Vehicle Performance

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Lesson 2
Performance criteria

Outline

- STEADY STATE PERFORMANCES
  - Maximum speed
  - Gradeability and maximum slope
- ACCELARATION AND ELASTICITY
  - Effective mass
  - Acceleration time and distance

References

- T. Gillespie. « Fundamentals of vehicle Dynamics », 1992, Society of Automotive Engineers (SAE)
- R. Bosch. « Automotive Handbook ». 5th edition. 2002. Society of Automotive Engineers (SAE)
- J.Y. Wong. « Theory of Ground Vehicles ». John Wiley & sons. 1993 (2nd edition) 2001 (3rd edition).
- W.H. Hucho. « Aerodynamics of Road Vehicles ». 4th edition. SAE International. 1998.
- M. Eshani, Y. Gao & A. Emadi. Modern Electric, Hybrid Electric and Fuel Cell Vehicles. Fundamentals, Theory and Design. 2nd Edition. CRC Press.

Max speed and gradeability

Vehicle performances

- Vehicle performance are dominated by two major factors:
  - The maximum power available to overcome the power dissipated by the road resistance forces
  - The capability to transmit the tractive force to the ground (limitation of tire-road friction)

- Performance indices are generally sorted into three categories:
  - Steady state criteria: max speed, gradebility
  - Acceleration and braking
  - Fuel consumption and emissions

Study of performances with tractive force diagrams

- The steady state performances can be studied using the tractive forces / road resistance forces diagrams with respect to the vehicle speed
- Newton equation

$$F_T - F_{AERO} - F_{RR} - F_{SLOPE} = m\frac{dv}{dt}$$

- Stationary condition $$a_x = \frac{dv}{dt} = 0$$
- Then equilibrium writes

$$F_T = F_{RES} = F_{AERO} + F_{RR} + F_{SLOPE}$$
$$\mathcal{P}_T = \mathcal{P}_{RES} = F_{RES}\, v$$

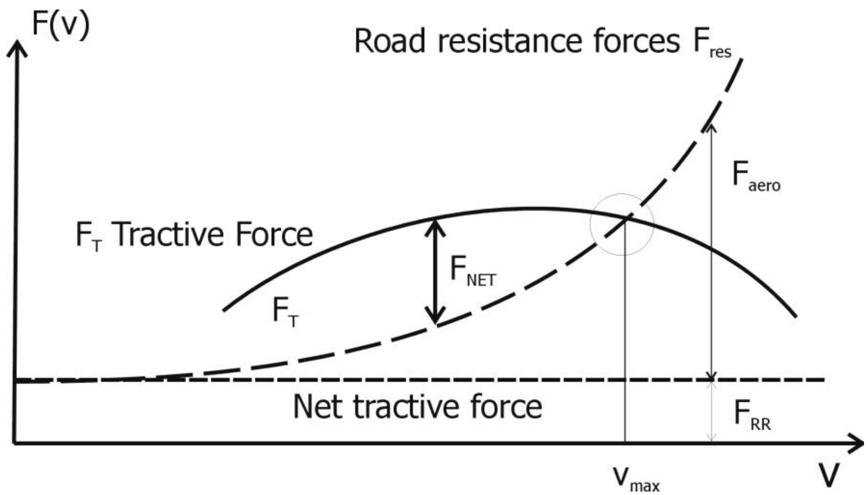
Study of performances with tractive force diagrams

- One generally defines the net force

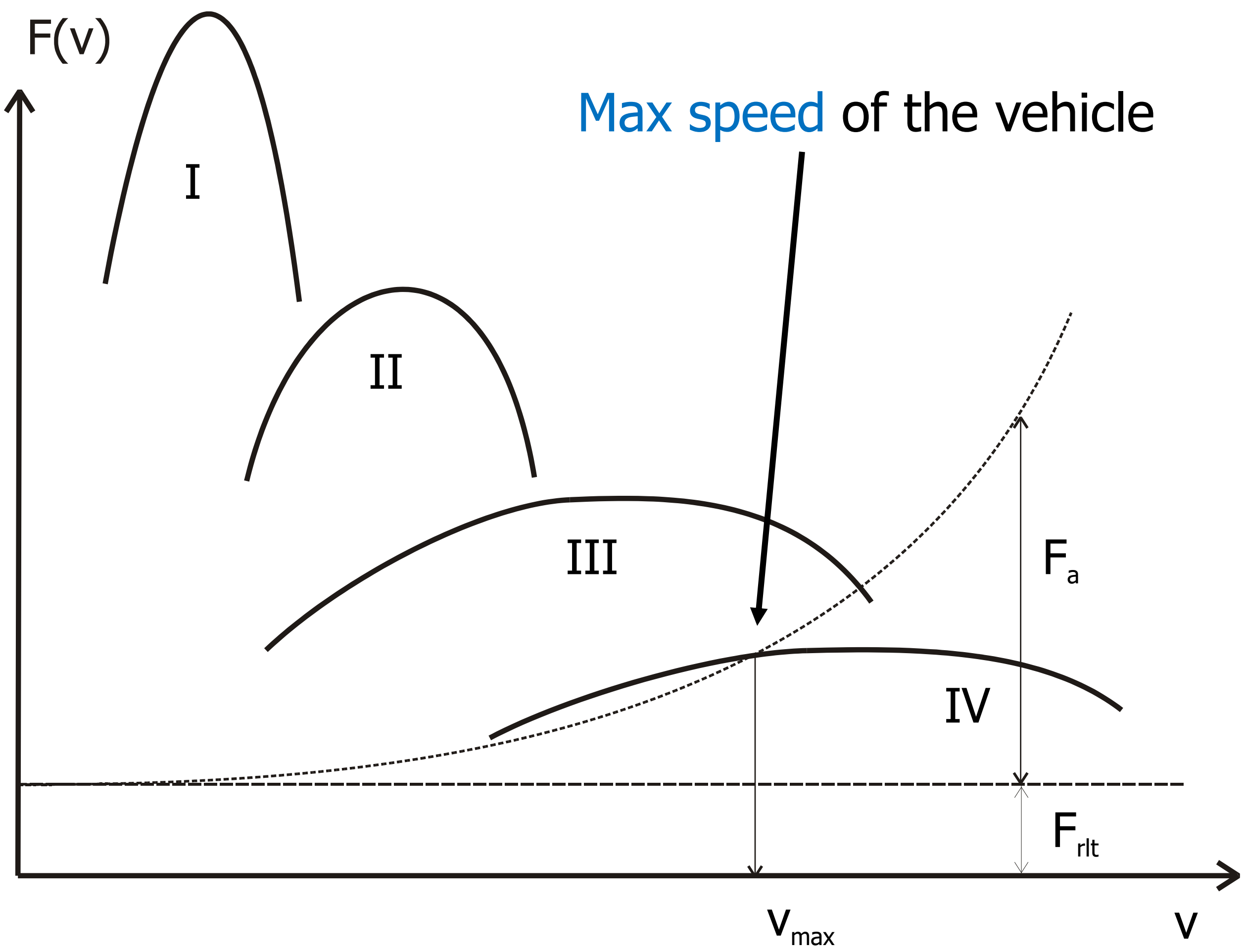
$$F_{NET} = F_T - F_{AERO} - F_{RR} - F_{SLOPE}$$

- One also can use the net force diagram to calculate
  - The maximum speed
  - The maximum slope
  - The reserve acceleration available

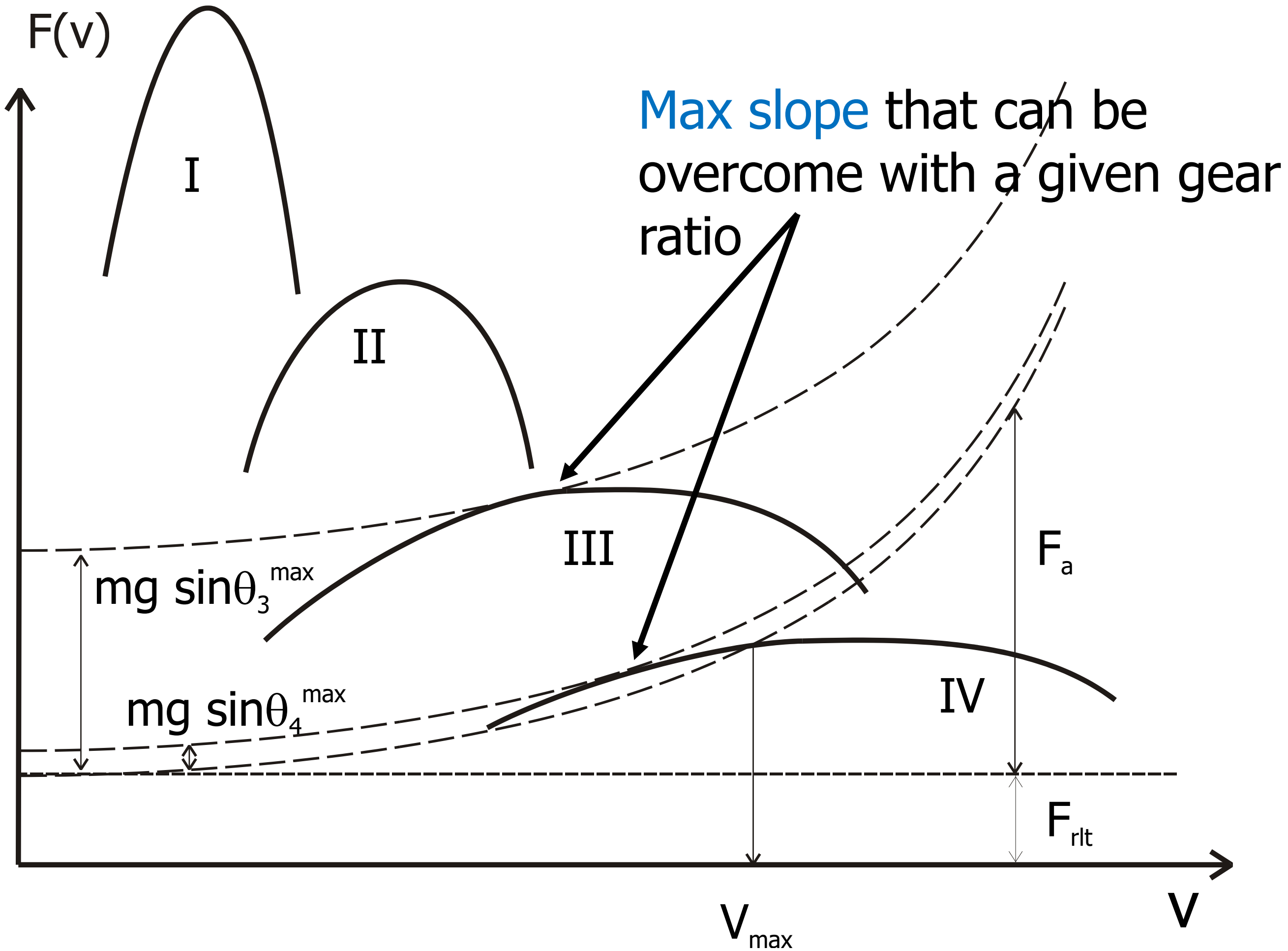
$$a_x = \frac{F_{NET}}{m}$$



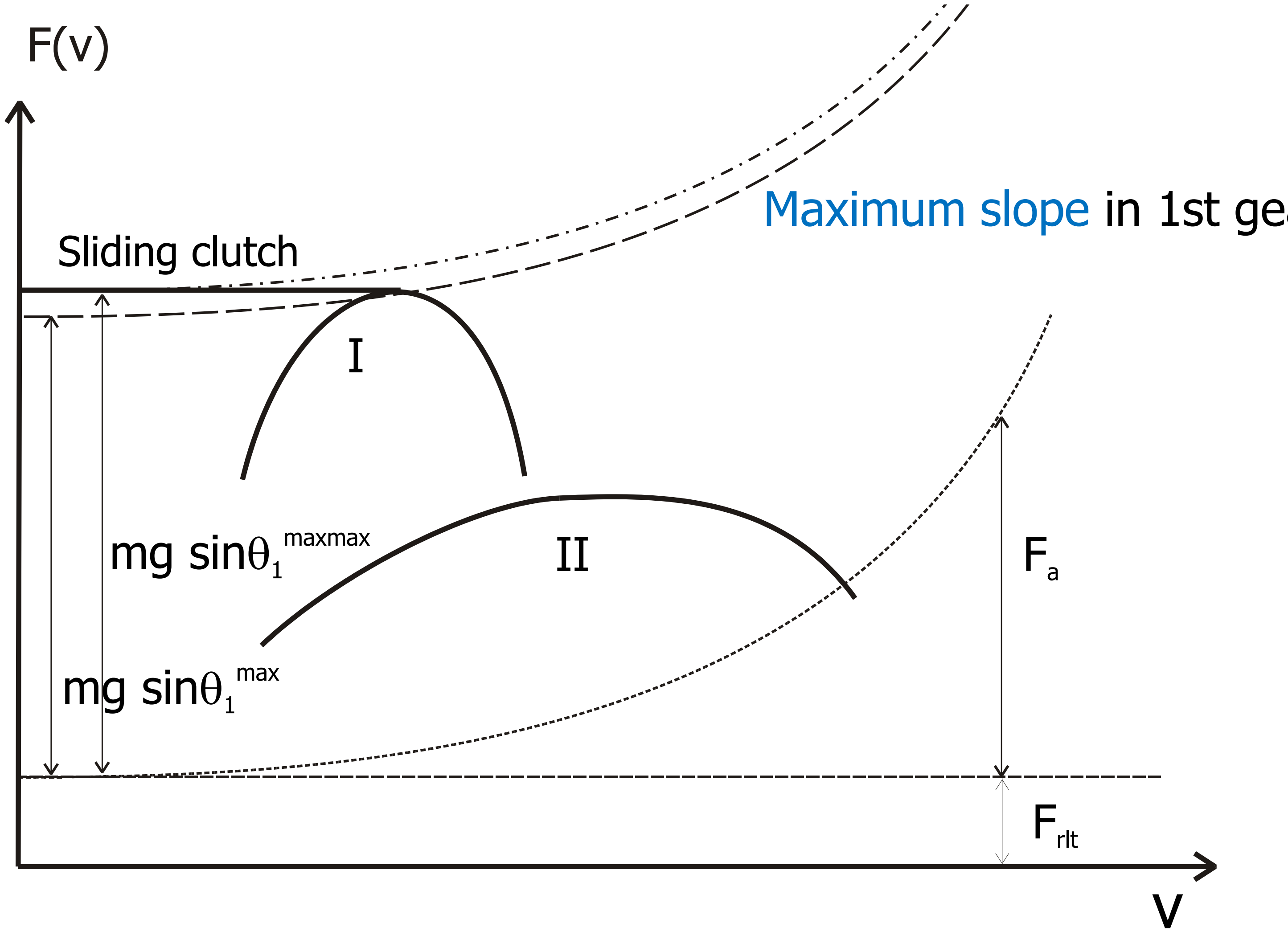
Study of performances with tractive force diagrams



Study of performances with tractive force diagrams



Study of performances with tractive force diagrams



Maximum speed

- For a given vehicle, tires, and engine, calculate the transmission ratio that gives rise to the greatest maximum speed
- Solve equality of tractive power and dissipative power of road resistance

$$\mathcal{P}_w = \mathcal{P}_{res}$$

  - with $\mathcal{P}_{res} = Av + Bv^3 \quad A,\ B > 0$

- As the power of resistance forces is steadily increasing, the maximum speed is obtained when using the maximum power of the power plant

$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

Maximum speed

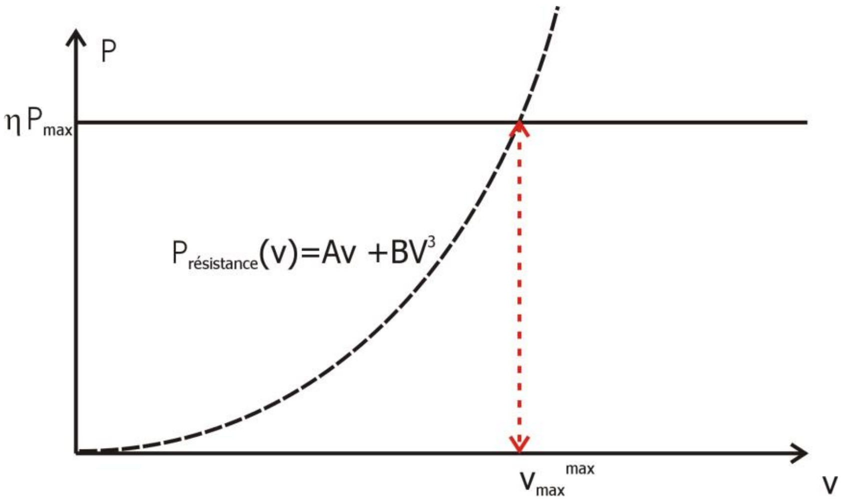
- Iterative scheme to solve the third order equation (fixed point algorithm of Picard)

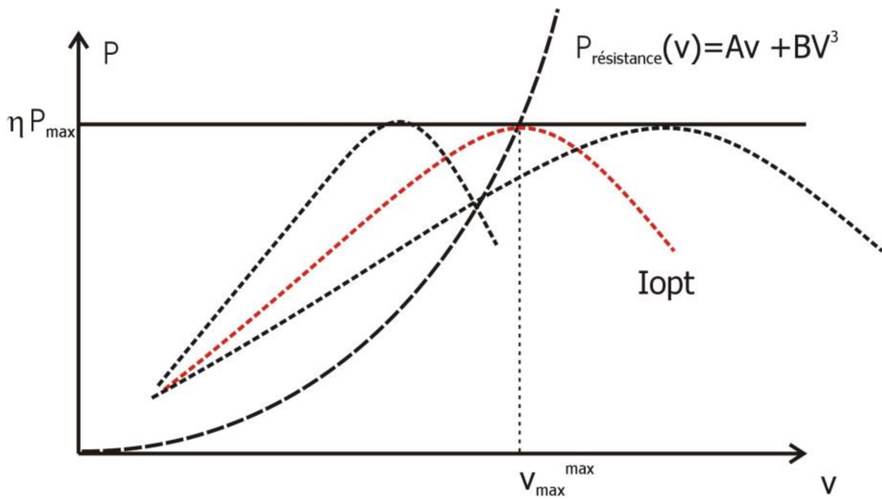
$$
\begin{aligned}
v_0 &= 0 \\
v_{n+1} &= \left(\frac{\eta \mathcal{P}_{max} - A v_n}{B} \right)^{1/3}
\end{aligned}
$$

- Once the maximum speed is determined the optimal transmission ratio can be easily calculated by since it occurs for the nom rotation speed:

$$
\left(\frac{R}{i} \right)^* = \frac{v_{max}^{max}}{\omega_{nom}}
$$

Maximum speed



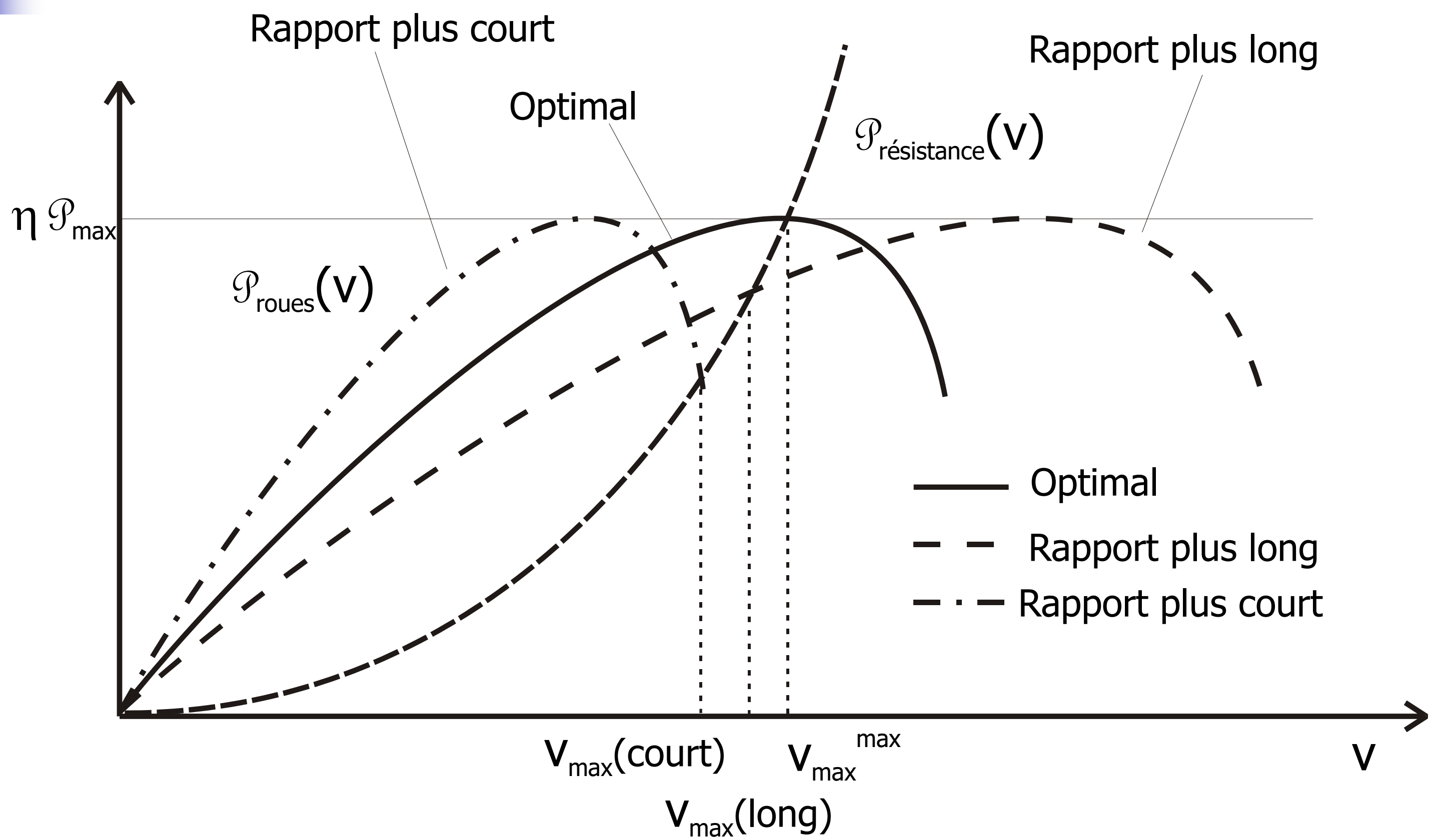


$$Av + Bv^3 = \eta \mathcal{P}_{max}$$

$$\mathcal{P}_w = \mathcal{P}_{res}$$

$$\left(\frac{R}{i}\right)^* = \frac{v_{max}^{max}}{\omega_{nom}}$$

Max speed for given reduction ratio



Max speed is always reduced compared to v_{max}^{max}

Max speed for given reduction ratio

- Solve equation of equality of tractive and resistance power, but this time, the plant rotation speed is also unknown.

$$\mathcal{P}_{res} = A v_{max} + B v_{max}^3 = \eta\, \mathcal{P}(\frac{i}{R} v_{max})$$

- Numerical solution using a fixed-point algorithm (Picard iteration scheme)

$$v_0 = 0 \quad \text{ou} \quad v_{max}^{max}$$

$$\mathcal{P}_0 = \eta \mathcal{P}_{max}$$

$$v_{n+1} = \left(\frac{\mathcal{P}_n - A v_n}{B} \right)^{1/3}$$

$$\mathcal{P}_{n+1} = \eta \mathcal{P}(\frac{i}{R} v_{n+1})$$

Selection of the top gear ratio

- Design specifications for the top gear ratio in connection with the topo speed criteria (from Wong)
  - To be able to reach a given top speed with the given engine
  - To be able to maintain a given constant speed (from 88 to 96 km/h) while overcoming a slope of at least 3% with the selected top gear ratio
- These specifications enable to select a proper top gear ratio
  - The first requirement enables to select a first gear ratio
  - The second condition enforces to select a gear ratio that gives rise to a engine rotation speed that is just above the nominal rotation speed (and the max power) in order to save a sufficient power reserve to keep a constant speed while climbing a small slope, overcoming wind gusts or accounting for loss of engine performance with ageing.

Maximum slope

- For the maximum slope the vehicle can climb, two criteria must be checked:

- The maximum tractive force available at wheel to balance the grading force

$$F_{slope} = mg \sin\theta$$

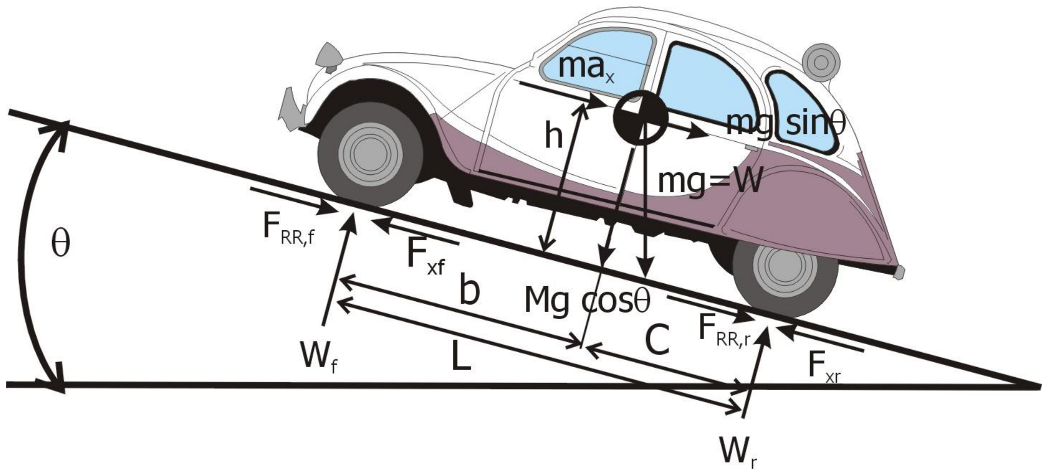
- The maximum force that can be transmitted to the road because of tire friction and weight transfer

$$F_{w,f} \leq \mu W_f \qquad F_{w,r} \leq \mu W_r$$

Maximum slope

Conditions	Coefficient de friction μ
Route sèche	0,8 à 1,2
Route mouillée - 0,2 mm d'eau	0,5 - 0,8
Gravier	0,4
Route mouillée - 2 mm d'eau	0,05 - 0,5
Neige	0,2
Glace	0,1 ou moins

Maximum slope



- Vertical equilibrium

$$m\,g\cos\theta = W_f + W_r$$

- Rotational equilibrium about rear wheels contact point

$$W_f\,L + mg\;\sin\theta\,h + m\,a_x\,h = mg\;\cos\theta\,c$$

- Rotational equilibrium about rear wheels contact point

$$W_r\,L = mg\;\cos\theta\,b + mg\;\sin\theta\,h + m\,a_x\,h$$

Maximum slope

- Limitation due to the friction coefficient

$$F_{w,f} \leq \mu\, W_f \qquad\qquad F_{w,r} \leq \mu\, W_r$$

- Normal forces under the front and rear wheel sets

$$W_f = mg\ \cos\theta\ \frac{c}{L} - mg\ \sin\theta\ \frac{h}{L} - m\, a_x\ \frac{h}{L}$$

$$W_r = mg\ \cos\theta\ \frac{b}{L} + mg\ \sin\theta\ \frac{h}{L} + m\, a_x\ \frac{h}{L}$$

- At low speed and constant speed (a_x=0)

$$W_f = mg\ \cos\theta\ \frac{c}{L} - mg\ \sin\theta\ \frac{h}{L}$$

$$W_r = mg\ \cos\theta\ \frac{b}{L} + mg\ \sin\theta\ \frac{h}{L}$$

Maximum slope

FOUR-WHEEL DRIVE with electronic power split

$$F_p = F_{w,f} + F_{w,r} \leq \mu\,(W_f + W_r)$$

$$mg\ \sin\theta + mg\ \cos\theta\ f_{RR} \leq \mu(mg\cos\theta\frac{c}{L} + mg\cos\theta\frac{b}{L})$$

$$\leq \mu\, mg\ \cos\theta$$

Maximum slope $\boxed{\tan\theta \leq (\mu - f_{RR})}$

Maximum slope

FRONT WHEEL DRIVE

$$F_{w,f} \leq \mu W_f$$

$$mg\ \sin\theta + mg\ \cos\theta\ f \leq \mu\ mg(\cos\theta\frac{c}{L} - \sin\theta\frac{h}{L})$$

Maximum slope

$$\boxed{\tan\theta\ \leq\ \frac{\mu c/L - f}{1 + \mu h/L}}$$

Maximum slope

REAR WHEEL DRIVE

$$F_{w,r} \le \mu \, W_r$$

$$mg\ \sin\theta + mg\ \cos\theta\ f \le \mu\, mg(\cos\theta \frac{b}{L} + \sin\theta \frac{h}{L})$$

Maximum slope

$$\boxed{\tan\theta \ \le\ \frac{\mu b/L - f}{1 - \mu h/L}}$$

Selection of first gear ration

- Maximum slope to be overcome, for instance θ_{max} = 25%

$$F_{\text{RES}} = mg\ \sin\theta_{max} + mg\ f_{RR}\ \cos\theta$$

- Tractive force at wheels

$$F_w \ = \ \eta \frac{i}{R_e} C_p$$

- Sizing of first gear ration

$$i_{max} \ = \ \frac{R_e\ F_{res}}{\eta\ C_{max}} \qquad i_{max} \ = \ \frac{R_e\ mg\ \sin\theta_{max}}{\eta\ C_{max}}$$

Selection of gear ratios

- Goal of the selected gear ratio: to adapt the characteristics of engine operation (rotation speed, torque) to the vehicle speed.
- The top and lowest gear ratios are selected to
  - Match a given top speed
  - To be able to drive over given grading conditions, that is to develop sufficiently high tractive forces at wheels
- The distribution of intermediate gear ratios in between the top and lowest gear ratio is made to span the full range of operating speeds more or less smoothl
- In principle, the different gear ratios should render as much as possible the maximum power curve

Accelerations and elasticity

Acceleration performance

- Estimation of acceleration and elasticity is based on the second Newton law

$$F_w - \sum F_{res} = F_{net} = m\frac{dV}{dt}$$

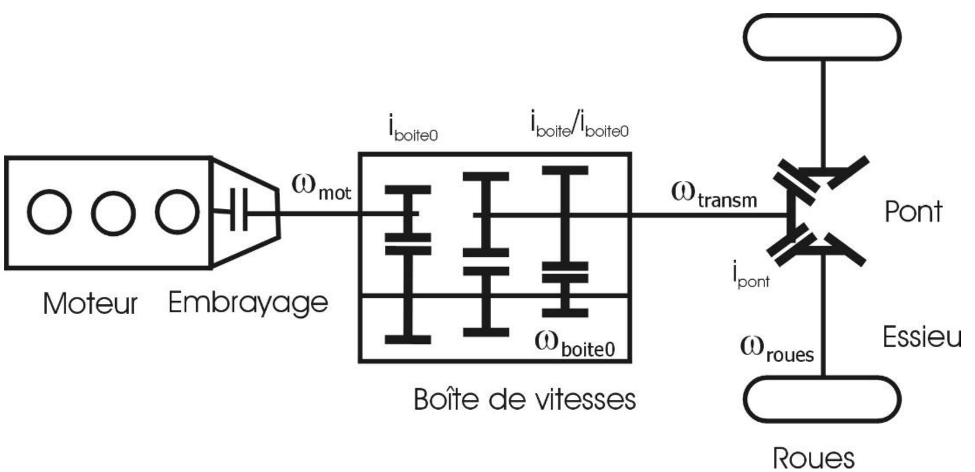
- Warning: when accelerating, the rotation speed of all driveline and transmission components is increasing: wheel sets, transmission shafts, gear boxes and differential, engine…

➔ Effective mass to account for the kinetic energy of all components (translation + rotation)

Effective mass

- Total kinetic energy of the vehicle and its driveline :

$$
\begin{aligned}
T =& 1/2 m\, v^2 + 1/2 (\sum I_{\mathrm{w}} + I_{\mathrm{axle}})\, \omega_{\mathrm{w}}^2 \\
& + 1/2 (I_{\mathrm{transm}} + I_{\mathrm{box2}})\, \omega_{\mathrm{transm}}^2 \\
& + 1/2 (I_{\mathrm{box0}})\, \omega_{\mathrm{box0}}^2 \\
& + 1/2 (I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, \omega_{\mathrm{p}}^2
\end{aligned}
$$



Effective mass

- The rotation speed of the driveline components is linked to the longitudinal speed of the vehicle

$$\omega_{\mathrm{w}} = v/R_e$$

$$\begin{aligned}\omega_{\mathrm{w}} &= \omega_{\mathrm{transm}}/i_{\mathrm{dif}} \\ \omega_{\mathrm{w}} &= \omega_{\mathrm{box0}}/(i_{\mathrm{dif}} * i_{\mathrm{box}}/i_{\mathrm{box0}}) \\ \omega_{\mathrm{w}} &= \omega_{\mathrm{p}}/(i_{\mathrm{dif}} * i_{\mathrm{box}})\end{aligned}$$

- The kinetic energy writes

$$\begin{aligned}T =& 1/2mv^2 + 1/2(\sum I_{\mathrm{w}} + I_{\mathrm{axle}})\, v^2/R_e^2 \\ &+ 1/2(I_{\mathrm{transm}} + I_{\mathrm{box2}})\, v^2 i_{\mathrm{dif}}^2/R_e^2 \\ &+ 1/2(I_{\mathrm{box0}})\, v^2 (i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2/i_{\mathrm{box0}}^2)/R_e^2 \\ &+ 1/2(I_{\mathrm{box1}} + I_{\mathrm{clutch}} + I_{\mathrm{crankshaft}})\, v^2 i_{\mathrm{dif}}^2 i_{\mathrm{box}}^2/R_e^2\end{aligned}$$

Effective mass

- One defines an effective mass $T = 1/2\, m_{\rm e}\, v^2$

$$m_{\rm e} = m + \frac{\sum I_{\rm w} + I_{\rm axle}}{R_e^2} + \frac{(I_{\rm transm} + I_{\rm box2})\, i_{\rm dif}^2}{R_e^2} + \frac{(I_{\rm box0})\, i_{\rm dif}^2 i_{\rm box}^2}{i_{\rm box0}^2\, R_e^2} + \frac{(I_{\rm box1} + I_{\rm clutch} + I_{\rm crankshaft})\, i_{\rm dif}^2\, i_{\rm box}^2}{R_e^2}$$

- The calculation of the effective mass requires the knowledge of the geometry of all the driveline components

- Empirical formula for preliminary design of cars by Wong

$$m_{\rm e} = m_0 + m_1\, i_{\rm box}^2 \qquad i = i_{\rm dif} * i_{\rm box}$$

Effective mass

- Empirical correction formula to estimate the effective mass of passenger car propelled by piston engines (Wong, 2001)

$$\gamma_m = \frac{m_e}{m} = 1.04 + 0.0025\, i^2$$

- This estimation formula puts forward the major factors of the corrections :
  - Nearly negligible for low reduction ratios (4th and 5th gear ratios)
  - Rather important for high gear ratios : 1st and 2nd gear ratios
- For railway systems, γ is of an order of magnitude 1,02 to 1,30 for classical train and from 1,30 to 3,50 for rack trains)

Effective mass

- Example: Peugeot 308 1.6 HDi with 5 gear ratios

	i_{boite}	i	γ_m
1	3,95	13,63	1,5043
2	1,87	7,39	1,1764
3	1,16	4,58	1,0925
4	0,82	3,24	1,0662
5	0,66	2,61	1,0570

$i_{dif}=3,95$

Velocity as a function of time

- We now proceed to time integration of Newton equation.

$$m_e \frac{dv}{dt} = F_w - \sum F_{res} = F_{net}(v)$$

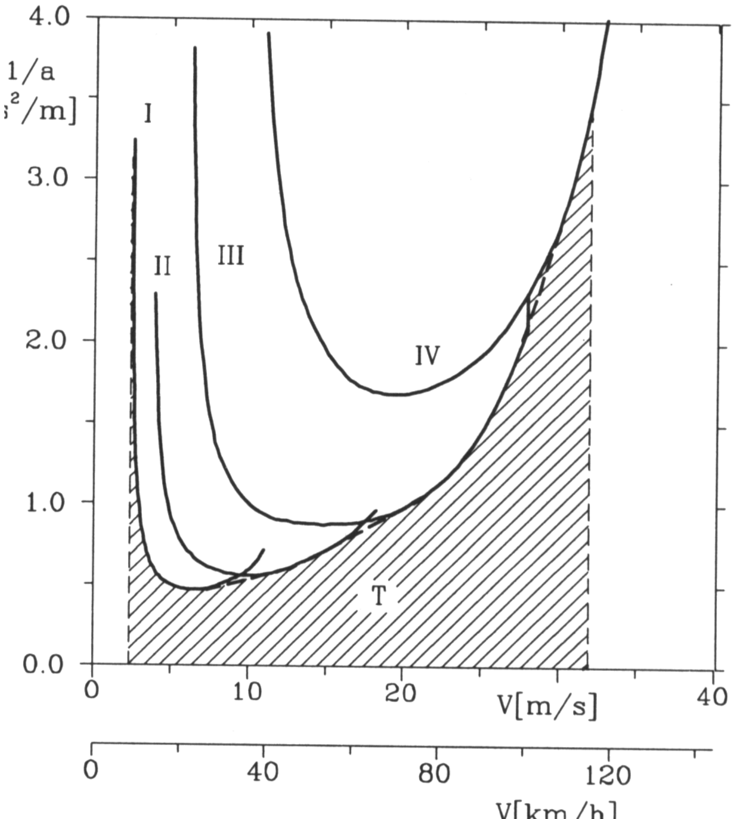
- Time to accelerate form V_1 to V_2.

$$dt = \frac{m_\mathrm{e}\, dv}{F_{net}(v)}$$

$$\Delta t_{V_1 \rightarrow V_2} = m_\mathrm{e} \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)}$$

Velocity as a function of time

$1/a = m_e/F_{net}$



Genta Fig 4.20 : 1/F as function of time

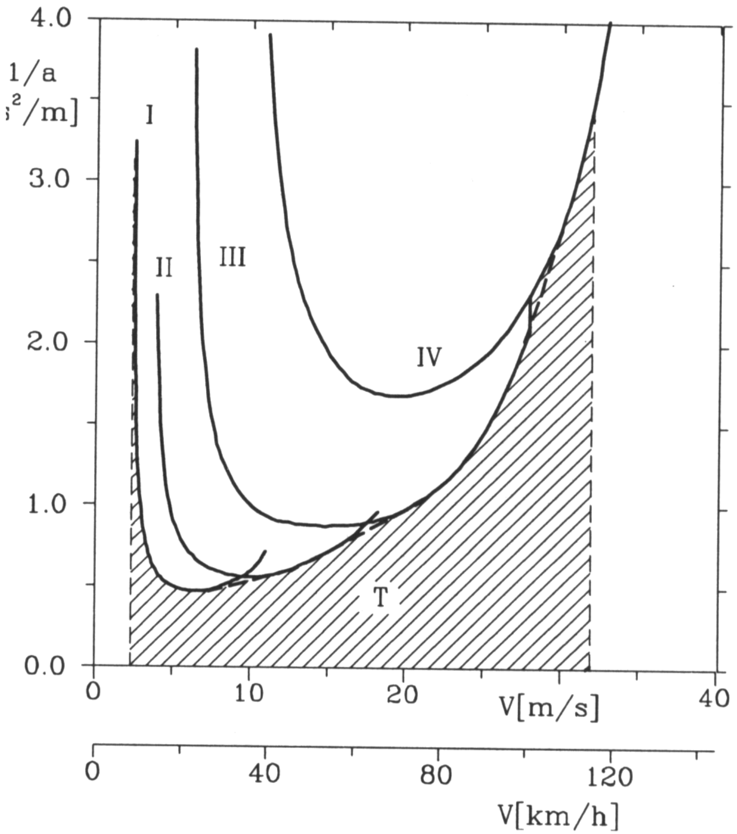
- Time to accelerate from V_1 to V_2:

$$\Delta t_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{dv}{F_{net}(v)}$$

- Alternatively

$$\Delta t_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{v\, dv}{\mathcal{P}_{net}(v)}$$

Velocity as a function of time



Genta Fig 4.20 : 1/F as function of time

- Criteria for gear ratio up shift in order to minimize the acceleration time
- If two curves intersects each other: **change the ratio at curve intersection**
- If there is no intersection, then it is necessary to push the ratio up to maximum rotation speed
- Lower limit is given by an infinite number of gear ratios, that is a Continuous Variables Transmission (CVT)

Velocity as a function of time

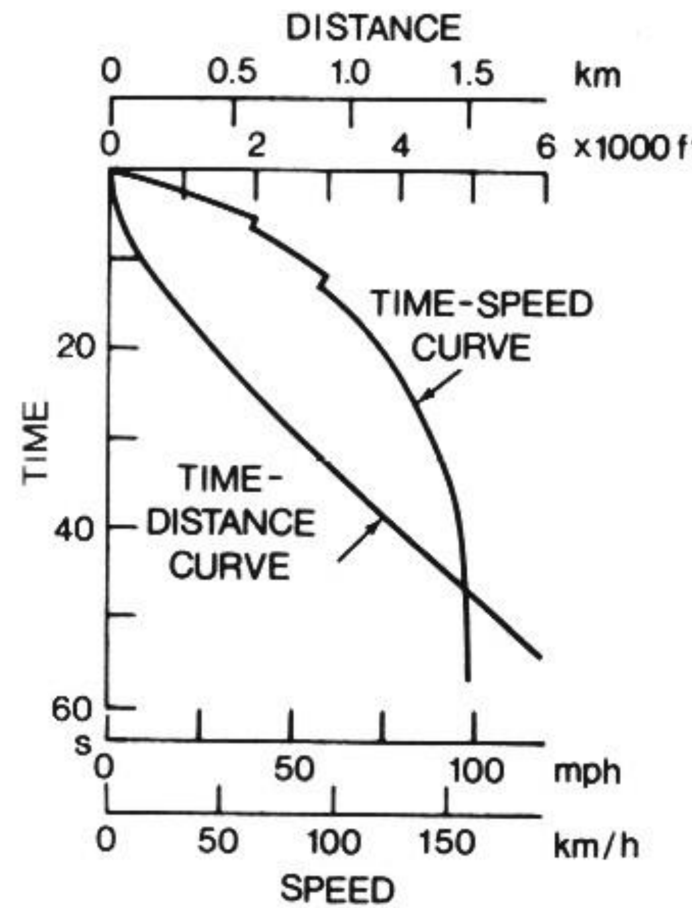
- The solution of differential equation yields the time t as a function of the velocity

$$t = f(v)$$

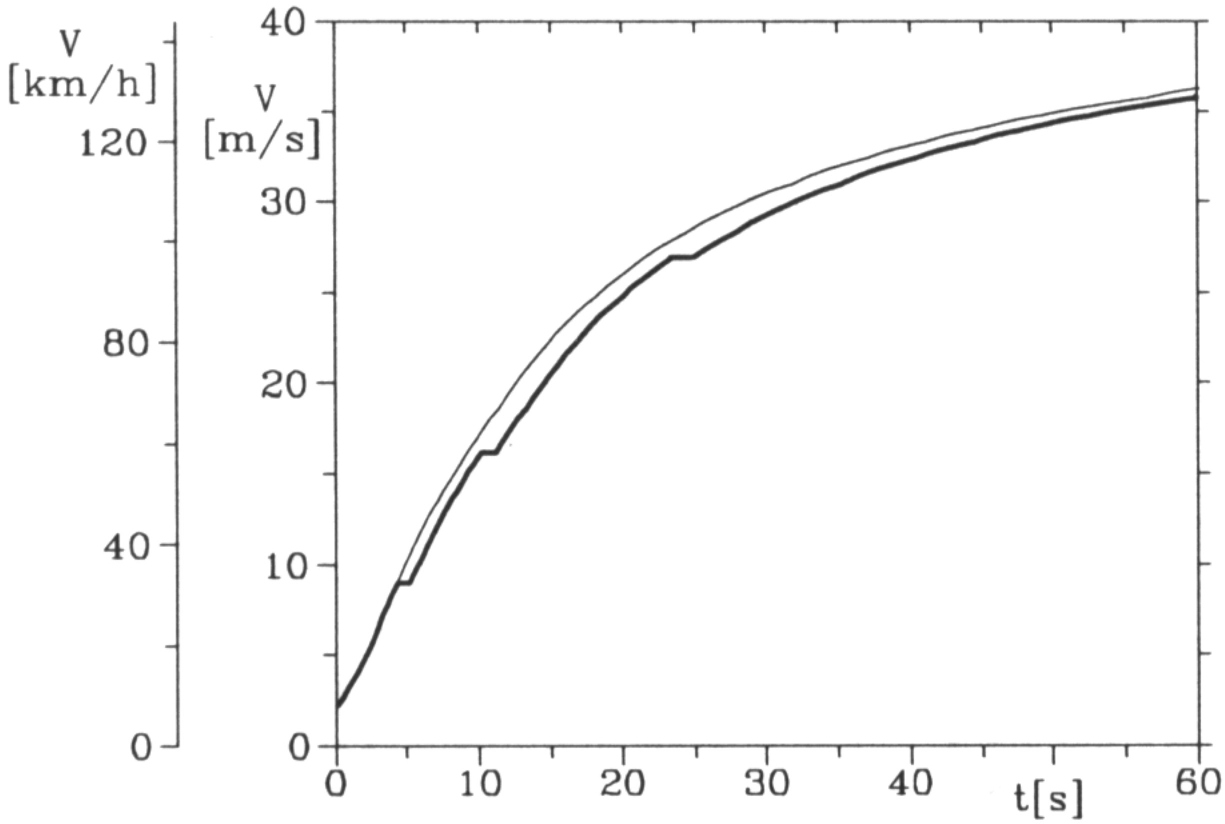
- The reciprocal function V=g(t) requires to invert the relation

$$g = f^{-1}$$

- The changes of gear ratio must be taken into account



Velocity as a function of time



G. Genta Fig 4.21

Distance as a function of the speed

- The distance from start can be evaluated by a second integration of the Newton equation

- Velocity and distance are linked by the kinematic relation

$$dx = v\,dt$$

- It comes

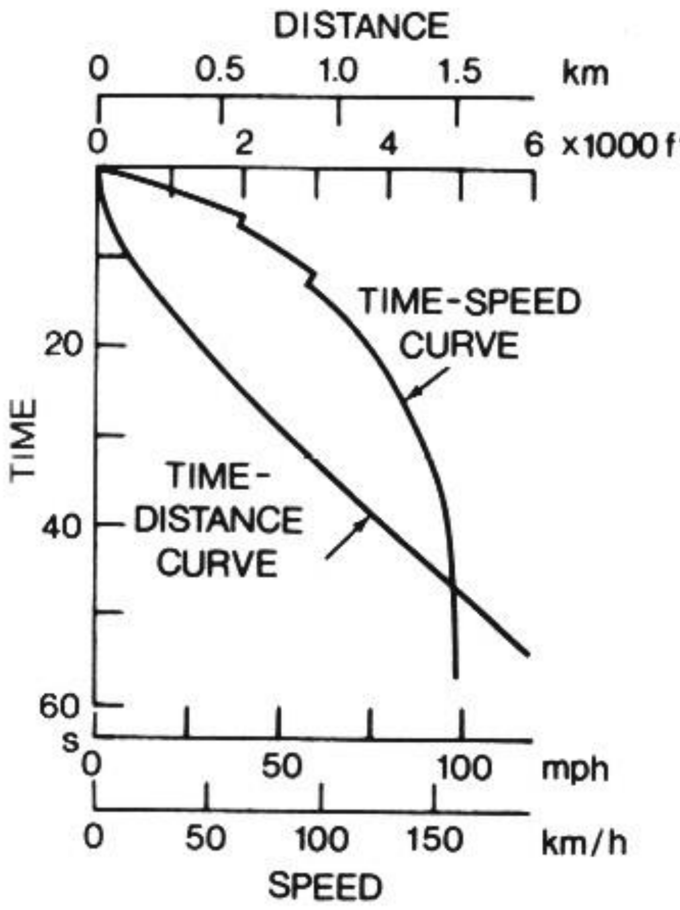
$$\Delta x_{V_1 \to V_2} = m_e \int_{V_1}^{V_2} \frac{v\,dv}{F_{net}(v)}$$

Distance as a function of the time

- One can eliminate the velocity V between the two curves t=f(V) and d=h(V)
- On gets the distance as a function of the time:

$$\Delta t = f(\Delta v) \qquad \Delta x = h(\Delta v)$$

$$\Delta x = h(f^{-1}(\Delta v))$$



Change of gear ratio

- Criteria for changing the gear ratio.
- Gear ratio changing is a delicate operation that needs being studied in details:
  - Changing the gear box ratio takes some time
  - Tractive force is interrupted
  - The vehicle is coasting and slows down
- For an expert driver
  - Small time to change the gear $\Delta t \approx 0,8 s.$
  - Reduction of the velocity can be estimated by the first order approximation

$$\Delta v \approx -\frac{F_{res}(v)}{m_e} \Delta t$$

Change of gear ratio

- When several gear change are necessary, the integration needs to be carried out by parts
- For instance

$$T_{V_1 \to V_2} = \int_{V_1}^{V_{I\to II}} \frac{m_e(i_1)\, dv}{F_{net}(v)} + \Delta t + \int_{V_{II}}^{V_{II\to III}} \frac{m_e(i_2)\, dv}{F_{net}(v)} + \Delta t + \int_{V_{III}}^{V_2} \frac{m_e(i_3)\, dv}{F_{net}(v)}$$

- with

$$V_{II} = V_{I\to II} - \frac{F_{res}(V_{I\to II})}{m_e} \Delta t$$

$$V_{III} = V_{II\to III} - \frac{F_{res}(V_{II\to III})}{m_e} \Delta t$$