MECA0063: Vehicle Architecture Suspension: anti squat / anti dive

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References

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Outline

- MOTIVATION

- ANTI DIVE GEOMETRIES
  - Longitudinal load transfer
  - Equivalent trailing arm analysis in the side-view plane
    - Equivalent model of suspension
  - Equilibrium of front rear suspension
  - Anti dive conditions : case of out-board brakes
    - Font suspension
    - Rear suspension
  - Anti dive conditions : case of in-board brakes
    - Front suspension
    - Rear suspension

Outline

- ANTI SQUAT SYSTEMS
  - Rear wheel drive and solid axle
  - Front wheel drive and independent suspensions
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INTRODUCTION

ANTI-DIVE / ANTI-SQUAT

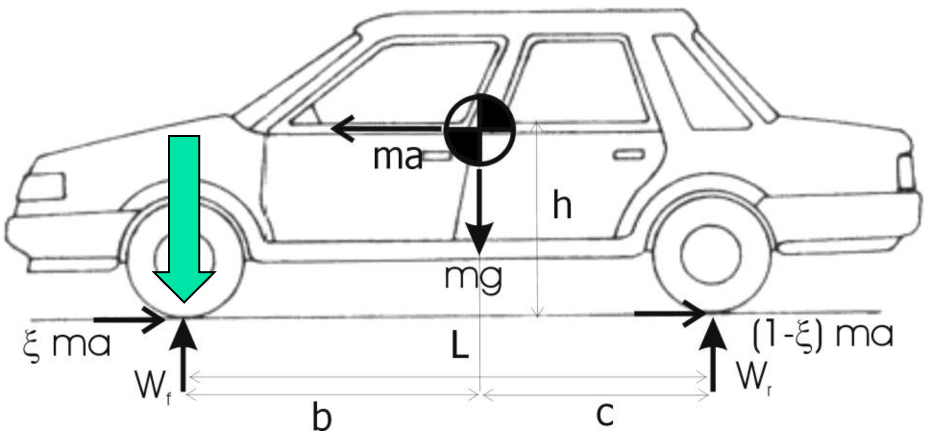
- Braking or acceleration effects cause load transfer between axles.
- This transfer leads to a pitching and pumping movement of the vehicle.
- An appropriate design of the suspension geometry in the lateral plane can limit or eliminate these phenomena.
- The crucial point is the position of the instantaneous center of rotation of the suspension in the side view projection plane.
- The suspension design does not change the load transfer in any way. It does, however, modify the reactions between the suspension (unsprung mass) and the body (suspended mass).

Longitudinal load transfer while braking

- Under braking conditions (a>0 if braking), the overall equilibrium of the vehicle writes

$$W_f + W_r = mg$$

$$mg\,b - ma\,h - W_r\,L = 0$$



It comes

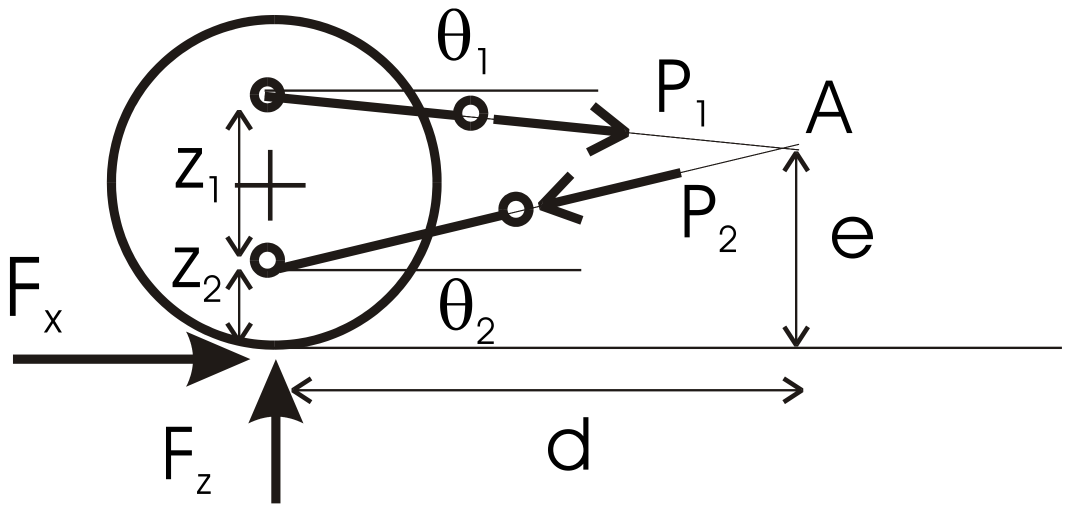
$$\begin{aligned} W_f &= mg\frac{c}{L} + ma\frac{h}{L} \\ W_r &= mg\frac{b}{L} - ma\frac{h}{L} \end{aligned}$$

If $\Delta W = ma\dfrac{h}{L}$ then

$$\begin{aligned} W_f &= mg\frac{c}{L} + \Delta W \\ W_r &= mg\frac{b}{L} - \Delta W \end{aligned}$$

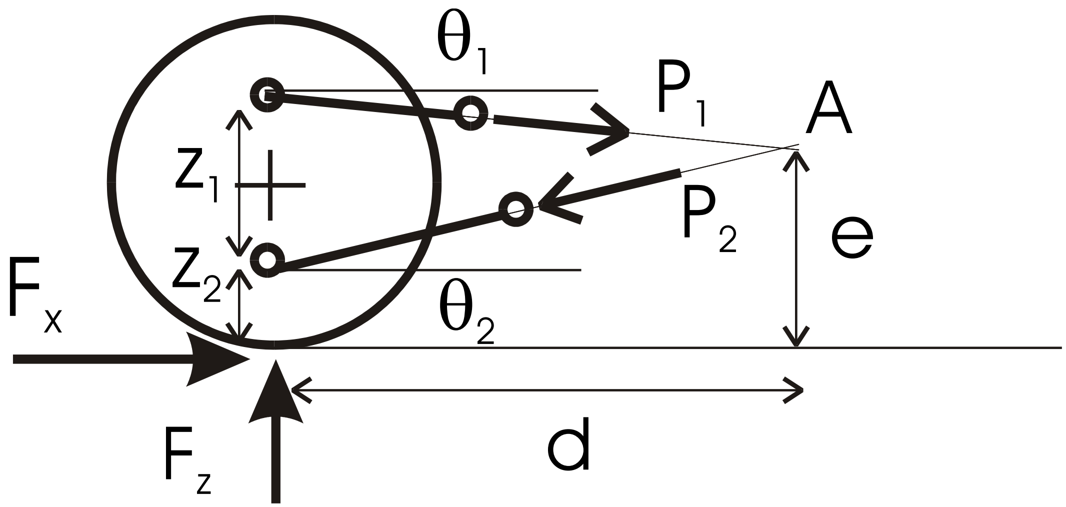
EQUIVALENT TRAILING ARM ANALYSIS

Equivalent trailing arm analysis



- Let’s consider the following model in which one considers the projection of the upper/lower arms in the lateral side-view plane passing through the wheel mid-plane
- IC center A is the intersection of the lines connecting the arms

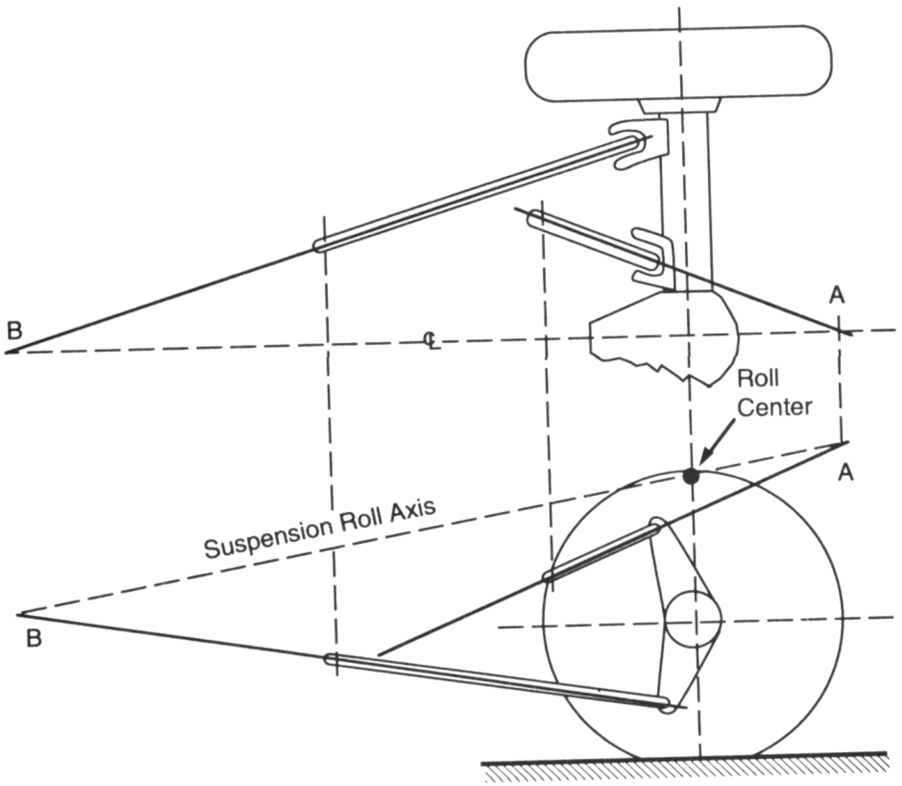
Equivalent trailing arm analysis



- The longitudinal force F_x is supposed to be given.
- Let's compute the internal loads transferred through the suspension arms P_1 and P_2 and the vertical ground reaction force F_z between the wheel and the ground to sustain the longitudinal force F_x.

Equivalent trailing arm analysis

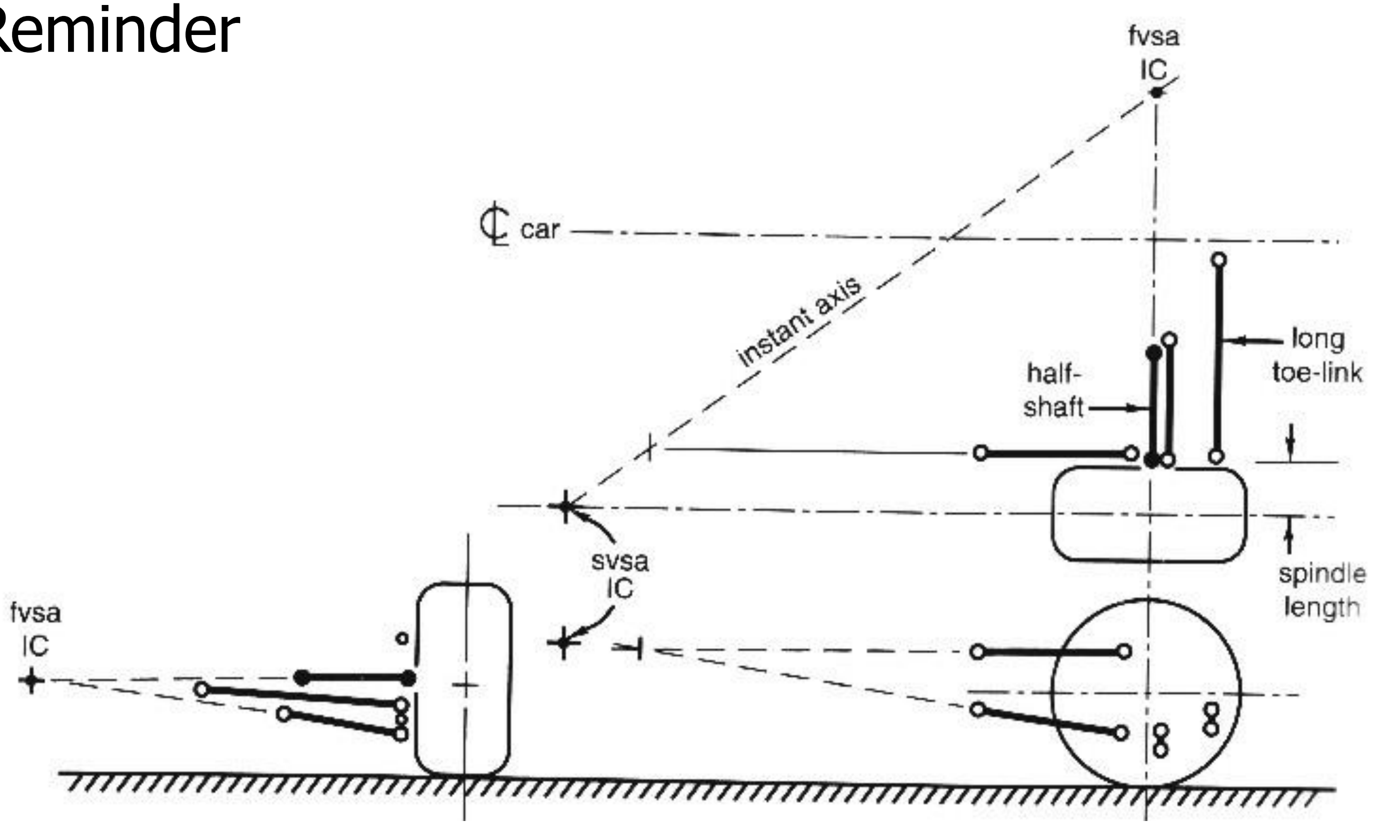
Reminder



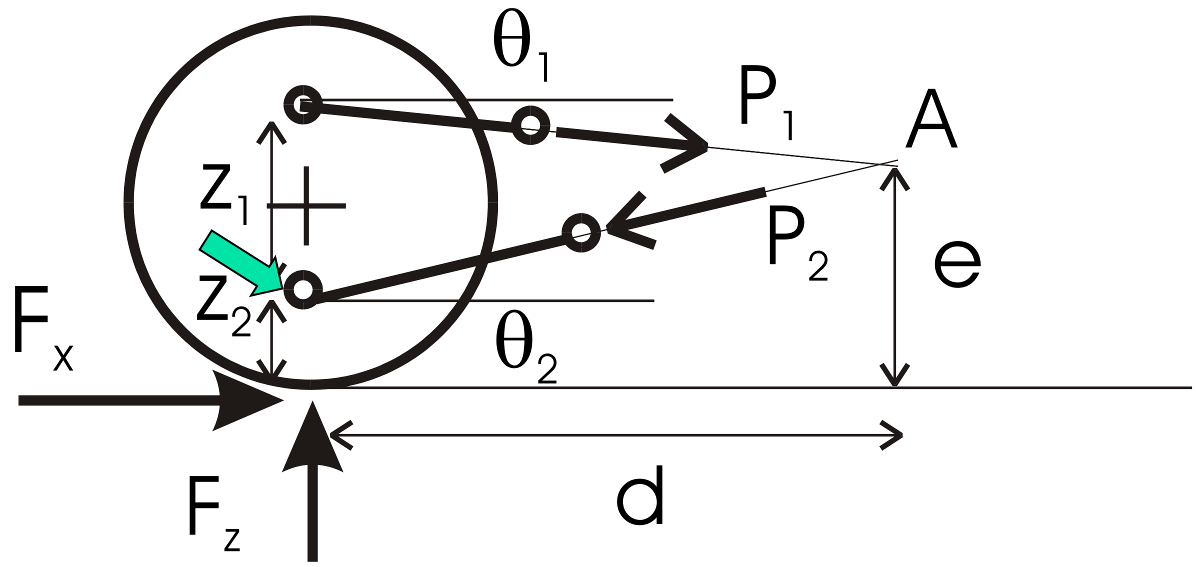
Gillespie Fig 7.15 Four arm suspension

Equivalent trailing arm analysis

Reminder



Equivalent trailing arm analysis



- Equilibrium

$$\begin{cases} F_x + P_1\cos\theta_1 - P_2\cos\theta_2 &= 0 \\ F_z - P_1\sin\theta_1 - P_2\sin\theta_2 &= 0 \\ F_x z_2 - P_1\cos\theta_1 z_1 &= 0 \end{cases}$$

Equivalent trailing arm analysis

- Equilibrium

$$\begin{cases} F_x + P_1 \cos\theta_1 - P_2 \cos\theta_2 & = 0 \\ F_z - P_1 \sin\theta_1 - P_2 \sin\theta_2 & = 0 \\ F_x z_2 - P_1 \cos\theta_1 z_1 & = 0 \end{cases}$$

- Internal loads in suspension arms

$$P_1 = F_x \frac{z_2}{z_1} \frac{1}{\cos\theta_1}$$

$$P_2 = F_x (1 + \frac{z_2}{z_1}) \frac{1}{\cos\theta_2}$$

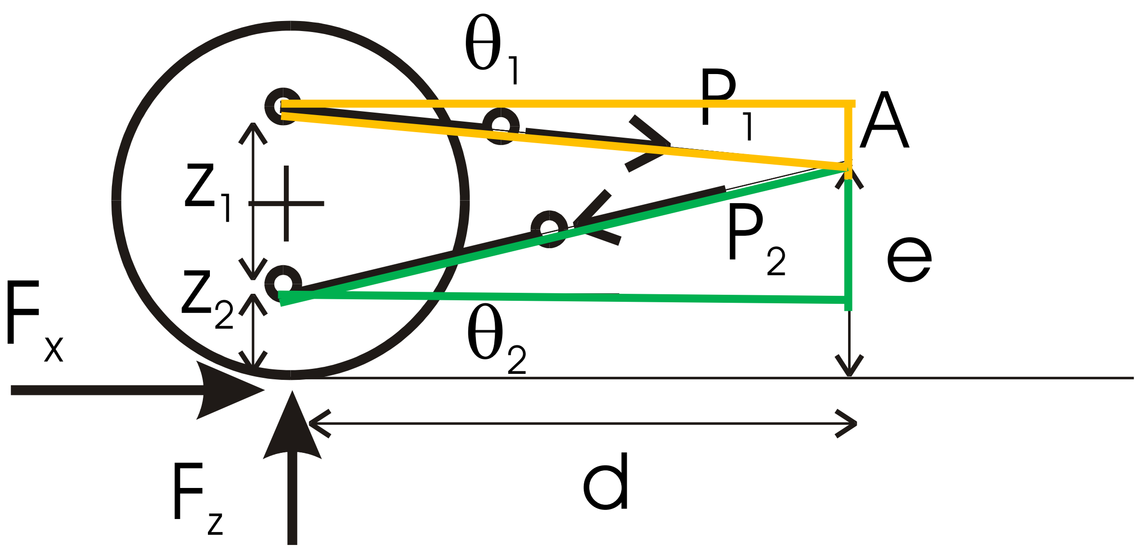
$$F_z = F_x \frac{z_2}{z_1} \tan\theta_1 - F_x (1 + \frac{z_2}{z_1}) \tan\theta_2$$

Equivalent trailing arm analysis

- Vertical reaction caused by the efforts in the suspension

$$F_z = F_x \frac{z_2}{z_1} \tan\theta_1 - F_x \left(1 + \frac{z_2}{z_1}\right) \tan\theta_2$$

$$\tan\theta_1 = \frac{z_1 + z_2 - e}{d} \qquad \tan\theta_2 = \frac{e - z_2}{d}$$



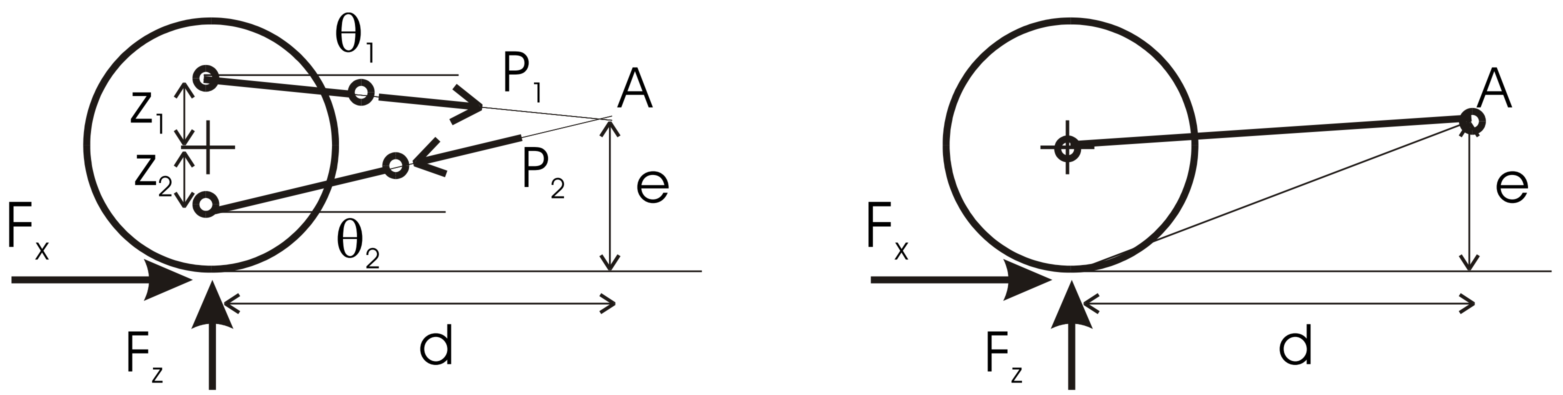
$$\boxed{F_z = F_x \frac{e}{d}}$$

Equivalent trailing analysis

- This expression is the same as the relation obtained when substituting the suspension mechanism by a lever arm hinged about the IC

$$\boxed{F_z = F_x \frac{e}{d}}$$

- IC is a virtual reaction point, about which the torque is null



ANTI DIVE GEOMETRY

Anti dive geometry

- Free body diagram of the front suspension with outboard brakes

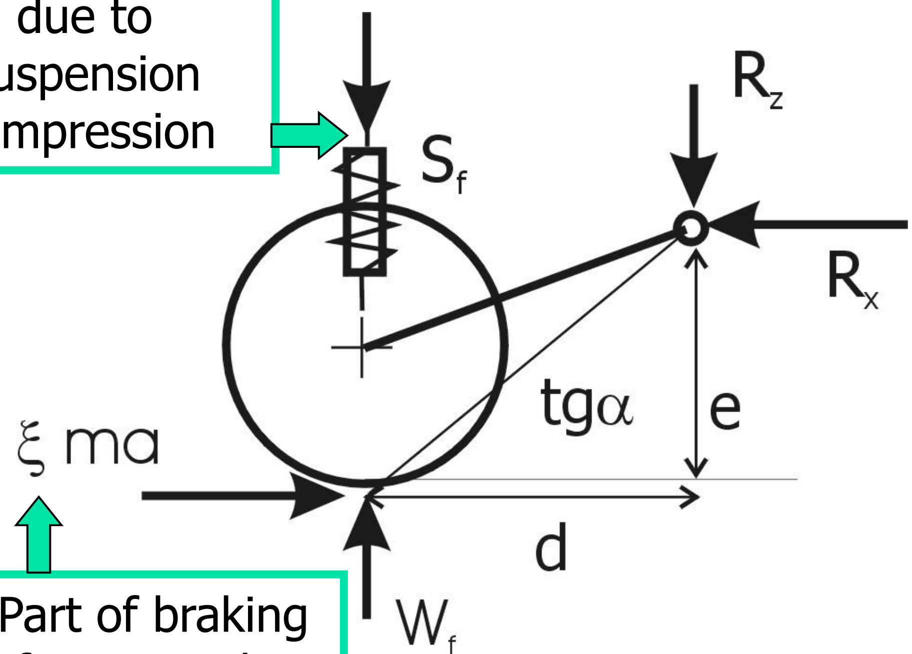
Reaction force due to suspension compression

- Equilibrium

$$\begin{cases} \xi\, ma &= R_x \\ W_f &= R_z + S_f \\ W_f d &= S_f d + \xi\, ma\, e \end{cases}$$

- It comes

$$\begin{cases} R_x &= \xi\, ma \\ S_f &= W_f - \xi\, ma\, e/d \\ R_z &= \xi\, ma\, e/d \end{cases}$$



Part of braking forces on the front wheels

Anti dive conditions

- If no movement of the body in relation to the suspension is desired, i.e. no additional deflection of the springs (in relation to static compression = weight absorption), here is the condition:

$$\Delta S_f = \Delta W - \xi \, ma \, e/d = 0$$

- This leads to the anti-dive condition $\Delta S_f = 0$

$$\Delta W \;=\; ma\frac{h}{L} \qquad\qquad \boxed{\frac{e}{d} = \frac{h}{\xi \, L} = \tan\alpha}$$

- Front suspension anti-dive rate

$$\text{Anti dive rate } (\%) = \frac{e_f/d_f}{h/(\xi \, L)} \, 100$$

Anti dive conditions

- The anti-dive condition of the rear suspension is achieved in a similar manner to the front suspension.
- It is written down:

$$\boxed{\frac{e}{d} = \frac{h}{(1-\xi)\,L} = \tan\beta}$$

- Rear Anti Dive Rate

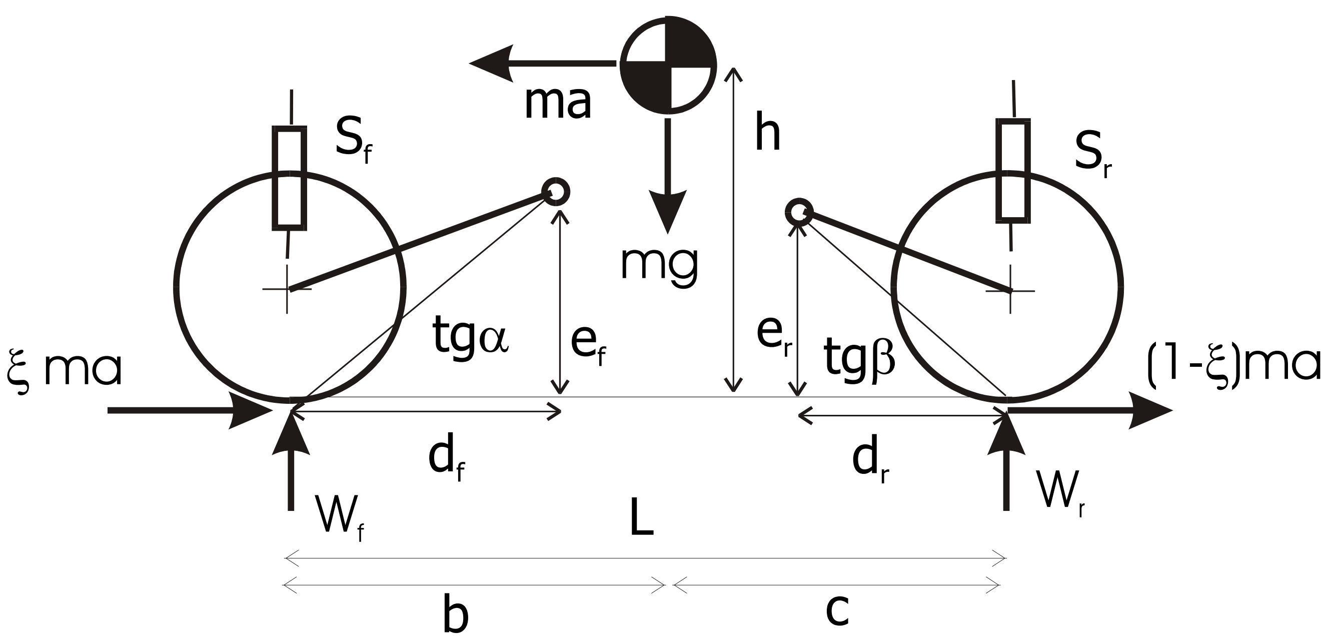
$$\text{Anti Dive Rate }(\%) = \frac{e_r/d_r}{h/((1-\xi)\,L)}\,100$$

Anti dive conditions

- The anti dive conditions for the front and rear axles yields

$$\boxed{\frac{e_f}{d_f} = \frac{h}{\xi\, L} = \tan\alpha}$$

$$\boxed{\frac{e_r}{d_r} = \frac{h}{(1-\xi)\, L} = \tan\beta}$$

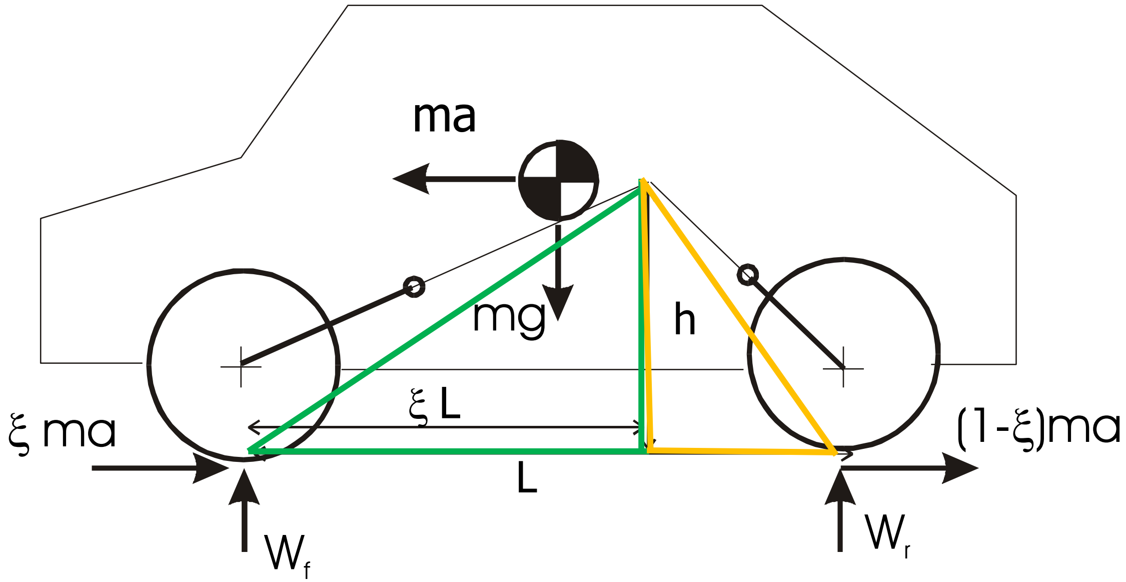


Anti dive conditions

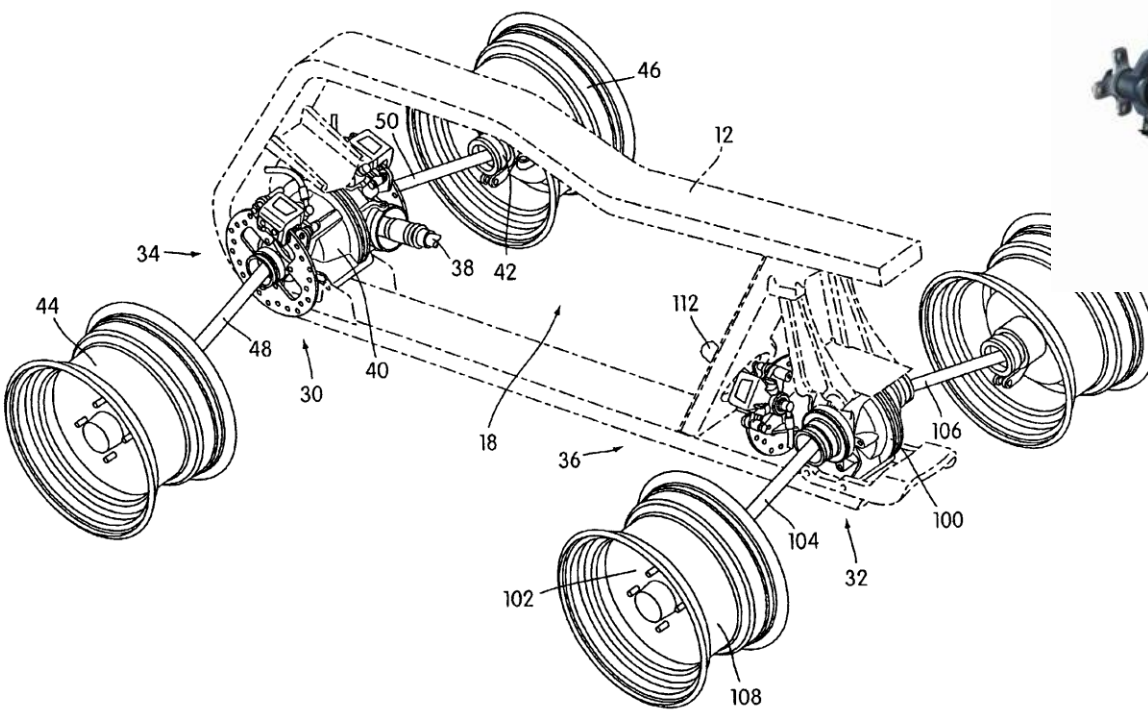
- Geometrical interpretation of the anti dive conditions

$$\boxed{\frac{e_f}{d_f} = \frac{h}{\xi\, L} = \tan\alpha}$$

$$\boxed{\frac{e_r}{d_r} = \frac{h}{(1-\xi)\, L} = \tan\beta}$$



In-board brakes



In-board brakes

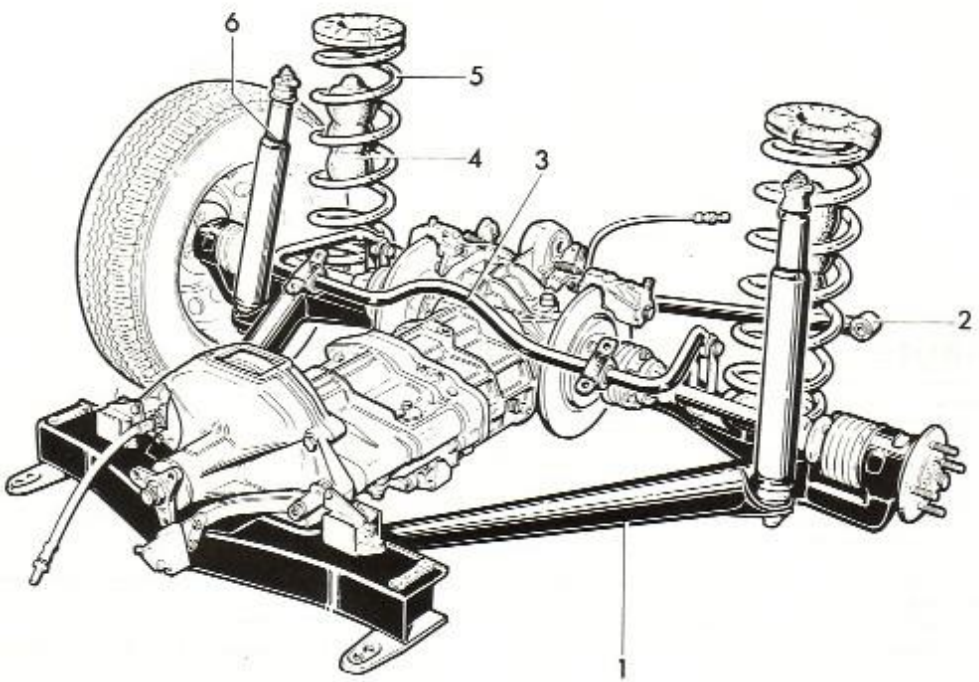


Fig. 11.1 Rear suspension components (Sec 1)

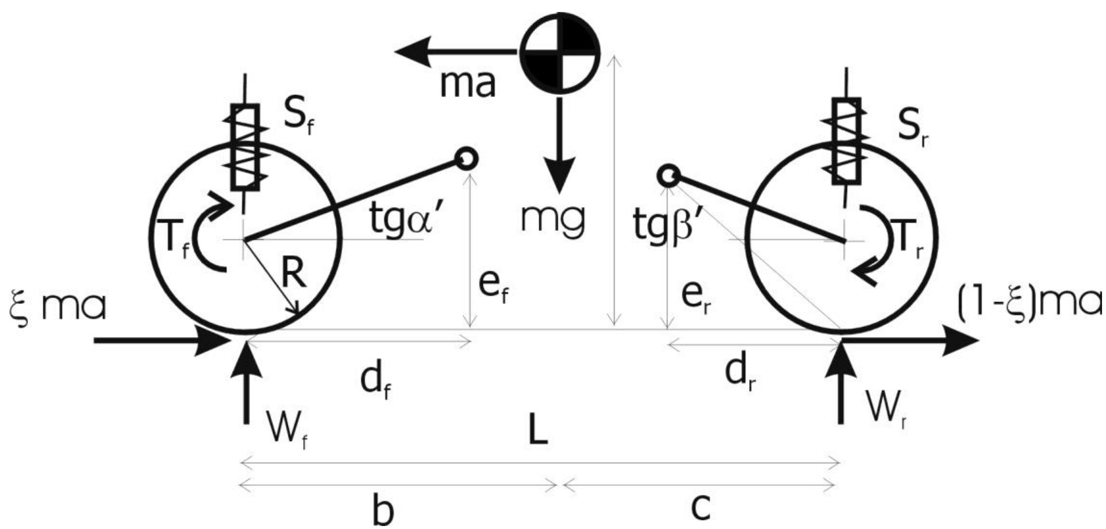
1 *Axle side tube* 3 *Anti-roll bar* 5 *Coil spring*
2 *Transverse link* 4 *Bump rubber* 6 *Shock absorber*

Alfa 75

Anti dive conditions: in-board brakes

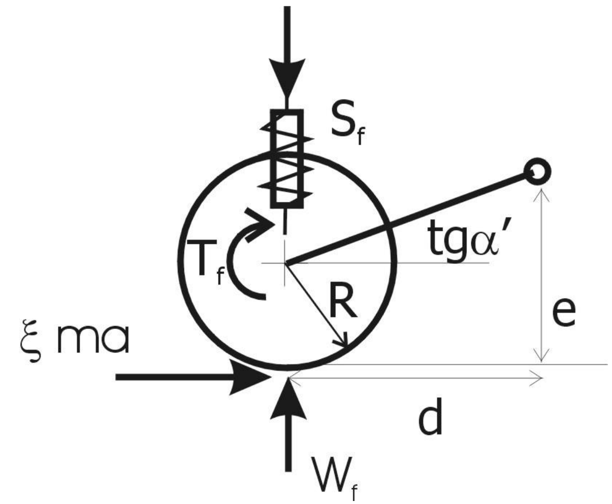
- In the case of inboard brakes (i.e. attached to the body), the braking torque must be considered in the free-body-diagram of the suspension. This changes the anti-dive conditions.
- Consider the torque developed by the brake system on front and rear wheels in the free body diagram

$$T_f = \xi\, ma\, R \qquad T_r = (1-\xi)\, ma\, R$$



Anti dive conditions: in-board brakes

- Free body diagram of the front suspension equipped with inboard brakes
- Equilibrium



$$T_f = \xi\, ma\, R \qquad \Delta W_f = ma\frac{h}{L}$$

$$T_f = \xi\, ma\, e - \Delta W_f\, d + \Delta S_f\, d$$

$$T_f = \xi\, ma\, e - ma\frac{h}{L}\, d + \Delta S_f\, d$$

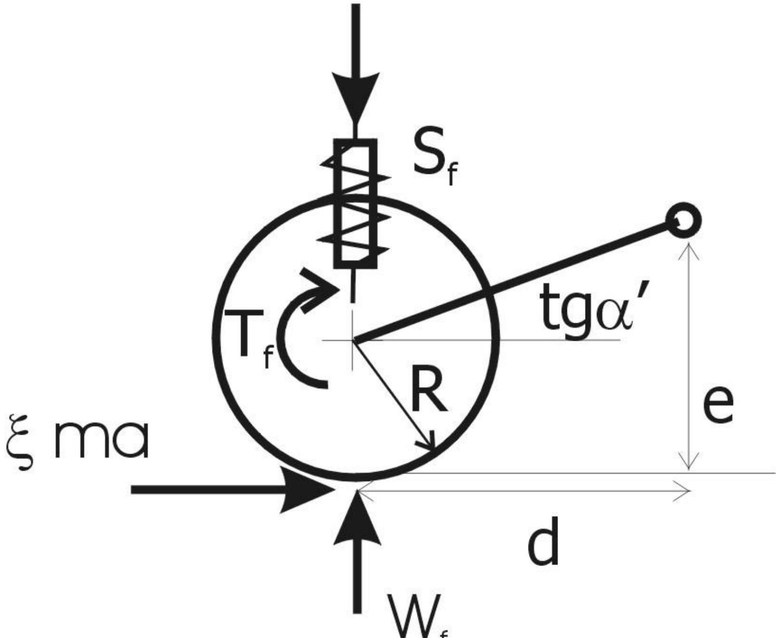
- The condition $\Delta S_f = 0$ gives the anti dive condition

$$\boxed{\frac{e - R}{d} = \frac{h}{\xi\, L} = \tan\alpha'}$$

Anti dive conditions: in-board brakes

- The anti dive rate on front axle writes

$$\text{Anti dive rate (\%)} = \frac{(e_f - R)/d_f}{h/(\xi\, L)}\, 100$$



Anti dive conditions: in-board brakes

- A similar development can be carried out for the rear axle and gives: The anti dive condition on the rear axle

$$\boxed{\frac{e_r - R}{d_r} = \frac{h}{(1-\xi)\,L} = \tan\beta'}$$

- The anti dive rate on front axle is

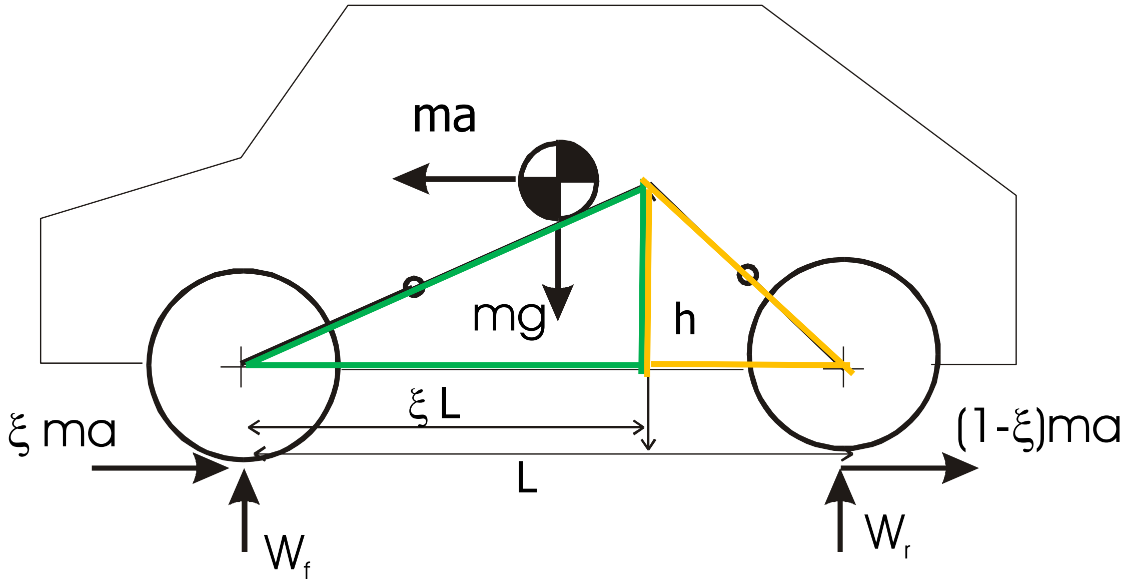
$$\text{Anti dive rate (\%)} = \frac{(e_r - R)/d_r}{h/((1-\xi)\,L)}\,100$$

Anti dive conditions

- Geometrical interpretation of the anti dive conditions for in-board brakes

$$\frac{e-R}{d} = \frac{h}{\xi\, L} = \tan\alpha'$$

$$\frac{e_r-R}{d_r} = \frac{h}{(1-\xi)\, L} = \tan\beta'$$



ANTI SQUAT GEOMETRY

ANTI-SQUAT

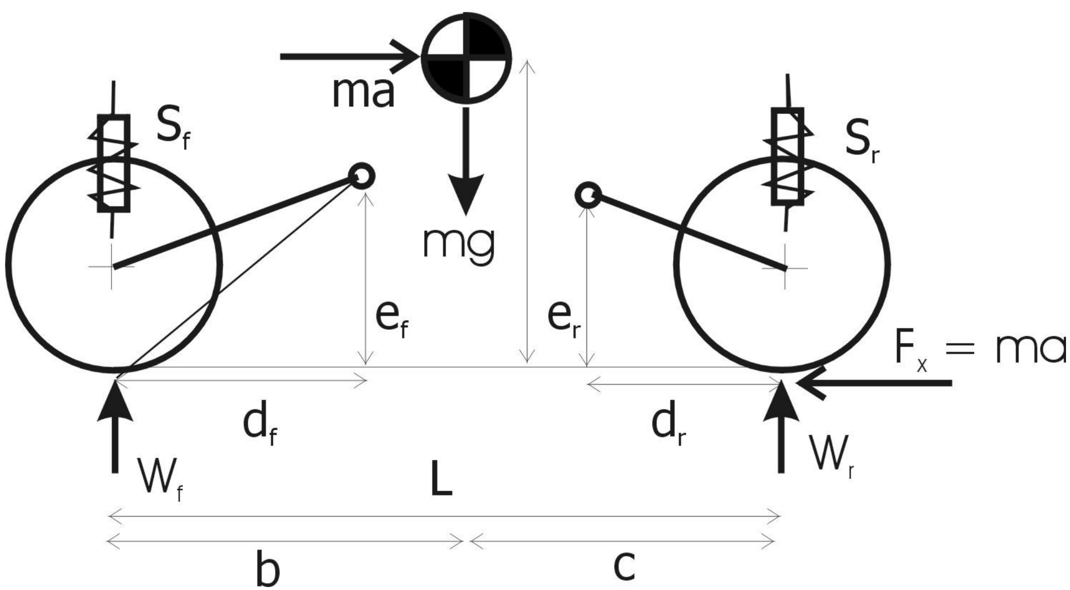
- Pitching occurs when the vehicle accelerates.
- The load transfer relieves the front axle which lifts up and loads the rear axle which bumps.
- The body is then subject to rearward pitching.
- This phenomenon can be reduced by a particular geometry of the suspension arms in the side-view plane.

Anti squat of rear-wheel-drive solid axles

- The situation is similar to the diving problem when braking except that here the whole longitudinal force may be generally developed by one axle, the live axle.
- For non-driven axle, the absence of longitudinal force does not allow generating a vertical reaction force that could counteract the load transfer. For this axle, compression of the suspension springs is unavoidable.
- In the case of rear wheel driven cars, the driving torque is external to the suspension in the case of independent suspension and must intervene in the free body diagram of the wheel.
- On the contrary, in the case of a rigid axle except for the Dion system, the driving torque coming from the axle is an internal torque and must therefore not be taken into account.

Anti squat of rear-wheel-drive solid axles

- Weight transfer during acceleration



$$W_f = mg\frac{c}{L} - ma\frac{h}{L}$$
$$W_r = mg\frac{b}{L} + ma\frac{h}{L}$$

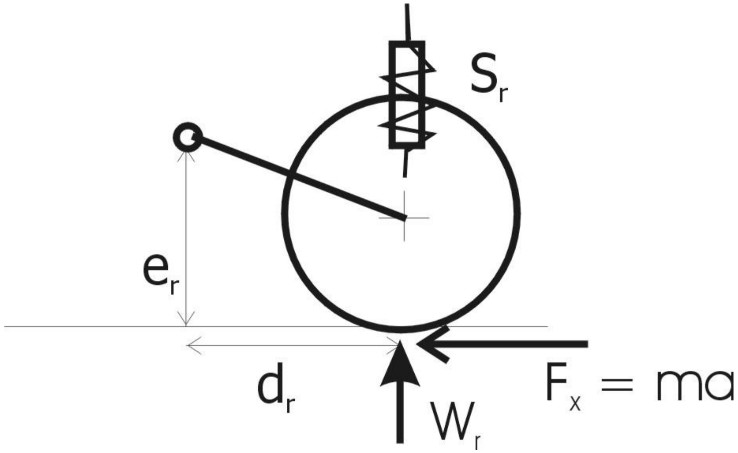
Anti squat of rear-wheel-drive solid axles

- Under static conditions, the compression of the springs is due to the part of the weight applied on the axle. For instance on the rear axle

$$S_r^{(0)} = mg\frac{b}{L}$$

- The equilibrium of the rear axle about its virtual reaction point

$$(mg\frac{b}{L} + ma\frac{h}{L})d - (S_r^{(0)} + \Delta S_r)\, d - F_x e = 0 \qquad F_x = ma$$



Anti squat of rear-wheel-drive solid axles

- The equilibrium of the rear axle about its virtual reaction point

$$mg\frac{b}{L}\,d + ma\frac{h}{L}d - \left(S_r^{(0)} + \Delta S_r\right)d - F_x e = 0$$

$$F_x = ma$$

$$S_r^{(0)} = mg\frac{b}{L}$$

- One gets the increase of the loads through the springs of the rear suspension:

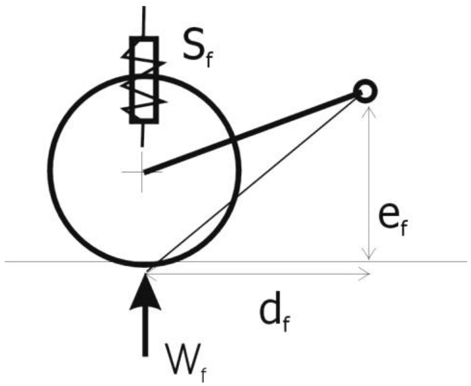
$$\Delta S_r = ma\frac{h}{L} - ma\frac{e}{d} = K_r\,\delta_r$$

- With K_r the stiffness of the spring and δ_r the compression of the spring

Anti squat of rear-wheel-drive solid axles

- For the front suspension, there is no longitudinal forces, so that the suspension lifts and the suspension bumps:

$$\Delta S_f = -ma\frac{h}{L} = K_f \delta_f$$



- The body pitch angle is given by

$$\theta_p = \frac{\delta_r - \delta_f}{L} = \frac{ma}{L}\left(\frac{1}{K_r}\frac{h}{L} - \frac{1}{K_r}\frac{e}{d} + \frac{1}{K_f}\frac{h}{L}\right)$$

- The pitch angle vanishes only if we have the relation

$$\frac{e}{d} = \frac{h}{L} + \frac{h}{L}\frac{K_r}{K_f}$$

Anti squat of rear-wheel-drive solid axles

- The first term refers to the anti-squat condition of the rear suspension, i.e., the absence of bump in the rear suspension.

$$\boxed{\frac{e}{d} = \frac{h}{L}}$$

- This condition is often partially met in practice:
  - Indeed usually h/L~ 0.2 (approx.) so that the suspension arm is 5 times longer than high!

- Anti-squat rate

$$\text{Anti-squat rate } (\%) = \frac{e_r/d_r}{h/L}\, 100$$

Anti squat of rear-wheel-drive solid axles

- Satisfying the condition with the second term means allowing the rear suspension to be raised slightly to the level of the front suspension.
- As generally $K_r/K_f \sim 1$, we have

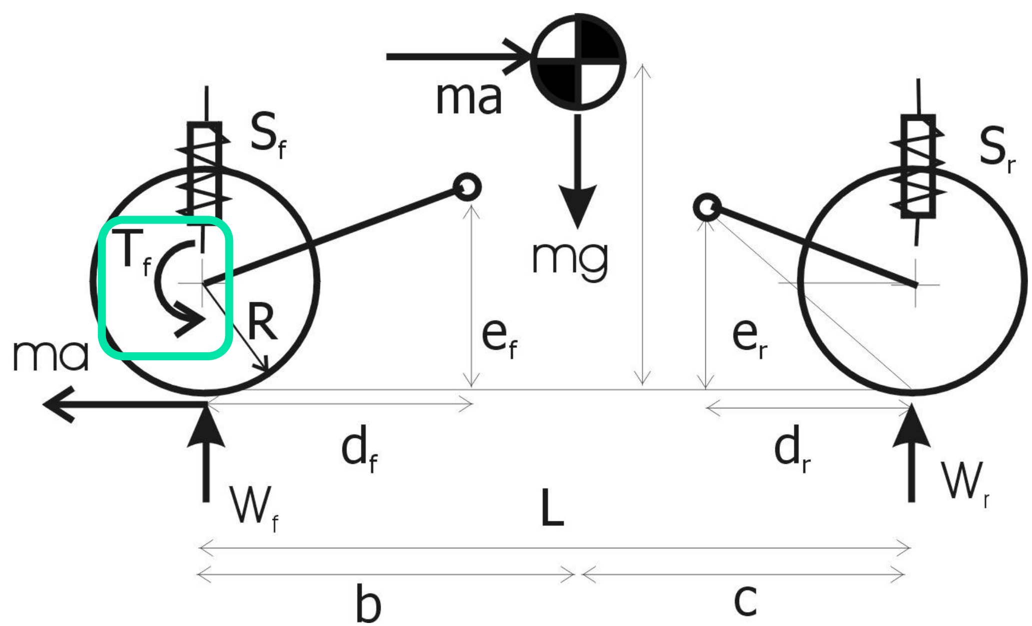
$$\frac{e}{d} \simeq 2\frac{h}{L} = \frac{h}{0,5 \cdot L}$$

- The condition corresponds to placing the IC on a line passing through the tire's contact center and a mid-wheelbase point at the height of the CG.
- Full compensation is rare, because it does not correspond to the anti-dive condition, or may be contrary to the road handling, for example: placing the IC above the center of the wheel produces oversteer roll (see lecture about influence of roll over understeer gradient).

Anti squat of front-wheel-drive independent suspension

- In the case of independent suspensions, it must be taken into account that the differential is now on the suspended mass and that the engine torque becomes an external load to the suspension.
- We have the following sketch

$$\boxed{T_f = F_x R}$$



Anti squat of front-wheel-drive independent suspension

- The propulsion torque

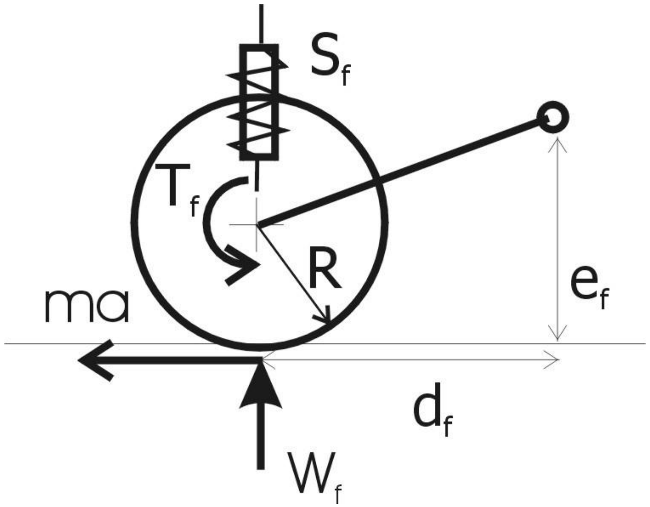
$$T_f = F_x\, R$$

- The equilibrium of the front suspension about the virtual reaction point

$$(S_f^{(0)} + \Delta S_f)\, d + F_x R - F_x e - (mg\frac{b}{L} - ma\frac{h}{L})d = 0$$

$$S_f^{(0)} = mg\frac{b}{L}$$

$$F_x = ma$$



Anti squat of front-wheel-drive independent suspension

- The equilibrium of the front suspension

$$(S_f^{(0)} + \Delta S_f)\, d + F_x R - F_x e - (mg\frac{b}{L} - ma\frac{h}{L})d = 0$$

- It comes

$$T_f = F_x\, R \qquad S_f^{(0)} = mg\frac{b}{L} \qquad F_x = ma$$

- Modification of the spring/damper effort of the suspension

$$\Delta S_f = ma\frac{e-R}{d} - ma\frac{h}{L} = K_f \delta_f$$

Anti squat of front-wheel-drive independent suspension

- The rear suspension can not benefit from any longitudinal force, so the change in the rear spring force due to weight transfer can not be modified

$$\Delta S_r = ma\frac{h}{L} = K_r\delta_r$$

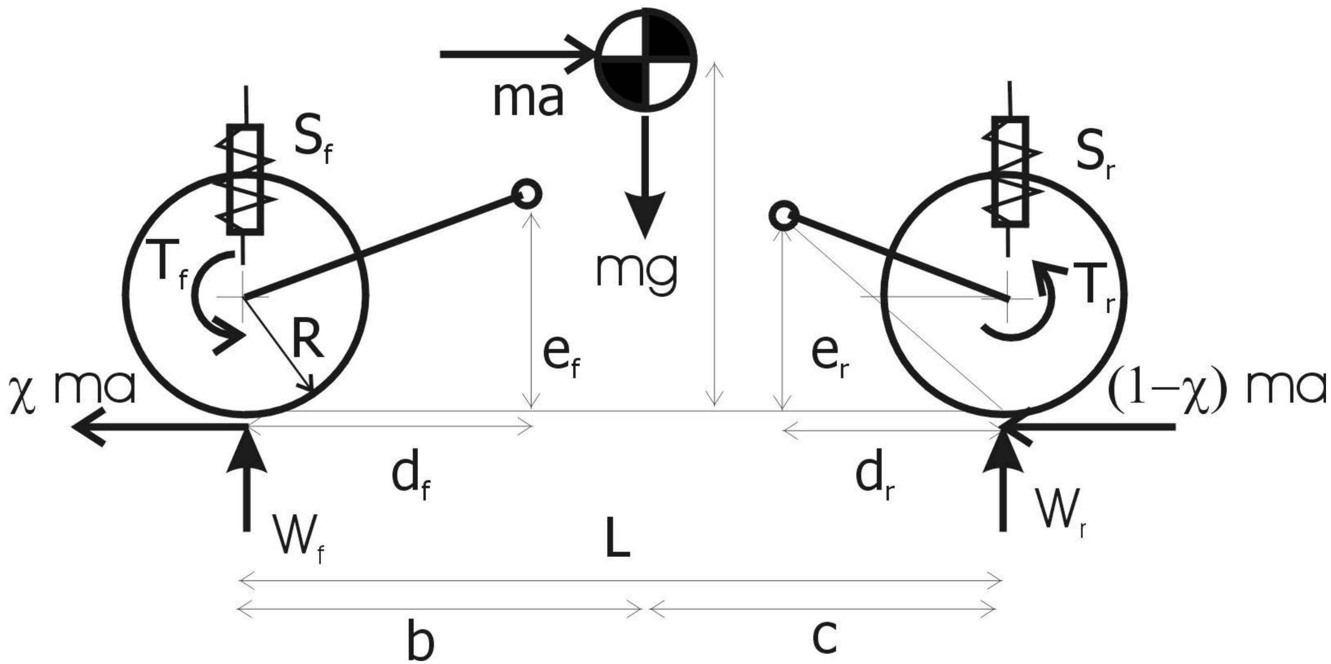
- The pitch angle θ:

$$\theta_p = \frac{\delta_r - \delta_f}{L} = \frac{ma}{L}\left(\frac{1}{K_r}\frac{h}{L} + \frac{1}{K_f}\frac{h}{L} - \frac{1}{K_f}\frac{e-R}{d}\right)$$

- The cancelling pitch gives the condition of complete anti-squat.

$$\boxed{\frac{e-R}{d} = \frac{h}{L} + \frac{h}{L}\frac{K_f}{K_r}}$$

Anti squat of 4-wheel-drive independent suspension



- Let assume a distribution of the tractive between the front and rear axles with ratios χ and (1-χ)

$$F_{xf} = \chi\, ma \quad F_{xf} = (1-\chi)\, ma$$

Anti squat of 4-wheel-drive independent suspension

- Equilibrium of the suspensions about their respective IC / virtual reaction enables to calculate the modifications of the loads in the spring/shock absorbers

$$\Delta S_f = ma\frac{e_f - R}{d_f} - \chi\, ma\frac{h}{L} = K_f \delta_f$$

$$\Delta S_r = ma\frac{e_r - R}{d_r} - (1-\chi)\, ma\frac{h}{L} = K_r \delta_r$$

- It comes the pitch angle of the car body

$$\theta_p = \frac{\delta_r - \delta_f}{L} = \frac{ma}{L}\left(\frac{1}{K_f}\frac{h}{L} - \frac{\chi}{K_f}\frac{e_f - R}{d_f} + \frac{1}{K_r}\frac{h}{L} - \frac{1-\chi}{K_r}\frac{e_r - R}{d_r}\right)$$