INTRODUCTION TO THE MECHANICS OF VEHICLE COLLISION AND CRASH

Pierre Duysinx

Research Center in Sustainable Automotive Technologies of University of Liege

Academic Year 2021-2022

Bibliography

- G. Genta. « Motor Vehicle Dynamics – Modeling and Simulation. World Scientific. 1997.
- M. Huang. Vehicle Crash Mechanics. CRC Press, 2002.
- J.-F. Debongnie. Cours de Véhicules Automobiles.

Introduction

- Safety issues: determine how collision against a fixed or a mobile obstacle will conduct to body damage and occupants injury

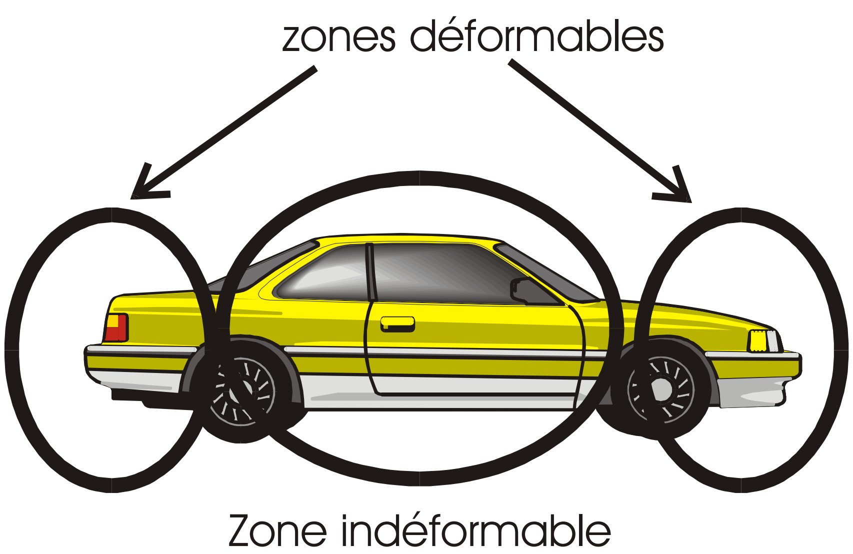
- Shocks introduce :
  - Large decelerations
  - Deformation of the vehicle structures

- The large accelerations are dangerous because
  - They lead to shocks for the passengers against the vehicle parts
  - They introduce internal efforts to human bodies which lead to important damages

- Car body deformations are dangerous because the possible contact of parts with the passengers and the intrusion of cutting and perforating parts

Introduction

- Consequence: configuration of the car with stiff compartment that remain intact and to preserve the passengers and with crushing zones at the front and at the back to absorb the energy and mitigate the decelerations



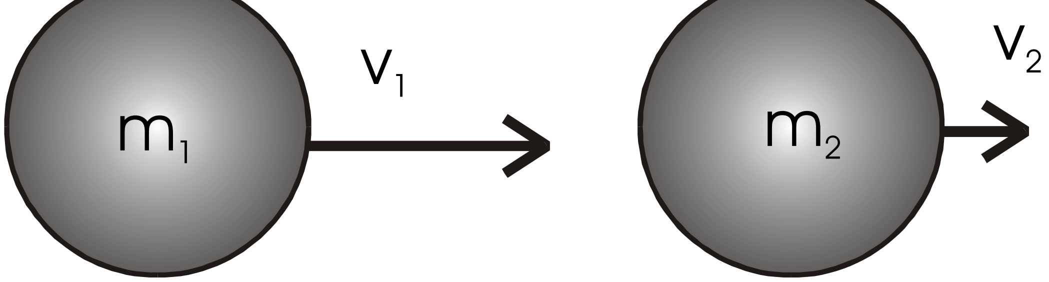
Introduction

- Conflicting targets of the vehicle design for crashworthiness is to limit simultaneously the deformation and the deceleration
- Because of the strongly conflicting nature of both criteria, one has to try to achieve compromise.

Source: http://www.euroncap.com/

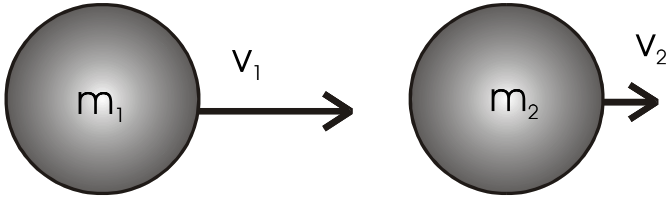
IMPULSIVE MODEL OF COLLISIONS

- Let consider two masses m_1 and m_2 with given velocities v_1 and v_2 in the same direction.
- Collision happens if $v_1 > v_2$
- Let's consider an impulsive model of the collision



Impulsive model of collisions

BEFORE THE SHOCK



- Linear total momentum

$$p = m_1 v_1 + m_2 v_2$$

- Average velocity (centre of mass)

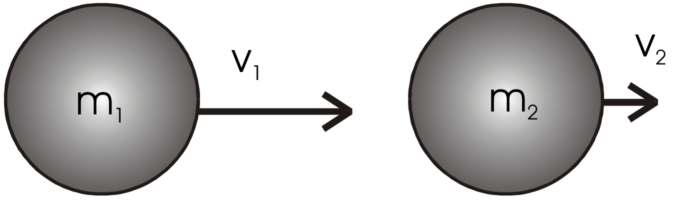
$$u = \frac{p}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

- Relative velocities w.r.t. the CM

$$w_1 = v_1 - u \quad w_2 = v_2 - u$$

Impulsive model of collisions

BEFORE THE SHOCK



- Relative linear momentum

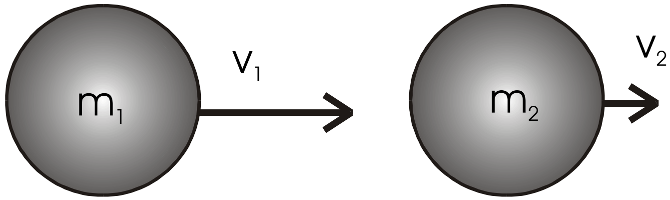
$$p_r = m_1\, w_1 + m_2\, w_2$$

Must be zero

$$\begin{aligned} p_r &= m_1\,(v_1 - u) + m_2\,(m_2 - u) \\ &= m_1\, v_1 + m_2\, v_2 - (m_1 + m_2)\, u \\ &= p - p = 0 \end{aligned}$$

Impulsive model of collisions

AFTER THE SHOCK



- The momentum is preserved since there is not external force

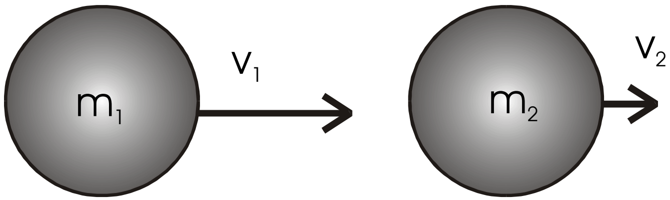
$$p = m_1\, v_1' + m_2\, v_2'$$

- The relative momentum remains zero

$$p_r = m_1\, w_1' + m_2\, w_2' = 0$$

Impulsive model of collisions

BEFORE THE SHOCK



- Kinetic energy

$$2T = m_1\, v_1^2 + m_2\, v_2^2 = m_1\, (u + w_1)^2 + m_2\, (u + w_2)^2$$
$$= (m_1 + m_2)\, u^2 + 2\, u\, (m_1\, w_1 + m_2\, w_2) + m_1\, w_1^2 + m_2\, w_2^2$$
$$= \frac{p^2}{m_1 + m_2} + m_1\, w_1^2 + m_2\, w_2^2 = 2T_0 + 2T_r$$

  - T_0 kinetic energy of the overall motion
  - T_r relative kinetic energy

Impulsive model of collisions

AFTER THE SHOCK

- The kinetic energy T_0 of the overall system is preserved

- No conservation of the relative kinetic energy T_r
  - Elastic shocks:

$$T_r = T_r'$$

  - Perfectly soft shocks

$$T_r' = 0$$

  - Soft shocks: a part of the kinetic energy is dissipated : ε the restitution coefficient

$$T_r' = \epsilon^2 \, T_r \quad 0 \leq \epsilon \leq 1$$

Impulsive model of collisions

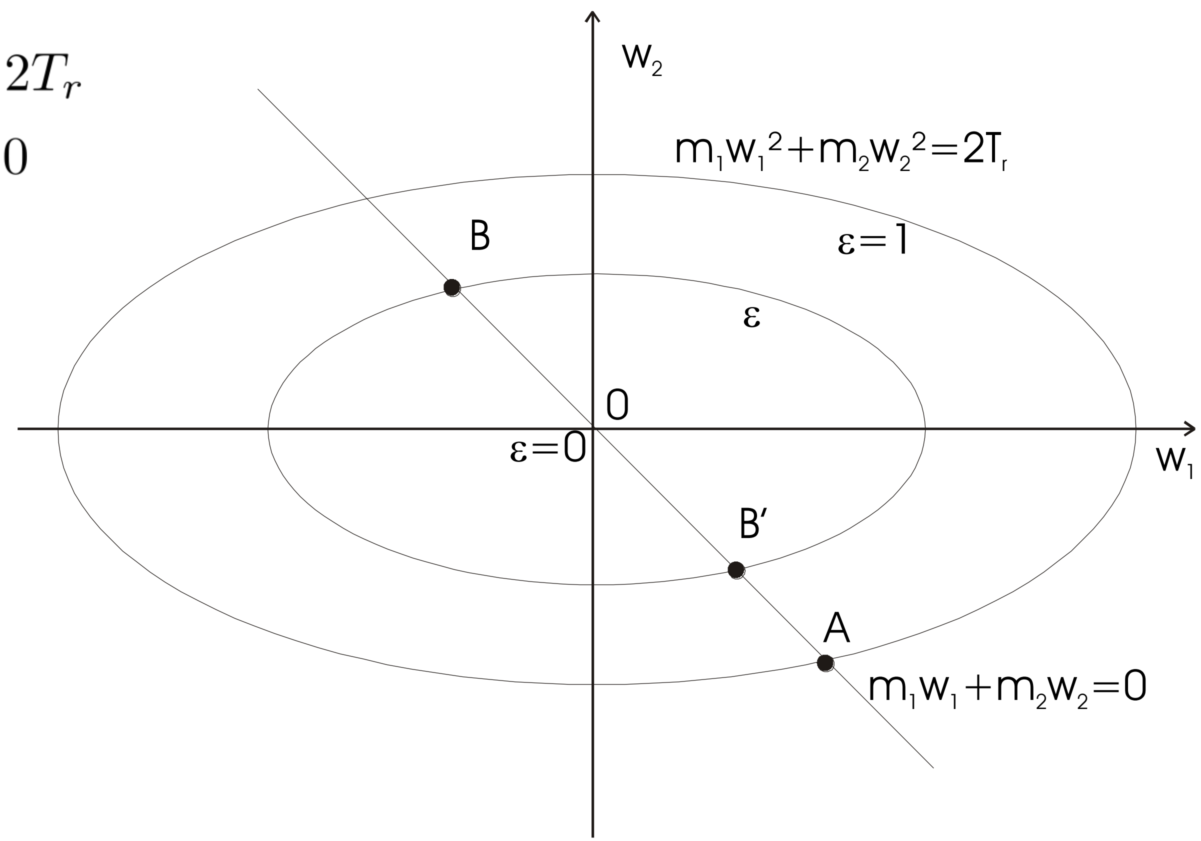
- For motor vehicle collisions, the value of ε is low, typically in the range of 0.05 to 0.2 in case of impacts with large permanent deformations
- The restitution coefficient ε depends on the relative velocity and can be higher in low speed collisions tending towards unity when no permanent deformation are left

Impulsive model of collisions: Solution

$$m_1\, w_1^2 + m_2\, w_2^2 = 2T_r$$
$$m_1\, w_1^2 + m_2\, w_2^2 = 0$$

$$w_1' = -\epsilon\, w_1$$
$$w_2' = -\epsilon\, w_2$$

$$\Delta T = (1 - \epsilon^2)\, T_r$$



Impulsive model of collisions: Solution

- Particular case study: vehicle against a stiff wall:

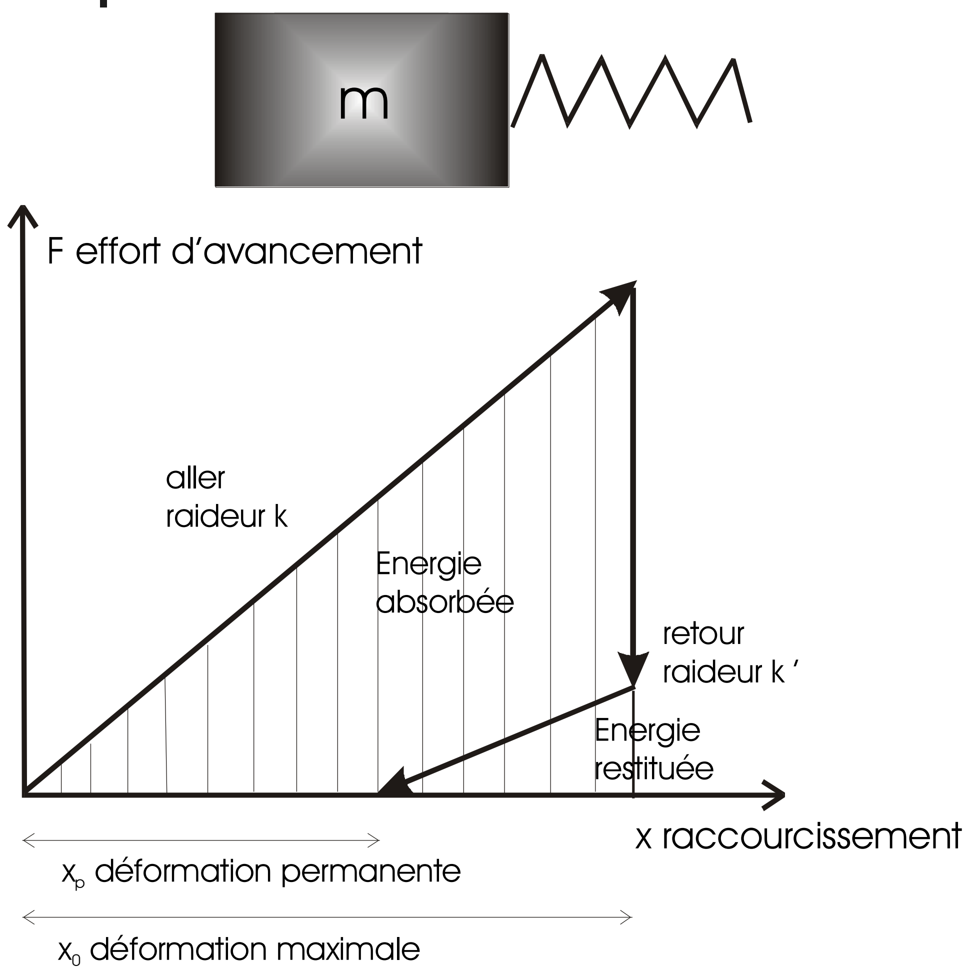
$$v_2 = 0 \qquad m_2 \gg m_1$$

- Solution

$$2T_0 = \frac{p_1^2}{m_1 + m_2} \approx 0 \qquad u = \frac{m_1\, w_1}{m_1 + m_2} \approx 0$$

$$T_r = \frac{1}{2}\, m_1\, w_1^2 \approx \frac{1}{2}\, m_1\, v_1^2 \qquad (1-\epsilon^2)T_r = (1-\epsilon^2)\frac{1}{2} m_1\, v_1^2$$

MODEL OF A FRONTAL CRASH AGAINST A RIGID WALL



- The vehicle is modelled as mass m representing the non deformable part and a crushing part aiming to absorb the energy.
- Assume a multi-linear model with two linear regions:
  - When crushing, the effort grows linearly with the deformation
  - Then an instantaneous dissipation phase at constant deformation
  - Finally an elastic return

Energy balance

- The balance of energies is the following:
  - The energy before crash $T = 1/2\, m\, v_0^2$
  - Stored energy at rest $W = 1/2\, k\, x_0^2$
  - The restituted energy $V = 1/2\, k'\, (x_0 - x_p)^2$

- When stopped, the kinetic energy is equal to the deformation energy: W=T

$$x_0^2 = \frac{m\, v_0^2}{k} \qquad \boxed{x_0 = \frac{v_0}{\omega}}$$

Where we have defined the pulsation $\omega = \sqrt{k/m}$

Energy balance

- The spring-back energy is transformed into kinetic energy

$$V = T' = \epsilon^2 \, T \qquad \frac{1}{2}\, m\, v_o'^2 = \frac{\epsilon^2}{2}\, m\, v_o^2 = \frac{1}{2}\, k' (x_0 - x_p)^2$$

It comes

$$\frac{\epsilon^2}{2}\, m\, \omega^2\, x_o^2 = \frac{1}{2}\, k' (x_0 - x_p)^2 \qquad \frac{\epsilon^2}{2}\, k\, x_o^2 = \frac{1}{2}\, k' (x_0 - x_p)^2$$

$$\epsilon \sqrt{k}\, x_o = \sqrt{k'} (x_0 - x_p)$$

So

$$\boxed{x_p = (1 - \epsilon \sqrt{k/k'})\, x_0 = (1 - \epsilon \sqrt{k/k'})\, v_0/\omega}$$

Equation of vehicle motion

FIRST PART OF THE SHOCK

- Equation of motion

$$m\,\ddot{x} + k\,x = 0$$

- Solution
  - displacement

$$x = x_0 \sin \omega t$$

  - velocity

$$\dot{x} = x_0\,\omega \cos \omega t$$

$$v_0 = \omega\, x_0$$

  - acceleration

$$\ddot{x} = -x_0 \omega^2 \sin \omega t = -\omega^2\, x$$

$$\gamma_0 = \omega^2\, x_0 = \omega\, v_0$$

Equation of vehicle motion

FIRST PART OF THE SHOCK

- End of the first part of the motion

$$t_c = \pi / 2\omega$$

SECOND PART OF THE MOTION

- Equation of motion

$$m\,\ddot{x} \;+\; k'\,(x - x_p) = 0 \qquad\qquad m\,(\ddot{x} - \ddot{x}_p) \;+\; k'\,(x - x_p) = 0$$

- Its solution

$$(x - x_p) = (x_0 - x_p)\cos\omega'(t - t_c)$$
$$\omega' = \sqrt{k'/m} = \omega\,\sqrt{k'/k}$$

Equation of vehicle motion

SECOND PART OF THE MOTION

- Given the value of x_p, the solution writes:

$$(x - x_p) = \epsilon \sqrt{\frac{k}{k'}}\, x_0 \ \cos \omega'(t - t_c) = \epsilon \sqrt{\frac{k}{k'}}\, \frac{v_0}{\omega} \ \cos \omega'(t - t_c)$$

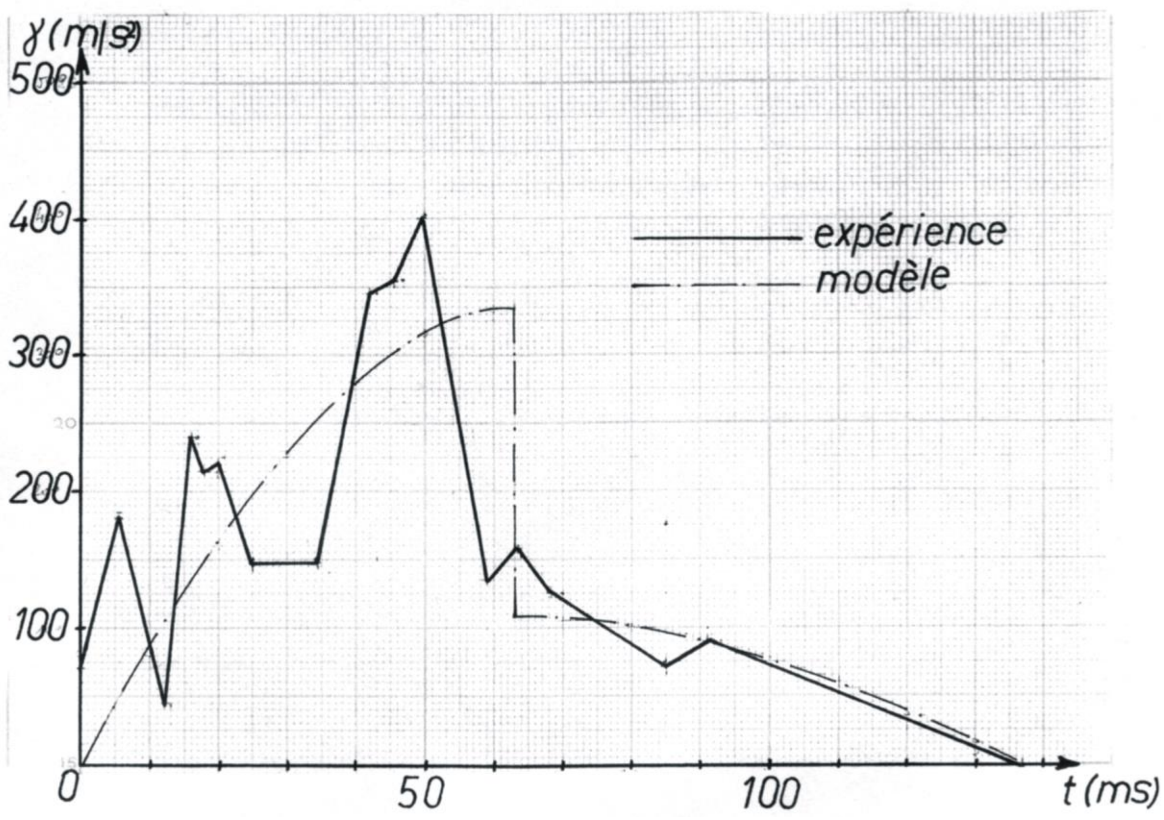
- One gets the displacement, the velocity, the acceleration

$$(x - x_p) = \epsilon \, \frac{v_0}{\omega'} \ \cos \omega'(t - t_c)$$

$$v = \epsilon \, v_0 \ \sin \omega'(t - t_c)$$

$$\gamma = -\epsilon \, \omega' \ v_0 \ \cos \omega'(t - t_c)$$

Parameter identification

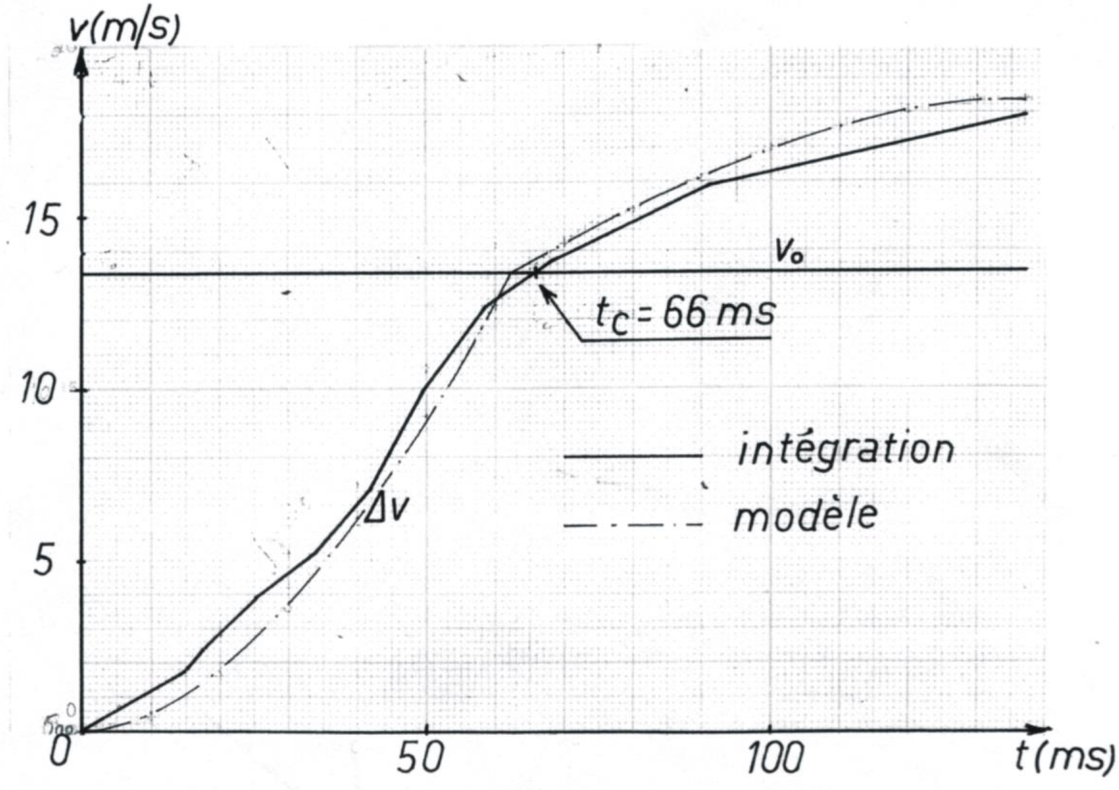


Accelerations as function of the time

Equation of vehicle motion

$$\Delta v = \int_0^t \gamma(\tau)\, d\tau$$

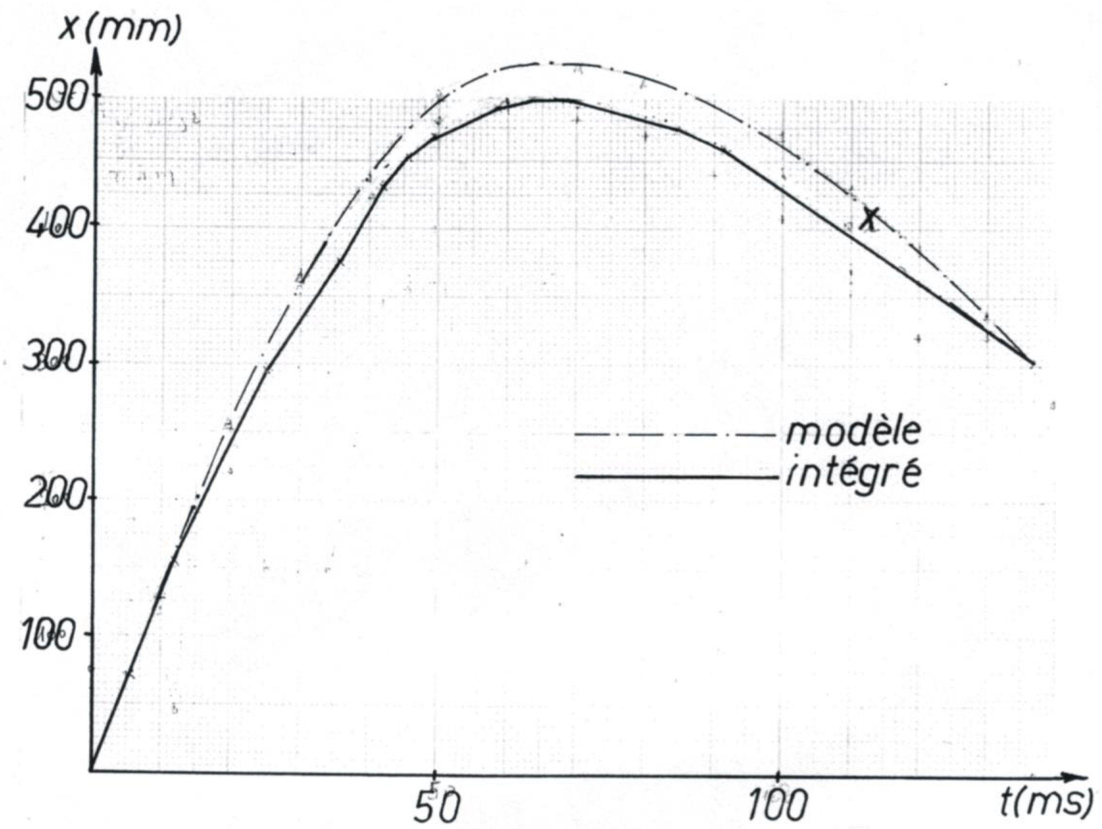
$$v = v_0 - \Delta v$$



Speed as function of the time (after time integration)

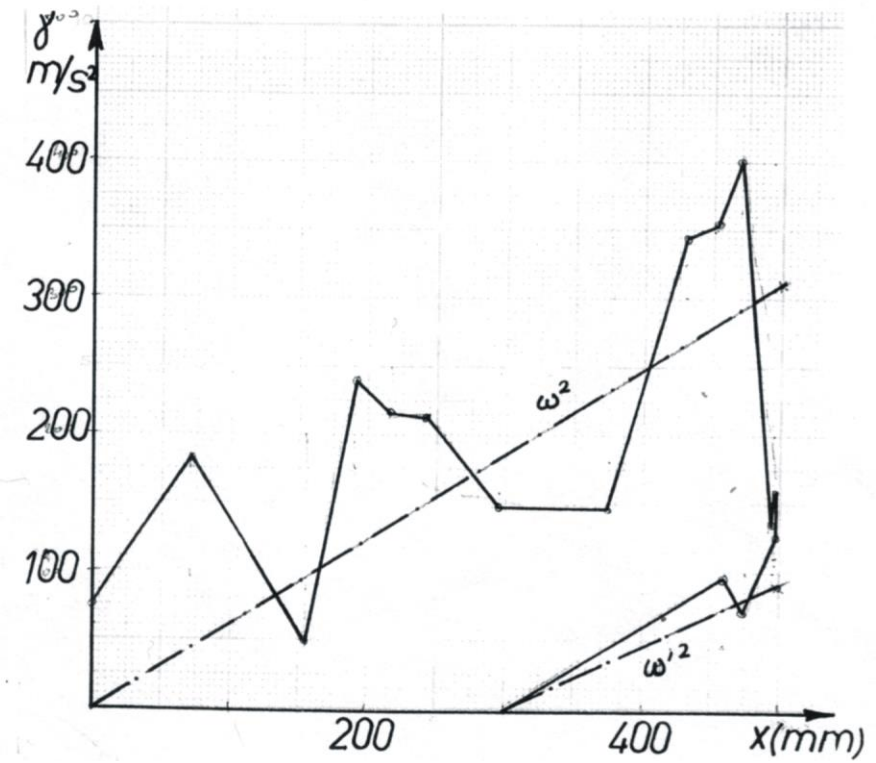
Equation of vehicle motion

$$x = \int_0^t v(\tau)\, d\tau$$



Displacement as function of the time (after time integration)

Equation of vehicle motion



$$\omega^2 = \frac{\sum_{v>0} \gamma \, x}{\sum_{v>0} x^2}$$

$$\omega'^2 = \frac{\sum_{v<0} \gamma \, (x - x_p)}{\sum_{v<0} (x - x_p)^2}$$

$$\omega^2 = 621,5 \qquad \omega'^2 = 449,3$$

$$\omega = 24,93 \; s^{-1} \qquad \omega' = 21,20 \; s^{-1}$$

Acceleration function of the position

Discussion

- It comes from the model that the maximum displacement and the accelerations are given by:

$$x_0 = v_0/\omega \qquad \gamma_{max} = \omega\, v_0$$

- So when reducing ω, one also reduces γ_{max}, but one increases the deformation, which is restricted by the length of the front of the car or at the price of increasing the length of the car. Conversely increasing ω increases the acceleration but gives a shorter deformation.
- One has to find a compromise between the maximum deformation and the deceleration rate

Discussion

- The following empirical rule is often mentioned

$$1km/h = 1cm = 1g$$

- Unfortunately it is incompatible with the physics as shown by the model:

$$v_0 = 0,2778\, m/S \qquad \omega = \gamma/v_0 = 35,51$$
$$x_0 = 0,01m \qquad \omega = v_0/x_0 = 27,78$$
$$\gamma = 9,81 m/s^2$$

- The model shows that :

$$\boxed{\gamma_{max} = \frac{v_0^2}{x_0}}$$

Discussion

- The rule

$$\gamma_{max} = \frac{v_0^2}{x_0}$$

means that the maximum acceleration is modified as:

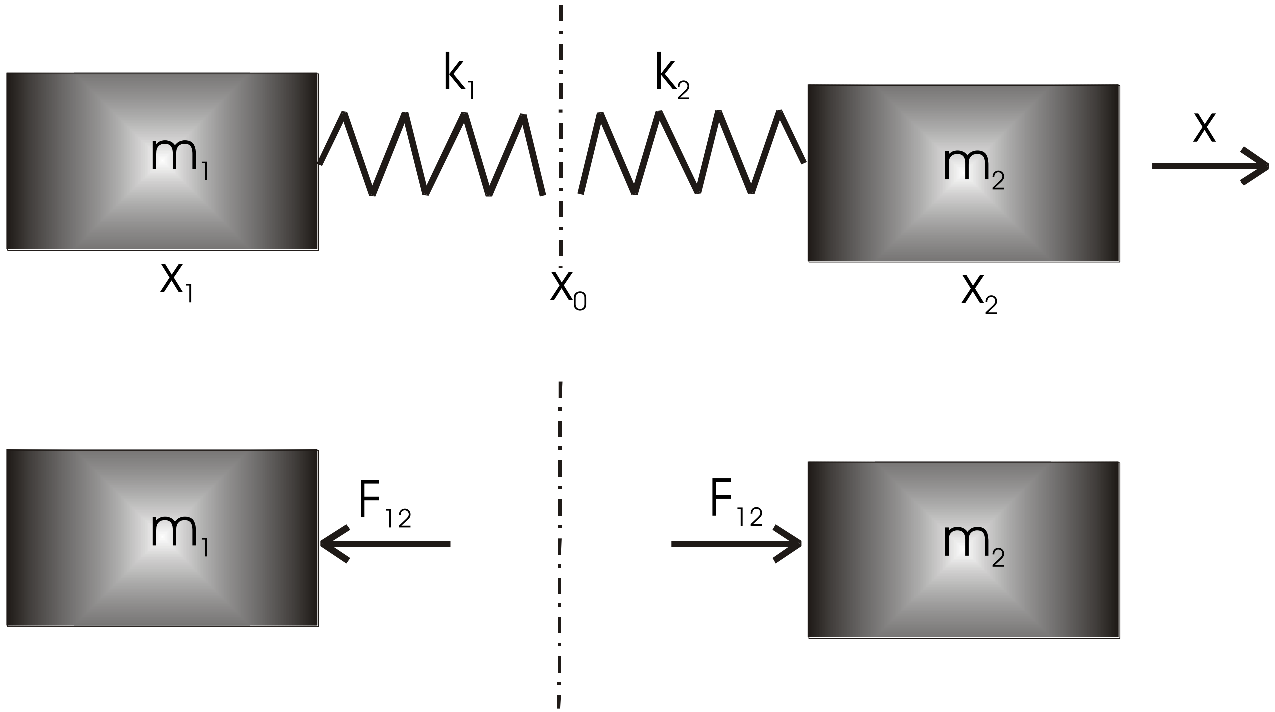
- The square of the initial velocity
- The inverse of the maximum deformation

This demonstrates the usefulness of sufficiently large crushing zones.

The following rule is consistent with the physics:

$$1km/h = 1cm = 0.8g$$

FRONTAL SHOCK BETWEEN 2 VEHICLES



Modelling of the situation

FRONTAL SHOCK BETWEEN 2 VEHICLES

- Let
  - x_0 the coordinate of the contact point between the two vehicles
  - x_1 and x_2 the coordinates of the centres of mass of the two vehicles
- The interaction forces between the two vehicles is given by:

$$F_{12} = k_1 \left(x_1 - x_0\right) = k_2(x_0 - x_2)$$

- So

$$\begin{cases} x_1 - x_0 = \frac{F_{12}}{k_1} \\ x_0 - x_2 = \frac{F_{21}}{k_2} \end{cases}$$

- Which gives

$$x_1 - x_2 = F_{12}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = F_{12}\frac{k_1 + k_2}{k_1.k_2} = \frac{F_{12}}{k_e} \qquad k_e = \frac{k_1.k_2}{k_1 + k_2}$$

FRONTAL SHOCK BETWEEN 2 VEHICLES

- The equation of motion

$$\begin{cases} m_1\ddot{x}_1 + k_e(x_1 - x_2) = 0 \\ m_2\ddot{x}_2 + k_e(x_2 - x_1) = 0 \end{cases}$$

- Let's look for a solution of the form

$$x_1 = X_1\, e^{st} \quad x_2 = X_2\, e^{st}$$

- One gets the conditions

$$\begin{cases} (m_1 s^2 + k_e)X_1 + k_e X_2 = 0 \\ -k_e X_1 + (m_2 s^2 + k_e)X_2 = 0 \end{cases}$$

FRONTAL SHOCK BETWEEN 2 VEHICLES

- The homogeneous system

$$\begin{aligned} (m_1 s^2 + k_e) X_1 - k_e X_2 &= 0 \\ -k_e X_1 + (m_2 s^2 + k_e) X_2 &= 0 \end{aligned}$$

admits solutions only of the determinant is zero

$$\begin{aligned} \Delta &= (m_1 s^2 + k_e).(m_2 s^2 + k_e) - k_e^2 \\ &= m_1 m_2 s^4 + k_e (m_1 + m_2) s^2 \\ &= m_1 m_2 s^2 \left(s^2 + \frac{k_e (m_1 + m_2)}{m_1 . m_2} \right) \end{aligned}$$

- The solutions are

$s = 0 \quad \text{(deux fois)}$

$s = \pm i \, \omega \quad \text{avec} \quad \omega^2 = \frac{k_e}{m_e} \qquad m_e = \frac{m_1 . m_2}{m_1 + m_2}$

FRONTAL SHOCK BETWEEN 2 VEHICLES

- The roots correspond to the following motion modes:

$$s = 0 \qquad X_1 = X_2$$

Rigid body mode

$$s = \pm i\,\omega \qquad \begin{aligned} (k_e - m_1\omega^2)X_1 &= k_e X_2 \\ -k_e X_1 &= (k_e - m_2\omega^2)X_2 \end{aligned}$$

$$-m_1\omega^2 X_1 = -m_2\omega^2 X_2$$

$$X_2 = -\frac{m_1}{m_2}X_1$$

FRONTAL SHOCK BETWEEN 2 VEHICLES

- One gets the final solution of the motion:

$$\begin{aligned} x_1 &= A_1 \sin \omega t + A_2 \cos \omega t + A_3 t + A_4 \\ x_2 &= -\frac{m_1}{m_2} A_1 \sin \omega t - \frac{m_1}{m_2} A_2 \cos \omega t + A_3 t + A_4 \end{aligned}$$

- Let's notice that it satisfies to:

$$m_1 x_1(t) + m_2 x_2(t) = (m_1 + m_2)(A_3 t + A_4)$$

- This means that A_3 and A_4 governs the motion of the centre of gravity of the overall system. If one is observing the motion in a reference travelling at the same speed as the centre of gravity of the centre of mass, one gets A_3=0 and A_4=0

FRONTAL SHOCK BETWEEN 2 VEHICLES

- In the frame work of the centre of gravity, we have:

$$\begin{aligned}\dot{x}_1 &= A_1\omega\cos\omega t + A_2\omega\sin\omega t\\ \dot{x}_2 &= -\frac{m_1}{m_2}A_1\omega\cos\omega t + \frac{m_1}{m_2}A_2\omega\sin\omega t\end{aligned}$$

- In time t=0, one knows the initial velocities of the two vehicles

$$\dot{x}_1(t=0) = w_1 \quad \text{et} \quad \dot{x}_2(t=0) = w_2$$

- With w_1 and w_2 the relative velocities of the vehicles with respect to the centre of mass

$$m_1 w_1 + m_2 w_2 = 0$$

- It comes

$$\dot{x}_1(t=0) = A_1\omega = w_1 \quad \text{et}$$

$$\dot{x}_2(t=0) = -\frac{m_1}{m_2}A_1\omega = -\frac{m_1}{m_2}w_1$$

FRONTAL SHOCK BETWEEN 2 VEHICLES

- If we further choose the reference frame on each vehicle so that

$$x_i(0) = 0 \quad i = 1, 2$$

- One has

$$A_2 = 0$$

- The motion is thus described by the following equations

$$x_1 = \frac{w_1}{\omega} \sin \omega t \qquad x_2 = -\frac{m_1}{m_2} \frac{w_1}{\omega} \sin \omega t$$

$$\dot{x}_1 = w_1 \cos \omega t \qquad \dot{x}_2 = -\frac{m_1}{m_2} w_1 \cos \omega t$$

$$\ddot{x}_1 = -\omega w_1 \sin \omega t \qquad \ddot{x}_2 = +\omega \frac{m_1}{m_2} w_1 \sin \omega t$$

FRONTAL SHOCK BETWEEN 2 VEHICLES

- Conclusion

$$x_1 = \frac{w_1}{\omega}\sin\omega t \qquad x_2 = -\frac{m_1}{m_2}\frac{w_1}{\omega}\sin\omega t$$

$$\dot{x}_1 = w_1\cos\omega t \qquad \dot{x}_2 = -\frac{m_1}{m_2}w_1\cos\omega t$$

$$\ddot{x}_1 = -\omega w_1\sin\omega t \qquad \ddot{x}_2 = +\omega\frac{m_1}{m_2}w_1\sin\omega t$$

- The study of the frontal shocks can be mapped back on the solution of a vehicle against a rigid wall that would follow the centre of mass of the two-mass system.