



PERFORMANCE & DESIGN OF BATTERY ELECTRIC VEHICLES

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References

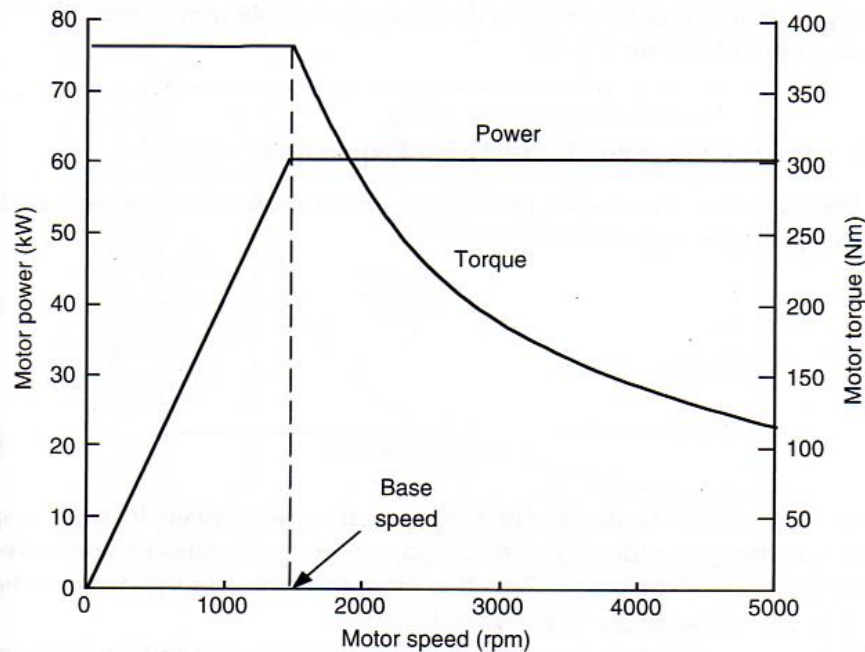
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Performances of Electric Vehicles

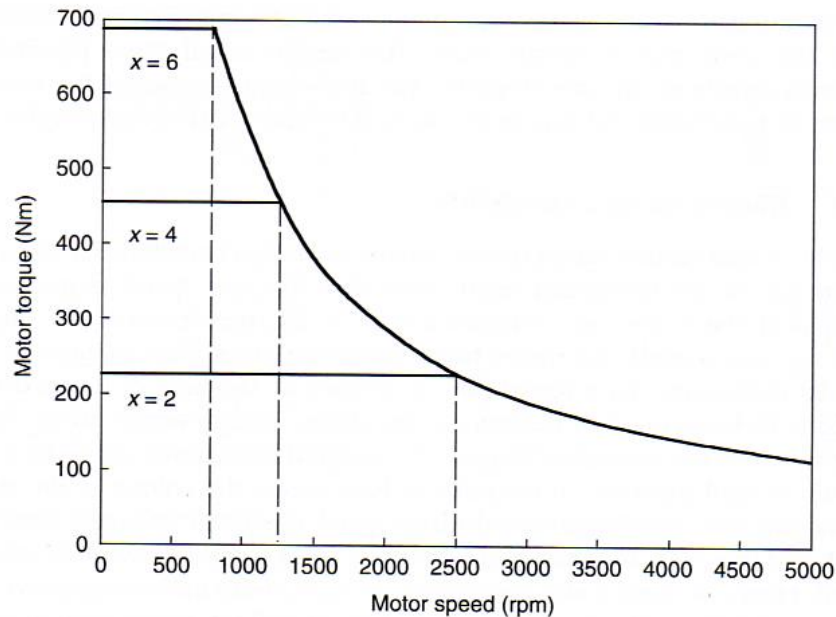
- Vehicle driving performance is assessed by
 - Acceleration time
 - Maximum speed
 - Gradeability
- In EV drivetrain design: the motor power rating and transmission parameters are selected to meet the performance specifications
- They depend mostly on the speed-torque characteristics of the traction motor

Traction motor characteristics



- At low speed: constant torque
 - Voltage supply increases with rotation speed through electronic converter while flux is kept constant
- At high speed: constant power
 - Motor voltage is kept constant while the flux is weakened,
 - Torque is reduced hyperbolically with the rotation speed
- **Base speed**: transition speed from constant torque to constant power regime

Traction motor characteristics



- Speed ratio X = ratio between maximum rotation speed to base speed

$$X = \frac{\omega_{max}}{\omega_B}$$

- $X \sim 2$ Permanent Magnet motors
 - $X \sim 4$ Induction motors
 - $X \sim 6$ Switched Reluctance motors
- For a given power, a longer constant power region (large X) gives rise to an important max constant torque, and so high vehicle acceleration and gradeability. Thus the transmission can be simplified.



Tractive efforts and transmission requirement

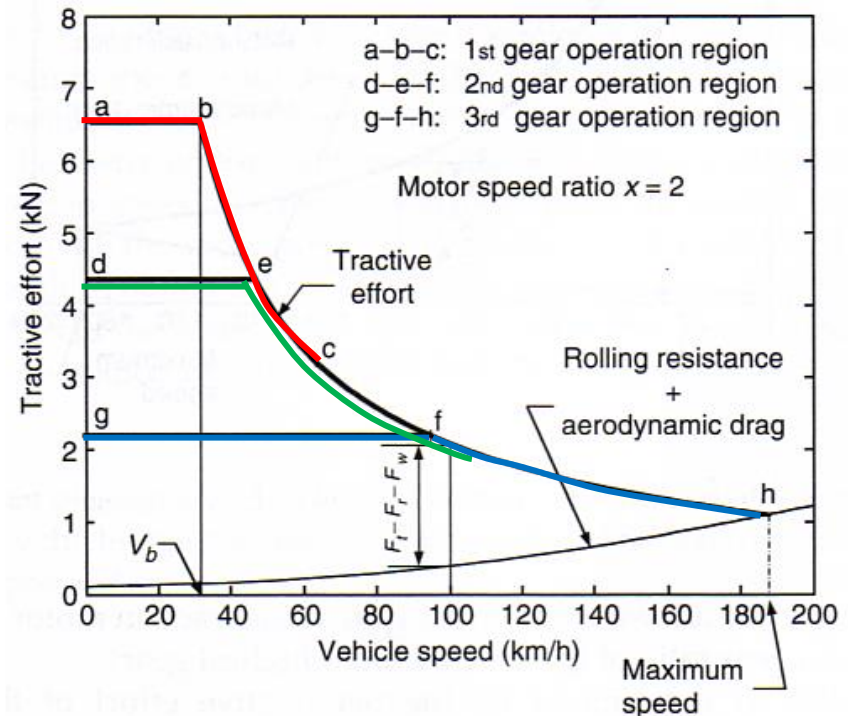
- Remind traction effort and vehicle speed

$$F_t = \eta_t \frac{C_m i}{R_e} \qquad v = \frac{\omega_m R_e}{i}$$

- The use of multi-gear or single gear transmission depends on the motor speed-torque characteristics.
 - For a given rated power, a long constant power region makes possible to use a single gear transmission, because of the high tractive efforts at low speeds.
 - For long constant torque plateau and a given rated power, the available maximum torque is sometimes not sufficient so that a multi gear is generally preferred.

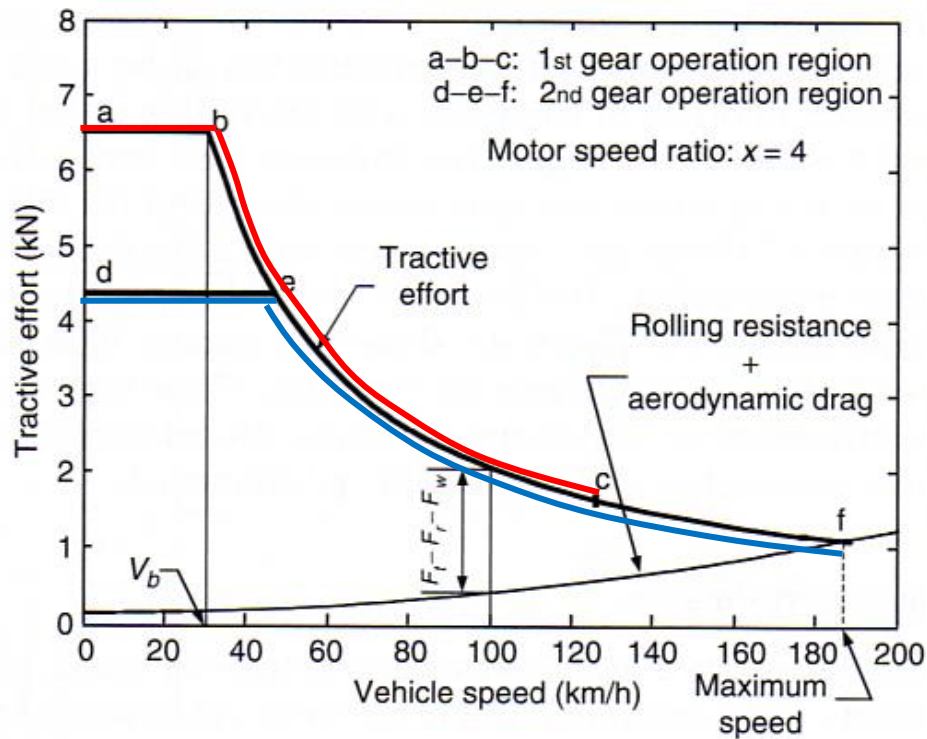
Tractive efforts and transmission requirement

- For a low x ($x=2$) motor, tractive effort is not large enough and 3-gear transmission may be chosen.
- For intermediate $x=4$, a two-gear transmission should be preferred.
- For a large $x=6$, a single gear transmission is possible.
- The 3 designs have the same tractive force / speed profiles, and so the same acceleration and gradeability performance.

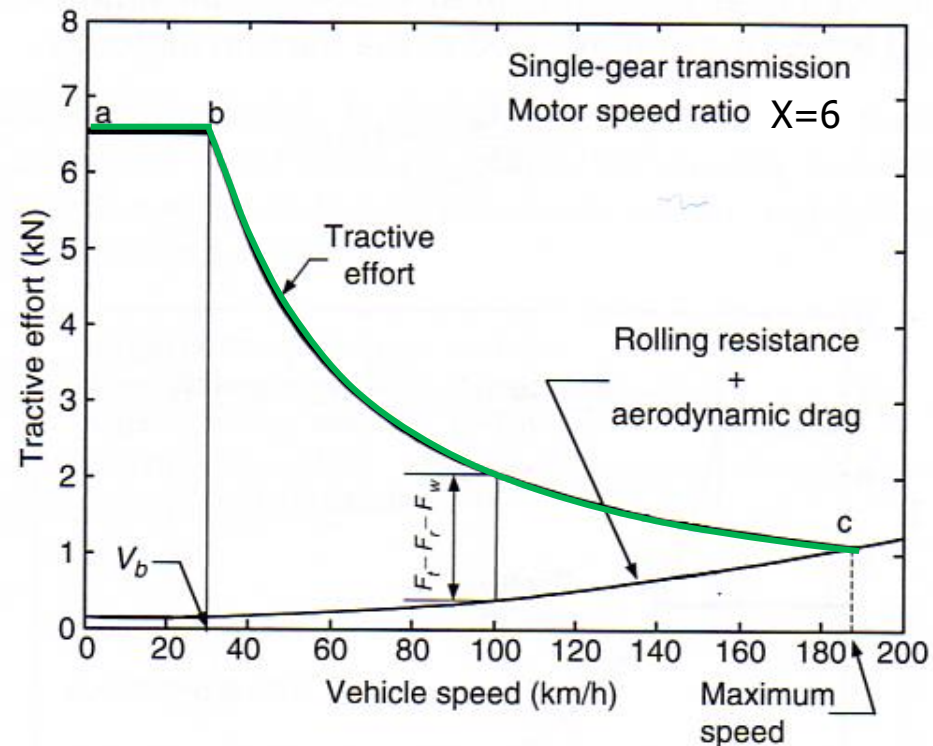


Ehsani et al. Fig 5.5. Low x ($x=2$) e-motor and 3-gear transmission

Tractive efforts and transmission requirement

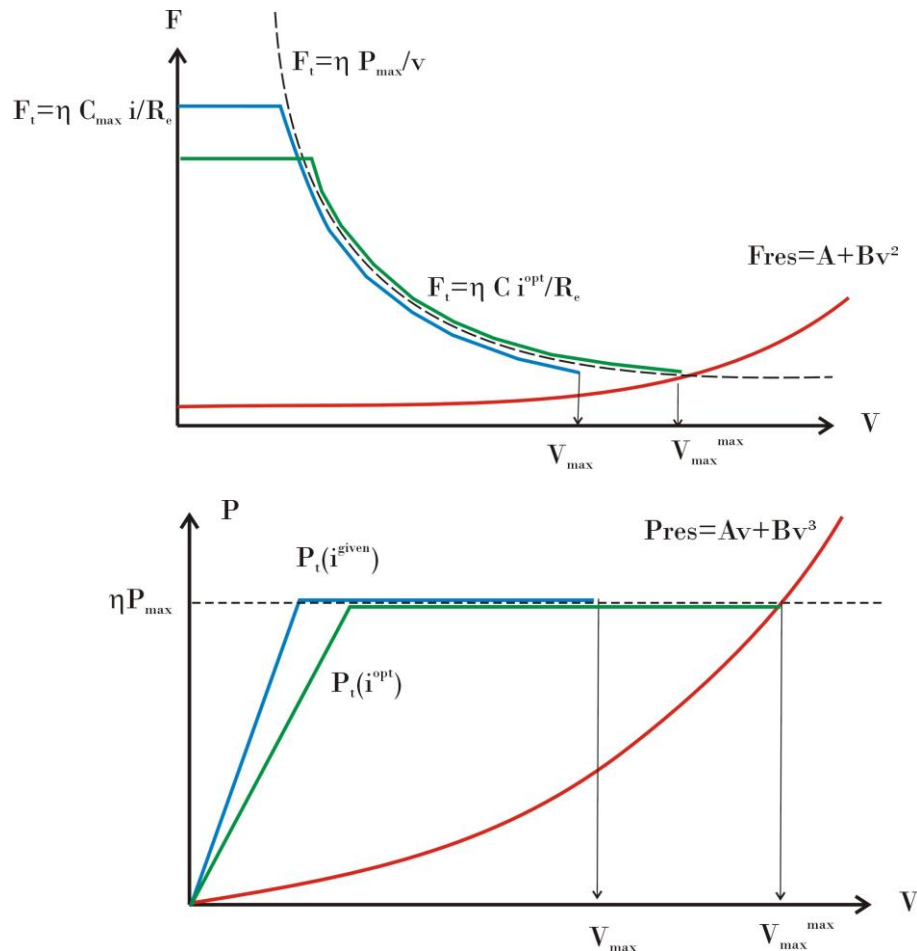


Ehsani et al. Fig 5.5. Induction motor ($x=4$) and 2-gear transmission



Ehsani et al. Fig 5.5. High torque e-motor (SRM) and single gear transmission

EV max speed



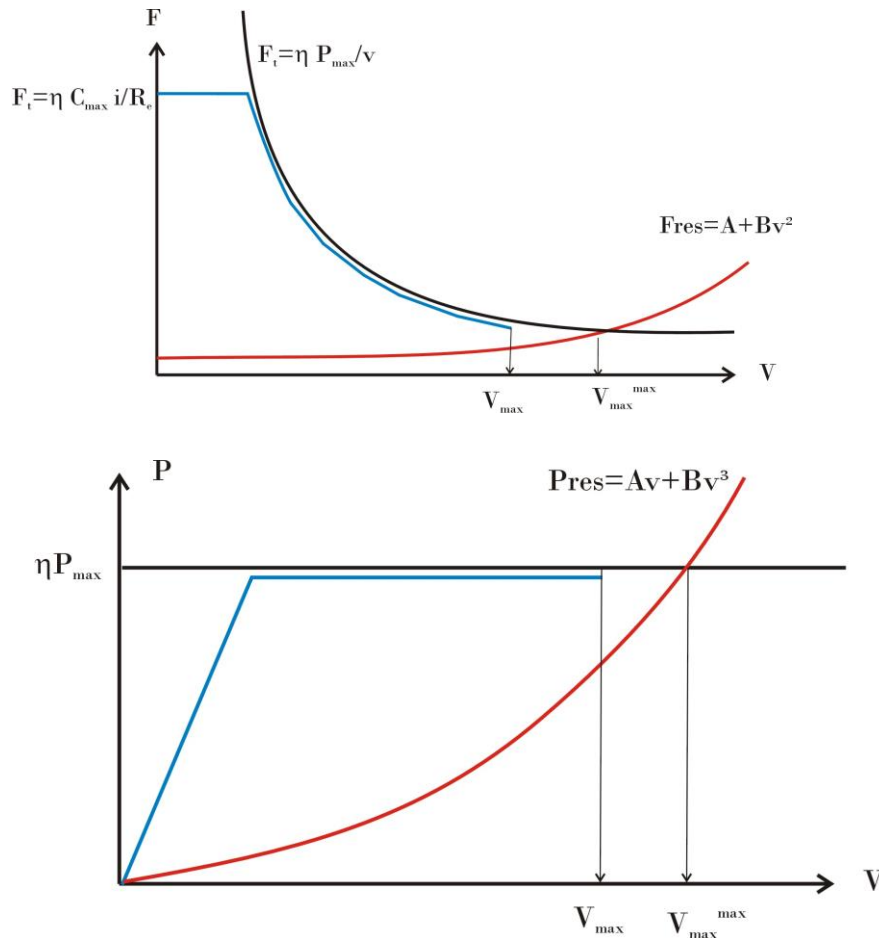
- Max speed can be evaluated by calculating the intersection between the tractive force curve and the resistance curve or alternatively the tractive power (constant) and the resistance forces power.

$$\eta_t P_m^{max} = A V_{max}^{max} + B (V_{max}^{max})^3$$

- Then we can define the gear ratio of the transmission so that max speed is reached at the motor max rotation speed

$$i^{opt} = \frac{\pi N_m^{max} R_e}{30 V_{max}^{max}}$$

EV max speed



- However sometimes, the intersection between the max power and the resistance power does not exist because it is over the maximum rotation speed of the motor

$$\eta_t P_m^{max} = A V_{max}^{max} + B (V_{max}^{max})^3$$

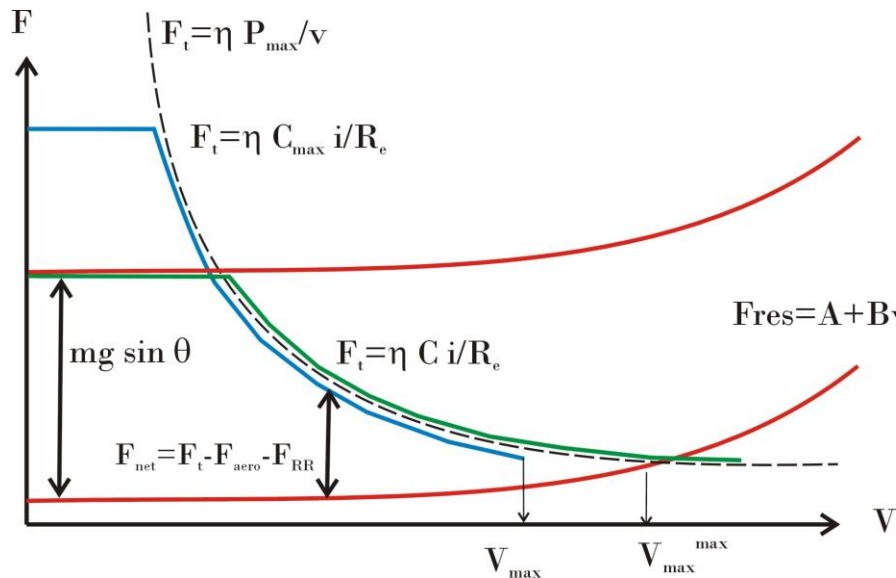
- This is the case when the V_{max} is prescribed

$$\bar{V}_{max} = \frac{\pi N_m^{max} R_e}{30 i} < V_{max}^{max}$$

- Or when the gear ratio is given

$$V_{max} = \frac{\pi N_m^{max} R_e}{30 \dot{i}} < V_{max}^{max}$$

Gradeability of EV



- Check also nonslip condition

$$F_t \leq \mu W_f \quad F_t \leq \mu W_r$$

- Gradeability is ruled by the net tractive force available

$$\begin{aligned} F_t^{\text{net}} &= F_t - F_{RR} - F_{\text{aero}} \\ &= F_t - mg f \cos \theta - 0,5 \rho S C_x V^2 \end{aligned}$$

- The maximum grade that can be overcome at a given speed is:

$$\sin \theta = \frac{F_t - F_{RR} - F_{\text{aero}}}{mg} = \frac{F_t^{\text{net}}}{mg}$$

- It comes :

$$\sin \theta = \frac{d - f \sqrt{1 + f^2 - d^2}}{1 + f^2}$$

$$d = (F_t - F_{\text{aero}}) / mg$$



Gradeability of EV

- The maximum slope can be evaluated as follows

$$\begin{aligned} F_t - F_{aero} - F_{RR} - mg \sin \theta &= 0 \\ \iff F_t - F_{aero} - mg \cos \theta f - mg \sin \theta &= 0 \end{aligned}$$

- If we set

$$d = (F_t - F_{aero})/mg$$

- It comes :

$$\begin{aligned} d - \sin \theta &= f \cos \theta \\ (d - \sin \theta)^2 &= f^2 \cos^2 \theta \\ \iff d^2 - 2d \sin \theta + \sin^2 \theta &= f^2 (1 - \sin^2 \theta) \\ \iff (1 + f^2) \sin^2 \theta - 2d \sin \theta + (d^2 - f^2) &= 0 \end{aligned}$$



Gradeability of EV

- It a second order equation in $\sin\theta$

$$(1 + f^2) \sin^2 \theta - 2d \sin \theta + (d^2 - f^2) = 0$$

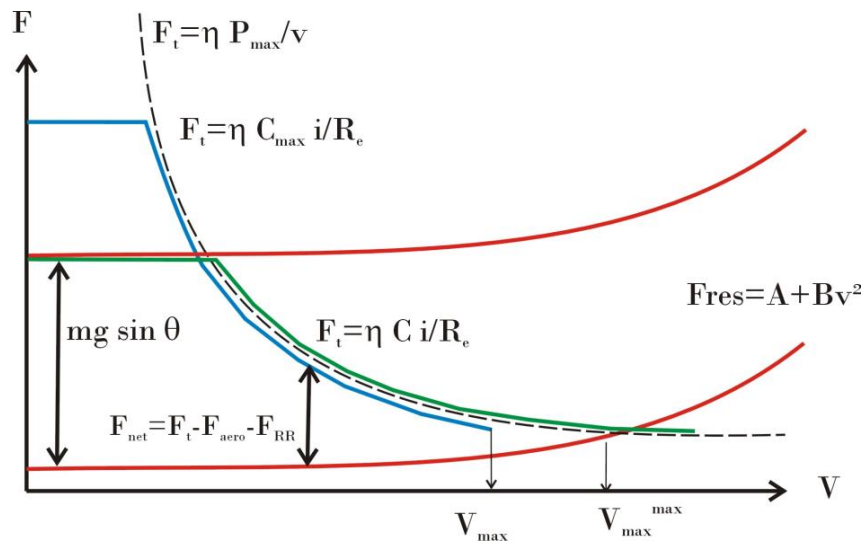
- Solving for $\sin\theta$ gives

$$\begin{aligned}\sin \theta &= \frac{2d \pm \sqrt{4d^2 - 4(1 + f^2)(d^2 - f^2)}}{2(1 + f^2)} \\ &= \frac{d \pm \sqrt{d^2 - d^2 + f^2 - f^2d^2 + f^4}}{1 + f^2} \\ &= \frac{d \pm \sqrt{f^2 - d^2 f^2 + f^4}}{1 + f^2} = \frac{d \pm f \sqrt{1 - d^2 + f^2}}{1 + f^2}\end{aligned}$$

- It comes :

$$\sin \theta = \frac{d - f \sqrt{1 + f^2 - d^2}}{1 + f^2}$$

Gradeability of EV



$$\sin \theta = \frac{d - f \sqrt{1 + f^2 - d^2}}{1 + f^2}$$

$$d = (F_t - F_{aero}) / mg$$

- One can increase the tractive force by increasing the transmission gear ratio i

$$F_t = \eta C_m^{max} \frac{i}{R_e}$$

- However increasing the reduction ratio is reducing the maximum speed (green line \rightarrow blue line)

$$V_{max} = \omega_{max} \frac{R_e}{i}$$

- It is also positive for the acceleration (see after)
- Verify that adherence is not saturated

$$F_t \leq \mu W_{f/r}$$



EV acceleration

- Acceleration can be evaluated by the time to accelerate from a given low speed (often zero) to a given high speed (e.g. 100 km/h).
- Acceleration performance is often more important for drivers than max speed and gradeability
- Acceleration performance dictates the **motor power rating**!



EV acceleration

- Newton second law:

$$F_t - \sum F_{res} = F_{net} = m_e \frac{dV}{dt}$$

- Effective mass m_e to account for the inertia of the wheel, the axle, the transmission, and the electric machine J

$$m_e = \gamma m$$

$$\gamma = 1. + (I_w / (m R_e^2)) + (J_m / (m R_e^2)) i^2 \simeq 1.04 + (J_m / (m R_e^2)) i^2$$

- Time to accelerate from 0 to V_f :

$$\Delta t_{0 \rightarrow V_f} = \int_0^{V_f} \frac{m_e}{F_{net}(v)} dv = \int_0^{V_f} \frac{m_e}{F_t(v) - m g f - 1/2 \rho S C_x v^2} dv$$

EV acceleration

- Tractive force expression of EV

$$F_t = \eta_t \frac{C_m i}{R_e}$$

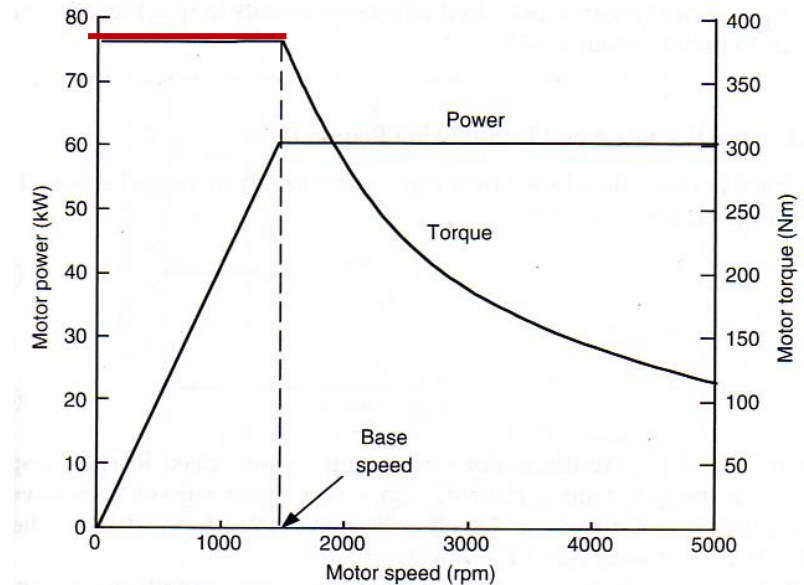
- Under the base speed

$$\omega \in [0, \omega_B] \quad C = C^{max}$$

$$C = C^{max} = \frac{P^{max}}{\omega_B}$$

$$F_t = \eta_t \frac{P^{max} i}{\omega_B R_e} = \eta_t \frac{P^{max}}{v_B} = \frac{P_t^{max}}{v_B} = \text{Cste}$$

$$v_B = \frac{\omega_B R_e}{i} \quad P_t^{max} = \eta_t P^{max}$$



EV acceleration

- Tractive force expression of EV

$$F_t = \eta_t \frac{C_m i}{R_e}$$

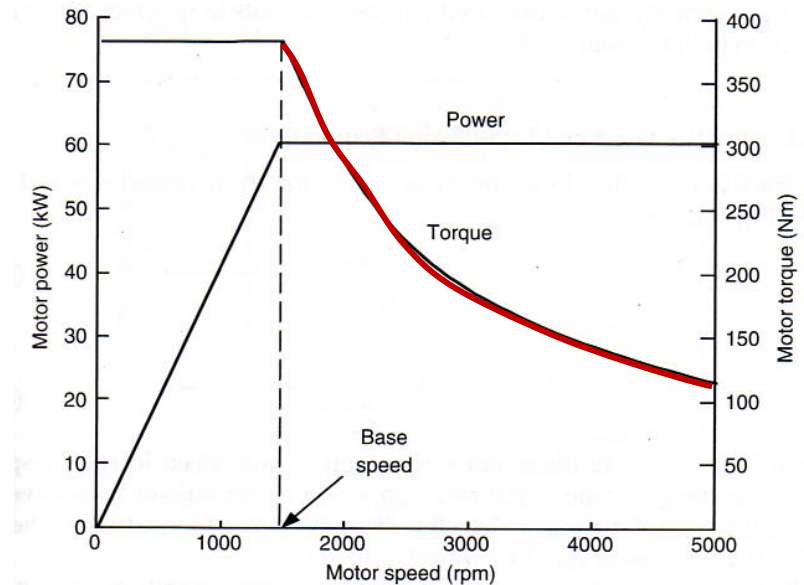
- Over the base speed

$$\omega \in [\omega_B, \omega_{max}] \quad P = P^{max}$$

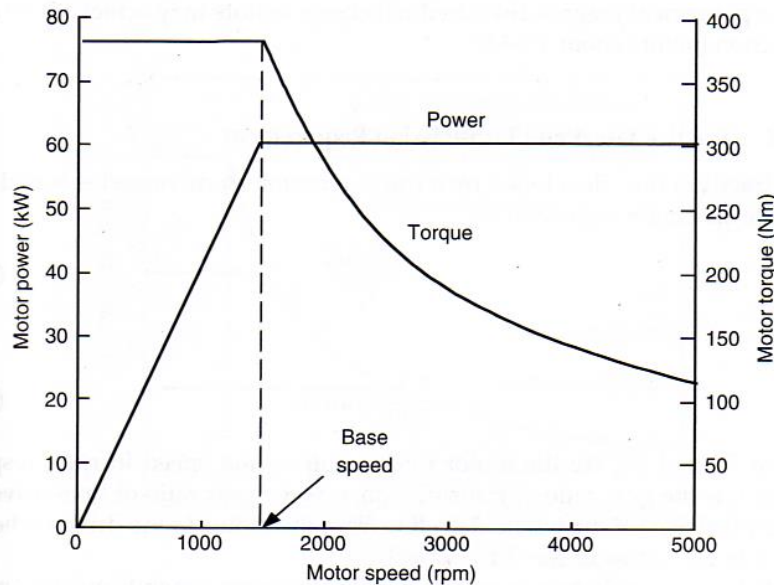
$$C = \frac{P^{max}}{\omega_m}$$

$$F_t = \eta_t \frac{P^{max} i}{\omega_m R_e} = \eta_t \frac{P^{max}}{v} = \frac{P_t^{max}}{v}$$

$$v = \frac{\omega_m R_e}{i} \quad P_t^{max} = \eta_t P^{max}$$



EV acceleration



- Acceleration time can be calculated by the integral

$$t_a = \int_0^{V_b} \frac{\gamma m}{P_t/V_b - m g f - 0,5 \rho S C_x V^2} dV + \int_{V_b}^{V_f} \frac{\gamma m}{P_t/V - m g f - 0,5 \rho S C_x V^2} dV$$

- Approximation solution: neglect the rolling and the drag resistances

$$t_a = \frac{\gamma m}{2 P_t} (V_f^2 + V_b^2)$$



EV acceleration

- Let's calculate the acceleration time when neglecting the aerodynamic and rolling resistance

$$t_{acc} = \int_0^{V_b} \frac{\gamma m}{P_t/V_b} dV + \int_{V_b}^{V_f} \frac{\gamma m}{P_t/V} dV$$

$$t_{acc} = \int_0^{V_b} \frac{\gamma m V_b}{P_t} dV + \int_{V_b}^{V_f} \frac{\gamma m V}{P_t} dV$$

$$= \frac{\gamma m V_B}{P_t} [V]_0^{V_B} + \frac{\gamma m}{P_t} \left[\frac{V^2}{2} \right]_{V_b}^{V_f}$$

$$= \frac{\gamma m}{P_t} \left(V_b^2 + \frac{V_f^2 - V_b^2}{2} \right)$$

$$t_{acc} = \frac{\gamma m}{P_t} \frac{V_f^2 + V_b^2}{2}$$



EV acceleration

- Sizing of rated power of electric motor:
 - If the acceleration time is given:

$$P_{max} = \frac{\gamma m}{2 t_a \eta_t} (V_f^2 + V_b^2)$$

- However to determine more accurately the rated power, one can add a correction that is the average power dissipated by the resistance forces during the acceleration phase

$$\bar{P}_{aver}^{Res} = \frac{1}{t_a} \int_0^{t_a} m g f V + 0,5 \rho S C_x V^3 dt$$

$$P_{max} = \frac{\gamma m}{2 t_a \eta_t} (V_f^2 + V_b^2) + \bar{P}_{aver}^{Res}$$



EV acceleration

- To compute the average power dissipation by the rolling and aerodynamic resistances during the acceleration, one generally assumes that it is performed at constant power all along the acceleration (actually it is only the case over V_B)

$$F = m \frac{dV}{dt} \qquad F = \frac{P}{V}$$

- It comes

$$\frac{P}{V} = m \frac{dV}{dt} \quad \Rightarrow \quad dt = \frac{m}{P} V dV$$

$$t_2 = \int_{t_1=0}^{t_2} dt = \int_{V_1=0}^{V_2} \frac{m}{P} V dV = \frac{m}{2P} (V_2^2 - V_1^2) = \frac{m}{2P} V_2^2$$

$$V(t) = V_f \sqrt{\frac{t}{t_a}}$$



EV acceleration

- Inserting into the integral,

$$V(t) = V_f \sqrt{\frac{t}{t_a}}$$

- It comes the estimated power of the motor

$$\begin{aligned}\bar{P}_{aver}^{Res} &= \frac{1}{t_a} \int_0^{t_a} mgf V(t) + 0,5\rho SC_x V(t)^3 dt \\ &= \frac{1}{t_a} \int_0^{t_a} mgf V_f \left(\frac{t}{t_a}\right)^{1/2} + 0,5\rho SC_x V_f^3 \left(\frac{t}{t_a}\right)^{3/2} dt \\ &= \frac{1}{t_a} \left\{ mgf \frac{2}{3} \frac{t_a^{3/2}}{t_a^{1/2}} V_f + 0,5\rho SC_x \frac{2}{5} \frac{t_a^{5/2}}{t_a^{3/2}} V_f^3 \right\} \\ &= mgf \frac{2}{3} V_f + \frac{1}{5} \rho SC_x V_f^3\end{aligned}$$



EV acceleration

- Power rating of the e-machine to ensure a given acceleration time

- Inertia forces

$$P_{max} = \frac{\gamma m}{2 t_a \eta_t} (V_f^2 + V_b^2)$$

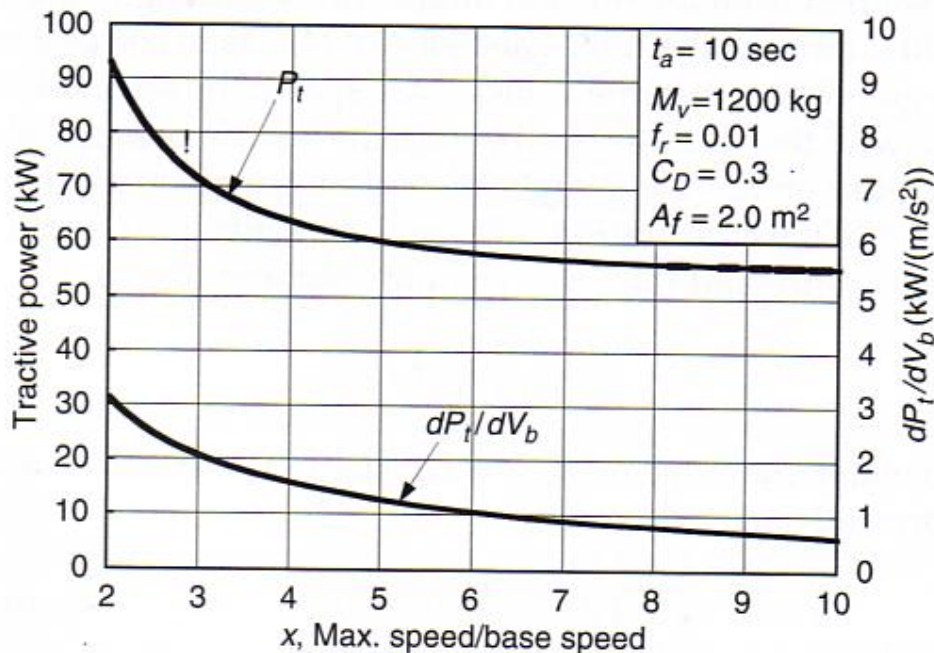
- Average power dissipated by resistance forces⁴

$$\bar{P}_{aver}^{Res} = \frac{2}{3} m g f V_f + \frac{1}{5} \rho S C_x V_f^3$$

- Estimated power of the motor

$$P_t = \frac{\gamma m}{2 t_a} (V_f^2 + V_b^2) + \frac{2}{3} m g f V_f + \frac{1}{5} \rho S C_x V_f^3$$

EV acceleration



Eshani Fig 5.9

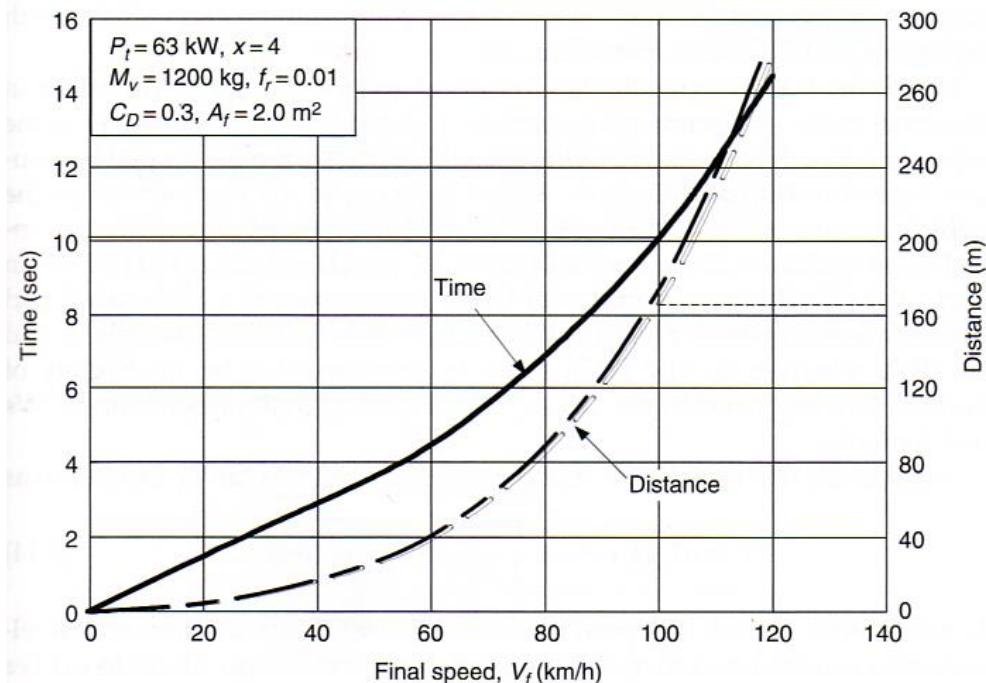
- The result shows that for a given acceleration performance, lower vehicle base speeds will result in smaller motor power rating
- However the power rating decline rate to the vehicle base speed reduction is not identical

$$\frac{dP_t}{dV_b} = \frac{\gamma m}{t_a} V_b$$

EV acceleration

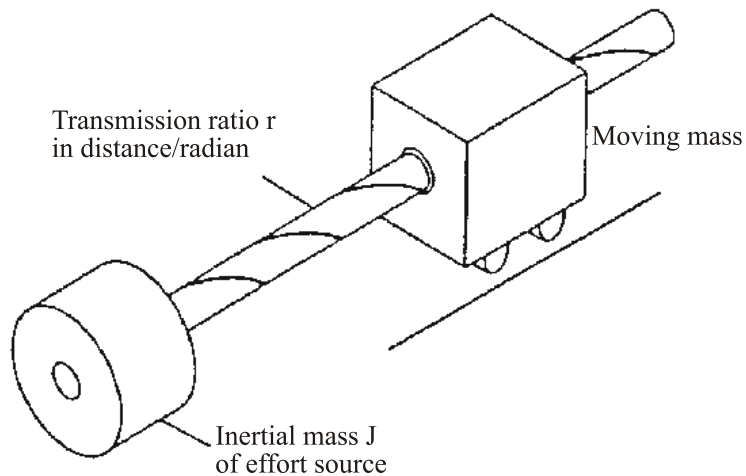
- Fine estimation of acceleration time requires solving exactly (numerically) the Newton equation

$$t_a = \int_0^{V_f} \frac{\gamma m}{F_t - mgf - 0,5\rho SC_x V^2} dV$$

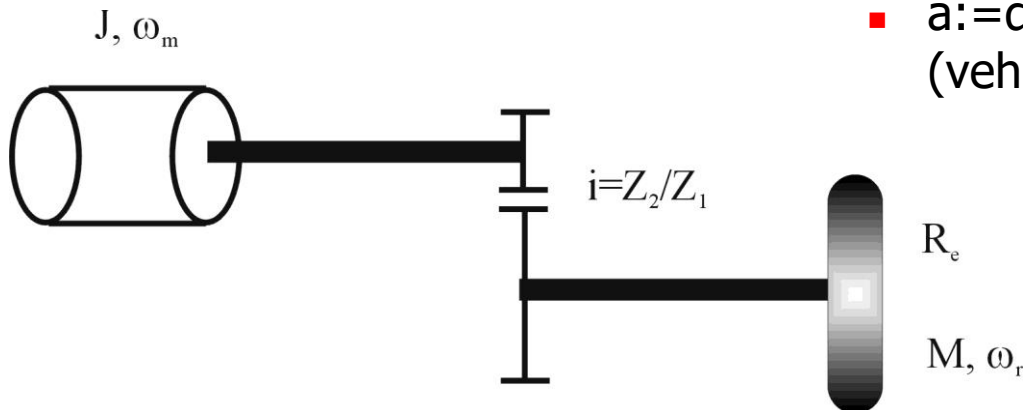


Eshani Fig 5.10

Acceleration capacity and impedance adaptation

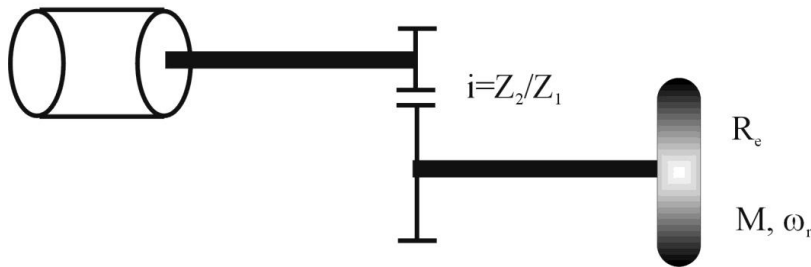


- 1D system with one electric motor connected to the mechanical load via a gear box or reduction ratio r
 - M mass of load (here the vehicle)
 - J inertia of electric motor
 - $i = Z_1/Z_2$, the gear ratio
 - R_e : tire rolling radius
 - $r = R_e/I$ transmission length
 - $a := dv/dt$ acceleration of the load (vehicle)



Acceleration capacity and impedance adaptation

J, ω_m



- Newton equation of the vehicle

$$F_t = M \frac{dV}{dt}$$

- Tractive force

$$F_t = \eta \frac{i}{R_e} \left(C_m - J_m \frac{d\omega_m}{dt} \right)$$

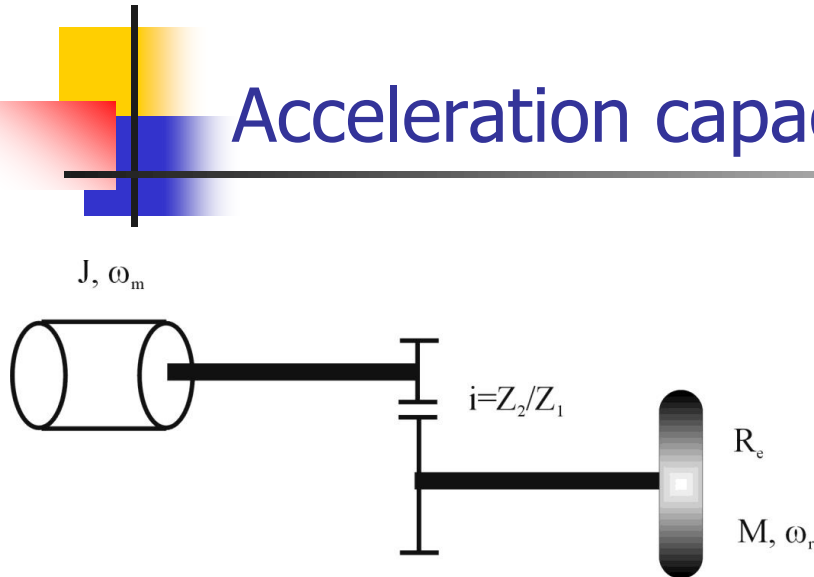
- Relation between engine rotation speed and velocity

$$V = \frac{\omega_m R_e}{i}$$

- Equation of motion

$$\eta \frac{i}{R_e} \left(C_m - J_m \frac{i}{R_e} \frac{dV}{dt} \right) = M \frac{dV}{dt}$$

Acceleration capacity and impedance adaptation



- Newton equation of the vehicle

$$C_m = \left(\frac{J_m}{(R_e/i)} + M(R_e/i) \right) \frac{dV}{dt}$$

$$= (M + J(i/R_e)^2) (R_e/i) \frac{dV}{dt}$$

- Acceleration

$$a = \frac{dV}{dt}$$

- Acceleration

$$a = \frac{C_m (i/R_e)}{M + J (i/R_e)^2}$$



Acceleration capacity and impedance adaptation

- Derivative of acceleration with respect to gear ratio

$$\frac{\partial a}{\partial (i/R_e)} = \frac{C_m}{M + J(i/R_e)^2} - \frac{C_m(i/R_e)}{(M + J(i/R_e)^2)^2} 2(i/R_e)J = 0$$

- Optimal gear ratio

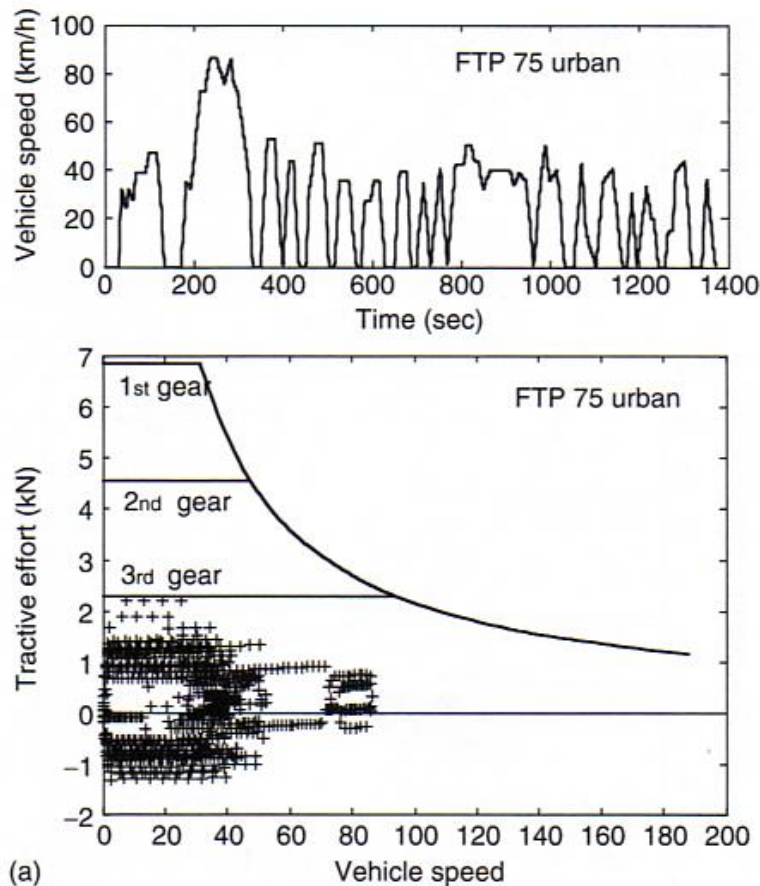
$$(i/R_e)^{opt} = \sqrt{\frac{M}{J}}$$

- Optimal acceleration power

$$a^{max} = \frac{1}{2} \frac{C_m}{\sqrt{JM}} = \frac{1}{2} \frac{C_m}{M} (i/R_e)^{opt}$$

- Conclusion: this is the maximum acceleration that can be given to the load by a motor with maximum torque C_m .

EV accelerations in normal operation



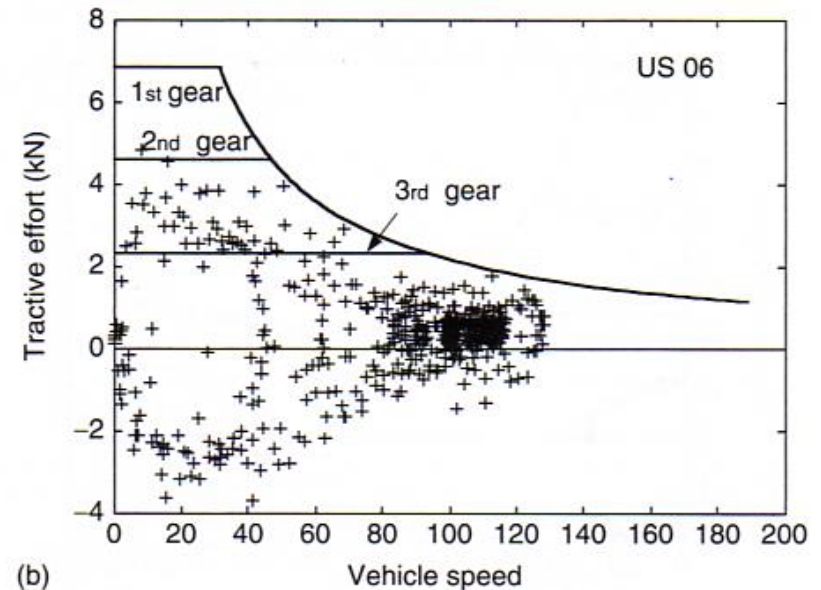
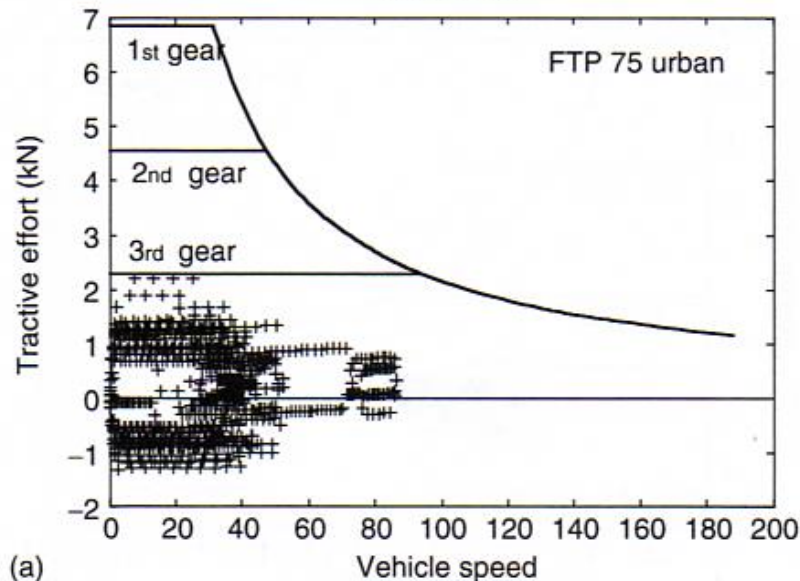
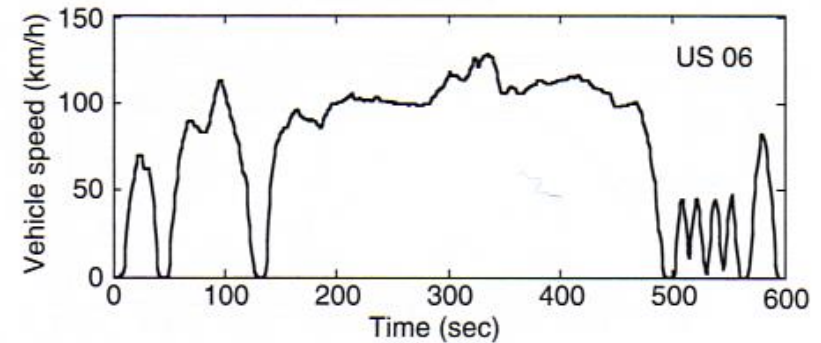
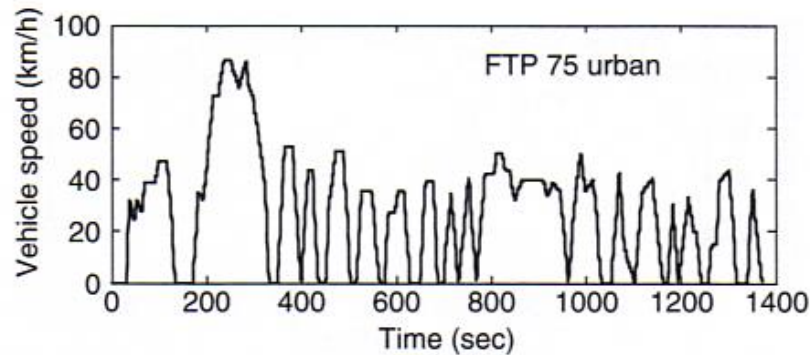
- Driving cycle $V(t)$ is given
- Evaluate the acceleration required: differentiating the velocity profile:

$$\frac{dV}{dt} \simeq \frac{V(t_{k+1}) - V(t_k)}{t_{k+1} - t_k}$$

- Tractive force is given by the net force necessary to follow the driving cycle

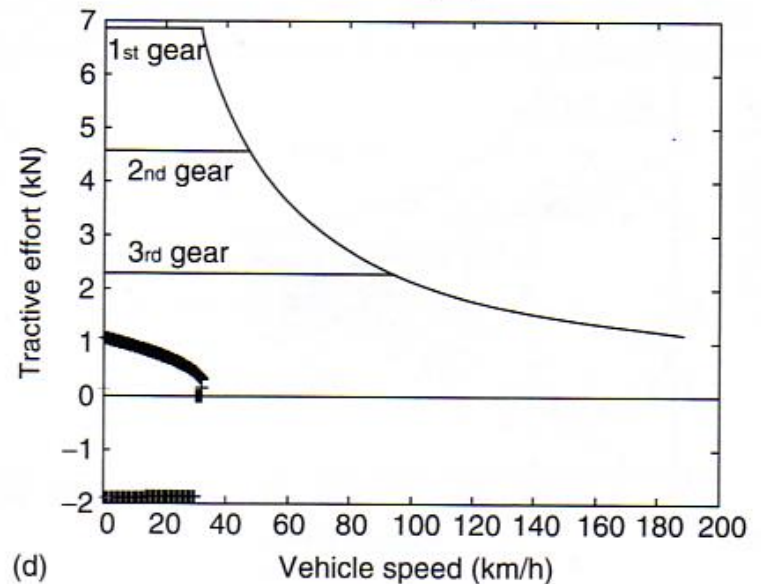
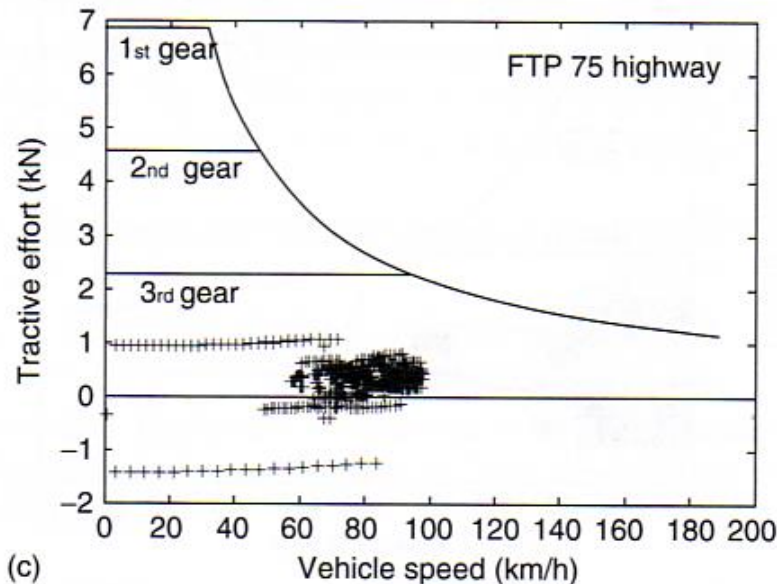
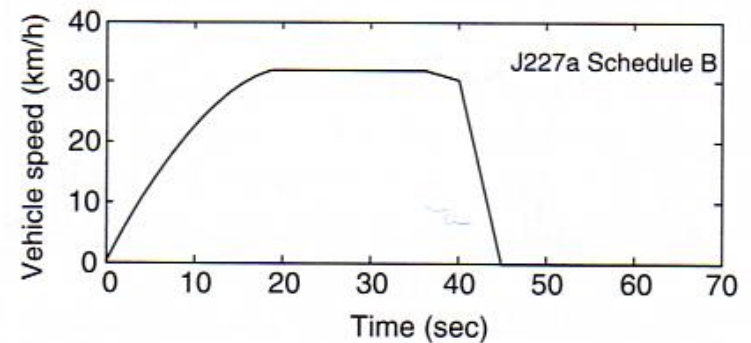
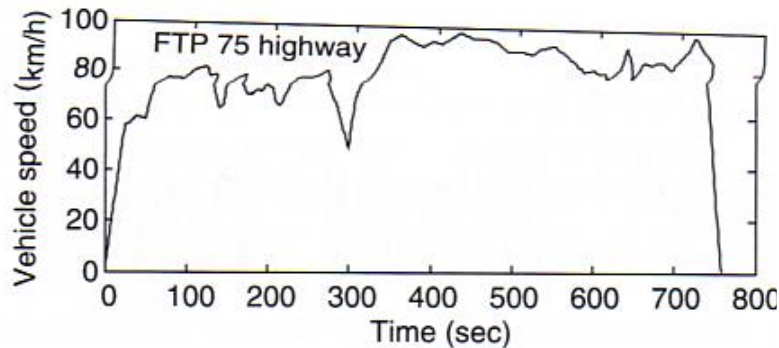
$$F_t = mgf \cos \theta + \frac{1}{2} \rho S C_x S V^2 + \gamma m \frac{dV}{dt}$$

EV accelerations in normal operation

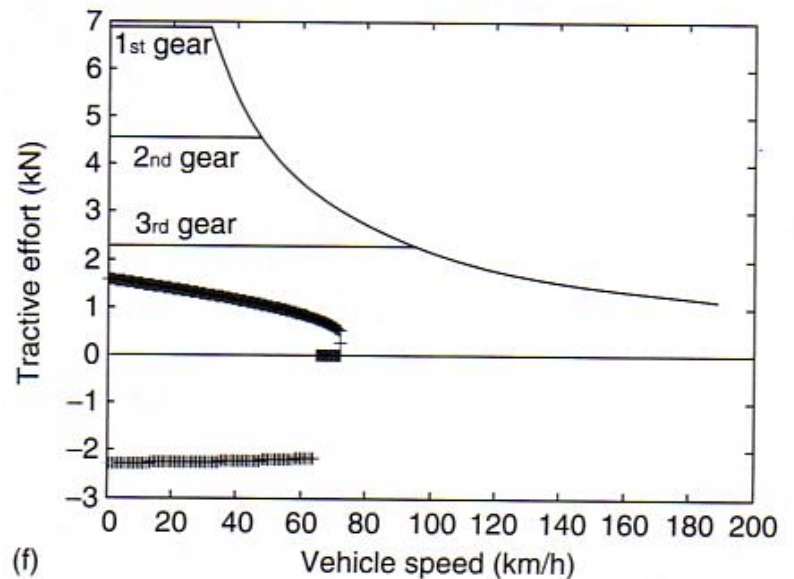
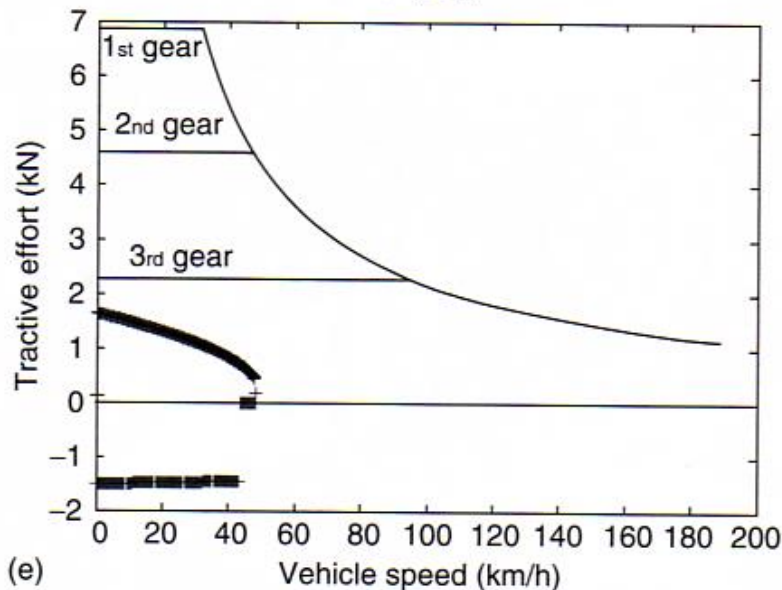
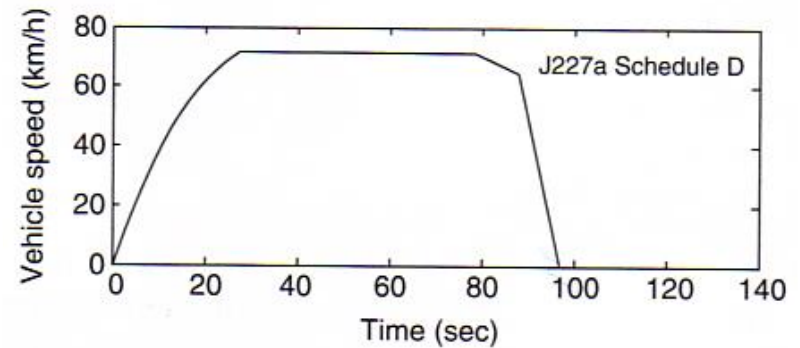
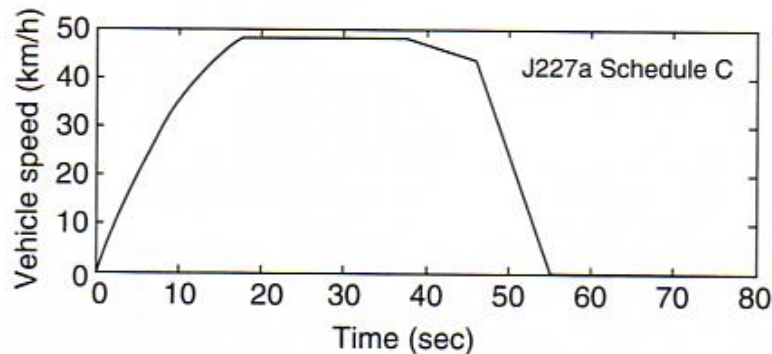


Eshani et al. Fig 5.12 Speed profile and tractive effort in FT75 and US06 drive cycle

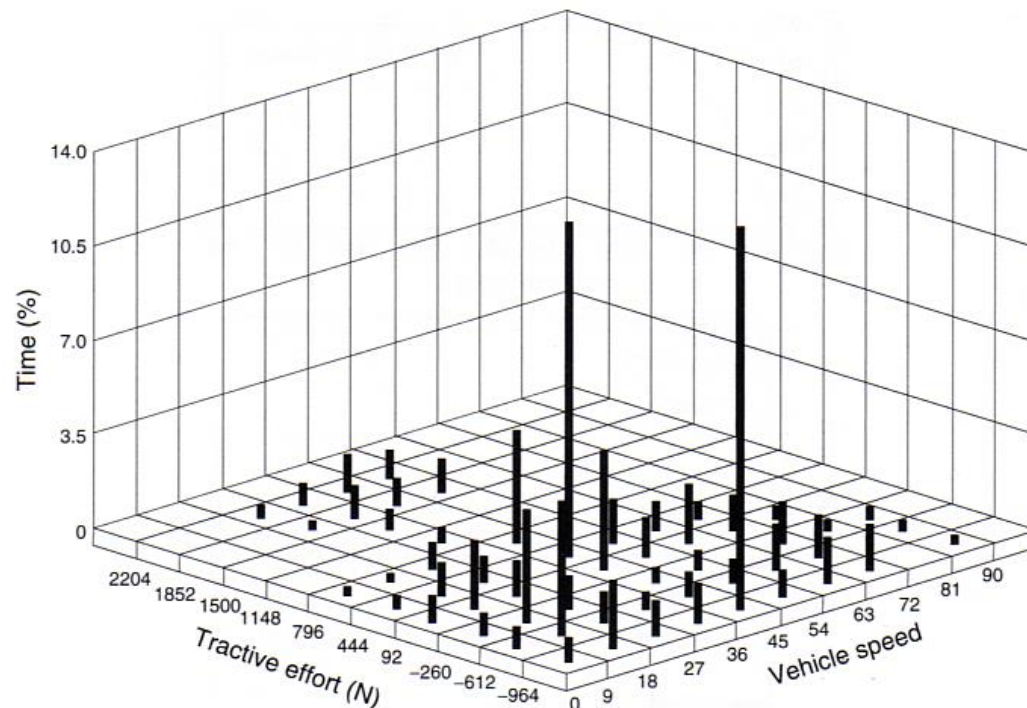
EV accelerations in normal operation



EV accelerations in normal operation



EV accelerations in normal operation



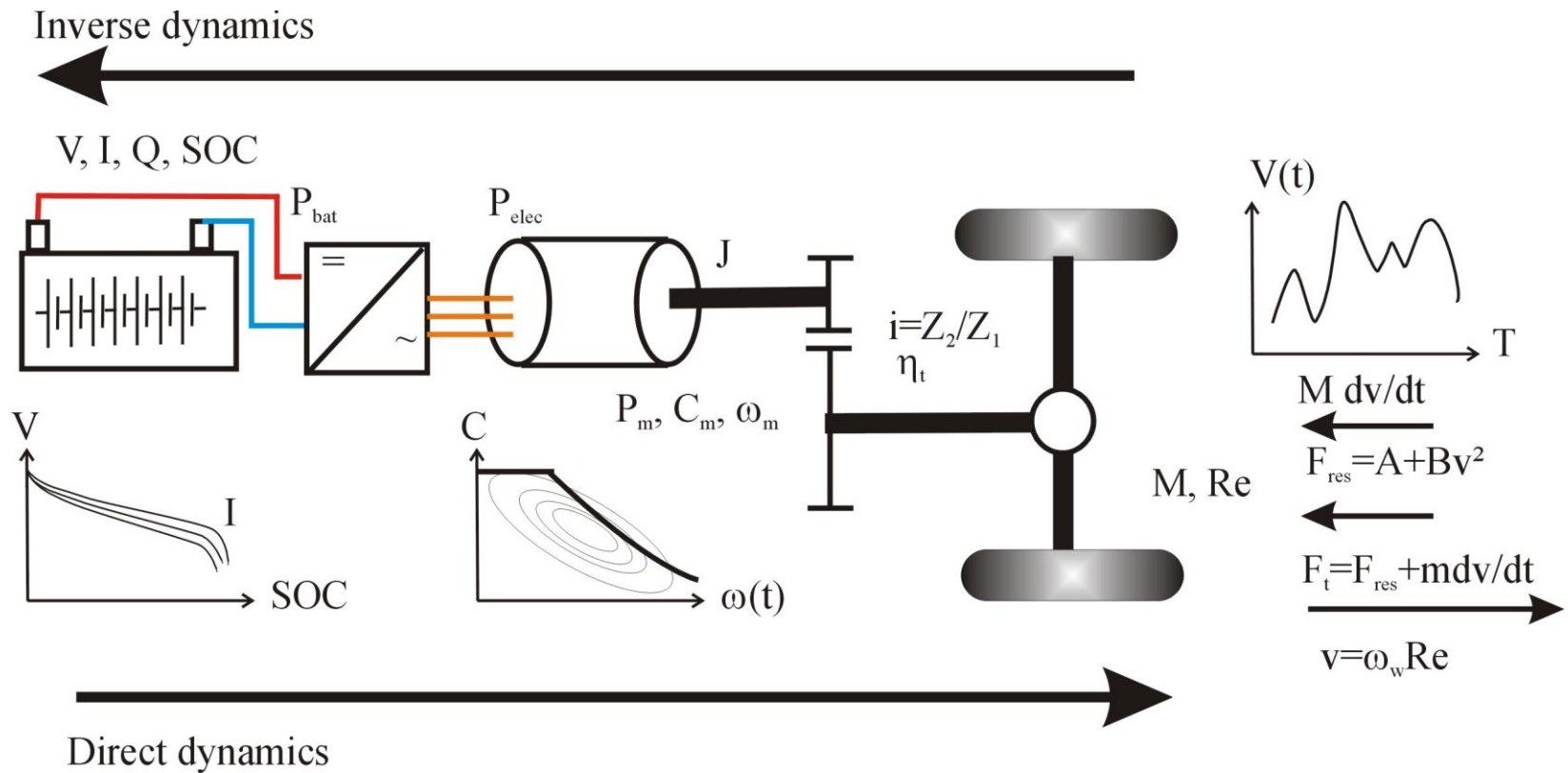
Eshani et al. Fig 5.13 Time distribution on vehicle speed and tractive effort in FT75 drive cycle



EV energy consumption

- In transportation the unit of energy is usually the kWh (kiloWatt hour) (preferred to J ou kJ)
 - ICE with liquid fuels: L/100 km or mpg
 - Gaseous fuel (CH_4 , H_2): kg/100 km
- Advantage: size of batteries given in kWh at battery ports so that the driving range can be calculated immediately.
- Energy consumption results from the time integration of the power output and input at battery terminal.

Inverse dynamics



EV inverse dynamics



$$F_{res} = mgf$$

$$+ 1/2 \rho S C_x V^2$$

$$mg \sin \theta$$

$$F_w = F_{res} + m_e \frac{dV}{dt}$$

$$v(t) \, dv/dt$$

$$P_w = \eta_t P_{mot}$$

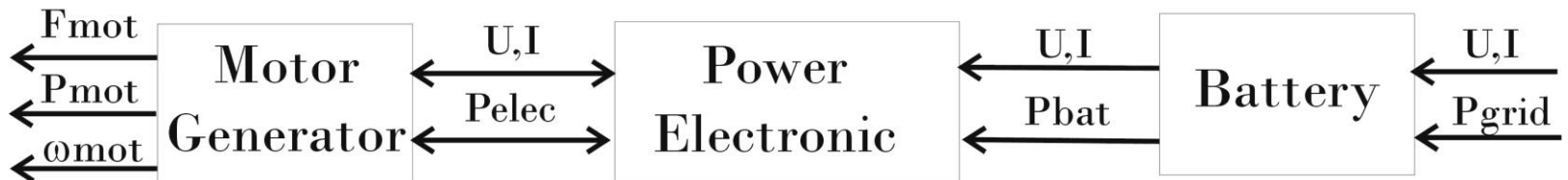
$$F_w = \eta C_{mot} i / R_E$$

$$v = \omega_w R_e \omega_{mot} R_e / i$$

$$P_m = \eta_{e-M} P_{elec}$$

$$\eta_{e-M} = \eta_{e-M}(C_m, \omega_m)$$

$$P_m = C_m \omega_m$$



$$P_{bat} = P_{elec} \text{ if } P_{elec} > 0$$

$$P_{bat} = -\alpha P_{elec} \text{ if } P_{elec} < 0$$

$$Q = Q_0 - \int i dt$$



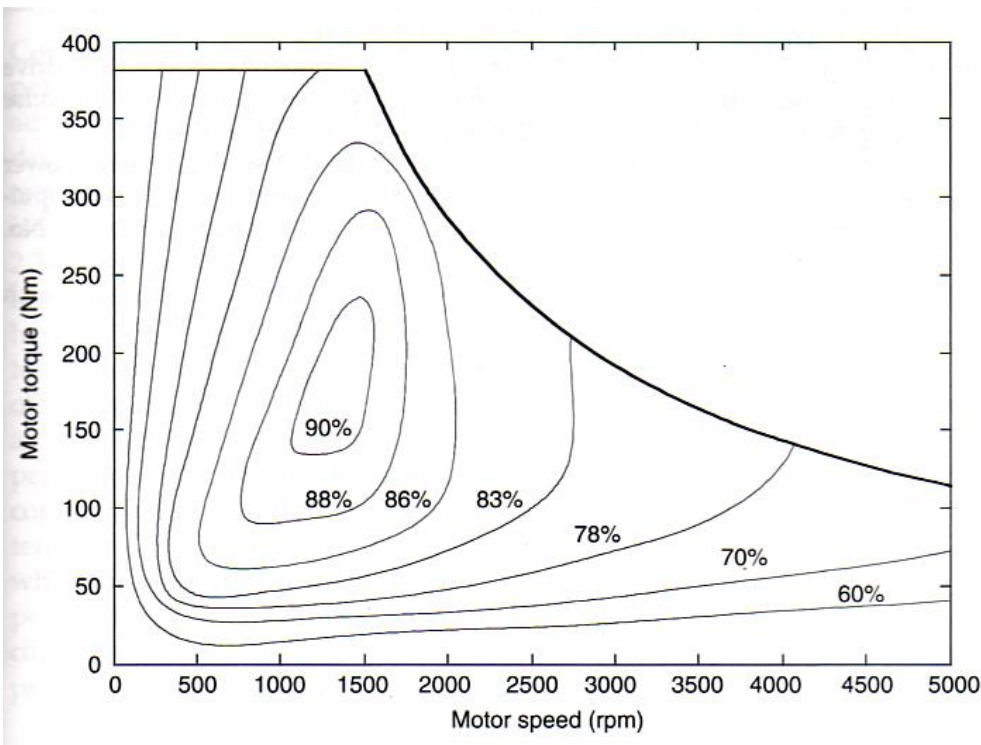
EV energy consumption

- Energy power output
 - Equal to the resistance power and the power losses in the transmission and motor drive including the power electronic losses

$$P_{bat}^{out} = \frac{V}{\eta_t \eta_m(C_m, \omega_m)} \left(mgf \cos \theta + mg \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma m \frac{dV}{dt} \right) > 0$$

- The non traction loads are not included (auxiliary loads) while they can be significant, and they should be added to the traction load.

EV energy consumption



Eshani et al. Fig 5.14 Typical electric motor efficiency characteristics

- The efficiency of the traction motor varies with the operating points on the speed-torque (speed-power) plane
- Good design: large overlap between the maximum efficiency region and the region visited by the most frequent operation points



EV energy consumption

- The regenerative braking power at battery can be evaluated as

$$P_{bat}^{in} = \alpha \eta_t \eta_m V \left(mgf \cos \theta + mg \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma m \frac{dV}{dt} \right) < 0$$

- In which
 - road slope $\sin \theta < 0$ and/or deceleration $dV/dt < 0$
 - $0 < \alpha < 1$ is the fraction of energy recovered during braking
The braking factor α is a function of the applied braking strength and the design and control of braking system
Typical α is around 0,3



EV energy consumption

- The **net energy consumption** from batteries is:

$$En^{out} = \oint_{P_t > 0} P_{bat}^{out} dt - \oint_{P_t < 0} P_{bat}^{in} dt$$

- When the net battery energy consumption reaches the total energy in the batteries, measured at terminal, the batteries are empty and need to be charged.
- The traveling distance between two charges is called the **effective travel range**.
- It is dependent on the battery capacity, the road resistance power, the driving cycle, the effectiveness of regenerative braking, the efficiency of the car and its powertrain.



Preliminary design procedure of EV



EV Design Procedure

- Let's give the vehicle specifications
 - Time t_a to accelerate from 0 to V_f
 - Maximum speed
 - Minimum gradeability
 - Range of the BEV

- Assume
 - Some vehicle characteristics: S C_x , f ...
 - Vehicle mass:
 - Body in blank
 - Mass of battery $M = M_V + M_{bat}$
 - Electric motor technology: IPM, Induction, SRM? $\rightarrow x=2, 4, \text{ or } 6$



EV Design Procedure

- First estimate the rating power of the e-motor. Acceleration time is generally the most critical criterion

$$P_{max} = \frac{\gamma m}{2 \eta_t t_a} (V_f^2 + V_b^2) \quad V_b = V_{max} / X$$

- V_b can be estimated using N_{max} and the aspect ratio of the selected e-motor technology (X factor)
- Then a reduction ratio of the transmission can be determined using the target top speed

$$i = \frac{2\pi N_m^{max} R_e}{60 V_{max}}$$

- Select one motor from the catalog



EV Design Procedure

- Check the top speed

$$\eta_t P_m^{max} = A V_{max} + B V_{max}^3$$

- And the grade ability

$$\sin \theta = \frac{d - f \sqrt{1 - d^2 + f^2}}{1 + f^2} \quad d = (F_t - F_{Aero})/mg$$

- If not satisfied, adapt the gear ratio i and if necessary, select a second gear ratio to satisfy both specifications



EV Design Procedure

- Adapt the base speed

$$V_b = \frac{2\pi N_m^{max} R_e}{60 i X}$$

- Compute a finer estimation of the acceleration power

$$P_t = \frac{\gamma m}{2t_a}(V_f^2 + V_b^2) + \frac{2}{3}mgfV_f + \frac{1}{5}\rho SC_x V_f^3$$

- And solve numerically the time integration of the Newton's second law to quote the acceleration time

$$t_a = \int_0^{V_f} \frac{\gamma m}{F_t - mgf - 0,5\rho SC_x V^2} dV$$



EV Design Procedure

- Repeat the design cycle till convergence
- Use simulation to estimate energy consumption and size the battery pack to reach the desired range
- Select the relevant driving cycles

$$P_{bat}^{out} = \frac{V}{\eta_t \eta_m (C_m, \omega_m)} \left(mgf \cos \theta + mg \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma m \frac{dV}{dt} \right)$$

$$P_{bat}^{in} = \alpha \frac{V}{\eta_t \eta_m} \left(mgf \cos \theta + mg \sin \theta + \frac{1}{2} \rho S C_x V^2 + \gamma m \frac{dV}{dt} \right)$$

$$En^{out} = \oint_{P_t > 0} P_{bat}^{out} dt - \oint_{P_t < 0} P_{bat}^{in} dt$$