MECA0063 : Introduction to Vehicle Stability Control

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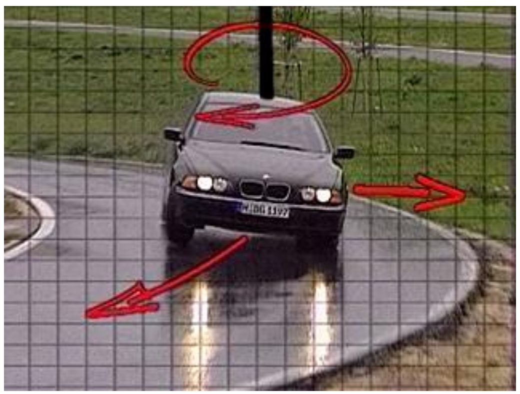
UNDERSTANDING THE ESP
Electronic Stability Program

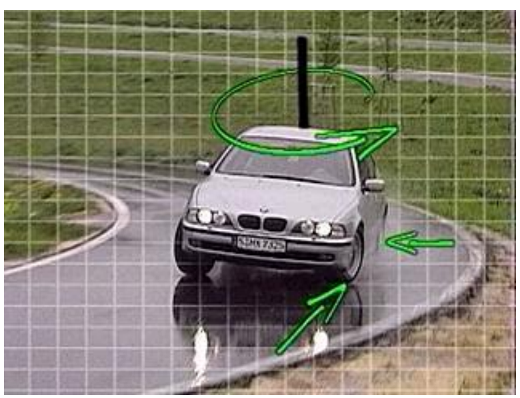
- DESCRIPTION OF THE ESP SYSTEM AND ITS OPERATION PRINCIPLES

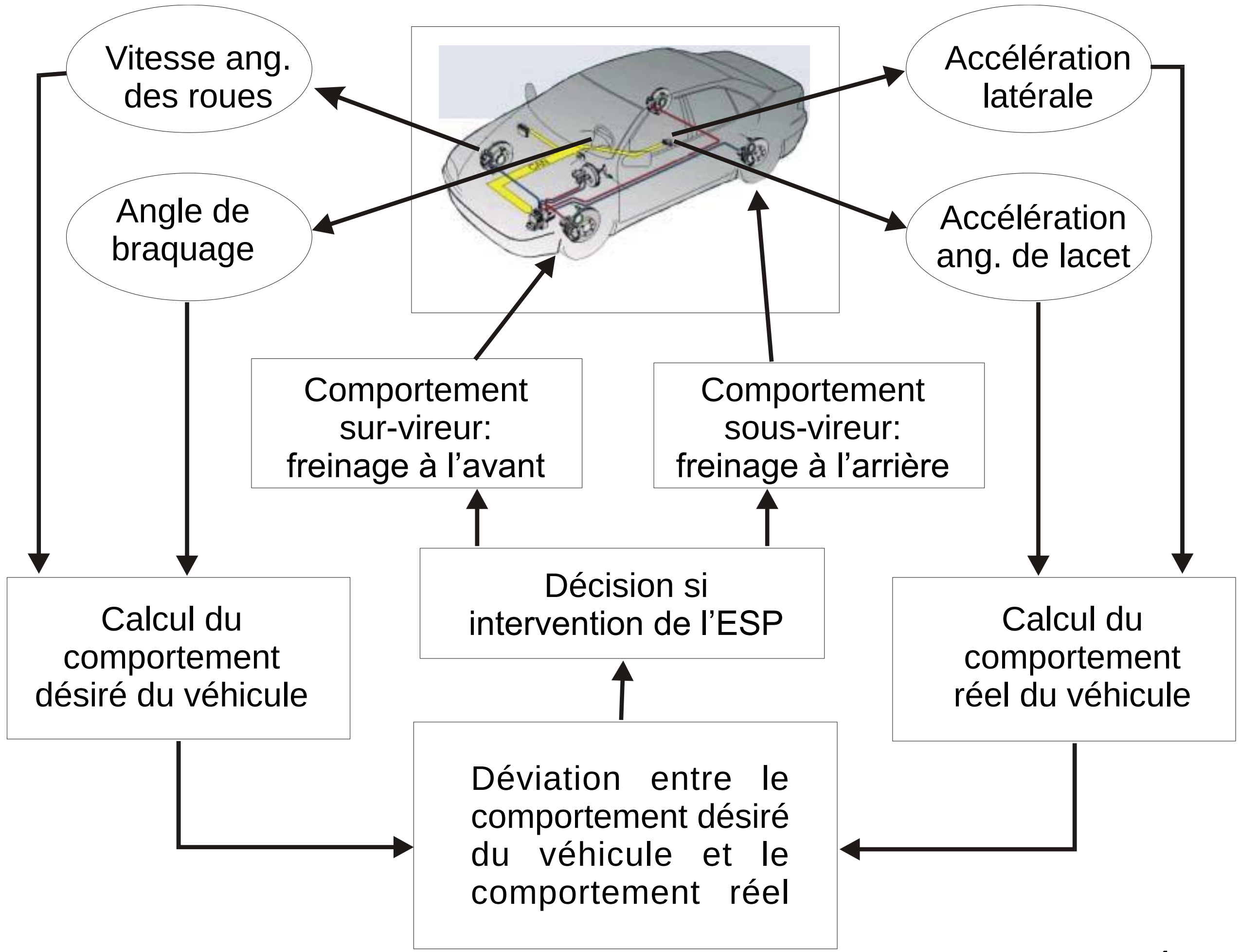
- SIMPLE MODEL:
  - 2-dof vehicle model
  - Equations of the transient behaviour (lateral equilibrium)
  - Extension of the model to include individual wheel brakings

- TOWARDS THE NUMERICAL SIMULATION :
  - The automobile as mechatronic system
    - Finite Element Multibody (SAMCEF-MECANO)
    - Integration of control systems
      - Sensors, control algorithms, actuators

FUNDAMETALS OF ESP



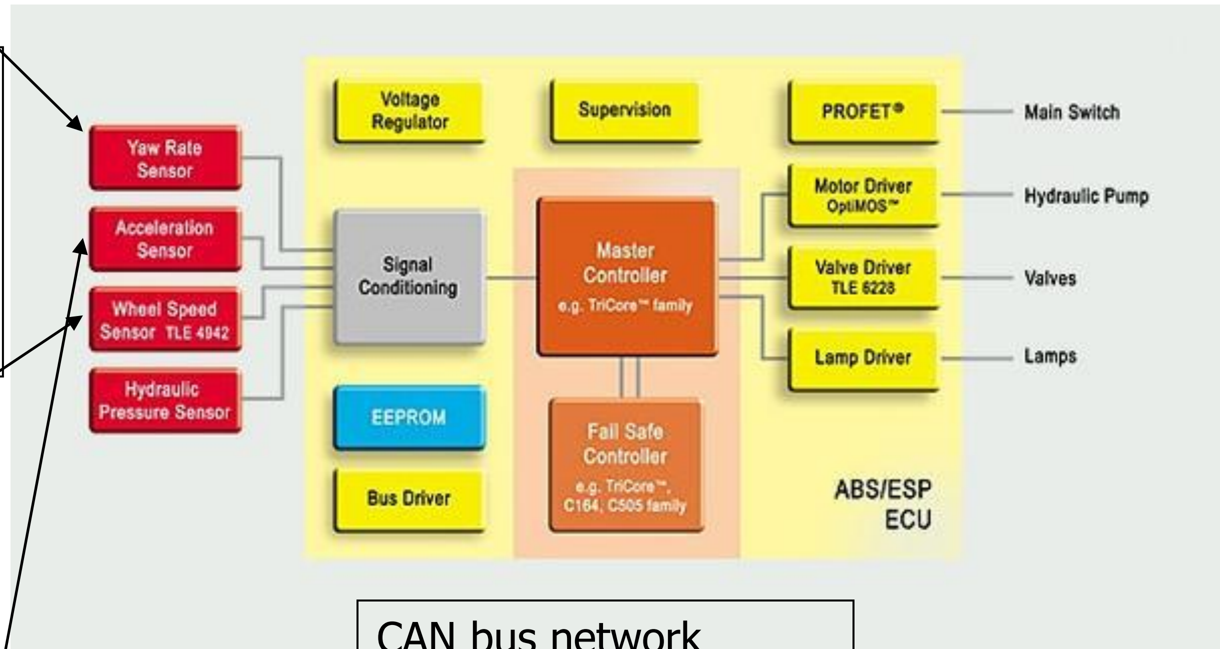




ESP = $(ABS+TCS)^2$

INSTRUMENTATION OF ESP

A micromechanical gyroscope (polySi surface micro machined MEMS) detects the rotations about the vehicle vertical axis



Miniaturized Sensors of the wheel rotation speed (based Hall effect)

CAN bus network

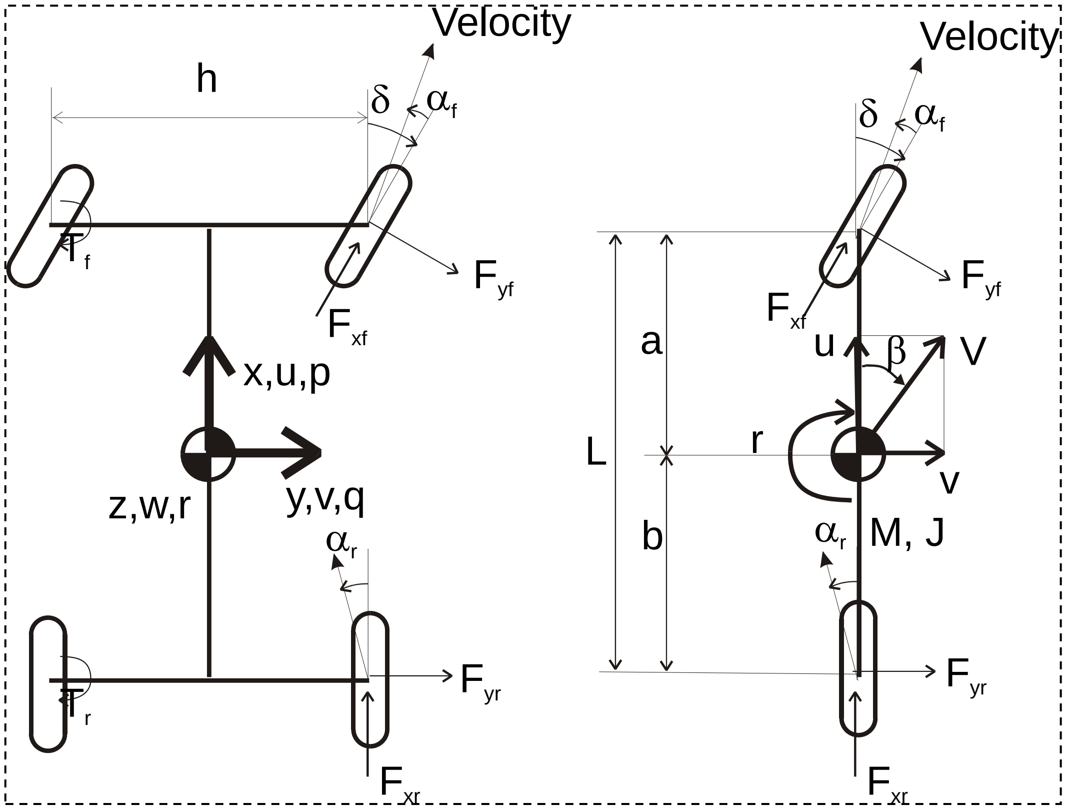
A highly sensitive MEMS accelerometer (polySi surface micromachined MEMS) Record the lateral acceleration

Steering wheel rotation measured by a contact less sensor

Source: Bosch

MODEL OF THE VEHICLE DYNAMICS

MODEL OF THE VEHICLE:



Bicycle model

<u>Assumptions:</u>

- Small slip and steering angles
- Vehicle is infinitely rigid in pitch q=0 and heave w=0
- No roll p=0
- 2 dof model β and r

MODEL OF THE VEHICLE DYNAMICS

- Newton-Euler equilibrium equation in the non inertial reference frame of the vehicle body

$$\sum \vec{F} = m\frac{d}{dt}\vec{V} + m\vec{\omega} \times \vec{V}$$
$$\sum \vec{T} = \frac{d}{dt}(J\vec{\omega}) + \vec{\omega} \times (J\vec{\omega})$$

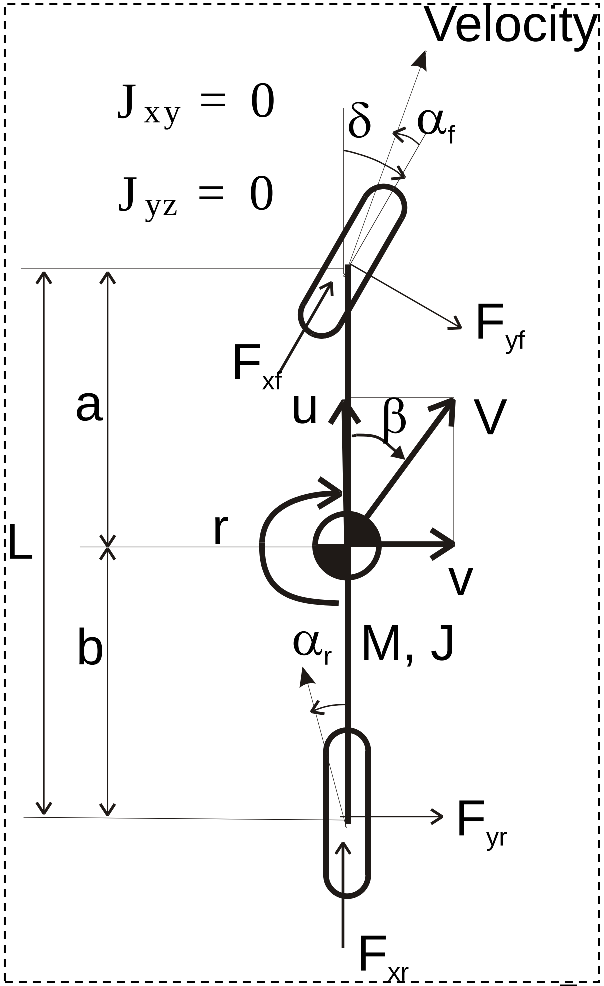
- Model with 2 dof β & r

$$\vec{V} = [u\ v\ 0]^T$$
$$\vec{\omega} = [0\ 0\ r]^T$$

- Equilibrium equations in F_y and M_z :

$$F_y = m(\dot{v} + ru)$$
$$N = J_{zz}\dot{r}$$

- Operating forces
  - Tyre forces
  - Aerodynamic forces (can be neglected here)



MODEL OF THE VEHICLE DYNAMICS

- The equilibrium equations along F_y and M_z writes:

$$\begin{aligned} m(\dot{v}+ru) &= F_{yr}+F_{xf}\sin\delta+F_{yf}\cos\delta \\ J_{zz}\dot{r} &= -F_{yr}\,c+F_{xf}\sin\delta\,b+F_{yf}\cos\delta\,b+T_{zf}+T_{zr} \end{aligned}$$

- Use small angles assumptions (linearized motion)

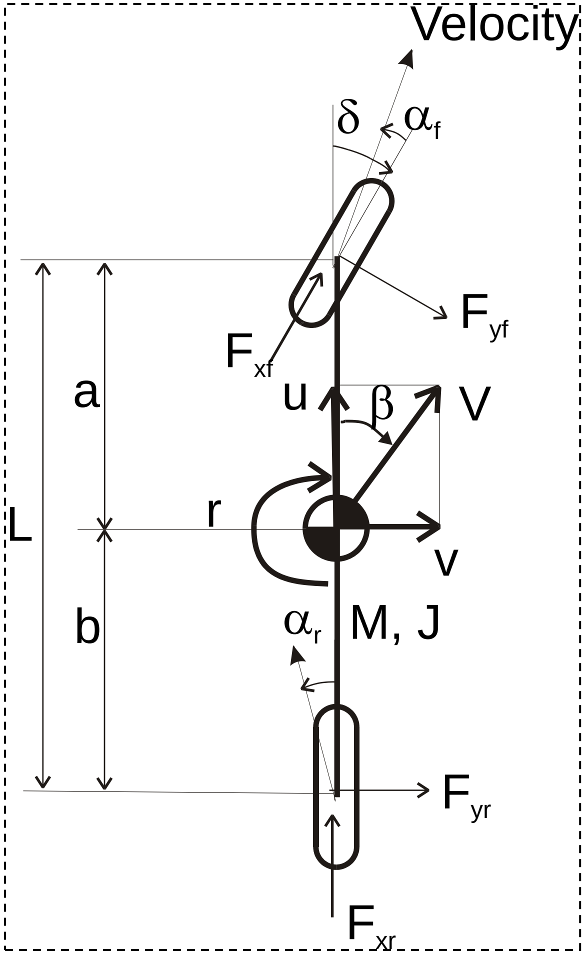
$$\beta\in[0^\circ,15^\circ] \quad \beta\simeq v/u \quad u = V\cos\beta\simeq V$$

$$\cos\delta\simeq 1 \quad \sin\delta\simeq\delta. \quad v = V\sin\beta\simeq V\;\beta$$

- Linearized equations of motion of the 2 dof model

$$\begin{aligned} mV(\dot{\beta}+r) &= F_{yr}+F_{yf}+F_{xf}\;\delta \\ J_{zz}\dot{r} &= -F_{yr}\,c+F_{yf}\,b+F_{xf}\delta\,b+T_{zf}+T_{zr} \end{aligned}$$

$$\begin{aligned} mV(\dot{\beta}+r) &= F_{yr}+F_{yf} \\ J_{zz}\dot{r} &= -F_{yr}\,c+F_{yf}\,b \end{aligned}$$



MODEL OF THE VEHICLE DYNAMICS

- Compatibility of slip angles and velocities:

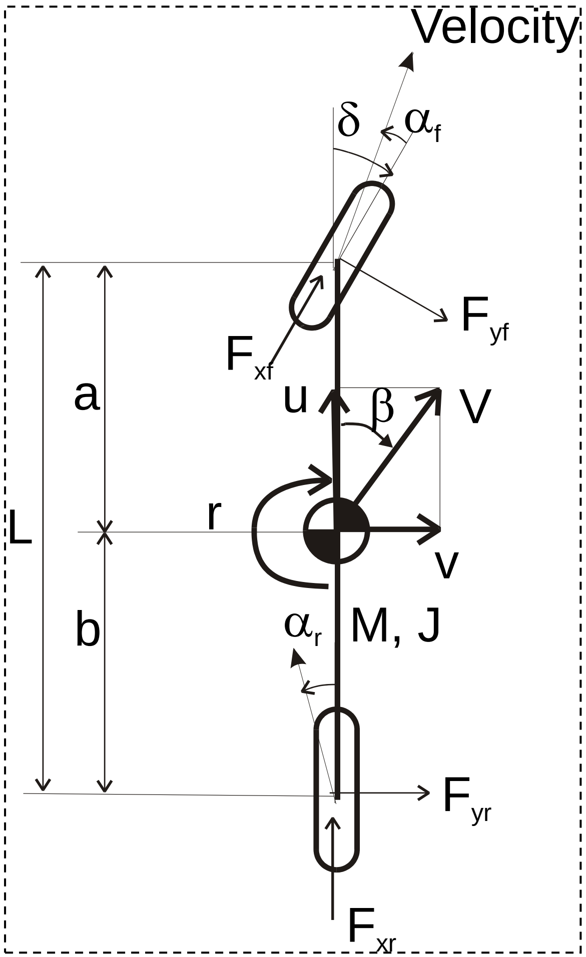
$$\tan(\delta - \alpha_f) = \frac{br + v}{u}$$
$$\tan \alpha_r = \frac{cr - v}{u}$$

- Use small angles assumption

$$\alpha_f \simeq \delta - \frac{br + v}{u}$$
$$\alpha_r \simeq \frac{cr - v}{u}$$
$$\beta \simeq v/u$$

- And so

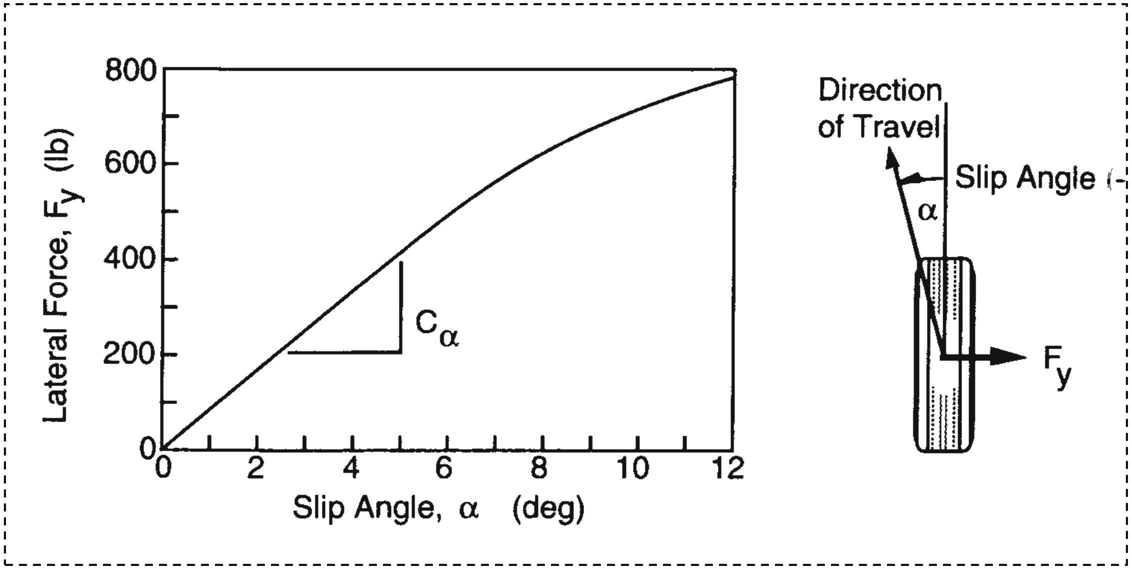
$$\alpha_f \simeq \delta - \frac{br}{V} - \beta$$
$$\alpha_r \simeq \frac{cr}{V} - \beta$$



MODEL OF THE VEHICLE DYNAMICS

Constitutive equations of the tyres

$$F_{yf} = C_{\alpha f}\alpha_f \qquad F_{yr} = C_{\alpha r}\alpha_r$$



Source: Gillespie (fig 6.2)

MODEL OF THE VEHICLE DYNAMICS

- Introducing the constitutive equations into the equilibrium equations

$$\begin{aligned} mV(\dot{\beta}+r) &= F_{yr} + F_{yf} \\ J_{zz}\dot{r} &= -F_{yr}\, c + F_{yf}\, b \end{aligned}$$

- It comes

$$\begin{aligned} mV(\dot{\beta}+r) &= C_{\alpha r}\alpha_r + C_{\alpha f}\alpha_f \\ J_{zz}\dot{r} &= -C_{\alpha r}\alpha_r\, c + C_{\alpha f}\alpha_f\, b \end{aligned}$$

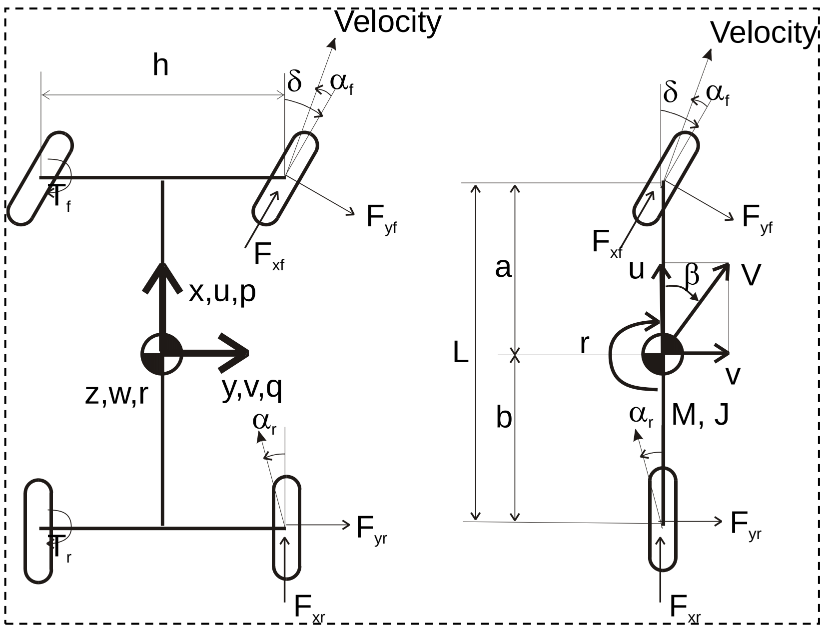
- Substitute the slip angles value by their value in terms in the velocities from compatibility relations

$$\begin{aligned} mV(\dot{\beta}+r) &= C_{\alpha r}(\frac{cr}{V} - \beta) + C_{\alpha f}(\delta - \frac{br}{V} - \beta) \\ J_{zz}\dot{r} &= -C_{\alpha r}(\frac{cr}{V} - \beta)\, c + C_{\alpha f}(\delta - \frac{br}{V} - \beta)\, b \end{aligned}$$

MODEL OF THE VEHICLE DYNAMICS

- Standard form of motion equations for the vehicle dynamics

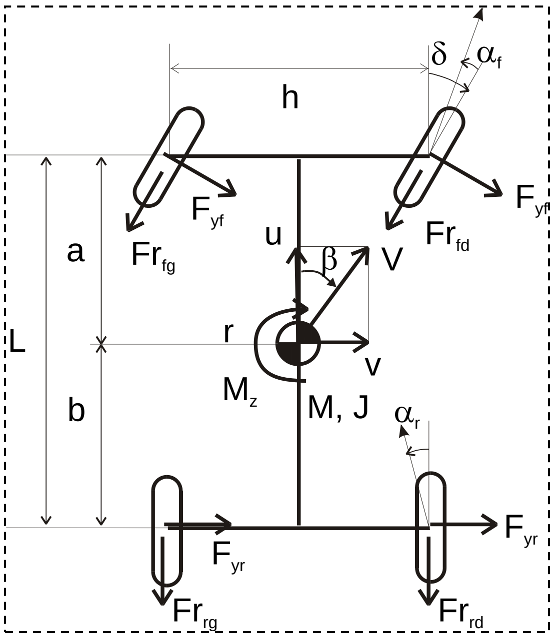
$$
\begin{aligned}
mV(\dot{\beta} + r) + (C_{\alpha f} + C_{\alpha r})\,\beta + (b\,C_{\alpha f} - c\,C_{\alpha r})\frac{1}{V}\,r &= C_{\alpha f}\,\delta \\
J_{zz}\dot{r} + (b\,C_{\alpha f} - c\,C_{\alpha r})\,\beta + (b^2\,C_{\alpha f} + c^2\,C_{\alpha r})\frac{1}{V}\,r &= b\,C_{\alpha f}\,\delta
\end{aligned}
$$



ADAPTATION OF VEHICLE DYNAMICS MODEL TO ESP

- ESP system (Electronic Stability Program) :
  - Apply individual braking forces in each wheel
  - Develop a yaw torque about the vertical axis

$$F_{xf} = -(F_{b,f,d} + F_{b,f,g})$$
$$M_z = +F_{b,f,d}\, t/2 - F_{b,f,g}\, t/2 + F_{b,r,d}\, t/2 - F_{b,r,g}\, t/2$$



- Equations of motion

$$mV(\dot{\beta} + r) + (C_{\alpha f} + C_{\alpha r})\beta + (bC_{\alpha f} - cC_{\alpha r})\frac{1}{V}\, r = C_{\alpha f}\,\delta \boxed{- (F_{b,f,d} + F_{b,f,g})\,\delta}$$

$$J_{zz}\dot{r} + (bC_{\alpha f} - cC_{\alpha r})\beta + (b^2 C_{\alpha f} + c^2 C_{\alpha r})\frac{1}{V} r = b\,C_{\alpha f}\,\delta \boxed{+ M_z} \boxed{- (F_{b,f,d} + F_{b,f,g})\, b\,\delta}$$

UNDERSTANDING THE ESP

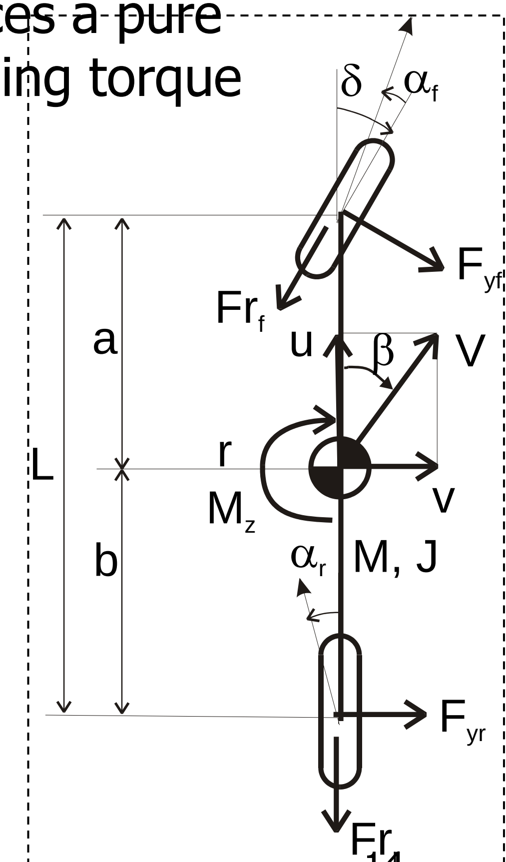
Brake a **<u>front wheel</u>** produces:

- –An aligning torque
- –A torque that tends to reduce the body slip
- –A lateral force that tends to deport the vehicle

Brake a **<u>rear wheel</u>** produces a pure realigning torque

$$
\begin{aligned}
mV(\dot{\beta}+r) \quad &+ \quad (C_{\alpha f}+C_{\alpha r})\beta + (bC_{\alpha f}-cC_{\alpha r})\frac{1}{V}\, r = \\
& \qquad C_{\alpha f}\,\delta - (F_{b,f,d}+F_{b,f,g})\,\delta \\
J_{zz}\dot{r} \quad &+ \quad (bC_{\alpha f}-cC_{\alpha r})\beta + (b^2C_{\alpha f}+c^2C_{\alpha r})\frac{1}{V}r = \\
& \qquad b\,C_{\alpha f}\,\delta + M_z - (F_{b,f,d}+F_{b,f,g})\,b\,\delta
\end{aligned}
$$

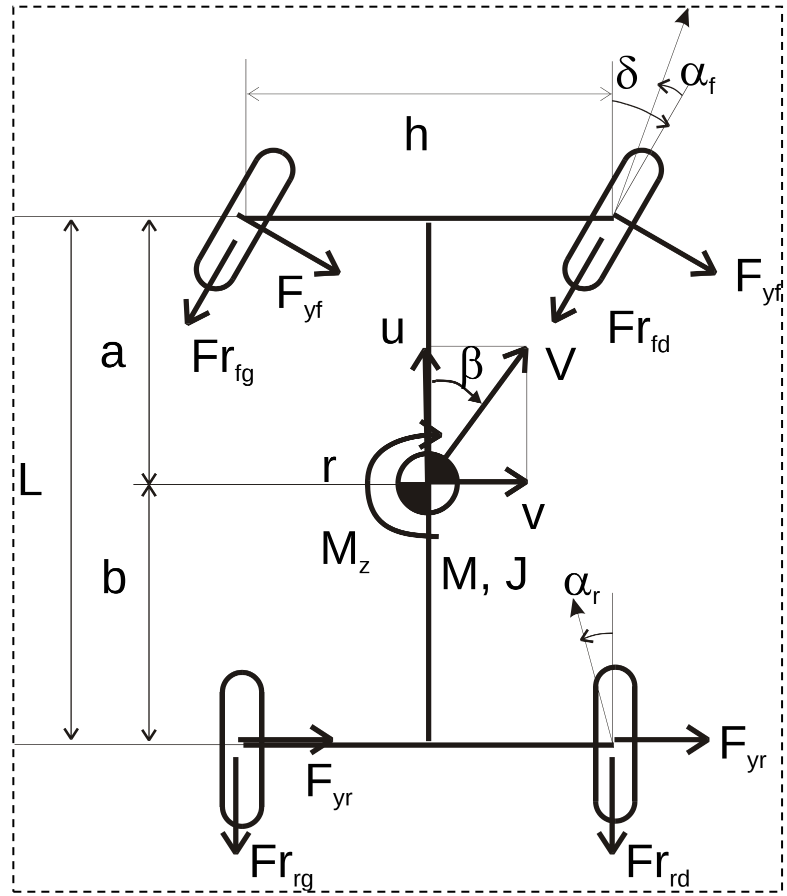
$$
\begin{aligned}
F_{xf} &= -(F_{b,f,d}+F_{b,f,g}) \\
M_z &= +F_{b,f,d}\,t/2 - F_{b,f,g}\,t/2 + F_{b,r,d}\,t/2 - F_{b,r,g}\,t/2
\end{aligned}
$$



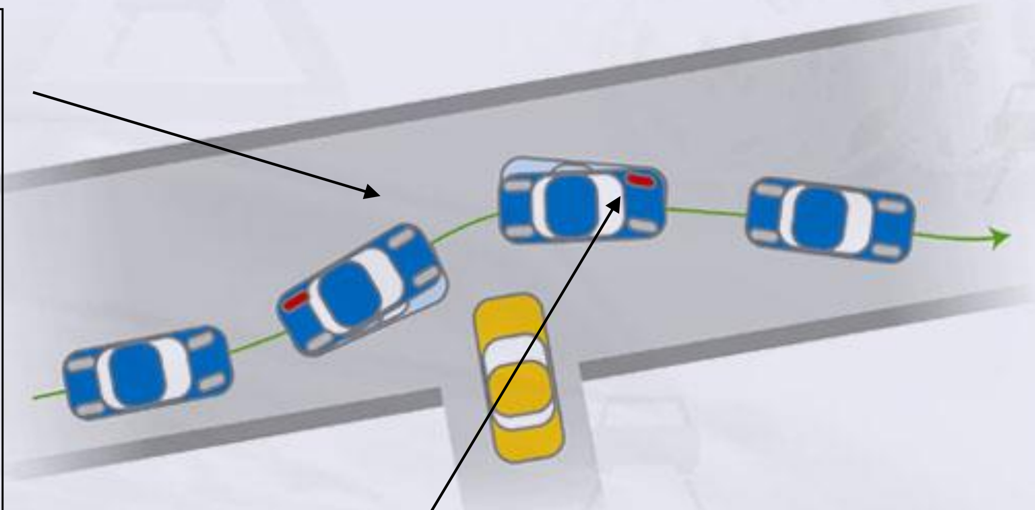
UNDERSTANDING THE ESP

- For an understeer trajectory:
  - The wheels of the front axle experience a larger side slip than rear wheels
  - The front axle is deported towards the outer of the turn
  - The ESP actuate the braking of the rear wheels, here the interior rear wheel to develop an aligning torque to come back on the desired trajectory

- For an oversteer trajectory:
  - The rear axles tends to be deported to the outer of the turn
  - The rear wheels have a higher side slip compared to the front wheels
  - The ESP uses the braking of the front wheels that is a braking on the outer front wheel in order to come back on the ideal trajectory.



UNDERSTANDING THE ESP



<u>Understeer trajectory:</u>

The front wheels are sliding and the front axle is deported to the outer of the turn.

The ESP actuates the rear wheel braking in the inner side of the turn.

<u>Oversteer trajectory:</u>

The rear axle tends to slide towards the outer of the turn.

The ESP systems reacts by braking the front wheel that is in the outer side of the turn.

NUMERICAL SIMULATION EXERCISE

INTEGRATION OF MOTION EQUATIONS IN MATLAB-SIMULINK

$$
\begin{aligned}
mV(\dot{\beta}+r) \quad &+ \quad (C_{\alpha f}+C_{\alpha r})\beta + (bC_{\alpha f}-cC_{\alpha r})\frac{1}{V}\, r = \\
& \quad\; C_{\alpha f}\,\delta - (F_{b,f,d}+F_{b,f,g})\,\delta \\
J_{zz}\dot{r} \quad &+ \quad (bC_{\alpha f}-cC_{\alpha r})\beta + (b^2C_{\alpha f}+c^2C_{\alpha r})\frac{1}{V} r = \\
& \quad\; b\,C_{\alpha f}\,\delta + M_z - (F_{b,f,d}+F_{b,f,g})\,b\,\delta
\end{aligned}
$$

- Phase I (t<0): V=30 m/s, R=100 m, M = 1000 Kg, J=1000 Kg/m^2, g=9.81 m/s^2, L=3 m, a=b=1.5, C_f et C_r = 100 000 N/rad
- Phase II (t=0): Loss of friction on the front wheel: C_f = 75 000N/rad
- Phase III (t>0): Find the braking torque able to re-establish the turn with the desired curvature.

  Check the feasibility of the braking torque (maximum friction under the braked wheels!)