rear wheels.
- However in practice, the CIR is often behind and above the wheel center for front suspensions and in front and above for rear wheels.



Suspension characteristics

ANTI SQUAT AND ANTI DIVE SYSTEMS

- These are suspension effects describing the coupling between the longitudinal and vertical forces. The coupling results in modification of the pitch angle and of the vertical position of the suspended mass during acceleration and braking.

- The anti squat / dive systems do not modify the longitudinal load transfer, but they alter the way the loads are transmitted from the rolling gear to the suspended mass.

- For a particular geometry of the suspension, the overall extra loads that is coming from the longitudinal weight transfer is taken by the suspensions arms instead of being taken by the springs. The suspension spring are not compressed.

Suspension characteristics

ANTI SQUAT AND ANTI DIVE CONFIGURATIONS

- The suspension geometries do not generally fully fulfil the conditions to perform anti squat and anti dive conditions. So we generally talk about percentage of partial accomplishment of the anti squat and anti dive conditions.

- The compensation of weight transfer in anti squat and anti dive conditions depends on the presence of longitudinal efforts to create a cancelling vertical force.

KINEMATIC PERFORMANCE CURVES

Kinematic Performance Curves

- The first performance curve concerns the variation of half-track according to the wheel travel.
- Definition: The half-track is the distance on the ground between the median plane of the chassis and the mid plane of the wheel.
  - That's not the half of it!
  - A rigid axle can have a half-track variation!
- The half-track variation is used to account for axle or tire scrubbing with respect to the chassis.

Kinematic Performance Curves

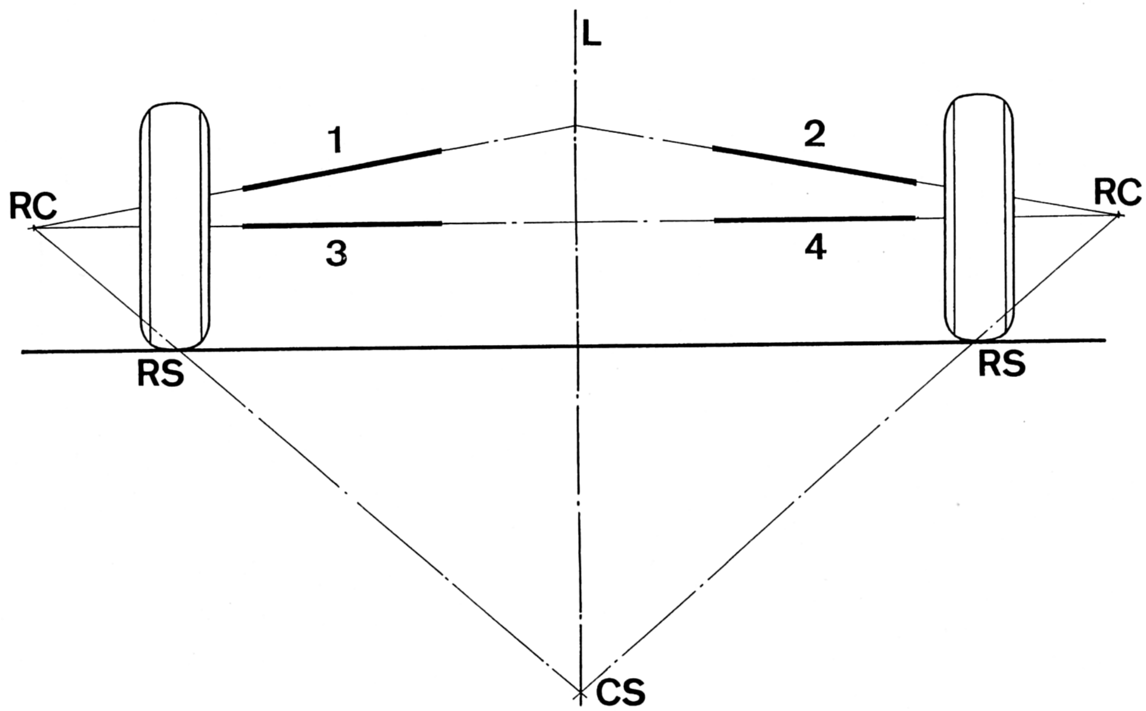


FIG. 3.17

Construction du centre instantané de roulis (cs).

(L) plan médian du véhicule.

1,2 : bras de suspension supérieur (gauche/droit).

3,4 : bras de suspension inférieur (gauche/droit).

T. Halconruy Fig 3.17: (CR) = IC between body and wheel, (RS)=IC between the wheel and the ground, (CS)=IC between the chassis and the ground

Kinematic Performance Curves

In roll:

- The half-track modification depends on the vertical position of the Roll Center = CIR of the chassis with respect to the ground (CS)
- It is reversed when the CS passes below the ground.

In pumping:

- The half-track variation is proportional to the product between the modification of vertical position of the wheel center and the tangent of the angle between the segments (RS,RC) and (RS,H) where H is the projection of RC on the ground.
- The variation is smaller when RC is far from RS and when RC is close to the ground.

In pumping as in roll, the half-track variation is null when the roll is on the ground

Kinematic Performance Curves

In roll:

- The half-track modification depends on the vertical position of the Roll Center = CIR of the chassis with respect to the ground (CS)
- It is reversed when the CS passes below the ground.

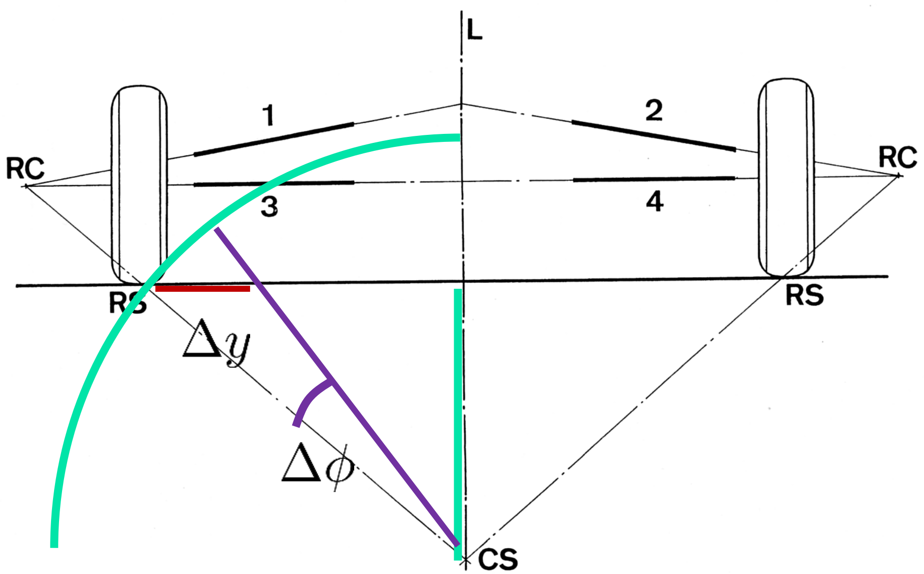


FIG. 3.17

Construction du centre instantané de roulis (cs).

(L) plan médian du véhicule.

1,2 : bras de suspension supérieur (gauche/droit).

3,4 : bras de suspension inférieur (gauche/droit).

T. Halconruy Fig 3.17

Kinematic Performance Curves

In pumping:

- The half-track variation is proportional to the product between the modification of vertical position of the wheel center and the tangent of the angle between the segments (RS,RC) and (RS,H) where H is the projection of RC on the ground.
- The variation is smaller when RC is far from RS and when RC is close to the ground.

$$\Delta y = \tan\alpha \, \Delta h$$

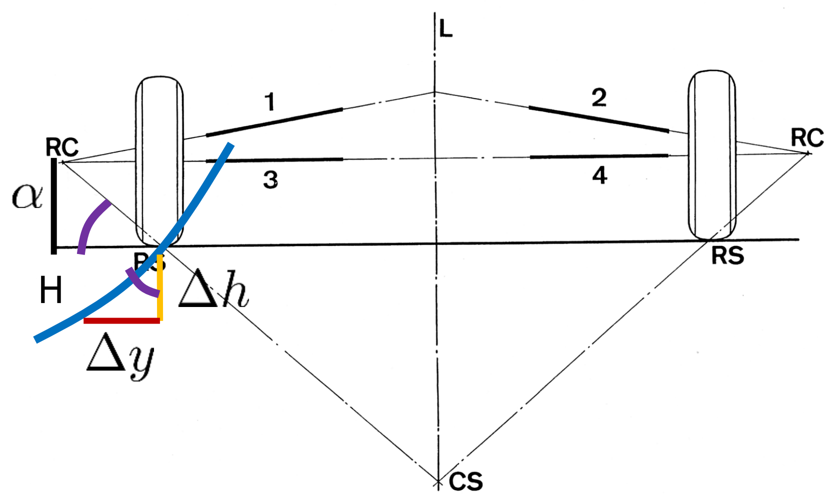


FIG. 3.17

Construction du centre instantané de roulis (cs).

(L) plan médian du véhicule.

1,2 : bras de suspension supérieur (gauche/droit).

3,4 : bras de suspension inférieur (gauche/droit).

T. Halconruy Fig 3.17

Kinematic Performance Curves

- The second type of kinematic curve characterizes the camber variations according to the wheel travel.

In pumping,

- The camber variation is inversely proportional to the distance between RS and H.
- The further RC is from RS in transverse direction, the smaller the camber variation will be in pumping.

In roll,

- It is necessary to calculate the body roll inclination, which is a superimposition of camber variation equal to the roll angle and the camber given by the relative displacement of the wheel with respect to the body.
- The camber variation linked to the wheel displacement is calculated as in pumping.

Kinematic Performance Curves

In pumping,

- The camber variation is inversely proportional to the distance between RS and H.
- The further RC is from RS in transverse direction, the smaller the camber variation will be in pumping.

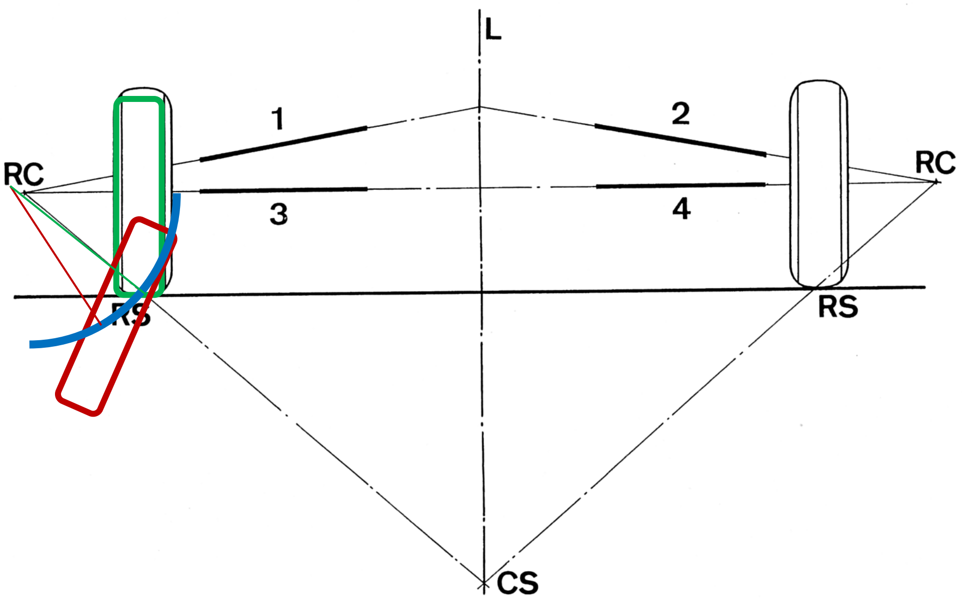


FIG. 3.17

Construction du centre instantané de roulis (cs).

(L) plan médian du véhicule.

1,2 : bras de suspension supérieur (gauche/droit).

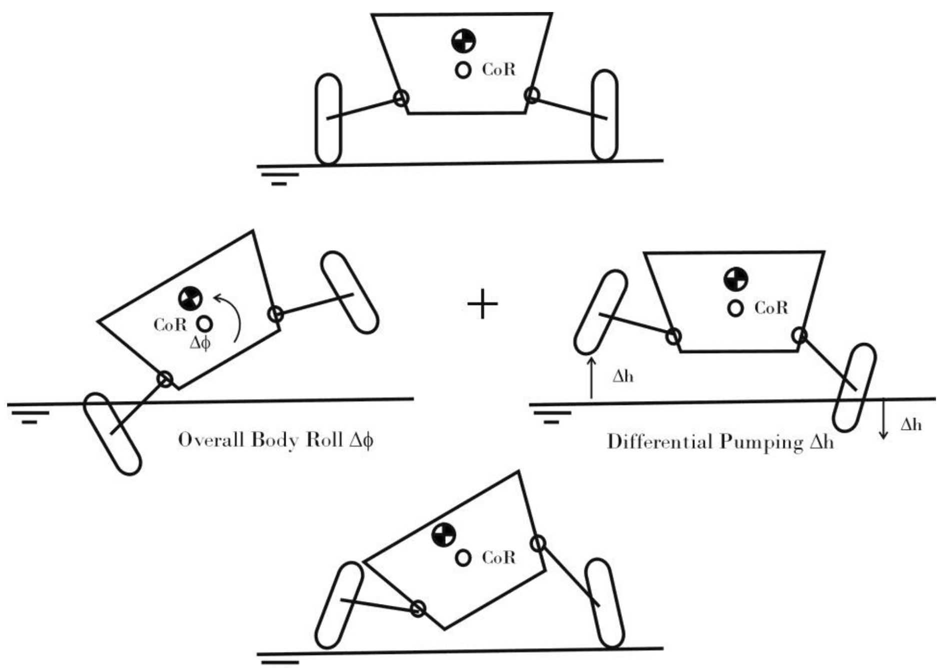
3,4 : bras de suspension inférieur (gauche/droit).

T. Halconruy Fig 3.17

Kinematic Performance Curves

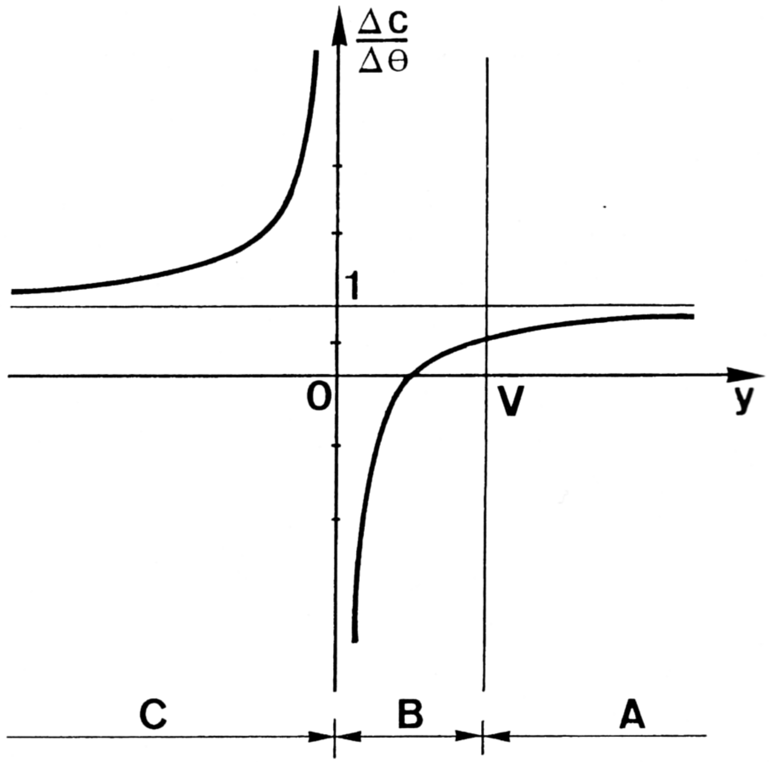
In roll,

- It is necessary to calculate the body roll inclination, which is a superimposition of camber variation equal to the roll angle and the camber given by the relative displacement of the wheel with respect to the body.



Kinematic Performance Curves

- In roll, we obtain a curve similar to the one sketched in the figure 3.18 by Halconruy.
- V= ground track of the axle and y= the position relative to the wheel.
- Zone A: RC is at a distance greater than V from the wheel and on the same side of the vehicle's median plane as the wheel.
- Zone B: RC is located in between the median plane and the wheel.
- Zone C: RC is on the opposite side of the median plane from the wheel.



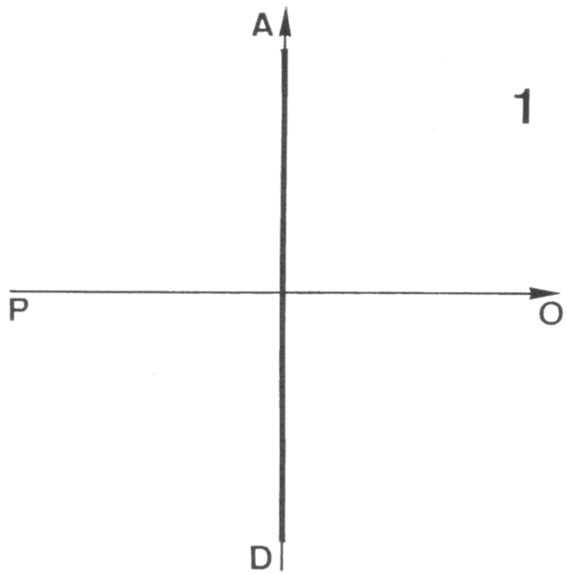
T. Halconruy Fig 3.18

FIG. 3.18 Variation de carrossage rapporté à l'angle de roulis en fonction de la position du centre instantané de rotation de la roue par rapport au châssis (RC).

Kinematic Performance Curves

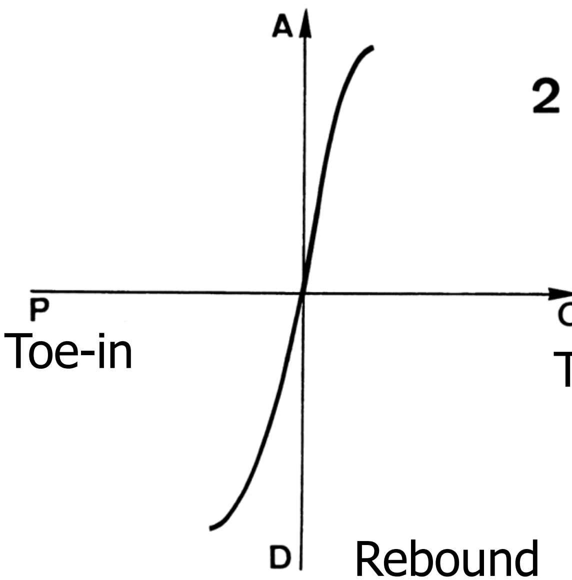
- The third curve represents the modification of the steering angle and of the resulting toe (toe-in/toe-out) with respect to the deflection.
- The steering variations are directly related to the arrangement of the connecting elements between the wheel and the frame. In the case where there is a steering device on the axle, the absence of induced-steering can only be achieved if there is full compatibility between the trajectory imposed by the kinematics of the axle and by the steering linkage.
- In practice, complete compatibility may not not always desirable, as it is often sought to induce steering / toe-in or toe-out to control the vehicle's dynamic behavior by means of induced steering → roll steer

Kinematic Performance Curves



Fully compatible

Compression



Toe-in

Toe-out

Rebound

Induced reduction of steering angle in rebound

FIG. 3.19
Épures cinématiques de variation de parallélisme en débattement vertical.

1. épure cinématiquement compatible
2. épure débraqueuse en débattement

O = ouverture
P = pincement
A = attaque
D = détente.

Halconruy Fig 3.19
Curve of toe modification

Kinematic Performance Curves

- Full compatibility between steering and axle:
  - Front wheel drive with a sharp performance
  - Not favorable for heading in dynamic conditions on uneven roads. On bumps, the suspension is compressed, the vertical load increases and steering power is increased.
  - If you have an asymmetrical road, this causes a different thrust to the left and right.
- Introduction of induced steering effects:
  - Induced steering is introduced to compensate for the increased steering power of the tire with the highest vertical load.
  - In the figure, the unsteering character is illustrated: the more you compress the suspension, the more you have induced opening (while remaining oriented in the direction of the steering wheel). It gives understeer effect.

Kinematic Performance Curves

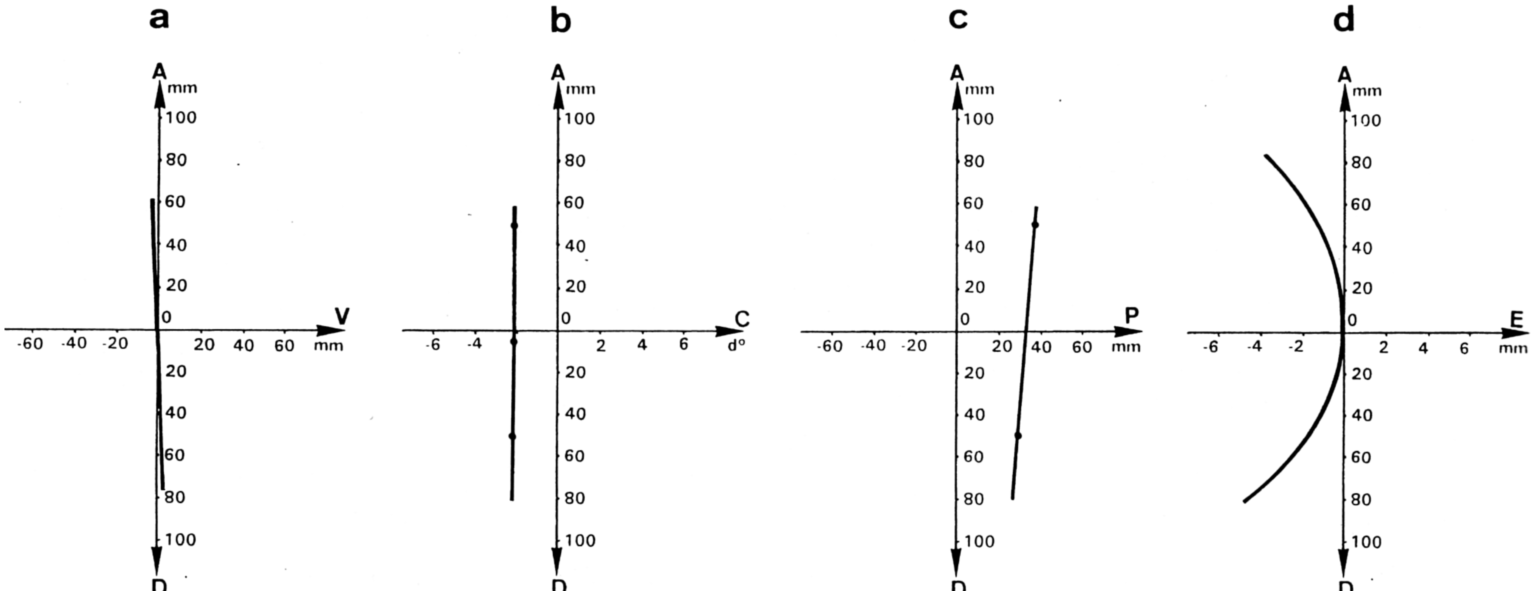
- The fourth response curve determines the variation in the longitudinal dimensions between the wheel center and a reference point on the chassis.
- This gives the wheelbase modification variation of the vehicle.
- Wheelbase modification will affect the under/oversteer character

$$\delta = \frac{L}{R} + \left(\frac{m\ c}{C_{\alpha f}\ L} - \frac{m\ b}{C_{\alpha r}\ L}\right)\frac{V^2}{R}$$

$$V_{\text{critical}} = \sqrt{\frac{L}{|K|}}$$

Kinematic Performance Curves

Half-track modification Camber modification Toe modification Wheel base modification



Fudamental kinematic performance curves of a suspension

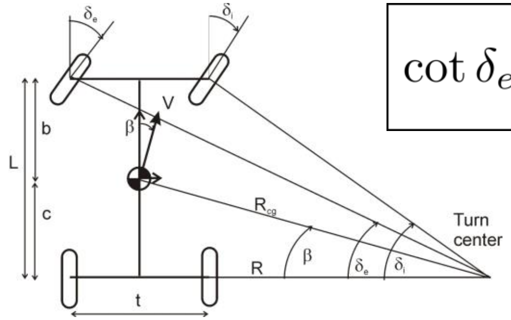
Halconruy Fig 3.20 *FIG. 3.20 Épures cinématiques fondamentales d'un essieu.*

a. variation de demi-voie.
b. variation de carrossage.
c. variation de parallélisme.
d. variation d'empattement.

Kinematic Performance Curves

- For steering axles, one must add two other kinematic performance curves
  - The Jeantaud drawing giving the compatibility of the steering mechanism with the Jeantaud conditions
  - The modification of steering angle with roll and vertical displacement

- For these kinematic performance curves, please refer to the section devoted to the steering mechanism



$$\boxed{\cot \delta_e - \cot \delta_i = \frac{t}{L}}$$

Elasto-Kinematic Performance Curves

- In kinematics, links and members are considered rigid.
- In practice, the axle components are at some point deformable and are mounted on kinematic joint exhibiting some flexibility such as rubber seals which enable a good filtering of shocks and vibrations. Under the effect of the forces, significant displacements may be observed.
- The basis of elastokinematics is to integrate the deformability of the members and the elasticity of the joints and to gather information on the position of the wheels as a function of the wheel travel and of the dynamic forces.
- It is common to draw the variation of the parameters during a turn taken under given static and kinematic conditions.

Elasto-Kinematic Performance Curves

- Illustration of the elasto-kinematic analysis with the variation of parallelism with the vertical displacement
- We consider the rear wheel outside the bend
- The kinematics of the wheel tends to make the wheel steer and to take toe-in geometry, which is positive to reinforce understeer (curve 1).
- With dynamic forces (curve 2), elastic deformations lead to effects leading the wheel to open up (toe-out effect), thus cancelling out the initial understeer effect.

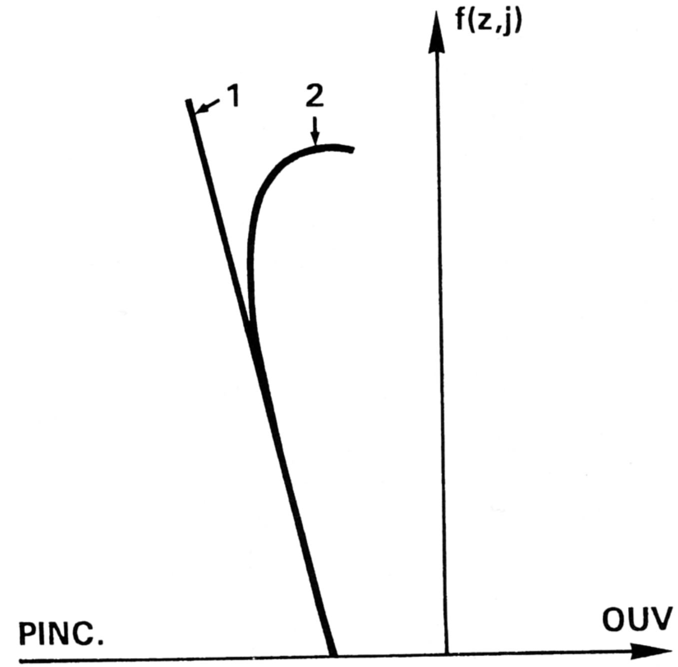


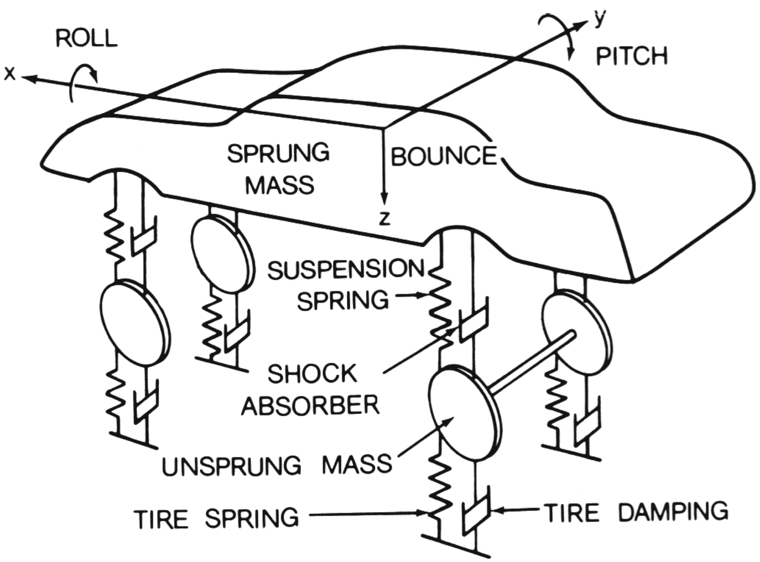
FIG. 3.22 Épure élastocinématique.

Le schéma représente la variation de parallélisme en fonction du débattement : la courbe (1) ne tient pas compte de la déformabilité des différents composants (épure cinématique) alors que la courbe (2) en tient compte (épure élastocinématique).

Halconruy Fig 3.22

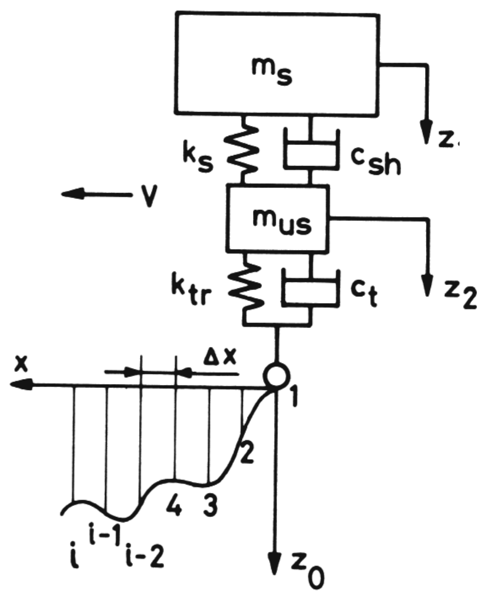
Appendix: Reminder of the Quarter car model

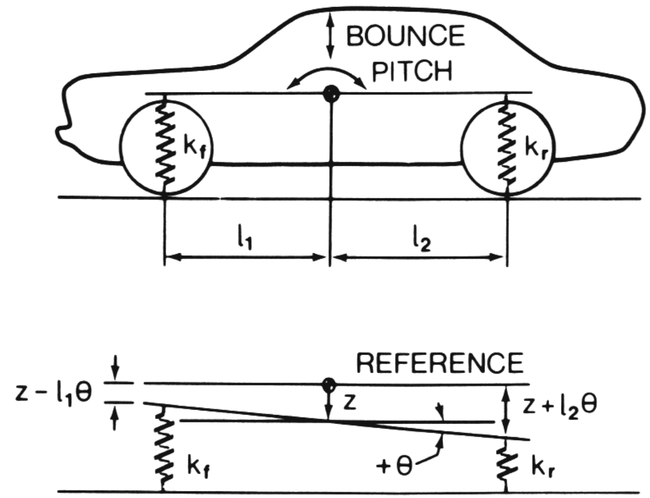
Simple models for vehicle comfort investigation



Wong : Fig 7.3
Model with 7 dof

Wong : Fig 7.6
Model with 2 dof, or quarter car model for vertical comfort

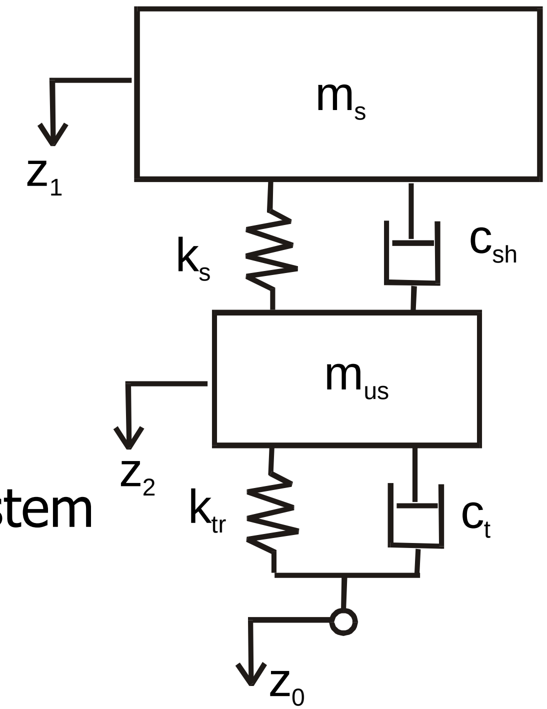




Wong : Fig 7.7
Model with 2 dof for pumping and pitch investigation

Two-dof model with suspended and non suspended masses

- Quarter car model



- Dynamic equations of the two dof system

$$m_s\,\ddot{z}_1 + c_{sh}\,(\dot{z}_1 - \dot{z}_2) + k_s\,(z_1 - z_2) \;=\; 0$$

$$\begin{aligned} m_{us}\ddot{z}_2 + c_{sh}\,(\dot{z}_2 - \dot{z}_1) + k_s\,(z_2 - z_1) + c_t\,\dot{z}_2 + k_{tr}\,z_2 &= F(t) \\ &= c_t\,\dot{z}_0 + k_{tr}\,z_0 \end{aligned}$$

Two-dof model with suspended and non suspended masses

- <u>Investigation of the system natural vibrations without damping</u>

$$
\begin{aligned}
m_s\,\ddot{z}_1 + k_s\,z_1 - k_s\,z_2 &= 0 \\
m_{us}\,\ddot{z}_2 + (k_s + k_{tr})\,z_2 - k_s\,z_1 &= 0
\end{aligned}
$$

- Harmonic solutions

$$
\begin{aligned}
z_1 &= Z_1 \cos \omega_n t \\
z_2 &= Z_2 \cos \omega_n t
\end{aligned}
$$

The system writes

$$
\begin{aligned}
(-m_s\,\omega_n^2 + k_s)\,Z_1 - k_s\,Z_2 &= 0 \\
-k_s\,Z_1 + (-m_{us}\,\omega_n^2 + (k_s + k_{tr}))\,Z_2 &= 0
\end{aligned}
$$

- Characteristic equation

$$
\omega_n^4\,(m_s\,m_{us}) - \omega_n^2\,(m_s\,k_s + m_s\,k_{tr} + m_{us}\,k_s) + k_s\,k_{tr} = 0
$$

Two-dof model with suspended and non suspended masses

- Natural frequencies

$$\omega_{n1}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \qquad \omega_{n2}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

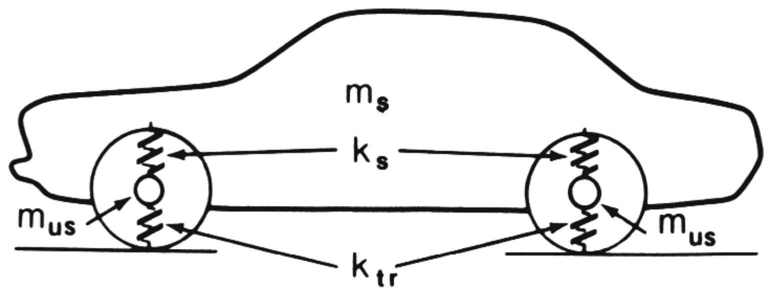
$$\begin{aligned} A_1 &= m_s\, m_{us} \\ B_1 &= m_s\, k_s + m_s\, k_{tr} + m_{us}\, k_s \\ C_1 &= k_s\, k_{tr} \end{aligned}$$

- Remark: given that $m_s > m_{us}$ and that $k_s < k_{tr}$, the eigenfrequencies are closed to the uncoupled frequencies

$$f_{n,s} = \frac{1}{2\pi}\sqrt{\frac{k_s\, k_{tr}/(k_s + k_{tr})}{m_s}} \qquad f_{n,us} = \frac{1}{2\pi}\sqrt{\frac{k_s + k_{tr}}{m_{us}}}$$

Two-dof model with suspended and non suspended masses

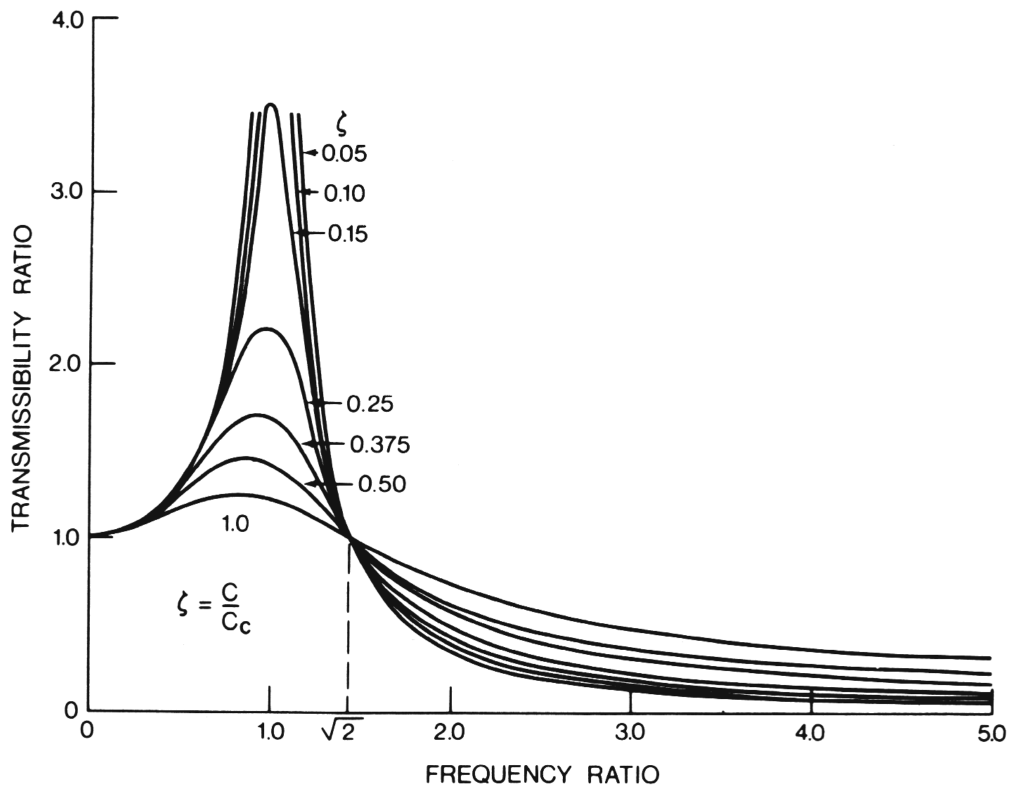
- Exercise:



m_s = 1814 kg, 4000 lb
m_{us} = 181 kg, 400 lb, COMBINED
k_s = 88 kN/m, 500 lb/in., COMBINED
k_{tr} = 704 kN/m, 4000 lb/in., COMBINED

- f_{n1} = 1.04 Hz
- f_{n2} = 10.5Hz

Two-dof model with suspended and non suspended masses



Wong: Fig 7.8: Transmissibility ratio as a function of the frequency ratio for a single dof system

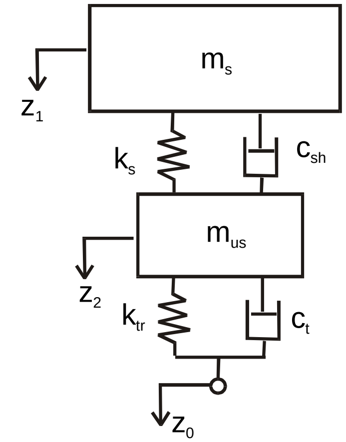
Two-dof model with suspended and non suspended masses

- <u>Sinusoïdal road profile</u>

$$z_0 = Z_0 \cos \omega t \qquad\qquad \omega = 2\pi\, V/l_w$$

- Transmissibility ratio between the road profile and the magnitude of the motion of the <u>suspended mass</u>

$$\frac{Z_1}{Z_0} = \frac{\sqrt{A_2}}{\sqrt{B_2 + C_2}}$$

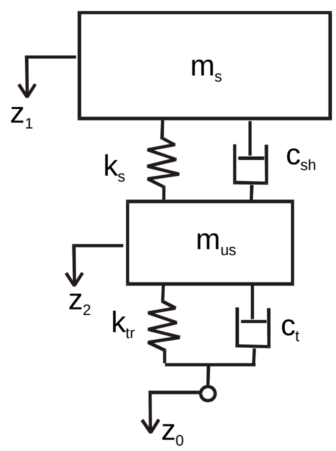


$$A_2 = (k_s\, k_{tr})^2 + (c_{sh}\, k_{tr}\, \omega)^2$$
$$B_2 = [(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2]^2$$
$$C_2 = (c_{sh}\, \omega)^2\, [m_s\, \omega^2 + m_{us}\, \omega^2 - k_{tr}]^2$$

Two-dof model with suspended and non suspended masses

- Transmissibilty ratio between the road profile and the motion of the non suspended mass

$$\frac{Z_2}{Z_0} = \frac{\sqrt{A_3}}{\sqrt{B_2 + C_2}}$$



$$\begin{aligned}
A_2 &= (k_s\, k_{tr})^2 + (c_{sh}\, k_{tr}\, \omega)^2 \\
B_2 &= [(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2]^2 \\
C_2 &= (c_{sh}\, \omega)^2\, [m_s\, \omega^2 + m_{us}\, \omega^2 - k_{tr}]^2 \\
A_3 &= [k_{tr}\, (k_s - m_s\, \omega^2)]^2 + (c_{sh}\, k_{tr}\, \omega)^2
\end{aligned}$$

Two-dof model with suspended and non suspended masses

- Transmissibility ratios when neglecting the damping i.e. c_{sh} =0

$$\frac{Z_1}{Z_0} = \frac{k_s\, k_{tr}}{(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2} = \frac{k_s\, k_{tr}}{m_s\, m_{us}\, (\omega_{n1}^2 - \omega^2)\, (\omega_{n2}^2 - \omega^2)}$$

$$\frac{Z_2}{Z_0} = \frac{k_{tr}\, (k_s - m_s\, \omega^2)}{(k_s - m_s\omega^2)(k_{tr} - m_{us}\omega^2) - m_s\, k_s\, \omega^2} = \frac{k_{tr}\, (k_s - m_s\, \omega^2)}{m_s\, m_{us}\, (\omega_{n1}^2 - \omega^2)\, (\omega_{n2}^2 - \omega^2)}$$

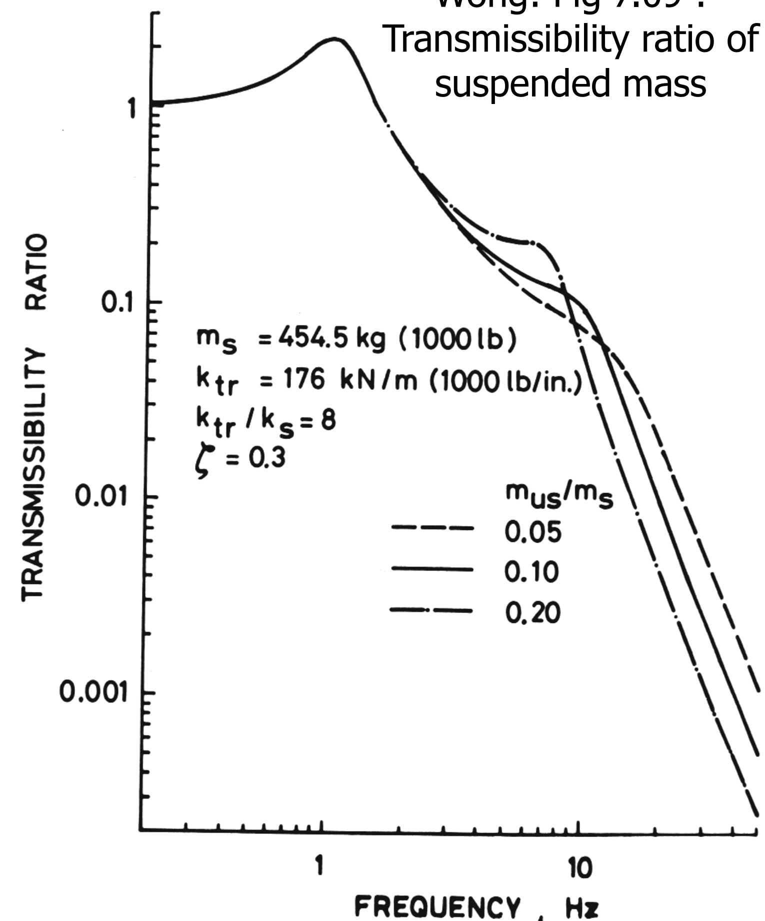
Two-dof model with suspended and non suspended masses

- To assess the performance of the suspension, one must consider three aspects :
  - Vibration isolation: Evaluating the response of the sprung mass to the excitation of the ground. This criterion is usually used to assess the vibration isolation characteristics of a linear suspension system.
  - The suspension travel: measured by the deflection of the suspension spring or by the relative displacement between the sprung and unsprung mass: z_2-z_1. It defines the space required to accommodate the suspension spring movement.
  - The road holding: when the vehicle vibrates, the normal contact force between the tire and the ground fluctuates. The longitudinal and lateral forces developed by the tire are function of the normal load, so its variation impacts directly the handling. The normal force is investigated through the dynamic deflection or by the displacement of the sprung mass relative to the road: z_2-z_0.

Two-dof model: Transmissibility Ratio

- <u>Effect of the non suspended mass</u>
  - Below the eigenfrequency (non suspended mass), the transmissibility is decreasing with lower unsprung masses. Over the second eigenfrequency the transmissibility is higher with low non suspended masses.
- Conclusion: <u>a low mass of the rolling gear and suspension (non suspended mass) is better</u> for the isolation of the suspended mass (passengers) even if there is small penalty a high frequencies

Wong: Fig 7.09 : Transmissibility ratio of suspended mass

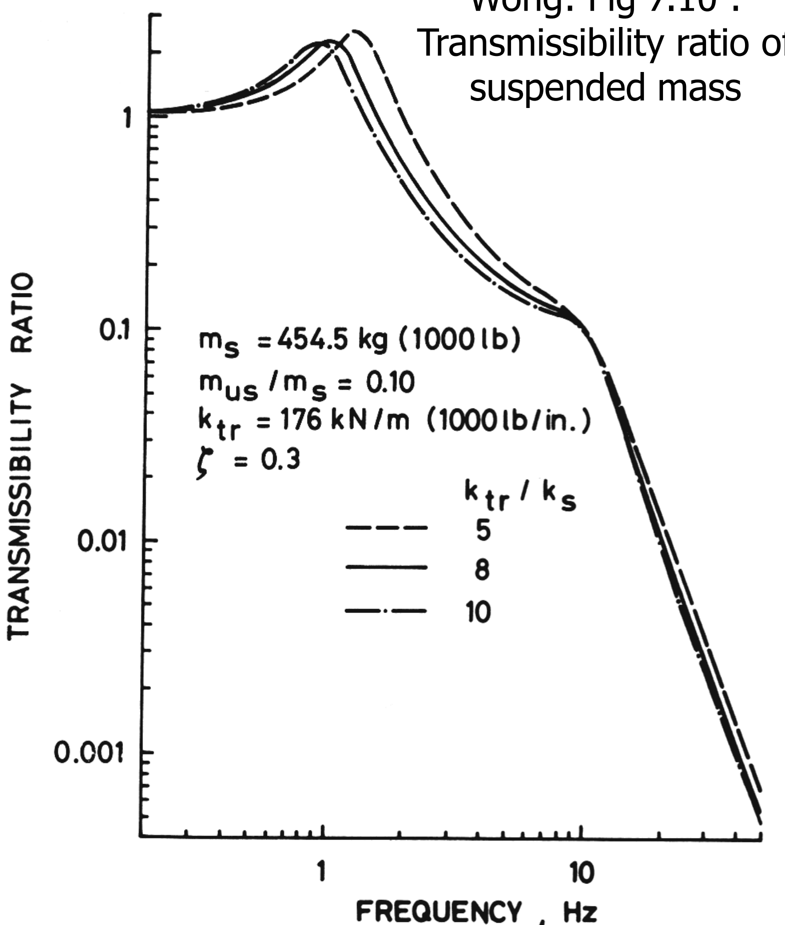


Two-dof model: Transmissibility Ratio

- Effect of the suspension stiffness
  - The tire stiffness is assumed to be given and is our reference

- Conclusion: One has to choose a soft suspension stiffness (k_{tr}/k_s high) to reduce the transmissibility between the two first eigenfrequencies, but there is a small penalty of higher frequencies.

Wong: Fig 7.10 : Transmissibility ratio of suspended mass



Two-dof model: Transmissibility Ratio

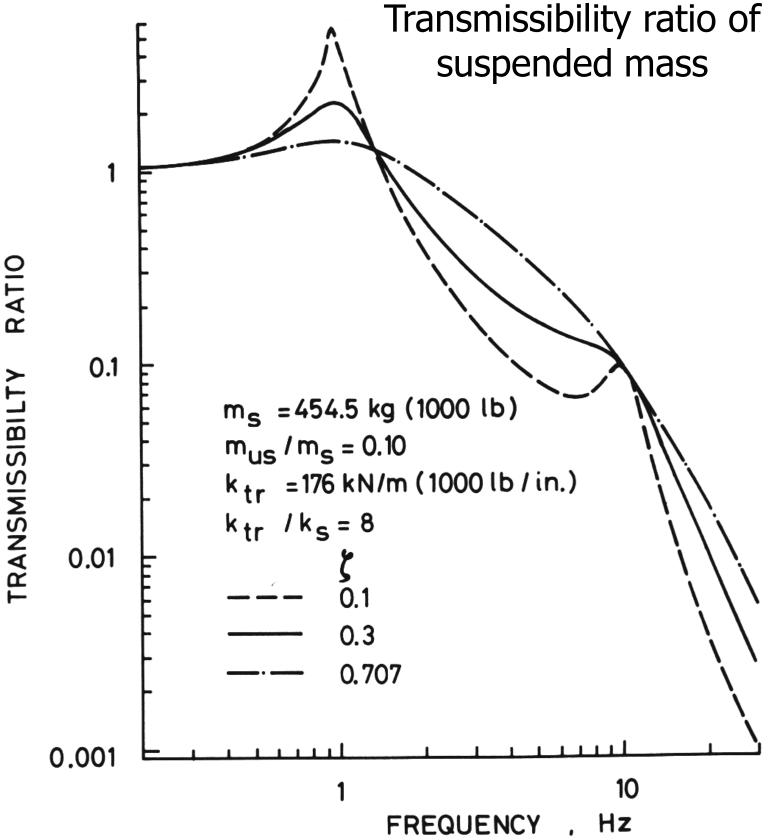
- Effect of the suspension damping ratio

- Conclusion: a medium damping ratio (below critical damping coefficient) is preferred.

  - A good isolation of the vibrations around the natural frequencies of the sprung mass would require increasing the damping

  - In the intermediate range between the natural frequencies, a lower damping ratio is favorable to reduce the transmissibility.

Wong: Fig 7.11 : Transmissibility ratio of suspended mass

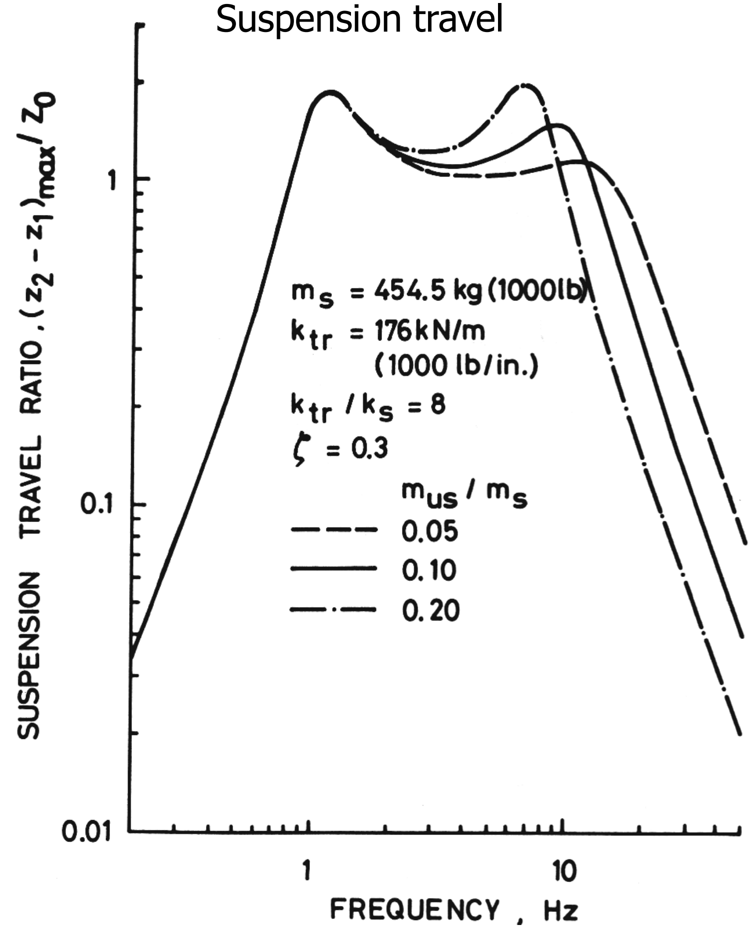


Two-dof model: Suspension travel

- <u>Effect of the non suspended mass</u>

- Conclusion: a <u>small mass of the unsprung mass</u> (rolling gear) is <u>better</u> to reduce the wheel travel even if the conclusion is opposite at higher frequencies.

Wong: Fig 7.12
Suspension travel

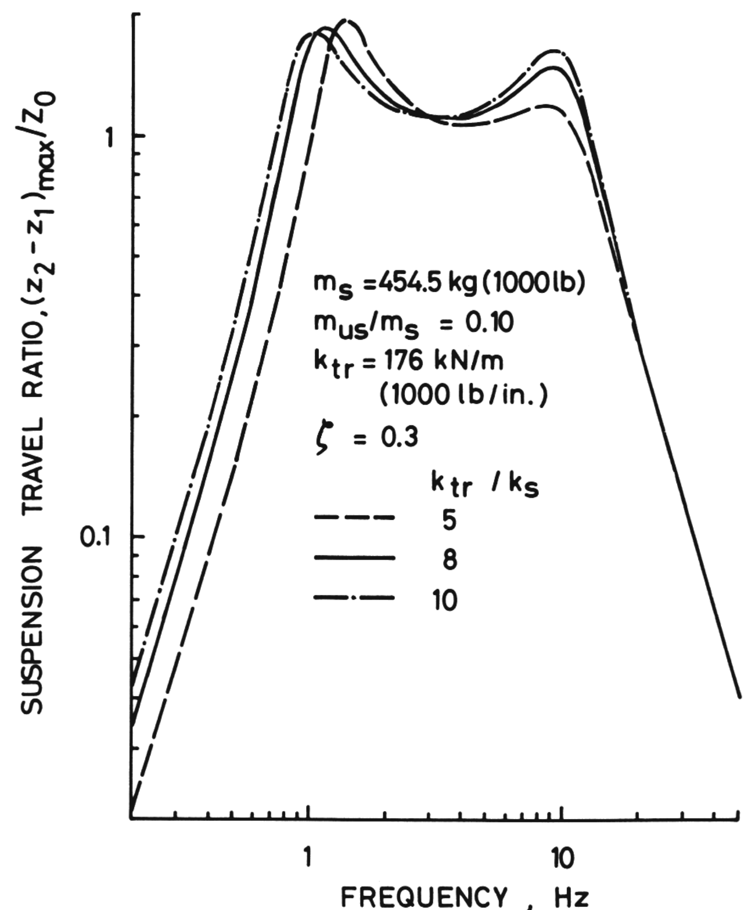


Two-dof model: Suspension travel

Wong: Fig 7.13
Suspension travel

- <u>Effect of the suspension stiffness</u>

- Conclusion:
  - Below the natural frequency of the suspended mass, a <u>stiff suspension</u> (k_{tr}/k_s low) can reduce the wheel travel
  - For the intermediate frequency range, there is crossing phenomenon: below the crossing, it is better to adopt a soft suspension stiffness. Over it is preferable to go for stiffer suspension springs
  - A high frequencies, stiffness has little influence

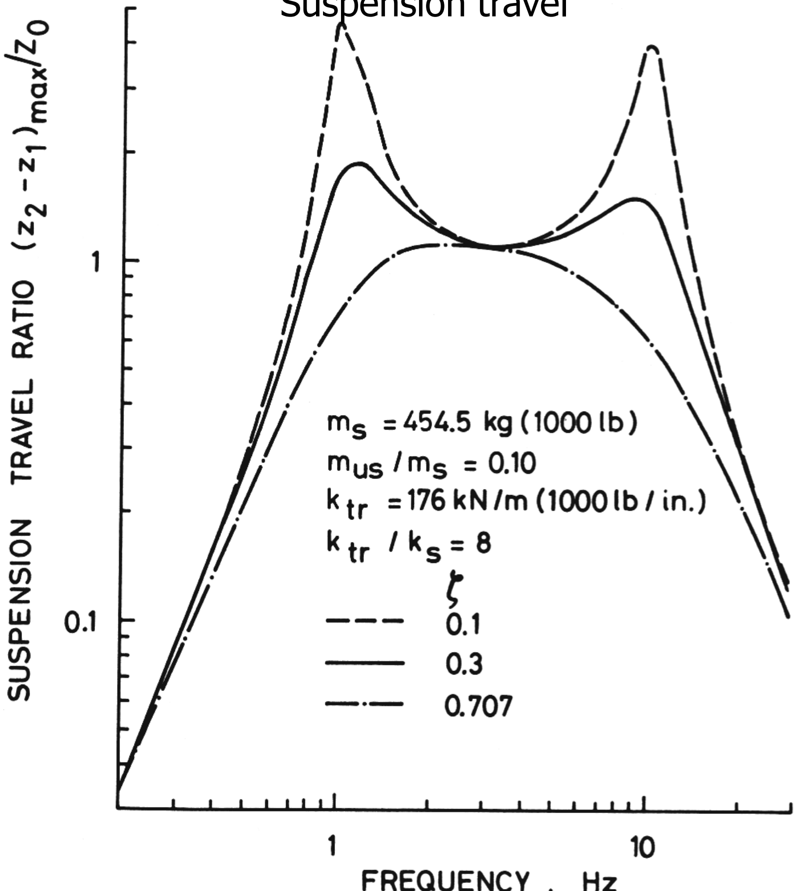


Two-dof model: Suspension travel

- <u>Effect of the suspension damping</u>

- Conclusion: in any cases, to reduce the suspension travel, it is better to adopt a <u>high damping coefficient</u>

Wong: Fig 7.14
Suspension travel



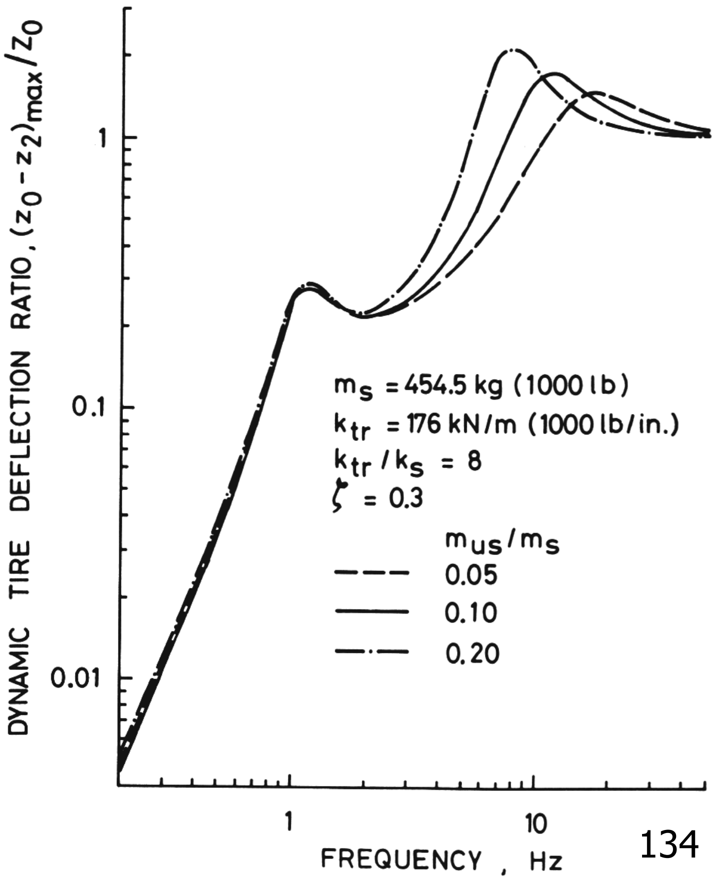
Two-dof model: Road Holding

- <u>Effect od the non suspended mass</u>

- Conclusion: <u>a low non suspended mass</u> reduces the tire dynamic deflection

- Warning: the tire leaves the ground if the dynamic deflection is exceeding the static deflection of the suspension

Wong: Fig 7.15 Dynamic tire deflection



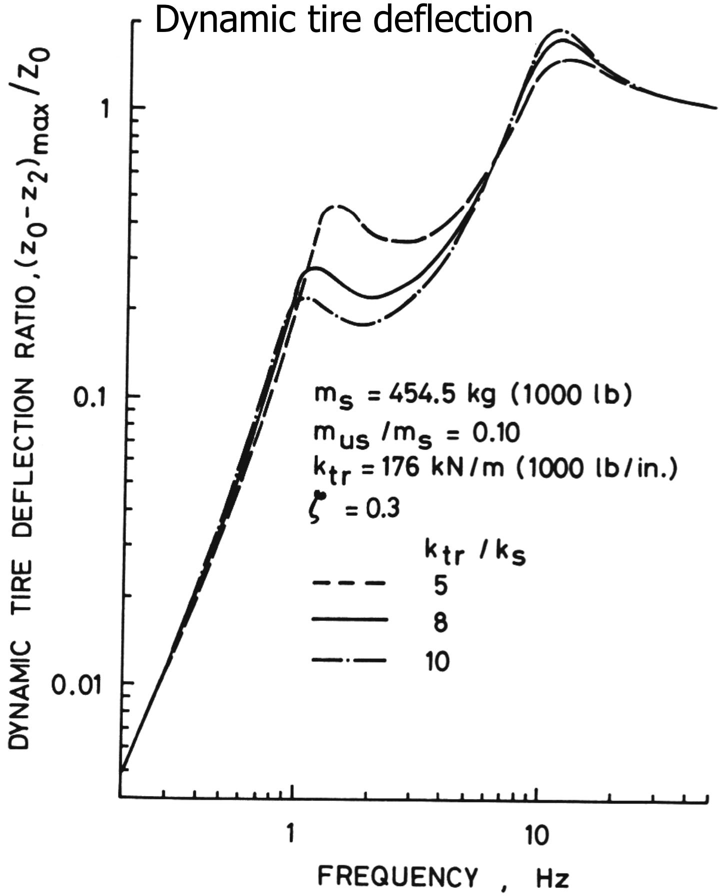
Two-dof model: Road Holding

- Effect of the suspension stiffness

- Conclusion: In the frequency range between the first natural frequency and the crossing frequency, a low stiffness (k_{tr}/k_s high) reduces the tire deflection. Over the crossing frequency and around the second frequency of unsprung mass, a stiff suspension (k_{tr}/k_s low) minimizes the tire deflection and so maximizes the road holding

- A high stiffness is better road holding, while the vibration isolation requires the opposite.

Wong: Fig 7.16
Dynamic tire deflection

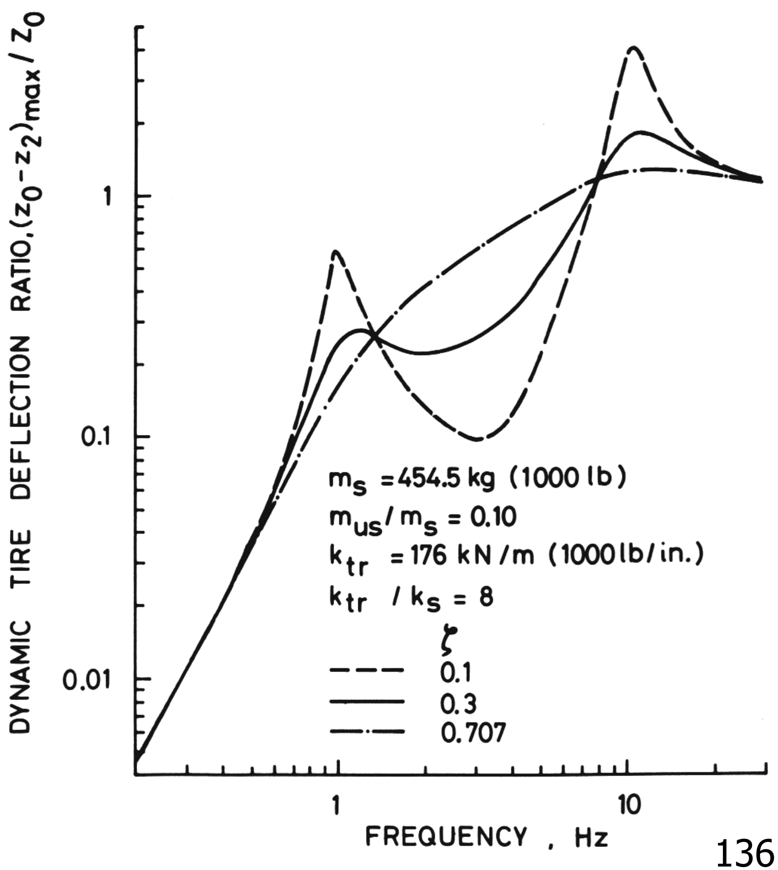


Two-dof model: Road Holding

- <u>Effect of the suspension damping</u>

- Conclusion: to reduce the tire dynamic deflection around both eigenfrequencies, one requires a <u>high damping ratio</u>. This penalizes the dynamic tire deflection in the intermediate range.

Wong: Fig 7.17 Dynamic tire deflection



Two-dof model: conclusions

- Suspended / Non Suspended mass:
  - In all cases, it is preferred to have <u>low non suspended mass</u> ($m_{us} < m_s/10$)
- Stiffness of the suspension springs
  - For vibration isolation: Soft suspension
  - For suspension travel: low stiffness is better at low frequencies but for higher frequencies it is recommended to have higher stiffness
  - For road holding, it is recommended to have high stiffness
  - Conclusion: Soft suspension = comfort favorized, Stiff suspension = focus on road holding
- Damping:
  - <u>Compromise between higher and low values</u> to reach a good comfort and in the same time a good road holding at all frequencies (ζ between 0.2 and 0.4)