Vehicle Performance

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Road Loads

Exercise 1 : Road Loads

- Let's consider the following car

$$m = 1000\ kg \qquad \eta_t = 0,9$$

$$S = 1,8\ m^2 \qquad C_x = 0,31 \qquad \rho = 1,2 kg/m^2$$

$$f = 0,01\ +\ 10^{-5}\, V^2$$

- Compute the road loads generated at speed in between 0 and 50 m/s

Exercise 1 : Road Loads

- Rolling resistance

$$F_{RR} = mg\ \cos\theta\, f_{RR} = mg\ \cos\theta\,(0.01\ +\ 10^{-5}\,V^2)$$

- It comes

$$V = 10\,m/s \qquad F_{RR} = 1000 \cdot 9,81 \cdot (0,01 + 10^{-5}\,10^2) = 107,91\,N$$

$$\mathcal{P}_{RR} = F_{RR}\,V = 1079,1\ W$$

$$V = 25\,m/s \qquad F_{RR} = 1000 \cdot 9,81 \cdot (0,01 + 10^{-5}\,25^2) = 159,91\,N$$

$$\mathcal{P}_{RR} = F_{RR}\,V = 39971,75\ W$$

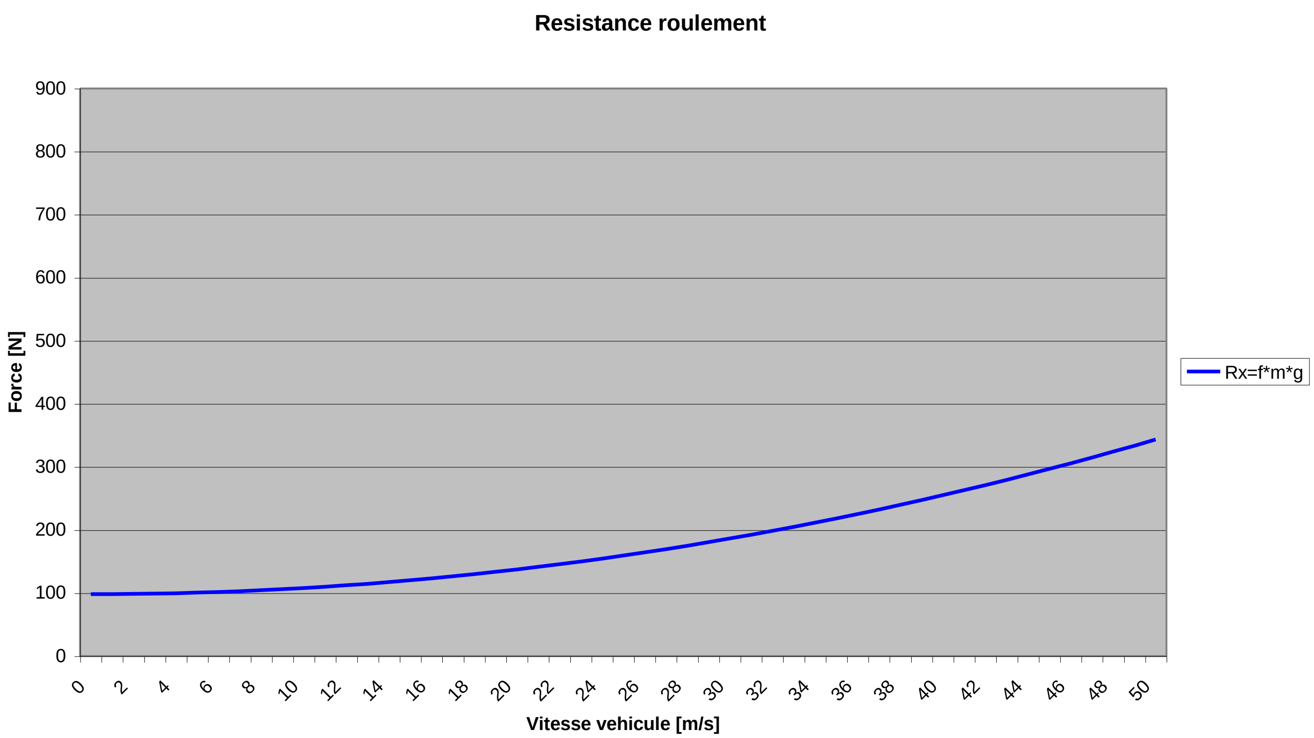
$$V = 50\,m/s \qquad F_{RR} = 1000 \cdot 9,81 \cdot (0,01 + 10^{-5}\,50^2) = 343,35\,N$$

$$\mathcal{P}_{RR} = F_{RR}\,V = 17167\ W$$

Exercise 1 : Road Loads

- Rolling resistance

$$F_{RR} = mg\ \cos\theta\ f_{RR} = mg\ \cos\theta\ (0.01\ +\ 10^{-5}\ V^2)$$



Exercise 1 : Road Loads

- Aerodynamic drag

$$F_{AERO} = \frac{1}{2}\rho\, S\, C_x\, V^2$$

- It comes

$$V = 10\, m/s \qquad F_{AERO} = 0,5 \cdot 1,2 \cdot 1,8 \cdot 0,31 \cdot V^2 = 33,48\ N$$

$$\mathcal{P}_{AERO} = F_{AERO} \cdot V = 334,8\ W$$

$$V = 25\, m/s \qquad F_{AERO} = 0,5 \cdot 1,2 \cdot 1,8 \cdot 0,31 \cdot 25^2 = 209,25\ N$$

$$\mathcal{P}_{AERO} = F_{AERO} \cdot V = 5231,25\ W$$

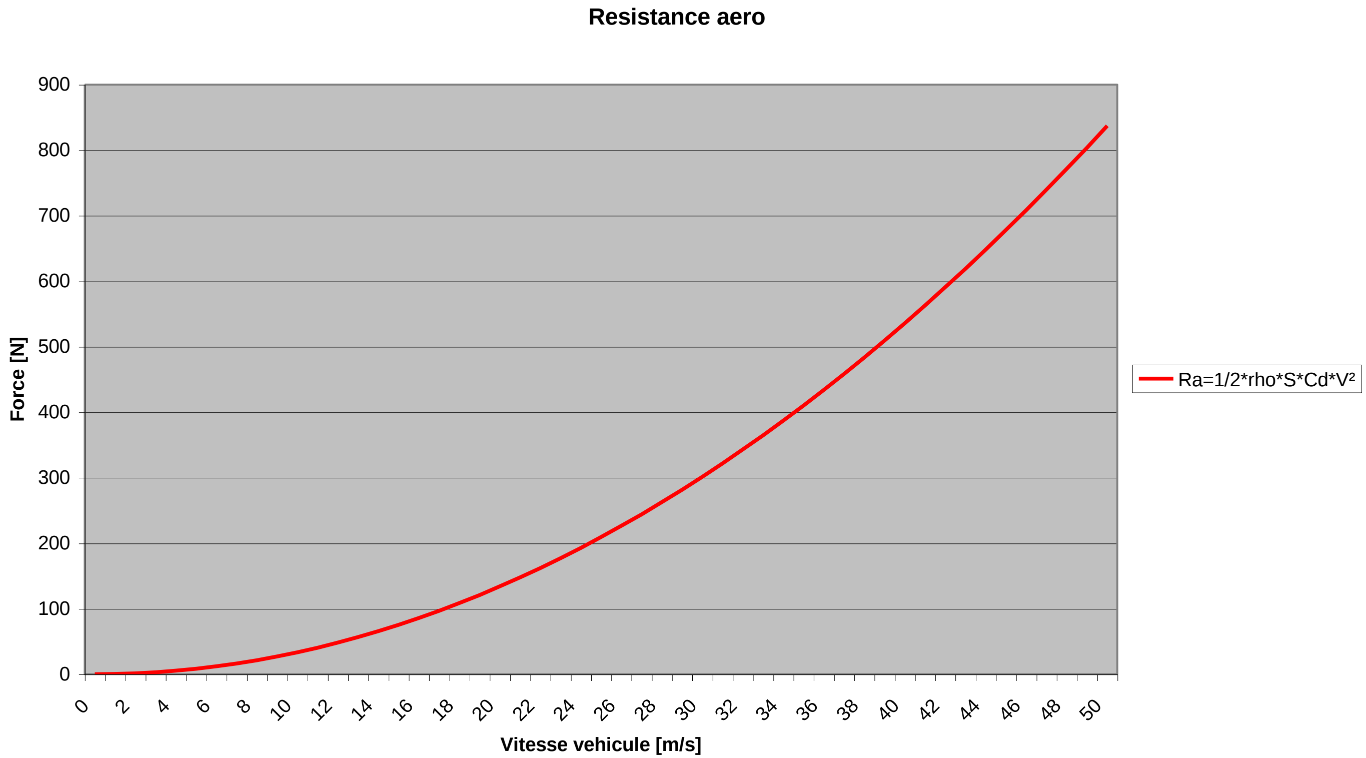
$$V = 50\, m/s \qquad F_{AERO} = 0,5 \cdot 1,2 \cdot 1,8 \cdot 0,31 \cdot 50^2 = 837\ N$$

$$\mathcal{P}_{AERO} = F_{AERO} \cdot V = 41850\ W = 41,850\ kW$$

Exercise 1 : Road Loads

- Rolling resistance

$$F_{AERO} = \frac{1}{2} \rho \, S \, C_x \, V^2$$



Exercise 1 : Road Loads

- Total road resistance forces

$$F_{RES} = F_{RR} + F_{AERO} = mg\left(f_0 + f_2 V^2\right)\,\cos\theta + \frac{1}{2}\rho\, S\, C_x\, V^2$$

- It comes

$$V = 10\, m/s \quad F_{RR} + F_{AERO} = 107,91 + 33,48 = 141,39\ N$$
$$\mathcal{P}_{RR} + \mathcal{P}_{AERO} = 1,4139\ kW$$

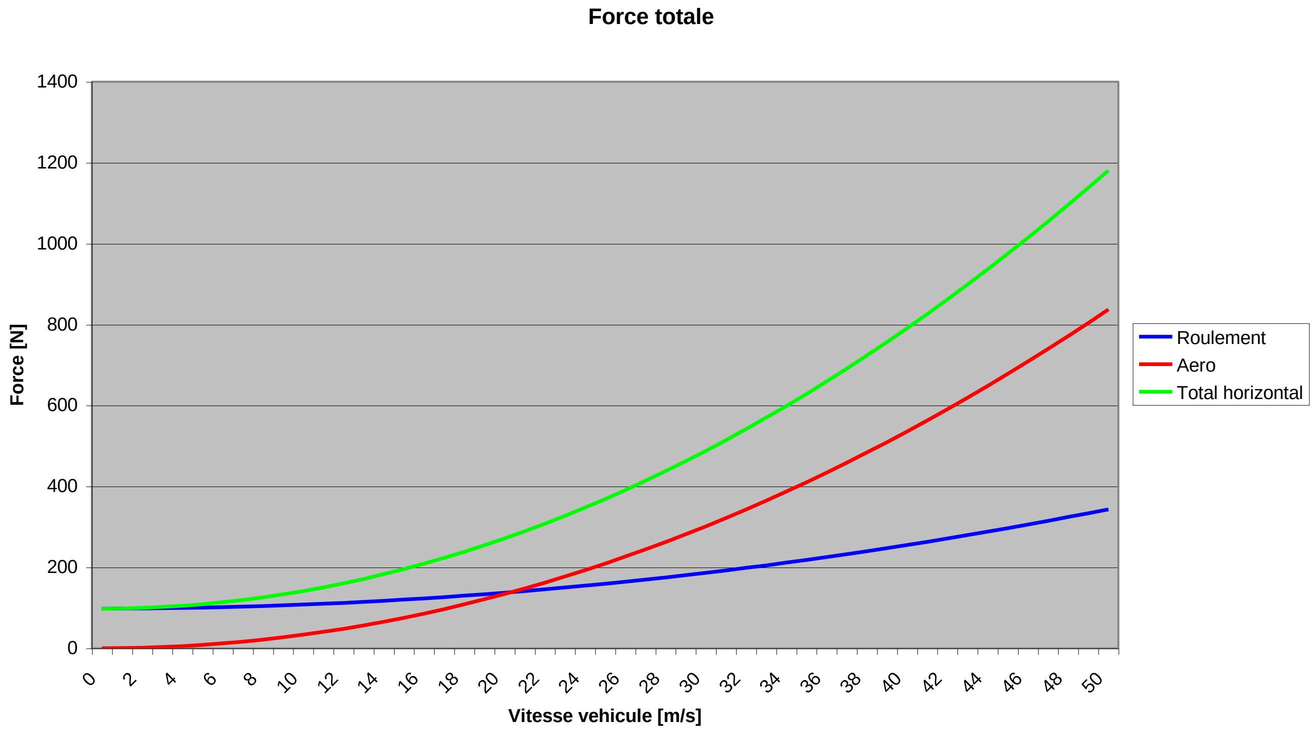
$$V = 25\, m/s \quad F_{RR} + F_{AERO} = 159,9 + 209,25 = 369,15\ N$$
$$\mathcal{P}_{RR} + \mathcal{P}_{AERO} = 9,225\ kW$$

$$V = 50\, m/s \quad F_{RR} + F_{AERO} = 343,35 + 837 = 1180,35\ N$$
$$\mathcal{P}_{RR} + \mathcal{P}_{AERO} = 59,0175\ kW$$

Exercise 1 : Road Loads

- Total road resistance forces

$$F_{RES} = F_{RR} + F_{AERO} = mg\left(f_0 + f_2 V^2\right)\,\cos\theta + \frac{1}{2}\rho\, S\, C_x\, V^2$$



Exercise 1 : Road Loads

- When aerodynamic drag and rolling resistance are equal?

$$F_{RR} = F_{AERO} \quad \Longleftrightarrow \quad mg\left(f_0 + f_2 V^2\right)\,\cos\theta = \frac{1}{2}\rho\, S\, C_x\, V^2$$

- We have

$$\left(\frac{1}{2}\rho\, S\, C_x - mg\, f_2\, cos\theta\right)\, V^2 = mg\, f_0\, cos\theta$$

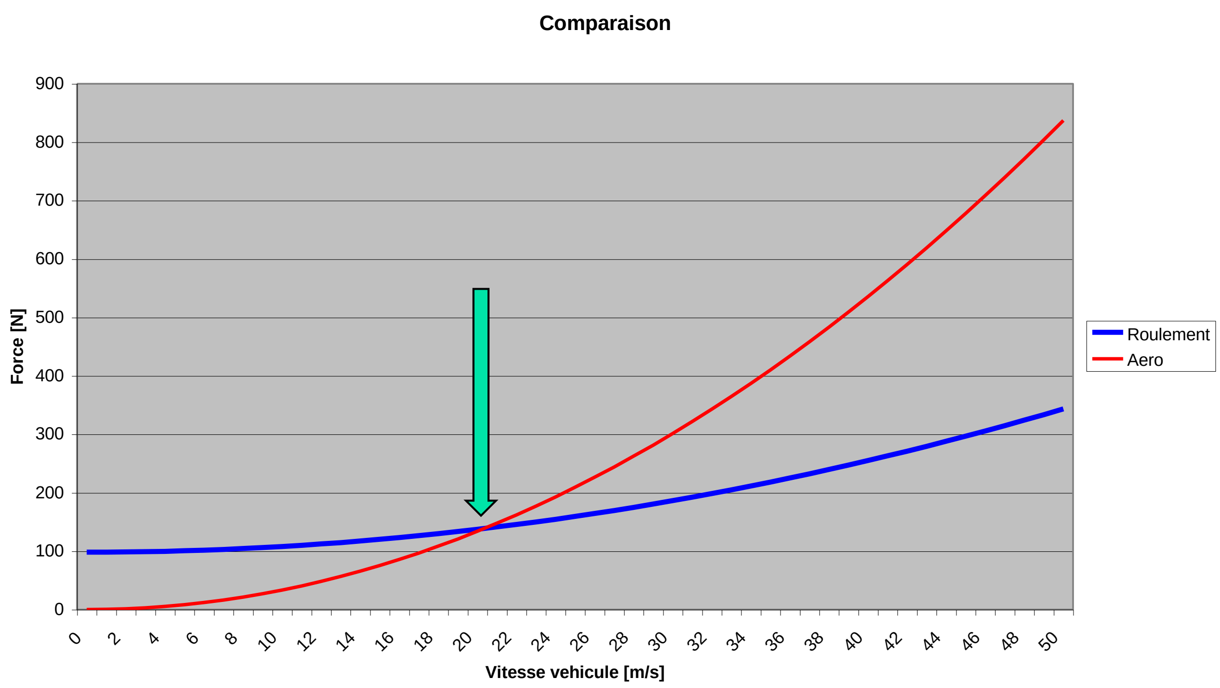
$$V = \sqrt{\frac{mg\, f_0\, cos\theta}{\frac{1}{2}\rho\, S\, C_x - mg\, f_2\, cos\theta}}$$

$$V = \sqrt{\frac{98,1}{0,2367}} = \sqrt{414,448} = 20,35\ m/s = 73,29 km/h$$

Exercise 1 : Road Loads

- When aerodynamic drag and rolling resistance are equal?

$$F_{RR} = F_{AERO} \qquad \Longleftrightarrow \qquad mg\left(f_0 + f_2 V^2\right)\,\cos\theta = \frac{1}{2}\rho\, S\, C_x\, V^2$$



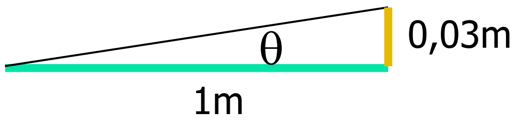
Exercise 1 : Road Loads

- Grading resistance

$$F_{GRADING} = mg\ \sin\theta$$

- We have

$$\begin{aligned} \text{slope} = 3\% \quad &\Longleftrightarrow \quad \tan\theta = 0,03 \\ &\Longleftrightarrow \quad \theta = 1,7183^\circ \\ &\Longleftrightarrow \quad \sin\theta = 0,0299 \end{aligned}$$



$$\theta = 3\% \quad F_{GRADING} = mg\ \sin\theta$$
$$= 1000 \cdot 9,81 \cdot 0,0299 = 294,3\ N$$

$$\begin{aligned} V &= 10\ m/S & \mathcal{P}_{GRAD.} &= F_{GRAD.} \cdot V & &= 2,941\ kW \\ V &= 25\ m/S & \mathcal{P}_{GRAD.} &= F_{GRAD.} \cdot V & &= 7,3542\ kW \\ V &= 50\ m/S & \mathcal{P}_{GRAD.} &= F_{GRAD.} \cdot V & &= 14,7084\ kW \end{aligned}$$

Exercise 1 : Road Loads

- Grading resistance

$$\text{slope} = 10\% \iff \theta = 5,71^\circ$$
$$\iff \sin\theta = 0,0995$$

$$F_{GRADING} = 1000 \cdot 9,81 \cdot 0,0995 = 976,13\,N$$

$$\text{slope} = 20\% \iff \theta = 11,3099^\circ$$
$$\iff \sin\theta = 0,1961$$

$$F_{GRADING} = 1000 \cdot 9,81 \cdot 0,1961 = 1923,89\,N$$

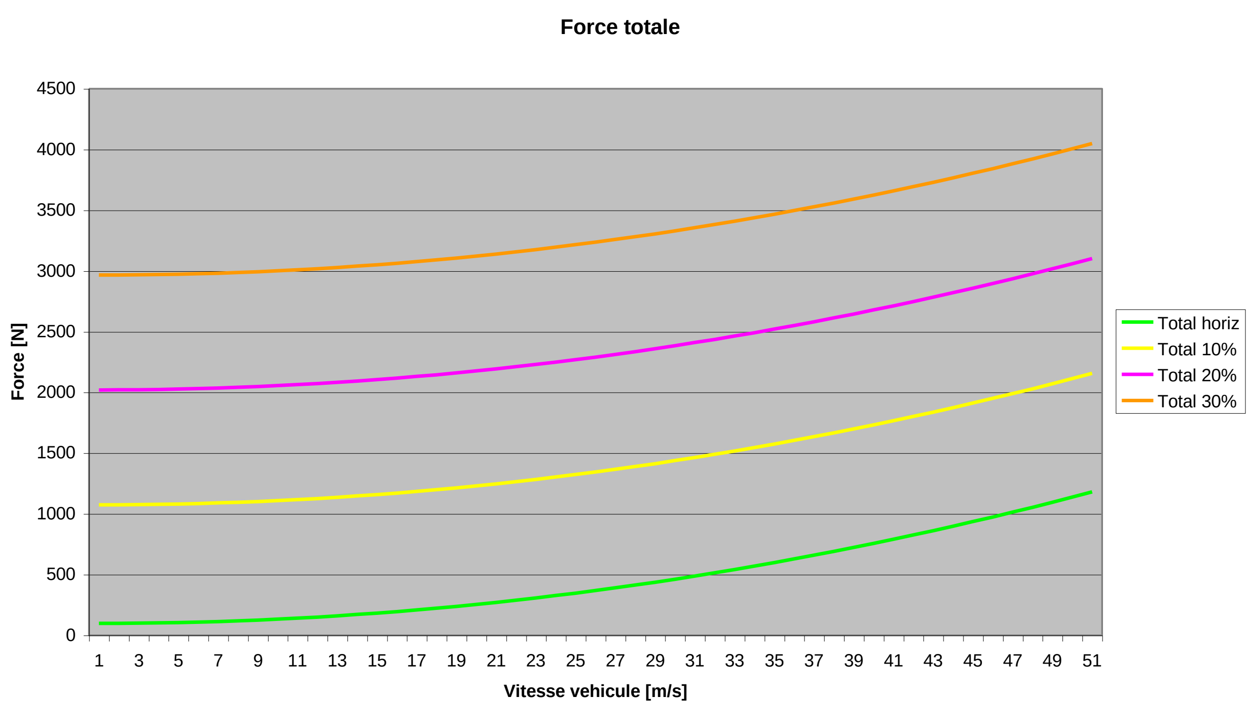
$$V = 10\,m/S \qquad \mathcal{P}_{GRAD.} = F_{GRAD.} \cdot V \quad = 19,2389\,kW$$
$$V = 25\,m/S \qquad \mathcal{P}_{GRAD.} = F_{GRAD.} \cdot V \quad = 48,0974\,kW$$
$$V = 50\,m/S \qquad \mathcal{P}_{GRAD.} = F_{GRAD.} \cdot V \quad = 96,194\,kW$$

Exercise 1 : Road Loads

- When aerodynamic drag and rolling resistance are equal?

$$F_{RES} = F_{RR} + F_{AERO} + F_{GRADING}$$

$$= mg\,(f_0 + f_2 V^2)\,\cos\theta + \frac{1}{2}\rho\,S\,C_x\,V^2 + mg\,\sin\theta = A + B\,V^2$$



Exercise 2 : Shell Eco Marathon

- Let's consider the following car

$$m = 145\ kg \qquad f = 0,01$$

$$S = 0,95\ m^2 \quad C_x = 0,28 \quad \rho = 1,22 kg/m^2$$

- Compute the road loads generated at speed V=8,33 m/s (30 km/h)
  - On level road
  - When θ=2,1°

Exercise 2 : Shell Eco Marathon

- Rolling resistance

$$F_{RR} = mg\,\cos\theta\, f_{RR}$$
$$= 145 \cdot 9,81 \cdot 1 \cdot 0,01 = 14,2245\ N$$
$$\mathcal{P}_{RR} = F_{RR} \cdot V = 14,2245 \cdot 8,33 = 118,4900\ W$$

- Aerodynamic resistance

$$F_{AERO} = \frac{1}{2}\rho S C_x\, V^2$$
$$= 0,5 \cdot 1,22 \cdot 0,95 \cdot 0,28 \cdot 8,33^2 = 11,2590\ N$$

$$\mathcal{P}_{AERO} = F_{AERO} \cdot V = 11,2590 \cdot 8,33 = 93,7878\ W$$

- Road resistance

$$F_{RES} = F_{RR} + F_{AERO} = 14,2245\ +\ 11,2590 = 25,4835\ N$$
$$\mathcal{P}_{RES} = \mathcal{P}_{RR} + \mathcal{P}_{AERO} = 118,49 + 93,7878 = 212,2779\ W$$

Exercise 2 : Shell Eco Marathon

- Grading resistance

$$\theta = 2,1^\circ \qquad \sin\theta \simeq \tan\theta = 3,6643\,10^{-2}$$

$$\begin{aligned} F_{GRADING} &= mg\,\sin\theta \\ &= 145 \cdot 9,81 \cdot \sin 2,1^\circ = 52,1238\,N \end{aligned}$$

- Rolling resistance → negligible modification!

$$\cos\theta = \cos 2,1^\circ = 0,9993$$

$$\begin{aligned} F_{RR} &= mg\,\cos\theta\, f_{RR} \\ &= 145 \cdot 9,81 \cdot 0,9998 \cdot 0,01 = 14,2149\ N \end{aligned}$$

- Road resistance

$$F_{RES} = F_{RR} + F_{AERO} + F_{GRADE} = 25,4835 + 52,1238 = 77,6073\,N$$

If V=8,33 m/s

$$\mathcal{P}_{RES} = F_{RES} \cdot V = 77,6073 \cdot 8,33 = 646,4688\ W$$

Exercise 2 : Shell Eco Marathon

- Let's consider the following car

$$m = 180\ kg \quad f = 0,0018 \quad (\text{Data: Michelin})$$

$$S = 1,049\ m^2 \quad C_x = 0,20 \quad \rho = 1,22 kg/m^2$$

- Compute the road loads generated at speed V=26 km/h
  - On level road
  - When θ= 0,35° and θ= 2%

Exercise 2 : Shell Eco Marathon

- Speed :

$$V = 26km/h = 26/3,6 = 7,23\,m/s$$

- Slope:

$$\theta = 0,35^\circ = 6,10\,10^{-3}\,rad$$

$$\cos 0,35^\circ = 0,9999 \qquad \sin 0,35^\circ = 6,1086\,10^{-3}$$

And

$$2\% \qquad \Longleftrightarrow \qquad \arctan(0,02) = 1,1457^\circ$$

$$\cos 1,1457^\circ = 0,9998 \qquad \sin 1,1457^\circ = 19,9949\,10^{-3}$$

Exercise 2 : Shell Eco Marathon

- Rolling resistance

$$F_{RR} = mg\ \cos\theta\ f_{RR} = 180 \cdot 9,81 \cdot 1,0 \cdot 0,0018 = 3,1784\ N$$

$$\mathcal{P}_{RR} = F_{RR} \cdot V = 3,1794 \cdot 7,23 = 22,9801\ W$$

- Aerodynamic resistance

$$F_{AERO} = \frac{1}{2}\rho S C_x\ V^2$$
$$= 0,5 \cdot 1,22 \cdot 1,049 \cdot 0,20 \cdot 7,23^2 = 6,8761\ N$$

$$\mathcal{P}_{AERO} = F_{AERO} \cdot V = 6,8761 \cdot 7,23 = 49,7143\ W$$

- Road resistance

$$F_{RES} = F_{RR} + F_{AERO} = 3,1784\ +\ 6,8761 = 10,0545\ N$$
$$\mathcal{P}_{RES} = \mathcal{P}_{RR} + \mathcal{P}_{AERO} = 22,9801 + 49,7143 = 72,6940\ W$$

Exercise 2 : Shell Eco Marathon

- Grading resistance

$$\theta = 2\% \quad \theta = 1,1457^\circ \quad \sin\theta = 1,9996\,10^{-2}$$

$$\begin{aligned} F_{GRADING} &= mg\,\sin\theta \\ &= 180\cdot 9,81\cdot 0,019996 = 35,316\,N \end{aligned}$$

- Rolling resistance → negligible modification!

- Road resistance

$$F_{RES} = F_{RR} + F_{AERO} + F_{GRADE} = 25,4835 + 52,1238 = 77,6073\,N$$

$$\mathcal{P}_{RES} = F_{RES}\cdot V = 77,6073\cdot 7,23 = 328,0287\ W$$

Exercise 2 : Shell Eco Marathon

- If the vehicle is equipped with two electric motors of the following characteristics

$$C_{max} = 0,9743\ N.m$$

$$i = 8,2 \qquad R_e = 0,2737\ m$$

$$\eta_{GearBox} = 0,9875 \qquad \eta_{Bearings} = 0,98$$

- What is the net force available?
- What is the maximum slope that be overcome?
- What is the maximum acceleration?

Exercise 2 : Shell Eco Marathon

- What is the net force available?
  - Transmission efficiency

$$\eta_t = \eta_{gear}\, \eta_{bearings} = 0,9875 \cdot 0,98 = 0,96775$$

  - Tractive force generated by one motor

$$F_t^{(1)} = \eta\, \frac{i}{R_e}\, C_{max} = 0,96775\, \frac{8,2}{0,2737}\, 0,9743 = 28,2638\ N$$

  - Net available tractive force

$$F_{net} = 2 \cdot F_t^{(1)} - F_{RES} = 2 \cdot 28,2658 - 10,0945 = 46,4371\ N$$

Exercise 2 : Shell Eco Marathon

- What is the maximum slope that be overcome?

$$F_{GRADE} = mg \ \sin\theta_{max} = F_{net} = F_t - F_{RES}$$

$$\sin\theta_{max} = \frac{F_{net}}{mg} = \frac{F_t - F_{RES}}{mg}$$

$$= \frac{46,4371}{180 \cdot 9,81} = 2,6298\ 10^{-2}$$

$$\theta_{max} = 2,6\ \%$$

Exercise 2 : Shell Eco Marathon

- What is the maximum acceleration?

$$m\frac{dV}{dt} = F_{net} = F_t - F_{RES}$$

$$\begin{aligned} a_{max} &= \frac{F_{net}}{m} = \frac{F_t - F_{RES}}{m} \\ &= \frac{46,4371}{180} = 0,2579\, m/s^2 \end{aligned}$$

Exercise 3 : Road Resistance of a Truck

- Determine the road resistance of a semi trailer truck when traveling at the cruise speed of V=90 km/h

$$m = 40000\,kg \qquad f_{RR} = 0.0085$$
$$S = 10\,m^2 \qquad C_x = 0.64$$
$$\eta_t = 0.90$$

- Grade resistance on a 3% road.

- Which slope can be overcome at 90 km/h if the truck is equipped with a 335 kW engine (Mercedes-Benz OM470)?

Exercise 3 : Road Resistance of a Truck

- Determine the road resistance of a semi trailer truck when traveling at the cruise speed of V=90 km/h

Truck type (1)	C_d (2)	Source (3)
Single unit	0.70	Fitch [1994]
Tractor-semitrailer	0.70	Fitch [1994]
Car hauler - cattle hauler	0.96 – 1.10	SAE J2188
Garbage	0.95 – 1.05	SAE J2188
No aerodinamic aids	0.78	SAE J2188
Aerodinamic aids on roof	0.64	SAE J2188
Full aerodinamic treatment	0.58	SAE J2188

Exercise 3 : Road Resistance of a Truck

- Speed :

$$V = 90km/h = 90 \cdot 1000/3600 = 25.0\, m/s$$

- Rolling resistance

$$\begin{aligned} F_{RR} &= mg\ \cos\theta\, f_{RR} \\ &= 40000 \cdot 9.81 \cdot 1 \cdot 0.0085 = 3335.40\ N \end{aligned}$$

$$\mathcal{P}_{RR} = F_{RR} \cdot 25.0 = 83.385\ kW$$

- Aerodynamic drag

$$\begin{aligned} F_{AERO} &= \frac{1}{2}\rho S C_x\, V^2 \\ &= 0,5 \cdot 1,2 \cdot 10.0 \cdot 0.64 \cdot 25^2 = 2400\ N \end{aligned}$$

$$\mathcal{P}_{AERO} = F_{AERO} \cdot V = 2400 \cdot 25 = 60.0\ kW$$

Exercise 3 : Road Resistance of a Truck

- Total road resistance on level road

$$F_{RES} = F_{RR} + F_{AERO} = 5735.4\ N$$

$$\mathcal{P}_{RES} = \mathcal{P}_{RR} + \mathcal{P}_{AERO} = 83.385 + 60.0 = 143.385\ kW$$

- Ratio between rolling resistance and aerodynamic drag

$$\frac{\mathcal{P}_{RR}}{\mathcal{P}_{AERO}} = \frac{83.385}{60.0} = 1.3898$$

- Power need at the engine

$$\mathcal{P}_{mot} = \frac{\mathcal{P}_{RES}}{\eta_t} = \frac{143.385}{0.9} = 159.51\ kW$$

$$\mathcal{P}_{mot} = \frac{159.51}{0.736} = 216.46\ HP$$

Exercise 3 : Road Resistance of a Truck

- For which speed aerodynamic drag is equal to the rolling resistance

$$F_{RR} = F_{AERO} \quad \Longleftrightarrow \quad mg\, f_{RR} = \frac{1}{2}\rho S C_x\, V^2$$

$$V = \sqrt{\frac{mg\, f_{RR}}{\frac{1}{2}\rho S C_x}} = \sqrt{\frac{40000 \cdot 9.81 \cdot \; 0.0085}{\frac{1}{2} \cdot 1.2 \cdot 10 \cdot 0.64}}$$

$$= 29.4719\, m/s = 106.0989\, km/h$$

Exercise 3 : Road Resistance of a Truck

- Determine net power available and max slope that can be taken

$$\mathcal{P}_{RR} + \mathcal{P}_{AERO} + \mathcal{P}_{GRADE} = \eta_t \, \mathcal{P}_{max}$$

$$83385 + 60000 + mg \, \sin\theta \, V = 0.9 \cdot 335000$$

$$\sin\theta = \frac{0.9 \cdot 335000 - 143385}{40000 \cdot 9.81 \cdot 25} = 0.0161$$

  - Maximum slope at 90 km/h is 1.61%

Tractive forces

Exercise 4

- Let's consider the following vehicle BMW Z3 (model 2000)

$$m = 1295\,kg$$
$$C_x = 0.37 \qquad S = 2.24\,m^2$$

$$f = 0.0136 + 0.4E - 7\,V^2[km/h]$$

$$\text{Tire } 225/50\,R\,16$$

$$P_{max} = 118\,HP \qquad \omega_{nom} = 5500\,rpm$$
$$C_{max} = 180\,N.m \qquad \omega_{Cmax} = 3900\,rpm$$

	1	2	3	4	5
i_g	4.23	2.52	1.66	1.22	1.00
i_d	3.38				

Exercise 4

- Determine missing parameters
- Define approximations of power and torque curves of the engine
- Compute the road resistance forces and draw their evolution from 0 to 200 km/h
- Compute the tractive forces for the five gear ratios and draw the diagram of tractive forces and road resistance of the vehicle

Exercise 4

- Let's estimate the missing parameters
- Rolling radius

$$\text{Tire } 225/50\, R\, 16$$

$$D = 16 \cdot 25,6 + 2 \cdot 0,50 \cdot 225 = 631.40\, mm$$

$$R_e \simeq 0.98 \cdot D/2 = 309,38\, mm$$

- Transmission efficiency
  - Engine with longitudinal mounting
  - Driven at rear axle
  - Manuel transmission with dry friction clutch
  - One direct drive gear ratio (5)

$$\eta_t(1,2,3,4) = 1.0\, 0,9875^2\, 0.975 = 0.95$$

$$\eta_t(5) = 1.0\, 1.0\, 0.975 = 0.975$$

Exercise 4 : Power curve approximations

- Let's find an approximation of the power curves

$$P_1 = P_{max} = 118\,HP = 118 \cdot 736 = 86.84\,kW$$

$$P_1 = P_{max} = 86840\,W \quad \omega_1 = \omega_{nom} = 5500\,rpm$$
$$C_2 = C_{max} = 180\,N.m \quad \omega_2 = \omega_{Cmax} = 3900\,rpm$$

$$\omega_1 = 5500\,\frac{2\pi}{60} = 575,9587\,rad/s \qquad \omega_2 = 408,4070\,rad/s$$

$$\mathcal{P}(\omega_2) = \mathcal{P}_2 = C_{max}\,\omega_{C_{max}}$$
$$= 180 \cdot 408,4070 = 75.513\,kW$$

$$\mathcal{P}_2/\mathcal{P}_1 = 0.84654$$
$$\omega_2/\omega_1 = 0.70909$$

Power approximation

- One looks for a power function of the type

$$\boxed{\mathcal{P} \;=\; \mathcal{P}_1 - a\,|\omega - \omega_1|^b} \quad \text{with } b > 0$$

- It comes

$$b \;=\; \frac{\frac{\omega_1}{\omega_2} - 1}{\frac{\mathcal{P}_1}{\mathcal{P}_2} - 1} = 2.2631 \qquad\qquad a \;=\; \frac{\mathcal{P}_1 - \mathcal{P}_2}{|\omega_1 - \omega_2|^b} = 0.12340$$

Polynomial approximation

- Polynomial approximation of order 3

$$\mathcal{P}(\omega)/\mathcal{P}_1 = a_0 + a_1\,(\omega/\omega_1) + a_2\,(\omega/\omega_1)^2 + a_3\,(\omega/\omega_1)^3$$

- Identification of the coefficients

$$\begin{aligned} & a_0 = 0 \\ & a_1 + a_2 + a_3 = 1 \\ & a_1\,n_2 + a_2\,n_2^2 + a_3\,n_2^3 = \mathcal{P}_2/\mathcal{P}_1 \\ & a_2 + 2\,a_3\,n_2 = 0 \end{aligned} \quad \Rightarrow \quad \left\{ \begin{aligned} a_1 &= 0.042187 \\ a_2 &= 3.248236 \\ a_3 &= -2.290423 \end{aligned} \right.$$

Polynomial approximation

- Polynomial approximation of order 4

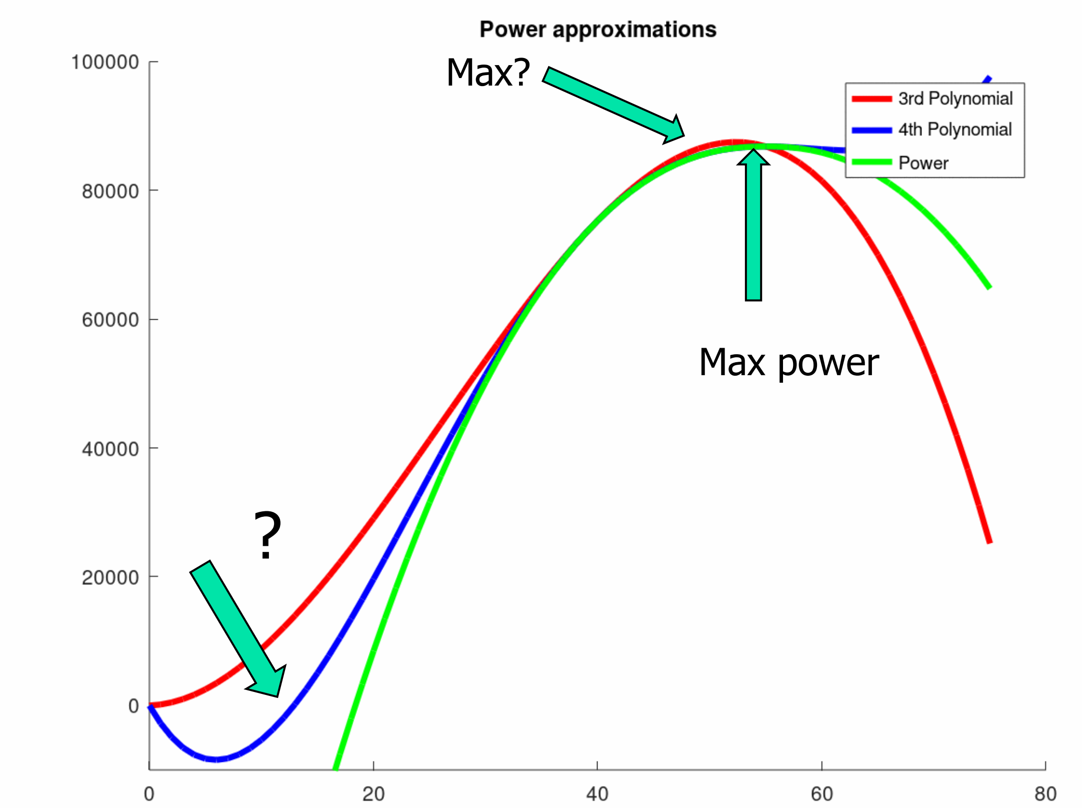
$$
\begin{aligned}
\mathcal{P}(\omega)/\mathcal{P}_1 \;&=\; a_0 + a_1\,(\omega/\omega_1) + a_2\,(\omega/\omega_1)^2 + a_3\,(\omega/\omega_1)^3 \\
&\quad + a_4\,(\omega/\omega_1)^4
\end{aligned}
$$

- Solution of the linear system

$$
\begin{aligned}
& a_1 + a_2 + a_3 + a_4 = 1 \\
& a_1 + 2\,a_2 + 3\,a_3 + 4\,a_4 = 0 \\
& a_1\,n_2 + a_2\,n_2^2 + a_3\,n_2^3 + a_4\,n_2^4 = \mathcal{P}_2/\mathcal{P}_1 \\
& a_2 + 2\,a_3\,n_2 + 3\,a_4\,n_2^2 = 0
\end{aligned}
\quad \Rightarrow \quad
\left\{
\begin{aligned}
a_1 &= -1.9340 \\
a_2 &= 10.7982 \\
a_3 &= -11.7945 \\
a_4 &= 3.9303
\end{aligned}
\right.
$$

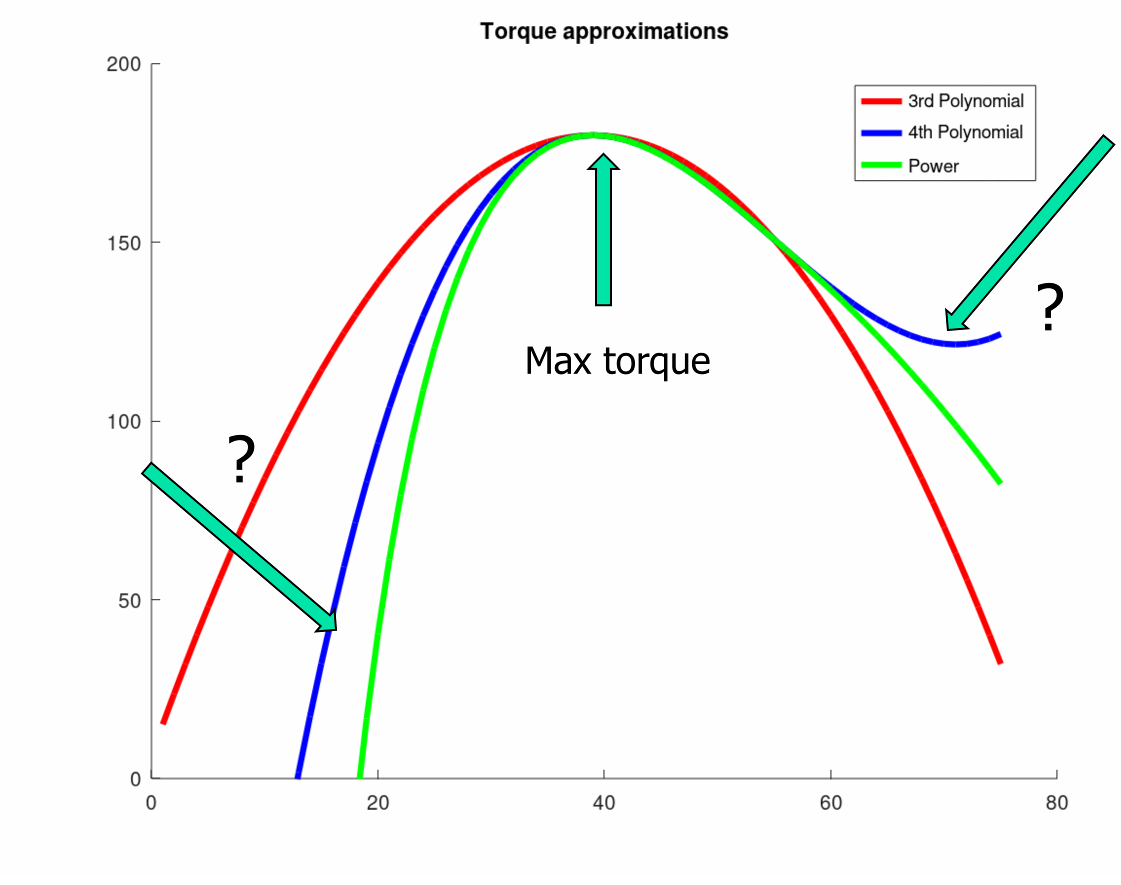
Comparison of Power approximations

- Power approximations



Comparison of Torque approximations

- Torque approximations



Road resistance

- Rolling resistance

$$F_{RR} = mg\ \cos\theta\ f_{RR}$$

$$f_{RR} = 0.0136 + 0.4E - 7\, V^2 [km/h]$$

$$f_0 = 0.0136$$

$$f_2 = 0.4E - 7\,(3.6)^2 = 5.184E - 7\ [V \text{ in } m/s]$$

- Aerodynamic drag

$$\begin{aligned} F_{AERO} &= \frac{1}{2}\rho S C_x\, V^2 \\ &= 0,5 \cdot 1,22 \cdot 2.24 \cdot 0.37 \cdot V^2 \\ &= 0.5056\, V^2\ [N] \end{aligned}$$

Road resistance

- Grading resistance

$$F_{GRADING} = mg\ \sin\theta$$

- Road resistance

$$F_{RES} = F_{RR} + F_{AERO} + F_{GRADE}$$
$$= mg\ \cos\theta(f_0 + f_2\,V^2) + 1/2\rho S C_x\,V^2 + mg\ \sin\theta$$

$$F_{RES} = A + B\,V^2$$

$$A = mg\ \cos\theta\ f_0 + mg\ \sin\theta$$

$$B = 1/2\rho S\,C_x + mg\ \cos\theta\ f_2$$

Road resistance

- If θ=0%

$$A = mg\ f_0 = (1295 + 75) \cdot 0.81 \cdot 0.0136 = 182.78\ N$$

$$\begin{aligned} B &= 1/2 \rho S\, C_x + mg\ \cos\theta\ f_2 \\ &= 0,5 \cdot 1,22 \cdot 2.24 \cdot 0.37 + 1370 \cdot 9.81 \cdot 5.184\, 10^{-7} \\ &= 0.50425 \end{aligned}$$

- If θ=3%

$$\begin{aligned} F_{GRADING} = \Delta A &= mg\ \sin\theta \\ &= 1370 \cdot 9.81 \cdot 0.03 = 403.1910 \end{aligned}$$

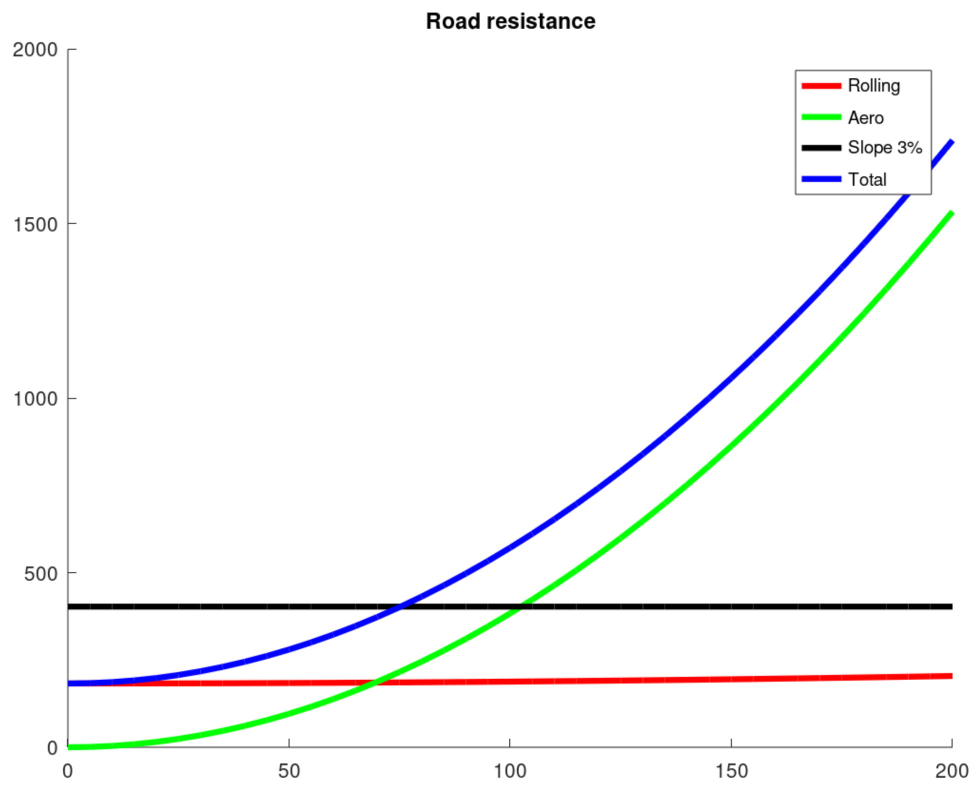
MATLAB code

```
%
% Force at wheels
%
vup=200.;
vlow=0.;
%
v=vlow:5:vup;
vms= v/3.6;
%
% Road resistance
%
% Re 0.309
% fRR = f0 + f2 V**2
% f0 = 0,0136
% f2 = 0.4E-7 *(3.6)**2 = 5.184E-7
% S = 2.24 m²
% Cx = 0.37
% eta tranmission: 0.95 (moteur longitudinal avant)
%
m = 1295.+75.;
grav=9.81;
Re = 0.309 ;
frr0 = 0.0136;
frr2 = 5.184e-7;
rho=1.2;
S = 2.24;
Cx = 0.37;
```

```
%
% FRES = A + B V**2
%
ares = m*grav*frr0*cos(theta) + m*grav*sin(theta)
bres = 0.5*rho*S*Cx + m*grav*frr2*cos(theta)
Fres= ares + bres*vms.^2;
%
FRLT= m*grav*frr0*cos(theta) + m*grav*frr2*cos(theta)*vms.^2;
FAERO = 0.5*rho*S*Cx*vms.^2;
% 3% = 1.7184 degres
theta = pi*1.7184/180.;
FGRAD = m*grav*sin(theta)*ones(length(vms),1);
%
figure
hold on
plot(v,FRLT,'LineWidth',3,'Color','red')
plot(v,FAERO,'LineWidth',3,'Color','green')
plot(v,FGRAD,'LineWidth',3,'Color','black')
plot(v,Fres,'LineWidth',3,'Color','blue')
ylim([0 2000])
title('Road resistance')
legend('Rolling', 'Aero', 'Slope 3%','Total')
hold off
```

Road resistance

- Plot the road resistance from v=0 to 200 km/h



Tractive forces

- Let's now plot the tractive forces for the different gear ratio
  - Speed in terms of the engine rotation speed

$$v = \omega_e \, \frac{R_e}{i}$$

  - Tractive force in terms of the engine speed and torque value

$$F_t = \eta_t \, \frac{i}{R_e} \, C(\omega_e)$$

$R_e = 0.3090$

$$\eta_t(1,2,3,4) = 1.0\, 0,9875^2\, 0.975 = 0.95$$

$$\eta_t(5) = 1.0\, 1.0\, 0.975 = 0.975$$

	1	2	3	4	5
i_g	4.23	2.52	1.66	1.22	1.00
i_d	3.38				
$i = i_g\, i_d$	14.2974	8.5176	5.6108	4.1236	3.38

MATLAB code

```
%
% Tractive forces
%
% Gear box: i1=4.23 / i2 = 2.52 / i3 = 1.66 / i4 = 1.22 / i5 = 1.0
% Differential : id= 3.38
id = 3.38;
i1 = 4.23*id;
i2 = 2.52*id;
i3 = 1.66*id;
i4 = 1.22*id;
i5 = 1.00*id;
%
etatrans = 0.95;
etatransdirect = 0.975;
%
NROT=1000:100:6000;
%
```

MATLAB code

```
NROT=1000:100:6000;
%
for i=1:length(NROT)
  omega = NROT(i)*pi/30.;
  Pcal = P1*(A3_0 + A3_1*(omega/O1) + A3_2*(omega/O1).^2 + A3_3*(omega/O1).^3);
  Ccal = Pcal/omega;
%
  vms1(i)=3.6*omega*Re/i1;
  FT1(i) = etatrans*i1*Ccal/Re;
%
  vms2(i)=3.6*omega*Re/i2;
  FT2(i) = etatrans*i2*Ccal/Re;
%
  vms3(i)=3.6*omega*Re/i3;
  FT3(i) = etatrans*i3*Ccal/Re;
%
  vms4(i)=3.6*omega*Re/i4;
  FT4(i) = etatrans*i4*Ccal/Re;
%
  vms5(i)=3.6*omega*Re/i5;
  FT5(i) = etatransdirect*i5*Ccal/Re;
%
end
%
```

MATLAB code

```
figure
hold on
plot(v,Fres,'LineWidth',3,'Color','black')
plot(vms1,FT1,'LineWidth',3,'Color','red')
plot(vms2,FT2,'LineWidth',3,'Color','magenta')
plot(vms3,FT3,'LineWidth',3,'Color','green')
plot(vms4,FT4,'LineWidth',3,'Color','cyan')
plot(vms5,FT5,'LineWidth',3,'Color','blue')
ylim([0 10000])
title('Tractive Forces')
legend('Road Resistance','Gear 1','Gear 2','Gear 3','Gear 4','Gear 5')
hold off
```

Tractive forces

- Plot the tractive force from v=0 to 200 km/h